

# A Novel Skewed-Type Iron Slot Wedge for Permanent Magnet Synchronous Generators for Improving Output Power and Reducing Cogging Torque

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**Abstract** – This paper proposes a novel skewed-type iron slot wedge that can improve both the cogging torque and the output power of a permanent magnet synchronous generator (PMSG). Generally the open slot structure is adopted in a PMSG due to its convenient winding work, but the high cogging torque is undesired. Firstly, an iron slot wedge was utilized to reduce the cogging torque of an open slot type PMSG. However, the output power of the machine decreased rapidly with this method. Thus, a proposed skewed type iron slot wedge is presented to improve the output power as well as the cogging torque as compared to the open slot type. Shape optimization of the skewed-type iron slot wedge is performed to simultaneously maximize the output power and reduce the cogging torque. The Kriging model based on the Halton sequence method and a genetic algorithm are used to optimize the design.

**Keywords:** Finite element analysis, Optimal design, Permanent magnet synchronous generator, Slot wedge.

## 1. Introduction

A low-speed direct drive PMSG for sports equipment has been designed as a multi-slot and multi-pole type in order to improve the output power [1]. Generally an open slot structure should be applied in a multi-slot-type machine to overcome winding difficulties, but this design has the drawback of a large cogging torque.

Therefore, a slot wedge with high permeability is used to reduce the cogging torque. Ferrite is typically used in slot wedges, as it has a slightly higher permeability than air. Ferrite slot wedges can decrease the cogging torque due to the reduction in the imbalance of the magnetic resistance between the slots and the stator teeth. However, these wedges do have drawbacks in terms of the vibration and frangibility [2]. Recently, soft magnetic composite (SMC) slot wedges have been studied. SMCs can be easily produced in various shapes. However, their applications are limited because of their higher costs and lower efficiency, which is a result of their low permeability in low frequency regions compared to cast steel [3]. Therefore, iron material, which can be formed into a greater variety of shapes than ferrite and can be used to produce slot wedges more cheaply

than SMC, has been utilized [4]. With the use of iron slot wedges in an open-slot-type PMSG, the cogging torque is lower than with ferrite or SMC. However, the output power also rapidly decreases because the high permeability of the iron material results in high flux leakage in the slot wedge.

Moreover, skewed stator teeth have also been used to reduce the cogging torque of permanent magnet synchronous machines [5]. However, when the stator teeth of a PMSG with multiple slots and multiple poles are skewed, the winding work becomes more difficult [6]. To facilitate the winding work and reduce the cogging torque, the stator core of the PMSG can be laminated. Laminations are rotated with slot pitch intervals [7]. However, this approach cannot increase the output power due to the same effective flux linkage path cross sectional area.

This paper proposes a novel skewed-type iron slot wedge that improves the output power and reduces the cogging torque, as it can increase the effective cross-sectional area of the flux linkage. Moreover, to maximize the output power, an optimal design using a Kriging model based on the Halton sequence method and a genetic algorithm is performed.

## 2. Characteristic Analysis of PMSGs According to Slot Type

### 2.1 Open and closed slot type

#### 2.1.1 Configuration and specification

Figs. 1 (a) and (b) show an open slot type PMSG, and (c)

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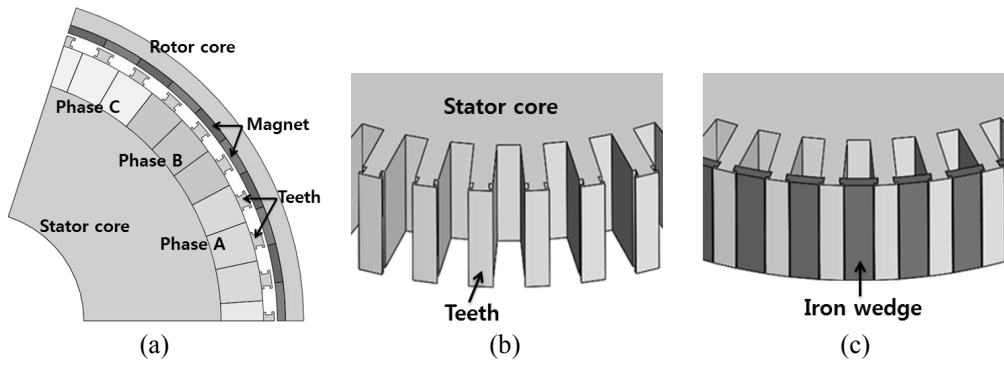
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Received: April 19, 2014; Accepted: September 6, 2014



**Fig. 1.** Structures of PMSGs: (a) An open slot type PMSG; (b) Open slot; (c) Closed slot

**Table 1.** Basic specifications of an open-slot-type PMSG

Item	Unit	Value
No. of slots / poles	-	45/50
Rated speed	rpm	70
No. of turns per slot	turns	72 (concentrated winding)
Resistance	$\Omega$	5.15
Outer diameter of stator	mm	199.5
Inner diameter of rotor	mm	200.5
Stack length	mm	50
Material of core (stator and rotor)	-	Silicon steel
Air gap	mm	1
Material of PM	-	Bonded NdFeB
Residual flux density of PM	T	1.142
Volume of PM	cm <sup>3</sup>	53.04

shows a closed slot type with an iron slot wedge. The outer rotor consists of surface-mounted type permanent magnets with 50 poles, whereas the inner stator consists of 45 slots. Table 1 gives the specifications of an open-slot-type PMSG.

Ferrite and SMC have been widely used for conventional slot wedges [2]. However, ferrite is vulnerable to vibrations and is limited in terms of the possible shape designs. On the other hand, although SMC can facilitate flexible shapes, its high cost is disadvantageous. However, iron material allows for flexible shape design and is cheaper than SMC, so the iron composite ss400 is utilized for the slot wedges in the proposed machine. The characteristics of ss400 are shown in Table 2.

**Table 2.** Characteristics of ss400 composite

Item	Unit	Value
Non-permeability	-	3000
Conductivity	S/m	$7.51 \times 10^6$

### 2.1.2 Characteristics of the two type PMSGs

Cogging torque is generated due to energy changes in electric devices with rotor rotation. Most of this energy change occurs in the airgap. The cogging torque is determined by (1) [8].

$$T_{\text{cog}} = \frac{\pi}{4} D L_{\text{stk}} \sum_n \frac{\sin(n\sigma L_{\text{stk}})}{n\sigma L_{\text{stk}}} A_n f_n \sin(n\zeta) \quad (1)$$

where  $n=ks$ ,  $k=1,2,\dots$ ,  $s$  is the lowest common multiple of the numbers of slots and poles,  $D$  is the outer diameter of the rotor,  $L_{\text{stk}}$  is the stack length,  $\sin(n\sigma L_{\text{stk}})/n\sigma L_{\text{stk}}$  is the effect of the skew,  $\sigma$  is the skew angle,  $A_n$  is the  $n$ -th harmonic of the permeance coefficient,  $f_n$  is the  $n$ -th space harmonic of the flux distribution, and  $\zeta$  is the rotational angle of the rotor.

As shown in (1), since the frequency of the cogging torque is determined from the lowest common multiple of the numbers of slots and poles, the period of the cogging torque (mechanical angle) can be calculated from (2) [9].

$$\text{period of cogging torque} = \frac{360^\circ}{\text{LCM}(s,p)} \quad (2)$$

where  $s$  is the number of slots,  $p$  is the number of poles, and  $\text{LCM}(s,p)$  is the lowest common denominator of the numbers of slots and poles. Using (2), the period of the cogging torque for 45 slots and 50 poles is  $0.8^\circ$ . The 3D-FEA results for an open-slot-type PMSG are shown in Table 3. The cogging torque of a closed-type PMSG with an iron slot wedge is decreased to less than that of an open slot type, because the closed-type PMSG reduces the imbalance of the magnetic resistance compared to the open slot type. However, the output power is almost zero.

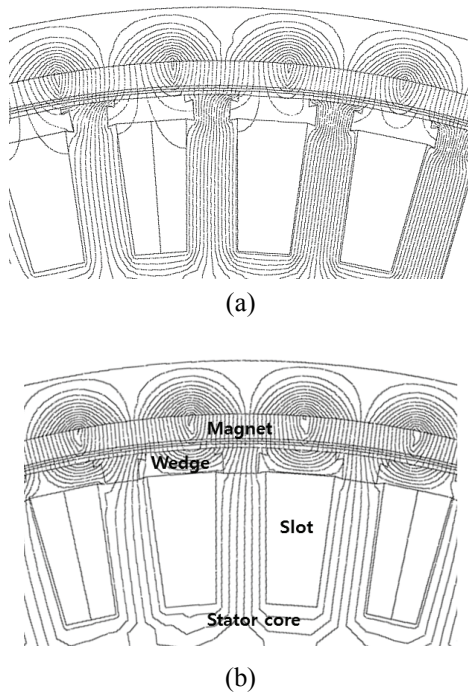
**Table 3.** Analysis results of open-slot-type and closed-type PMSGs

Item	Unit	Open slot type	Closed slot type
Cogging torque (pk-pk)	Nm	0.131	0.00098
Output power (average)	W	48.3	0.02

Fig. 2 shows the flux lines of open-slot-type and closed-type PMSGs. The rotation of magnet causes the flux linkage of windings to alternate and so generate an EMF.

$$e = \frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial \theta} \cdot \frac{d\theta}{dt} = \omega \frac{\partial \psi}{\partial \theta} \quad (3)$$

where  $\omega$  is the angular velocity,  $\theta$  is the rotor position, and  $\psi$  is the flux linkage of the winding. The first expression is faraday's law which is described correlation



**Fig. 2.** Flux lines of PMSGs: (a) Flux lines of an open-slot-type PMSG; (b) Flux lines of a closed-type PMSG

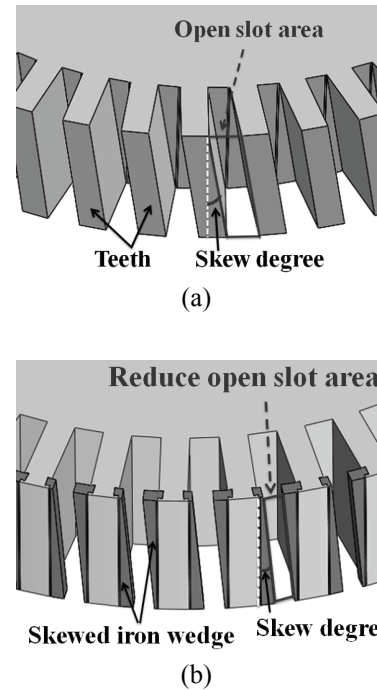
of flux linkage, velocity and EMF. As shown in (3), The EMF is proportional to rotational velocity and increasing flux linkage on the rotor position change. It means the imbalance in the magnetic resistance between slot area and teeth increase the EMF.

The imbalance in the magnetic resistance of the closed type was reduced more than that of the open slot type because the iron slot wedge has a higher permeability than air. Thus, the flux linkage of the closed type was also reduced. In addition, the flux leakage in the slot wedge of the closed type was increased. Therefore, the output power of the closed type was almost zero.

## 2.2 Proposed skewed-type iron slot wedge

### 2.2.1 Configuration

To improve the output power as well as reduce the cogging torque, a skewed type iron slot wedge is proposed. Fig. 3 (a) shows a stator with the conventional skew method, which can be applied to stator teeth or permanent magnets. Although the conventional skew method cannot decrease the open slot area, the proposed skewed slot wedge can decrease the open slot area in proportion to the skew angle. Therefore, the cogging torque of a skewed type PMSG can be less than that of the open slot type, and the output power is increased. The skew angle of the analysis model was determined to be  $2^\circ$ . Fig. 3 (b) shows the slot structure of the skewed-type PMSG with the proposed slot wedge, which has a  $2^\circ$  skew angle.

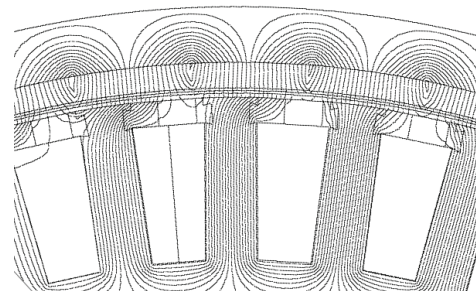


**Fig. 3.** Structure of skewed type PMSG: (a) skewed stator; (b) skewed slot wedge

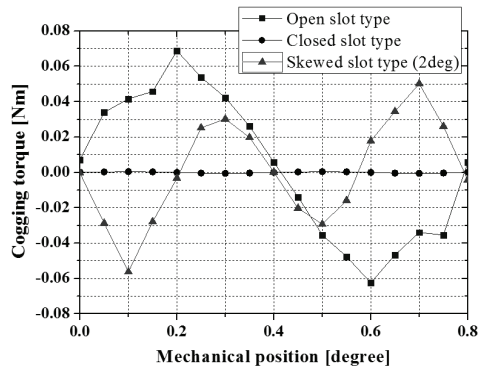
### 2.2.2 Characteristics of the three type PMSGs

Figs. 4 and 5 show the results of the finite element analysis. The output power is from dynamic analysis of 3D-FEM and used average value on the stable section of the output power. The rotor of PMSG rotates 70 rpm, and is connected to converter for saving DC voltage from AC. And the output power vary with the load of circuit. The three slot type PMSGs are compared with same load condition. As seen in Fig. 4, the flux linkage of the skewed-type PMSG increased more than that of the open slot type (Fig. 2 (a)) due to the decreased open slot area. Therefore, the output power of the skewed type PMSG increased because of the increase in the flux linkage. In addition, the cogging torque was reduced due to a reduction in the imbalance of the magnetic resistance.

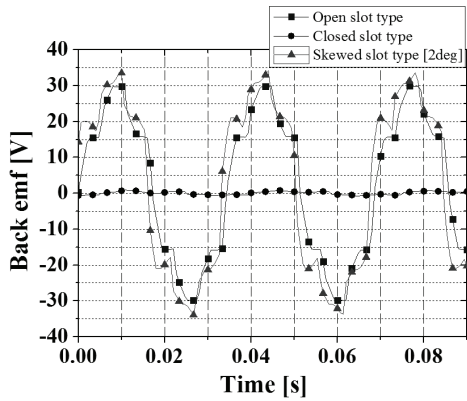
The period of the cogging torque with 45 slots and 50 poles is  $0.8^\circ$ , which has been verified by the analysis result in Fig. 5 (a). It also shows that the cogging torque values of



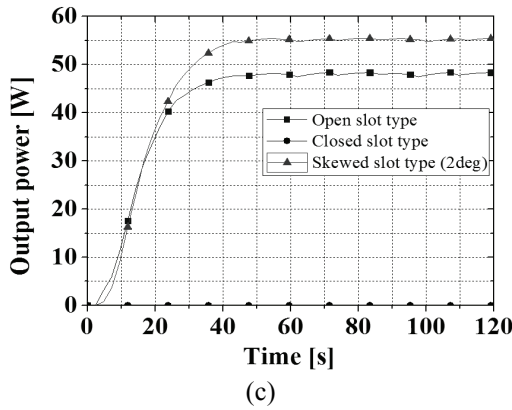
**Fig. 4.** Flux lines of the skewed type PMSG



(a)



(b)

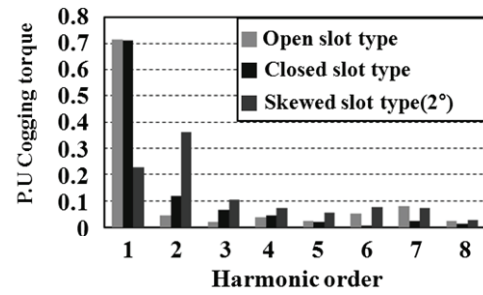


(c)

**Fig. 5.** Results of the finite element analysis: (a) Cogging torque; (b) Back EMF; (c) Output power.

the closed type and skewed type PMSGs are reduced due to the reduced imbalance of the magnetic resistance compared to the open slot type. However, although the back-EMF of the closed-type PMSG was reduced more than that of the open slot type, the back-EMF of the proposed skewed-type PMSG increased up to even more than that of the open slot type, because of the increase in the flux linkage to the iron slot wedge as shown in Fig. 5 (b). Therefore, the output power of the proposed skewed-type PMSG increased more than that of the open slot type as shown in Fig. 5 (c).

The analysis results show that the output power of the



**Fig. 6.** Harmonic analysis of the cogging torques

skewed-type PMSG is 54.9 W, which is an increase of 13.7% compared to the output power of the open slot type. In addition, the cogging torque of the skewed-type PMSG is 0.106 Nm, which is a decrease of 19.1% compared to the open slot type due to the reduction in the imbalance of the magnetic resistance by the skewed-type iron slot wedge.

Fig. 6 show the harmonic analysis of three slot type PMSGs. The skewed slot type PMSG increases the harmonics after second compared by the open and closed slot type PMSGs. The harmonics increase can be generated the shape of the slot wedge and permeance coefficient difference between the stator core and the slot wedge [9]. However the skewed slot type PMSG decrease the peak to peak value of cogging torque in that the skewed slot wedge also highly decrease fundamental harmonic.

### 3. Optimal Design of a Skewed-type Iron Slot Wedge

#### 3.1 Optimization techniques

The Kriging method, which is one of the interpolation methods, is composed of the global model and the localized deviation in a random field. The estimated equation of the Kriging model was defined to eliminate bias and thereby minimize any error variance [10]. The Kriging model is a weighted linear combination as follows.

$$z^* = \sum_{i=1}^n \lambda_i z_i \quad (3)$$

where  $z^*$  is an estimated point using the Kriging model,  $n$  is the total number of experiments,  $\lambda_i$  is the weight value function, and  $z_i$  is the experimented point.

The minimized error deviation can be expressed as

$$\sigma_{OK}^2 = \sigma^2 - 2 \sum_{i=1}^n \lambda_i \sigma_{oi}^2 + \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j \sigma_{ij}^2 \quad (4)$$

where a constraint with Eq. (4) is

$$\sum_{i=1}^n \lambda_i = 1 \quad (5)$$



where  $\sigma_{OK}^2$  is the error variance of the Kriging model,  $\sigma^2$  is the variance of  $z_0$ ,  $\sigma_{0i}^2$  is the covariance of  $z_0$  and  $z_i$ ,  $\sigma_{ij}^2$  is the covariance of  $z_i$  and  $z_j$ , and  $z_0$  is a real value for the prediction, for the unweighted situation in which all elements have equal weight.

When the error deviation is minimized, it can be expressed as

$$L(\lambda_1, \lambda_2, \dots, \lambda_n; \omega) = \sigma^2 - 2 \sum_{i=1}^n \lambda_i \sigma_{0i}^2 + \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j \sigma_{ij}^2 + 2\omega \left( 1 - \sum_{i=1}^n \lambda_i \right) \quad (6)$$

where  $L(\lambda_1, \lambda_2, \dots, \lambda_n; \omega)$  is a Lagrange objective function,  $\omega$  is a Lagrange factor, and the coefficient 2 is used for convenience.

The objective function is determined by a partial derivative of the Lagrange factor with respect to  $\lambda$  and  $\omega$  as follows.

$$\sum_{i=1}^n \lambda_i \sigma_{0i}^2 - \omega = \sigma_{0l}^2, l = 1, 2, \dots, n \quad (7)$$

The error deviation of the Kriging model can be expressed as

$$\sigma_{OK}^2 = Var(z) - \sum_{i=1}^n \lambda_i Cov(z_0, z_i) + \omega = \sigma^2 - \sum_{i=1}^n \lambda_i \sigma_{0i}^2 + \omega \quad (8)$$

where  $\sigma_{0i} = Cov(z_0, z_i)$ . The parameter can be estimated by minimizing the Lagrange object function. Applying Eq. (8), the Kriging method effectively corresponds to a real function with a sampling point.

The Halton sequence is one of the most popular low discrepancy sequences. It is constructed based on a deterministic method that uses a prime number. To effectively construct the Kriging model, the Halton sequence is also used for improved accuracy over random sampling and stratified sampling to estimate the means, deviations, and distribution functions of an output. Moreover, it ensures that each of the input variables represent all portions of its range.

### 3.2 Optimal design process

To maximize the output power and to reduce the cogging torque, the optimal design process for a skewed type iron slot wedge was performed as shown in Fig. 7. The Halton sequence method was applied as a method for the design of the experiment, which can perform sampling in one design variable [11]. The Kriging model was used to approximate the objective and constraints, and a genetic algorithm was utilized as the optimization algorithm. The objective function of the optimal design was to maximize the output power, and the constraint was to decrease the cogging

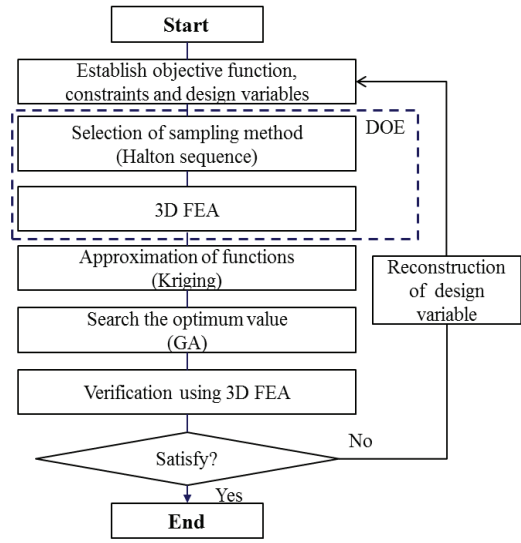


Fig. 7. Optimal design process

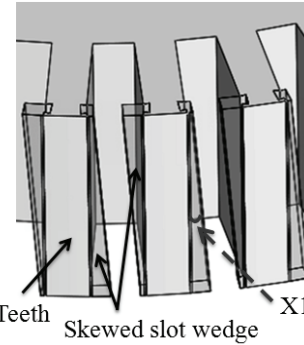
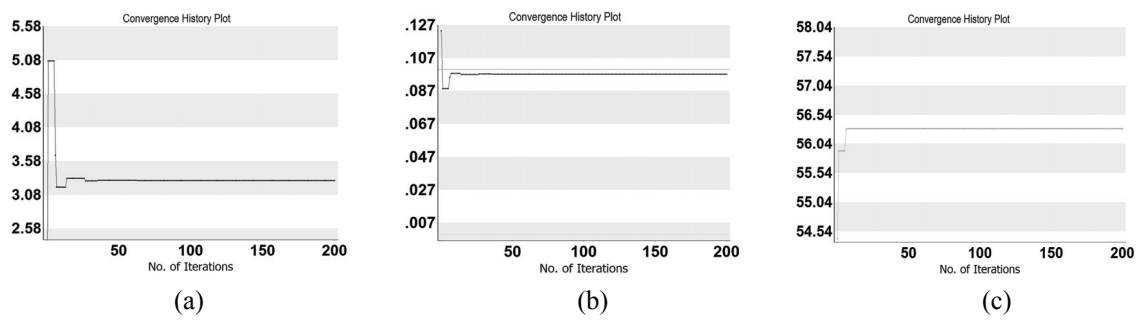


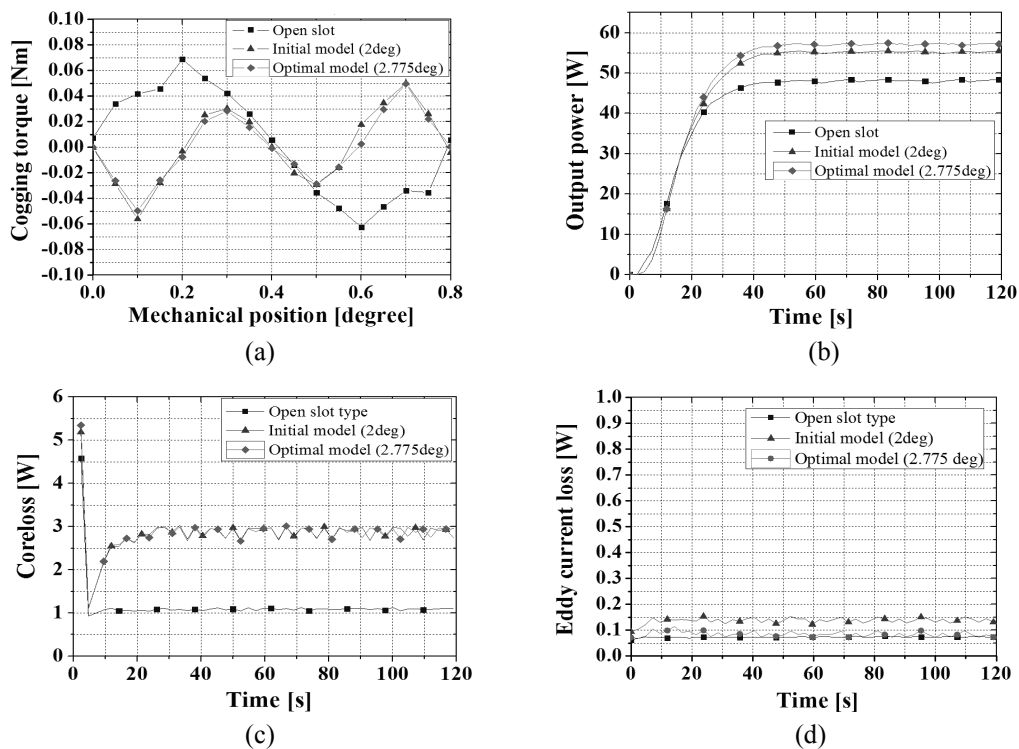
Fig. 8. Optimal design variables

torque by 20% compared to that of the open slot type. To satisfy the objective function and the constraint, a design variable (X1) was established. X1 represents the skew angle of the slot wedge as shown in Fig. 8. In the optimal design process, the prime number of halton sequence method was determined by two and the design variable range is divided by two. Using the prime number, the deviated ranges are continuously deviated by the prime number. Then the sampling points have been calculated to 1.25°, 2.5°, 5°, 6.25°, 7.5° in the design variable range. The approximation of function and searching optimum value are performed by the optimization software. Then the results are verified using 3D-FEA.

- *Objective function*  
Maximize output power
- *Constraint*  
Cogging torque  $\leq 0.1$  Nm  
(cogging torque of skew type should be decreased by 20% compared with the open slot type)
- *Design variable*  
 $0^\circ \leq X1$  (Skew angle)  $\leq 10^\circ$



**Fig. 9.** Convergence progress of optimal point by using GA: (a) Skew angle; (b) Cogging torque, and (c) Output power



**Fig. 10.** Results of FEA: (a) Cogging torque; (b) Output power; (c) Core loss; (d) Eddy current loss

### 3.3 Optimal design results

The convergence progresses for obtaining the optimal points are conducted by using GA as shown in Fig. 9. After the design process, the optimized skew angle is obtained.

The optimal results obtained by the optimization algorithm are very similar to the optimal results verified by 3D-FEA as shown in Fig. 10 and Table 4. Compared to the initial model, the cogging torque was decreased by 10.4% and the output power of the optimized model was increased by 2.9%. The cogging torque was also decreased, because the open slot area of the optimized model was reduced to less than that of the initial model. In addition, the improvement in the output power can be attributed to the increased flux linkage of the optimized model versus that of the initial model. Therefore, in the optimal model, the cogging torque was decreased by 27.5% compared to the open-slot-type PMSG, and the output power was improved

**Table 4** Analysis results of open-slot-type and optimal model PMSGs

Item	unit	Open slot type	Skewed iron wedge		
			Initial Model (by FEA)	Optimal Model (by algorithm)	Optimal model (by FEA)
Skew angle	°	-	2	3.293	3.293
Mechanical input power	W	59	70.5	-	70.7
Voltage	V	44.9	47.3	-	47.5
Current	A	1.07	1.17	-	1.19
Electrical output power (average)	W	48.1	54.9	56.3	56.5
Generation efficiency	%	77.5	77.8	-	79.9
Cogging torque (pk - pk)	Nm	0.131	0.106	0.096	0.095
Core loss (average)	W	1.09	2.81	-	2.86
Eddy current loss (average)	W	0.072	0.093		0.083

by 17.5%.

The iron slot wedge is formed by a solid magnetic substance, which results in the eddy current loss. Fig. 10 (c) and (d) show the core loss and the eddy current loss of the PMSGs, respectively. Although these losses of the skewed type PMSG increased more than those of the open slot type, the output power of the skewed type PMSG also increased.

#### 4. Conclusion

This paper proposes a novel skewed-type iron slot wedge to reduce the cogging torque and improve the output power of the PMSG. When the skewed type iron slot wedge is used, the open slot area is reduced to less than that of the open slot type. Therefore, the cogging torque of the skewed type PMSG is less than that of the open slot type due to the reduction in the imbalance of the magnetic resistance. Furthermore, the output power is improved due to an increase in the flux linkage of the slot wedge.

To maximize the output power and reduce the cogging torque, optimization of a skewed-type iron slot wedge was performed. The verified results using 3D-FEA were similar to the optimal design results obtained by the optimization algorithm. As a result, the output power of the skewed-type PMSG was increased by 17.5% compared to that of the open slot type, and the cogging torque was decreased by 27.5%. These results show that the proposed skewed type iron slot wedge is useful in reducing the cogging torque and improving the output power.

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