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# Data Interpolation and Design Optimisation of Brushless DC Motor Using Generalized Regression Neural Network

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**Abstract** – This paper proposes a generalized regression neural network (GRNN) based algorithm for data interpolation and design optimization of brushless dc (BLDC) motor. The procedure makes use of magnet length, stator slot opening and air gap length as design variables. Cogging torque and average torque are treated as performance indices. The optimal design necessitates mitigating the cogging torque and maximizing the average torque by varying design variables. The data set for interpolation and ensuing design optimisation using GRNN is obtained by modeling a standard BLDC motor using finite element analysis (FEA) tool MagNet 7.1.1. The performance indices of the standard motor obtained using FEA are validated with an experimental model and an analytical method. The optimal design is authenticated using particle swarm optimization (PSO) algorithm and the performance indices of the optimal design obtained using GRNN is validated using FEA. The results indicate the suitability of GRNN as an interpolation and design optimization tool for a BLDC motor.

**Keywords**: BLDC motor, Cogging torque, FEA, GRNN, PSO

## 1. Introduction

Brushless DC (BLDC) motors are considered superior to brushed DC motors as they are exceedingly efficient and require less maintenance due to absence of brushes. They are also more versatile, mainly because of their ability in the speed and torque domain. Since BLDC motors are available in compact packages, they are used in number of automotive and electronic applications [1-4]. Besides BLDC motors need a complex and expensive electronic controller to keep the motor running, they also experience a serious problem termed as cogging torque which is detrimental to its suitability in industrial and drive applications [6, 7]. Various design modifications and optimization procedures have been reported in the literature to lessen the effect of cogging torque and improve the performance of BLDC motors [7-9]. Variety of methods to assuage the cogging torque during the machine design phase itself has been reported [1]. These methods include skewing of the stator laminations or rotor magnets, varying slot width, varying magnet width, shifting alternate pair of poles and notching of teeth [13-16].

These design variations and optimization procedures attract a great attention as they could solve different machine design problems that are defiant to conventional programing techniques [17, 18]. Multi-objective optimization techniques to improve the performance of BLDC motors are also discussed in the recent past [10]. Related literatures reveal that modern swarm based optimization techniques like predator- prey algorithm [11] and bat inspired optimisation approach [12] are proposed for design optimization of brushless DC wheel motor considering mass and efficiency as performance indices. Among all the vital performance indices of BLDC motors, cogging torque and its effect on average torque dictate the application domain. Therefore, an optimal design procedure for mitigating the cogging torque and thus maximizing the average torque needs to be explored.

Having reviewed the magnitude of work done in the design and optimization province of BLDC motors, it is understood that finite element analysis (FEA) is used to predict the performance of BLDC motor since the geometry and the materials used are highly non-linear. However, in order to carry out optimization with ample accuracy, abundant field solutions are required. Hence FEA method is combined with either interpolation methods or heuristic approaches to determine the optimal design. While heuristic methods are time consuming, the function approximation obtained from interpolation methods like non-linear least square method result in significant error and hence the optimization may not be accurate paving way to artificial neural network (ANN) based optimization procedures [20]. Since the data from FEA is meager, even ANN based procedures may not result in an accurate solution [19]. Therefore this paper explores the application of generalized regression neural network (GRNN) for data interpolation and design optimization of BLDC motor. Though other types of neural networks like multi layer perceptron (MLP) networks can be considered for this application, GRNN seems to be a better option due to the

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fact that i) it has only one design parameter (spread factor); ii) it is easy to train since it is a one- pass algorithm and iii) it can easily approximate functions from sparse and noisy data [21].

This paper comprises of five sections wherein an overview of BLDC motors and the necessity for an optimal design is illustrated in section 1. While section 2 briefs problem formulation, section 3 vividly elucidates the performance analysis of BLDC motor using FEA. Section 4 enlightens how a GRNN is used for interpolation and design optimisation. The concluding remarks are provided in section 5.

## 2. Design Optimization of BLDC-Problem Formulation

The structure of BLDC is presented in Fig. 1 [2]. Literature review reveals that the cogging torque in BLDC motors is very much affected by the machine design variables and manufacturing related issues [1]. This work of optimal design is applied to a 120W BLDC motor by considering the magnet length, stator slot opening and air gap length as design variables with a view to minimize cogging torque and maximize average torque. The design variables are chosen after a detailed investigation of the variables affecting cogging torque [5]. The lower and upper bound of the design variables [5, 11, 12] are given in Table 1.Since the proposed work is for a 120W motor cited in the Appendix 1, the bounds of design variables are chosen to meet the average torque requirement of 0.34 N-m. The other dimensions [2] of the motor are also given in Appendix 1.

The objectives of the optimization procedure are defined as follows.

Maximization of average torque

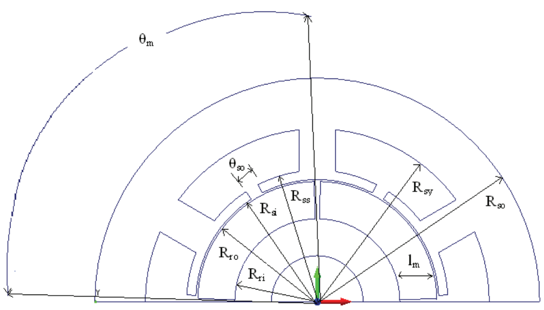


**Fig 1.**Structure of BLDC motor

**Table 1.** Bounds of design variables

| Design variable | Lower limit | Upper limit |
|---|---|---|
| Magnet length(mm) | 7.2 | 8.8 |
| Stator slot opening(Deg) | 5 | 13 |
| Air gap length (mm) | 0.45 | 0.55 |

$$f_1 = \max(T_{av}) \tag{1}$$

Minimization of cogging torque

$$f_2 = \min(T_{cog}) \tag{2}$$

## 3. Performance Analysis Using Finite Element Analysis

The optimization procedure makes use of FEA to predict the performance of the BLDC motor. A standard motor with dimensions listed in Appendix 1 is modeled using MagNet 7.1. 1 and the static 2D analysis is performed to predict the performance indices of the machine. While the flux lines of the model is shown in Fig. 2, the flux linkage variation for one rotation of rotor through 360 degrees is shown in Fig. 3. The variation in cogging torque as a function of rotor position is depicted in Fig. 4. The results

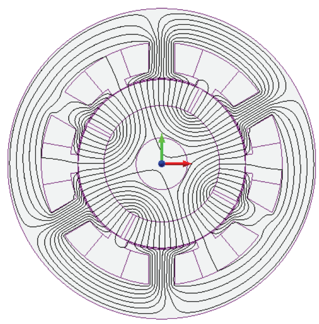

**Fig. 2.** Flux lines in BLDC

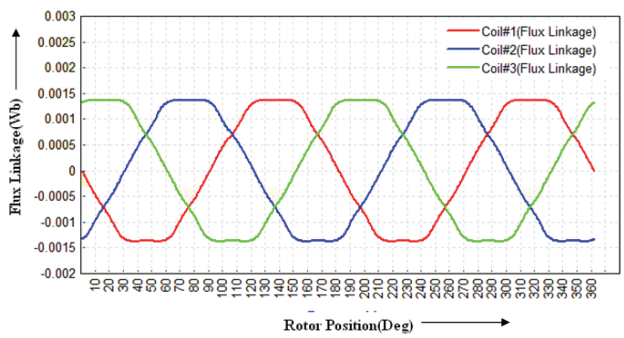


**Fig. 3.** Flux linkage Vs Rotor position

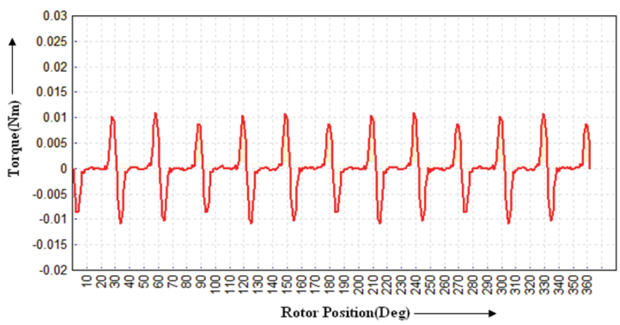


**Fig. 4.** Cogging torque Vs Rotor position

of FEA are validated with an experimental model and an analytical method.

An experimental setup shown in Fig. 5 is used to calculate the cogging torque and validate the result obtained using FEA. The procedure to work out the cogging torque is illustrated through the flow chart shown in Fig. 6.

Table 2 gives the cogging torque values obtained using FEA and experimentally. The closeness of the results validates the FEA based modeling of the prototype motor.

The average torque is computed analytically using the procedure described in [2]. Table 3 summarizes the average torque obtained using FEA and analytically. It is clear that the two values are near perfect thus validating the design of standard BLDC motor.

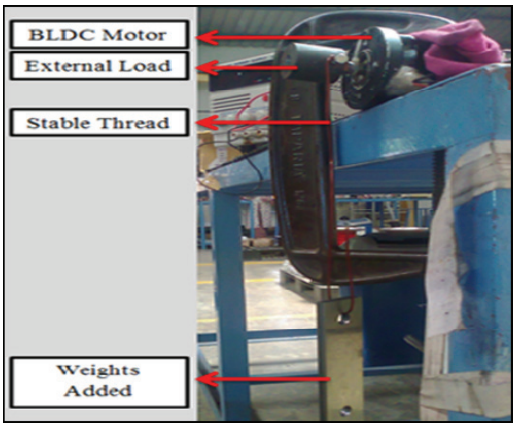


**Fig. 5.** Experimental setup for cogging torque measurement

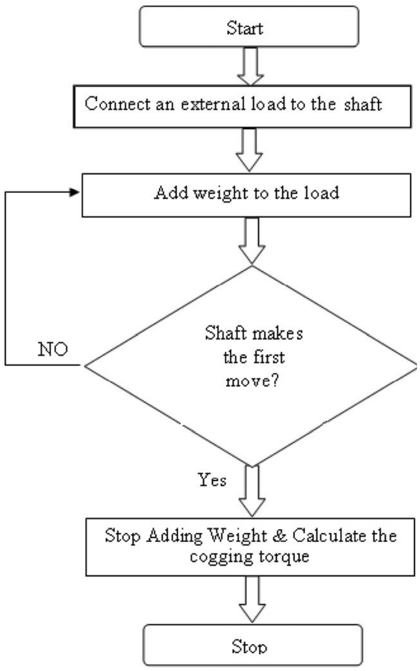


**Fig. 6.** Flow chart to measure cogging torque

**Table 2.** Comparison of cogging torque

| Performance parameter | Finite element analysis | Experimental value |
|---|---|---|
| Cogging torque (N-m) | 0.012 | 0.0094 |

**Table 3.** Comparison of average torque

| Performance parameter | Finite element analysis | Experimental value |
|---|---|---|
| Cogging torque (N-m) | 0.363 | 0.37 |

## 4. Design Optimization Approach Using GRNN

### 4.1. GRNN

This work involves application of GRNN for data interpolation and design optimization of BLDC motor. From the literature [22] it is understood that GRNN is a variation of radial basis function networks and has spread factor as the only design and varying parameter. It is denoted as $\sigma = \frac{d_{\max}}{\sqrt{2n}}$ where '$d_{\max}$' is the maximum distance between training points and 'n' is the number of training points. The spread factor can be tuned to result in a better performance of GRNN which means larger the value of '$\sigma$' smoother is the response [21].

The architecture of GRNN shown in Fig. 7 consists of three layers namely the input layer, the hidden layer, and the output layer [21].

The hidden layer has radial basis neurons, while neurons in the output layer have a linear transfer function. In GRNN, the number of radial basis neurons in the hidden layer is equal to the number of training samples. The distance between the input vector and the training sample form the input to each of the radial basis neurons. The RBF of the input scaled by the spread factor is the neurons output.

For a given number of input-output pairs 'm' and $\{x_i, y_j\} \in \Re^n \times \Re^1,\ i = 1,2,...m$ as the training samples, the GRNN output [21] for a test point $x \in \Re^n$ is given by

$$\hat{y}(x) = \sum_{i=1}^{n} W_i y_i ; \tag{3}$$

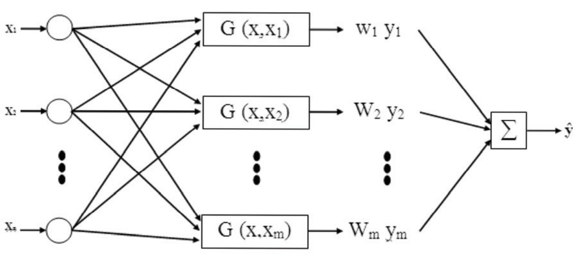


**Fig. 7.** GRNN structure

Where, $$w_i = \frac{\exp\left(\frac{-\left(\left\|x - x_i\right\|\right)^2}{2\sigma^2}\right)}{\sum_{k=1}^{m}\exp\left(\frac{-\left\|x - x_k\right\|^2}{2\sigma^2}\right)}$$

### 4.2. Interpolation using GRNN

In order to achieve the objectives stated in section 2, the effect of varying the design variables on cogging toque and average torque is analysed using FEA. The analysis reveals that both cogging toque and average torque are sensitive and vary non- linearly to the changes in the design variables. Hence, an optimization process is needed to ascertain the behaviour of cogging torque and average torque for each set of design variables. The performance parameters are evaluated using analytical method or FEA. While, analytical method involves approximations, FEA is more accurate but time consuming. Hence, in the optimisation routine GRNN based interpolation is involved.

Also the data available from FEA is discrete and sparse in the design space which may result inaccurate optimisation process as accuracy demands a continuous search space. This is achieved using GRNN that is found to be suitable for the problem mentioned above [17]. Besides GRNN has accurate prediction ability, it is also suitable to represent the objective function in an optimization process.

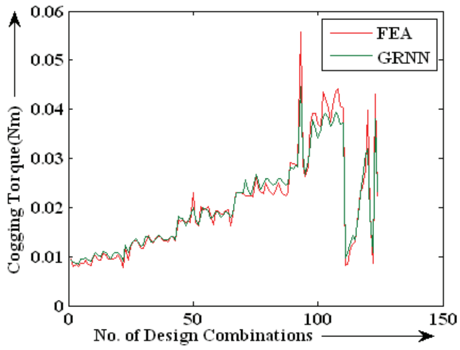


**Fig. 8.** Cogging torque Vs No. of design combinations

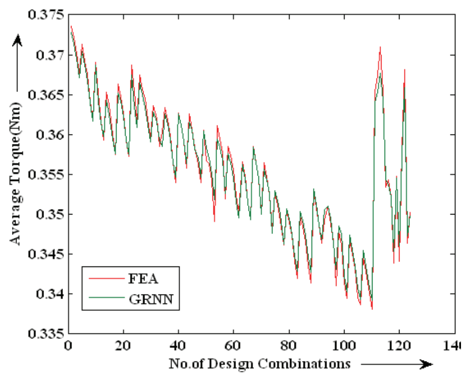


**Fig. 9**. Average torque Vs No. of design combinations

Other advantages of GRNN are that they are fast, accurate and minimize the programming effort once the network is trained.

To validate the accuracy of GRNN and subsequent application of GRNN in the optimization process, the variations of cogging torque and average torque for 120 design combinations are shown in Figs. 8 and Fig. 9 respectively. From the variations it is evident that GRNN results go with the ones obtained using FEA.

### 4.3. Optimization using GRNN

The optimization problem is formulated as

$$F(x) = (W1 * f_1(x) + W2 * f_2(x)) \tag{4}$$

Where

$f_1(x)$ = Maximization of average torque
$f_2(x)$ = Minimization of cogging torque

The constants W1 and W2 are weight factors for the objectives average torque and cogging torque respectively.

To determine the optimal design of BLDC motor with minimum cogging torque and maximum average torque, an optimization procedure using GRNN is included in the computation itself that follows training. The optimal design algorithm is explained using the flowchart in Fig. 10. The GRNN algorithm a spread factor of 0.5 is executed using MATLAB and it searches for the optimal values of cogging torque and average torque that are tabulated in Table 4. Further, the results obtained using GRNN are validated using PSO algorithm [23, 24]. The following parameter settings are used in PSO; population size=30; C1=1.5; C2= 1.5; maximum iterations=100. For efficient performance of algorithm, the parameters of PSO and GRNN are selected carefully after performing several simulation runs. It is to be noted that GRNN is used as data interpolation tool in PSO based optimization. The results conclude that GRNN based optimization routine effectively toils as interpolation

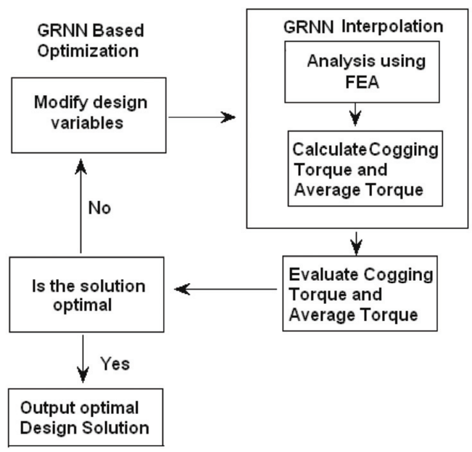


**Fig. 10.** Flow chart for optimization

**Table 4.** Optimal design obtained using GRNN

| Design variables and performance parameters | Initial model | GRNN based optimal design W1=0;W2=1 | PSO based optimal design W1=0;W2=1 |
|---|---|---|---|
| Magnet length (mm) | 8.0 | 7.80 | 7.78 |
| Stator slot opening (Deg) | 6 | 5 | 5 |
| Air gap (mm) | 0.5 | 0.46 | 0.45 |
| Average torque (N-m) | 0.363 | 0.379 | 0.371 |
| Cogging torque (N-m) | 0.012 | 0.0084 | 0.0084 |

**Table 5.** Design and performance measure for different weight factors

| Parameter | W1=1, W2=0 | W1=0.2, W2=0.8 |
|---|---|---|
| Magnet length (mm) | 7.2 | 7.3 |
| Stator slot opening (Deg) | 5 | 5 |
| Air gap (mm) | 0.45 | 0.45 |
| Average torque (N-m) | 0.373 | 0.372 |
| Cogging torque (N-m) | 0.009 | 0.0093 |

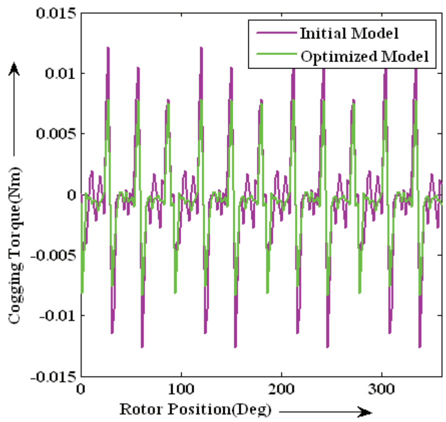


**Fig. 11.** Cogging torque variations between standard and optimized models

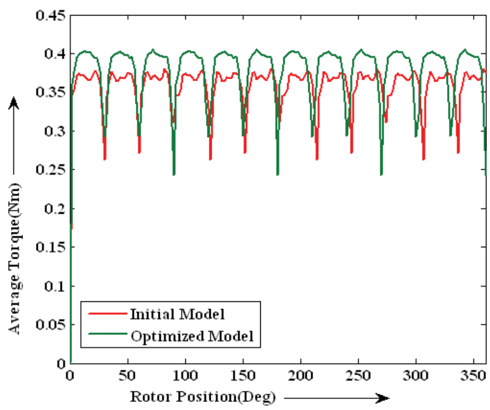


**Fig. 12.** Average torque variations between standard and optimized models

and optimization algorithm.

Further, the performance parameters of the optimal design are predicted using FEA. The variations in the cogging torque and average torque for the standard and optimized models are shown in Figs. 11 and Fig. 12 respectively. The variations reveal that there is significant improvement in the torque profile which validates the GRNN based optimization procedure.

## 5. Conclusion

In this paper a GRNN based interpolation and design optimization algorithm for a BLDC motor is proposed. The design variables are varied and the effect of varying the design variables on the performance indices is studied by performing FEA. Having obtained discrete data from FEA, GRNN is used for training the data and further optimization. The results of GRNN disclose an optimized BLDC motor design with a significant improvement in the torque profile. The same problem is solved using PSO to validate the GRNN based design solution and the results of PSO prove the precision of the design variables. The performance of the optimal design is analysed using FEA and the results indicate that the optimal design yields the estimated outcome.

## Appendix 1

Specifications of BLDC motor

| Parameters | Symbol | Value |
|---|---|---|
| Magnet length (mm) | $l_m$ | 8 |
| Magnet arc (deg) | $\Theta_m$ | 87 |
| Number of phases | $N_{ph}$ | 3 |
| Number of magnet poles | P | 4 |
| Number of armature slots | S | 6 |
| Air gap length (mm) | G | 0.5 |
| Slot opening arc (degree) | $\Theta_{so}$ | 6 |
| Inner radius of rotor (mm) | $R_{ri}$ | 18 |
| Outer radius of rotor (mm) | $R_{ro}$ | 26 |
| Inner radius of stator (mm) | $R_{si}$ | 26.5 |
| Radius of stator shoe (mm) | $R_{ss}$ | 28.5 |
| Radius of stator yoke (mm) | $R_{sy}$ | 37.5 |
| Outer radius of stator (mm) | $R_{so}$ | 48.5 |
| Tooth width of stator | W | 8 |
| Stack length (mm) | L | 43 |

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