

A Substation-Oriented Approach to Optimal Phasor Measurement Units Placement

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Abstract – State Estimation (SE) is the basis of a variety of advanced applications used in most modern power systems. An SE problem formed with enough phasor measurement units (PMUs) data is simply a linear weighted least squares problem requiring no iterations. Thus, designing a minimum-cost placement of PMUs that guarantees observability of a power system becomes a worthy challenge. This paper proposes an equivalent integer linear programming method for substation-oriented optimal PMU placement (SOOPP). The proposed method uses an exhaustive search to determine a globally optimal solution representing the best PMU placement for that particular power system. To obtain a more comprehensive model, contingencies and the limitation of the number of PMU measurement channels are considered and embodied in the model as changes to the original constraints and as additional constraints. The proposed method is examined for applicability using the IEEE 14-bus, 118-bus and 300-bus test systems. The comparison between SOOPP results and results obtained by other methods reveals the excellence of SOOPP. Furthermore, practical large-scale power systems are also successfully analyzed using SOOPP.

Keywords: Phasor measurement units, OPP, Contingency, Measurement channels limitation, Substation oriented, Linear programming method

1. Introduction

State estimation (SE) is the basis of almost all advanced power system applications and has played a significant role in power system monitoring and controlling over the past several decades. As modern power systems are constantly becoming more and more complex, new approaches to accelerate SE are strongly needed. For this reason, when PMUs were first introduced into power systems as mature devices [1], both utilities and research institutions soon began to investigate the potential for using the complex measurement data provided by PMUs for SE. With the growing number of PMUs planned for installation in the near future, it is becoming even more important to find optimal solutions for PMU placement [2].

One major problem with SE based on conventional measurements is that measurements of the system are unsynchronized. Thus, the system state can only be inferred from the unsynchronized power flow measurements from one scan (such as data gathering by SCADA). In this case, state estimators are obliged to make compromises. The most significant one is assuming the system did not change during the scan – that the system was static [3]. This assumption is not required in an SE formed with only PMU

data.

Another major problem of conventional measurements-based SE – which optimal PMU placement (OPP) attempts to solve – is the nonlinear equivalent SE model. Existing SE algorithms [4] use conventional measurements of line flows and injections of both real and reactive power to estimate all bus voltage angles and magnitudes. Thus, functions consisting of the power system state and conventional measurements are nonlinear. However, an SE problem populated with only PMU data is linear because PMUs gather the complex voltage and current measurement data directly.

Therefore, the system observability could be assessed by considering the topology of the network along with the types and locations of the measurements. There are two strict definitions of system observability: one from a numerical view and one from a topological view. Numerical observability is defined as the ability of a system model to be solved for a state estimate, if the gain or measurement Jacobin matrix has full rank and is well conditioned. Topological observability is defined as the existence of at least one spanning measurement tree of full rank in the network [5].

Because each PMU has more than one measurement channel, each PMU can be used to gather several measurements such as the bus voltage, the line current and injections simultaneously. Thus, to make an N-bus system observable usually requires fewer than N PMUs. A basic OPP problem is to determine the least number of PMUs required and their locations in the network to ensure

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system observability.

Previously reported approaches could be classified into meta-heuristic and deterministic methods. The techniques for solving OPP problems by using one or a combination of meta-heuristic optimization methods are developed in [5-18], and include simulated annealing, practical heuristic algorithms, genetic algorithms, particle swarm optimization, Tabu search, differential evolution, an immunity algorithm, decision trees, an iterated local search, spanning tree searches, greedy algorithms, and recursive security N algorithms. The major weakness of using intelligent search algorithms is that globally optimal solutions are not guaranteed. Approaches based on deterministic methods as proposed in [19-31] attempt to build equivalent integer problems (EIP) to describe how the PMUs installed in the power system cause the whole network to be observable according to the observability rules and then solve the EIP using local or general optimization techniques. The method described in [6] was a breakthrough that used an EIP model to describe the task of PMU placement at strategic buses to make the entire system observable. The existence of constraints that consider PMU measurements, injections and flows is also discussed. However, the consideration of zero injections has made the formulations nonlinear. The method provided in [24] built an EILP model for the OPP problem that considered PMU and line outages as well as PMU channel limitations, and obtained globally optimal solutions through an exhaustive search. However, this method did not address single PMU outages. The method in [32] considered this gap in the method of [24] and built an almost perfect model, except for the one tiny flaw that the bus observability provided by neighboring buses is not fully formulated.

Until recently, all OPP models have assumed that PMUs must be installed at buses and that the PMUs only gather the voltage of the installed bus and the current of branches connected to the installed bus. In this work, we classify these OPP algorithms as “bus-oriented” methods (BOM). However, because of the additional development of PMU devices and communication techniques, the maximum number of a PMU's measurement channels is usually more than the number of bus voltages and currents of the lines attached to the bus. Because a substation is a basic unit for electrical power companies and utilities management organizations, as well as of power system infrastructure constructions, this paper presents a substation oriented optimal PMU placement (SOOPP) approach through an equivalent integer formulation. Most of the BOMILPs were actually binary ILPs, which means the solutions only indicated whether a PMU was installed at a particular bus or not and allowed only one PMU installed at one bus. This is not a fatal problem to BOM because usually one PMU has enough channels to gather all of the measurements of one bus and the branches attached to the bus. However, this is crucial for SOOPP in that it is very possible that one PMU could not gather all of the measurements of one

substation in a practical system. Therefore we use integer variables to represent the number of PMUs installed in one substation. We introduce two sets of auxiliary variables to linearize the originally nonlinear model associated with zero injections and observability rules. Furthermore, this model handles constraints due to the limitation of the number of PMU measurement channels as first additional constraints because a substation usually requires many more measurement channels than does a bus. Moreover, SOOPP can be extended to satisfy the additional constraints of line and PMU outages. Because PMUs are highly reliable devices, and considering the features of substation-oriented PMU installations, we replace a full PMU outage with an outage of an individual PMU measurement channel. This change will be explained in detail in the remainder of this work.

The SOOPP model has three advantages comparing with the conventional OPP models:

- a. SOOPP could make better use of PMUs with a large maximum number of measurement channels. This could bring a significant cost reduction in practical power systems.
- b. Substation oriented engineering projects and plans are more welcomed by power system operators and employee.
- c. Devices in one substation are very geographically concentrated. Therefore the placements of SOOPP results are usually more engineering implementable.

The rest of this paper is organized as follows. Section II provides a short review of the observability rules and their expressions in BOM formulations. Note that classical observability rules are still strictly applied in SOOPP, but are written in a new form. In Section III, the proposed SOOPP is formulated with basic and additional conditions. Simulation results on IEEE standard test systems and practical large scale systems are presented in Section IV along with the comparisons between the results using the SOOPP model and other methods.

2. Observability Rules

Conventional BOM without consideration of zero injections formulates the basic observability rules as follows:

$$\text{minimize } \sum_{j=1}^N \gamma_j b_j \quad (1)$$

$$\text{subject to: } O_i \geq 1, \quad \forall i \quad (2)$$

where γ_j is the cost of a PMU installed at bus j , b_j is the binary variable indicating whether a PMU is installed at bus j , and O_i is the observability function of bus i . The value of O_i is equal to or greater than 1 if bus i is

observable, and 0 otherwise. Here O_i is written as:

$$O_i = b_i + \sum_{j=1, j \neq i}^N a_{j,i} b_j \geq 1 \quad \forall i \quad (3)$$

where N is the number of network buses (including buses and star points of three-winding transformers) and $a_{j,i}$ is the $j-i$ entry of the connectivity matrix where:

$$a_{j,i} = a_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or bus } i \text{ is connected with bus } j \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

To formulate the help of zero injections mathematically, suppose z_s is the index of the s th zero injection bus and the auxiliary binary variable x_{i,z_s} is defined so that $x_{i,z_s} = 1$ implies that the calculation of the voltage of bus i is assigned to the equation associated with zero injection s [32]. Accordingly, the formulations of the observability constraints with inclusion of zero injections are as follow:

$$O_i = b_i + \sum_{j=1, j \neq i}^N a_{i,j} b_j + \sum_{s=1}^S a_{i,z_s} x_{i,z_s} \geq 1 \quad \forall i \quad (5)$$

$$\sum_{i=1}^N a_{i,z_s} x_{i,z_s} = 1 \quad \forall s \quad (6)$$

$$\sum_{s=1}^S a_{i,z_s} x_{i,z_s} \leq 1 \quad \forall i, \quad (7)$$

where S is the number of network zero injection buses (including buses and star points of three-winding transformers). In simple terms, (6) guarantees that every zero injection has a contribution that changes a previously unobservable bus into an observable bus, whereas (7) guarantees that, to any bus, there is no more than one neighbor zero injection (including the bus itself) contributing to its observability. The strict proof for this form of O_i is provided in [32]. In the next section, the SOOPP formulation inspired by BOM is proposed.

3. Integer Linear Soopp Formulation Approach

In this section, the basic SOOPP formulations in which zero injections are both included and excluded are first proposed. Then, the case considering a limitation of the number of PMU measurement channels is discussed. Because one substation usually requires more measurements than a PMU's maximum number of channels to satisfy observability requirements, the case without consideration of the limitation on channels is discussed only to help describe and explain the thought process of the formulation. Furthermore, cases with constraints on branch outages and PMU measurement channel outages are considered.

3.1 SOOPP with basic conditions

In a fashion similar to BOM models, the basic SOOPP formulation is as follows:

$$\text{minimize} \quad \sum_{k=1}^K \omega_k q_k \quad (8)$$

$$\text{subject to:} \quad O_i \geq 1, \quad \forall i \quad (9)$$

where K is the number of power system substations, ω_k is the cost for each PMU installed in substation k and q_k is an integer variable that represents the number of PMUs installed in substation k . Within this SOOPP, O_i is defined as:

$$O_i = q_g + \sum_{j=1, j \neq i}^N a_{j,i} q_k \geq 1 \quad \forall i \quad (10)$$

where k and g indicate that bus i belongs to substation g and bus j belongs to substation k . Note that if bus i is not connected with any buses belonging to other substations, the only way to make bus i observable is to install a PMU in substation g .

With a change similar to that for the BOM approach, and including zero injections into (6) and (7), we can then write the observability constraint formulation considering zero injections as follows:

$$O_i = q_g + \sum_{j=1, j \neq i}^N a_{j,i} q_k + \sum_{s=1}^S a_{i,z_s} x_{i,z_s} \geq 1 \quad \forall i \quad (11)$$

Constraints (10) and (11) guarantee that observability of any bus is first provided by PMUs installed in the substation to which the bus belongs. If there is not a PMU installed in the substation, then one of the observable neighbors of the unobserved bus will provide the observability. Furthermore, (11) dictates that even if there is no neighboring bus directly observable, observability still can be provided by one of the neighboring zero injection buses, whereas (6) and (7) guarantee that all zero injections have contributed to the system observability while still obeying the observability rules.

3.2 SOOPP with inclusion of PMU measurement channels limitations

To include a limitation on the number of PMU channels into SOOPP, the basic observability constraints dictated by (10) need to be reformulated to use individual channels as variables, as follows:

$$O_i = p_i + \sum_{j=1, j \neq i}^N a_{j,i} (l_{j,i} \| l_{i,j}) p_j \geq 1 \quad \forall i \quad (12)$$

where p_i, p_j are the binary decision variables that are

equal to 1 if a PMU voltage measurement channel is used to measure the voltage of bus i or bus j and are 0 otherwise; and $l_{i,j}, l_{j,i}$ are the binary decision variables that are equal to 1 if a PMU current measurement channel on the side of bus i or bus j is used to measure the current of the branch between bus i and bus j and are 0 otherwise. The operator “ \parallel ” serves as a logical “OR” operation. The constraints of (12) guarantee the observability of bus i either by a PMU measurement channel gathering the voltage of bus i , or through a calculation based on the voltage measurement of a neighboring bus gathered by a PMU channel and a current measurement of branch $i-j$ on either side gathered by another PMU channel.

Formulation (12) perfectly describes the observability constraints. However, the expression $(l_{j,i} \parallel l_{i,j})p_j$ is nonlinear, and thus a globally optimal solution is not guaranteed. Therefore, another set of binary auxiliary variables $h_{j,i}$ is introduced to linearize (12), where $h_{j,i}$ is defined as follows:

$$h_{j,i} \leq p_j \quad (13)$$

$$h_{j,i} \leq l_{j,i} + l_{i,j} \quad (14)$$

$$h_{j,i} \geq l_{j,i} + p_j - 1 \quad (15)$$

$$h_{j,i} \geq l_{i,j} + p_j - 1. \quad (16)$$

Take $l_{j,i} = l_{i,j} = p_j = 0$ as an example. From Eqs. (13) and (14), we can deduce that $h_{j,i} \leq \min(p_j, l_{j,i} + l_{i,j}) = 0$. And from Eqs (15) and (16), we can similarly deduce that $h_{j,i} \geq \max(l_{i,j} + p_j - 1, l_{j,i} + p_j - 1) = -1$. Because $h_{j,i}$ is a binary variable, we have $h_{j,i} = (l_{j,i} \parallel l_{i,j})p_j = 0$. From Table 1, we find that the value of $h_{j,i}$ is always equal to $(l_{j,i} \parallel l_{i,j})p_j$ with the above constraints. Thus (12) becomes:

Table 1. Value table for linearization trick

Expression	Value							
p_j	0	0	0	0	1	1	1	1
$l_{i,j}$	0	0	1	1	0	0	1	1
$l_{j,i}$	0	1	0	1	0	1	0	1
$\min(p_j, l_{j,i} + l_{i,j})$	0	0	0	0	0	1	1	1
$\max(l_{i,j} + p_j - 1, l_{j,i} + p_j - 1)$	-1	0	0	0	0	1	1	1
$h_{j,i}$	0	0	0	0	0	1	1	1
$(l_{j,i} \parallel l_{i,j})p_j$	0	0	0	0	0	1	1	1

$$O_i = p_i + \sum_{j=1, j \neq i}^N a_{j,i} h_{j,i} \geq 1 \quad \forall i. \quad (17)$$

With this alteration, (13) - (17) comprise a completely linear model for solving a BASIC SOOPP problem.

To address the limitation of the number of PMU measurement channels, an additional limitation constraint formulation is added as follows:

$$\sum_{\text{bus } i}^{\text{in substation } k} (p_i + \sum_{j=1, j \neq i}^N a_{i,j} l_{i,j}) \leq MCL_k q_k, \quad \forall k, \quad (18)$$

where MCL_k is the limit of the number of measurement channels of each PMU installed in substation k .

Note that if all MCL_k are extremely large numbers, the model is equivalent to the SOOPP model without the constraint on the number of channels.

The effect of zero injections on the basic SOOPP model is the same as the effect on the BOM model. In addition to using (6), (7), (13) - (16) and (18), the observability constraint is added as follows:

$$O_i = p_i + \sum_{j=1, j \neq i}^N a_{j,i} h_{j,i} + \sum_{s=1}^S a_{i,z_s} x_{i,z_s} \geq 1 \quad \forall i. \quad (19)$$

3.3 Impact of contingencies

Line outages and PMU malfunctions are inevitable in practical power systems. In this subsection, formulations described in the last two subsections are revised to handle these contingencies. One aspect of the SOOPP model is that a substation usually requires many more measurement channels than the maximum number of channels available on one PMU so that constraints limiting the number of channels must be considered. Note that constraints (13)-(16) and (18) may not be mentioned for the remainder of this work, although they are indispensable parts of the complete model.

3.3.1 Considering just line outages

Without the aid of zero injections, the observability formulation is converted to the following inequality to ensure observability of the whole system in the case of any one line outage:

$$O_i = 2p_i + \sum_{j=1, j \neq i}^N a_{j,i} h_{j,i} \geq 2 \quad \forall i. \quad (20)$$

Inequality (20) is easy to understand. If bus i is equipped with a measurement channel, this guarantees observability of the bus for any line outage. If the bus is not equipped with a channel, it needs two or more neighboring buses to ensure its observability in case of one of the neighboring buses loses connection with bus i .

If we do include zero injections into the model, the observability formulation instead becomes:

$$O_i^m = p_i + \sum_{j=1, j \neq i}^N a_{j,i}^m h_{j,i} + \sum_{s=1}^S a_{i,z_s}^m x_{i,z_s}^m \geq 1, \quad \forall i, \forall m, \quad (21)$$

where superscript m denotes the outage of line m , and $a_{j,i}^m$ (a_{i,z_s}^m) is equal to 0 if line m connects bus i with bus j (bus z_s) and $a_{j,i}^m = a_{j,i}$ ($a_{i,z_s}^m = a_{i,z_s}$) otherwise.

The most significant change is x_{i,z_s}^m in this equation. This variable indicates that, for each line outage, the placements (which illustrate different ways in which zero injections make other buses observable) of the set x_{i,z_s} may be - and usually are - different. Due to this change, (6) and (7) become as follows:

$$\sum_{i=1}^N a_{i,z_s}^m x_{i,z_s}^m = 1, \quad \forall s, \forall m \quad (22)$$

$$\sum_{s=1}^S a_{i,z_s}^m x_{i,z_s}^m \leq 1, \quad \forall i, \forall m. \quad (23)$$

3.3.2 Considering both line and measurement channel outages

For almost all of the previous approaches, the so-called “PMU (at bus i) outage” was equivalent to “bus i has no PMU installed.” However, this interpretation is not suitable in the SOOPP model, for it has little practical meaning to infer that “substation k has no PMU installed” because for most practical substations, this is inferring that there is no way to make the buses in the substation observable. Therefore, given that PMUs are quite reliable, this paper replaces the whole PMU outages concept with PMU measurement channel outages for the SOOPP model.

PMU measurement channel outages consist of two types. The first type is an outage of a current measurement channel. The current measurement channel is represented as $l_{i,j}$ in the model. If a system remains observable when line m (which connects bus i and bus j) is out, it will also remain observable when channels $l_{i,j}$ and $l_{j,i}$ are both out at the same time. This means that the SOOPP model's consideration of all line outages is stricter than the approach in which all of the current channel outages are considered, and the increased number of formulations of the former is only that of half of the latter. Therefore, this type of PMU measurement channel outage could be replaced by the equations proposed in subsection 1) that address line outages.

The second type of PMU measurement channel outage is an outage of voltage measurement channels. The voltage measurement channel is represented as p_i or p_j in the model. If we assume that p_n , the measurement channel gathering the voltage of bus n , is out, the system remains observable if one of the following observability constraints is satisfied:

- a. Without consideration of zero injections

$$O_i = p_i + \sum_{j=1, j \neq i}^N a_{j,i} h_{j,i} \geq 2, \quad \forall i; \quad (24)$$

- b. With consideration of zero injections

$$O_i^n = \zeta_{i,n} p_i + \sum_{j=1, j \neq i}^N a_{j,i} \zeta_{j,n} h_{j,i} + \sum_{s=1}^S a_{i,z_s} x_{i,z_s}^n \geq 1, \quad \forall i, \forall n \quad (25)$$

where $\zeta_{i,n}$ ($\zeta_{j,n}$) is a binary parameter whose value is 0 if $i \neq n$ ($j \neq n$) and is 1 otherwise. Like the formulations of (22) and (23), the additional constraints for x_{i,z_s}^n are as follows:

$$\sum_{i=1}^N a_{i,z_s}^n x_{i,z_s}^n = 1, \quad \forall s, \forall n \quad (26)$$

$$\sum_{s=1}^S a_{i,z_s}^n x_{i,z_s}^n \leq 1, \quad \forall i, \forall n. \quad (27)$$

Thus, Eqs (21) - (23) and (25) - (27) with (13) - (16) and (18) provide an equivalent integer linear model for SOOPP that takes into account zero injections, measurement channels limitations, and line and channel outages.

3.4 Considering existing measurements and system stability monitoring demand

SOOPP could also take into account the already installed measurements including PMUs and conventional measurements such as injections and flows.

3.4.1 Installed PMUs

In practical power systems, usually there are already PMUs installed.

To include the already installed PMUs in the power system, a set of constraints as follows should be added:

$$q_k = 1, \quad \forall \text{substation } k \in \Lambda.$$

where set Λ is the set of substations in which PMUs are already installed. Similarly, if considering PMU measurement channels, the following constraints should be added:

$$p_i = 1, \quad \forall \text{bus } i \in \Gamma$$

$$l_{i,j} = l_{j,i} = 1, \quad \forall \text{branch } i - j \in \Gamma.$$

where set Γ is the set of system buses and branches whose voltages or currents are gathered by PMU measurement channels. The equivalent expressions above usually will be pre-solved before solving the optimal problem.

If branch $i - j \in \Gamma$, the auxiliary variables $h_{j,i}$ and $h_{i,j}$ could be removed from the model. Furthermore, Eqs. (13) - (16) corresponding to $h_{j,i}$ and $h_{i,j}$ could be eliminated as well. Both numbers of variables and equations are reduced.

3.4.2 Installed conventional injection measurements

It is known that zero injection buses could help to achieve system observability. Suppose a bus is a zero injection one, the set including that bus and all its incident buses is defined as a zero injection cluster (ZIC). For each ZIC in the power network, a linear relation equation

between the voltage phasors of its buses can be obtained based on the KCL [32]. Thus, the only unknown bus voltage could be calculated by solving this equation. Similarly, suppose bus i is not a zero injection one, but there is a conventional measurement equipment installed at this bus. With the gathered injection flow measurement, the nodal power flow equations for bus i could be obtained. Thereby, bus i could be treated as a zero injection bus when building the SOOPP model according to the observability rules.

3.4.3 Installed conventional power flow measurements

Suppose set Ω is a set of system branches, and the power flows through these branches are gathered by conventional measurements. Thus we rewrite observability constraints formulation (19) as follow:

$$O_i = p_i + \sum_{\substack{j=1 \\ \text{branch } i-j \notin \Omega}}^N a_{j,i} h_{j,i} + \sum_{\substack{j=1 \\ \text{branch } i-j \in \Omega}}^N a_{j,i} P_j + \sum_{s=1}^S a_{i,z_s} x_{i,z_s} \geq 1 \quad \forall i.$$

Similar to subsection 1), if $\text{branch } i-j \in \Omega$, the auxiliary variables $h_{j,i}$, $h_{i,j}$ and the corresponding Eqs. (13)-(16) could be removed from the model.

3.4.4 System stability monitoring demand

PMUs are not installed for systems state estimation purpose only, another important task of PMUs is to monitor the systems stability. Theoretically, if the whole system observability is achieved, the complex voltages of all buses are directly gathered by PMU measurement channels or indirectly calculated using KCL and KVL. However, in many cases, the crucial substations, buses and branches are request to be monitored directly by PMUs to meet the demand of system stability monitoring demand.

In this case, we could assume that the crucial substations, buses and branches are already equipped by PMUs or PMU measurement channels before generate the optimization model, and then use the same constraints proposed in subsection 1) to obtain the optimal PMUs placement of the rest of the system.

3.5 Other constraints

In this subsection, several additional constraints are discussed. Some of the constraints are necessary to properly model SOOPP problems, whereas some others are optional.

3.5.1 No Channel Installed at star points of three-winding transformers

The star points of three-winding transformers do not

really exist as physical buses, but are instead only virtual buses in the equivalent power system networks. Thus, a set of constraints should be added into the SOOPP model guaranteeing that no PMU channel is used to gather the voltage of a nonexistent bus or the current of a branch at the nonexistent side:

$$p_{c_d} = 0, \quad \forall d \quad (28)$$

$$a_{c_d,j} I_{c_d,j} = 0, \quad \forall d, \forall j, \quad (29)$$

where c_d is the bus index of the virtual buses. If there are D virtual buses in the equivalent network, (29) contains $\sum_{d=1}^D (\sum_{j=1}^N a_{c_d,j} I_{c_d,j})$ formulations, which is a large number. Whereas we can replace (29) with an equivalent expression as follows:

$$\sum_{j=1}^N a_{c_d,j} I_{c_d,j} = 0, \quad \forall d, \quad (30)$$

the former is still better than the latter because (29) will usually be pre-solved before solving the optimal problem. Constraints (28) and (29) are included in the simulations described later in this work.

3.5.2 No channel at zero injection buses

The forbiddance of a measurement channel gathering the voltage of zero injection buses could be guaranteed by using constraints:

$$p_{z_s} = 0, \quad \forall s.$$

In most cases, this constraint reduces the search space, which improves the speed of finding a solution. However, in some specific cases, the constraint may also result in a sub-optimal solution. As a result, this constraint is not included in the following simulations.

4. Simulation Results

The models were generated by an application built in the C++ language, whereas the commercial software CPLEX was In this section, the proposed model is used to analyze IEEE 14-bus, 118-bus and 300-bus standard test systems as well as some real-world systems. All of the cases were implemented on a 2.80 GHz quad-core CPU with 4GB RAM. utilized as the solver. All of the presented SOOPP solutions are globally optimal solutions in that all of the optimality gaps are equal to zero.

First, we used two figures to show the contrast between the results and placements of BOM and SOOPP using an IEEE 14-bus system. Then, we conducted basic SOOPP and SOOPP considering single contingencies, both with

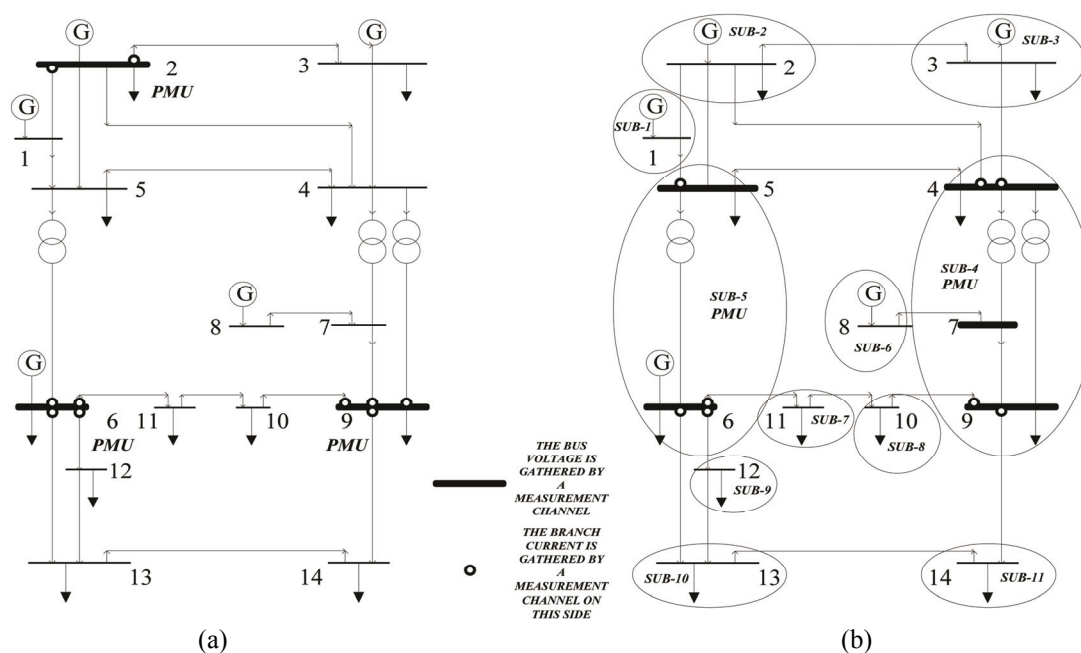


Fig. 1. (a) BOM result and placement three PMUs installed respectively at bus-2, bus-6 and bus-9; (b) SOOPP result and placement two PMUs installed respectively at substation-4 and substation-5

and without consideration of zero injections for the IEEE 118-bus test system. We also compared these results with the results of other approaches that are able to handle single contingencies. Next, PMU measurement channel limitations are considered, and the results are compared with one of the best BOM results. Finally, SOOPP is used to analyze two real-world power systems, the results of which show the strength and efficiency of the proposed method.

Because SOOPP is a substation-oriented method, we distributed the buses of the IEEE 14-bus test system into 11 substations, using the rule that only buses linked by transformer windings belong to the same substation. This system contains three windings, and bus 4 has two windings attached. Thus there is one substation with three buses, another substation with two buses and nine substations with one bus. Applying the same rule, the IEEE 118-bus test system contains 109 substations, and the largest substation has two buses; the IEEE 300-bus test system contains 193 substations, and the largest substation has nine buses. Note that these seemingly awkward distributions are not likely to occur in practical systems.

4.1 Brief comparison between the BOM and SOOPP results

The results and placements of BOM and SOOPP using the IEEE 14-bus system are shown in Figs. 1(a) and Fig. 1(b), respectively. The help of zero injections is considered, and PMUs are supposed to have unlimited numbers of channels. A possible confusion is that branch 7-9 is more like a switch in practical systems than a line. However, this

has no influence on our discussion.

According to the BOM result, we need to install three PMUs, one each at buses 2, 6 and 9. On the other hand, the SOOPP result requires only two PMUs, one installed in each of substations 4 and 5. Note that the two placements both used 13 channels, but the numbers of required PMUs are different. The contrast brings out the essence of SOOPP: that SOOPP could make better use of PMUs which have more channels. Furthermore, BOM placements usually involve more substations than SOOPP placements, whereas substations are the base management units of most power systems companies and utilities. Thereby, in contrast with BOM, SOOPP is a more applicable and more acceptable methodology.

In Fig. 1(b), if we use the same channel gathering the voltage of bus 7 to gather the current of branch 7-9, the system is still completely observable. Additionally, because the number of a PMU's channels is unlimited, we could add channels to gather measurements such as the currents of branch 7-8 and winding 4-7 without additional PMUs. Thus we could conclude that SOOPP problems could have more than one best solution. Spontaneously, we could revise the equations to achieve some other subgoals. For instance, we could add some channel variables terms into the object function to minimize the number of channels to increase the convergence velocity. On the other hand, some other channel variables terms could maximize the effect of the PMUs by using as many channels as possible (constrained by channels limitations) while still guaranteeing that the number of PMUs is minimized. The optimal placement of these redundant measurement channels is a very interesting issue.

4.2 Basic and single contingency constrained SOOPP

Results of the SOOPP method without consideration of measurement channel limitations for the IEEE 118-bus standard test system under different conditions are shown in Tables 2 and 3.

Comparison of the left and right columns in Table 2 demonstrates that the model achieves full system observability using less PMUs when zero injections are

Table 2. Number of PMUs for the IEEE 118-BUS TEST SYSTEM under different conditions

condition \ SOOPP	WITHOUT zero injections	WITH zero injections
Basic	31	27
Single lines outage	55	53
Single lines & channels outage	65	61

Table 3. Optimal pmu locations for the IEEE 118-bus test system under different conditions

SOOPP condition	WITHOUT zero injections	WITH zero injections
Basic	1, 9, 12, 15, 21, 23, 29, 35, 40, 43, 45, 49, 53, 56, 71, 75, 77, 85, 86, 90, 94, 102, 105, 110, 115, (17/30), (5/8), (59/63), (65/66), (68/69), (80/81)	3, 12, 15, 21, 23, 28, 34, 40, 45, 49, 53, 56, 62, 71, 75, 77, 85, 87, 90, 94, 102, 105, 110, 115, (17/30), (5/8), (80/81)
Single lines outage	1, 7, 10, 11, 12, 15, 19, 21, 23, 27, 29, 32, 36, 40, 42, 43, 44, 46, 49, 50, 51, 53, 56, 62, 70, 72, 73, 75, 76, 78, 83, 85, 87, 88, 90, 92, 94, 96, 100, 101, 105, 107, 109, 111, 112, 114, 116, 117, (17/30), (25/26), (37/38), (5/8), (59/63), (65/66), (80/81)	1, 7, 10, 11, 12, 15, 18, 20, 22, 24, 28, 29, 32, 36, 40, 41, 43, 45, 46, 49, 52, 54, 57, 58, 60, 67, 73, 74, 77, 79, 83, 85, 87, 89, 91, 92, 94, 96, 100, 102, 105, 106, 109, 111, 112, 115, 116, 117, 118, (17/30), (25/26), (59/63), (80/81)
Single lines & channels outage	1, 3, 6, 9, 10, 11, 12, 15, 19, 21, 22, 24, 28, 29, 32, 34, 36, 40, 41, 44, 45, 46, 49, 50, 52, 53, 56, 58, 62, 70, 71, 73, 75, 76, 77, 78, 84, 85, 86, 87, 89, 91, 92, 94, 96, 100, 102, 105, 106, 109, 110, 111, 112, 114, 115, 116, 117, (17/30), (25/26), (37/38), (5/8), (59/63), (65/66), (68/69), (80/81)	1, 3, 7, 9, 10, 11, 12, 15, 19, 21, 22, 24, 27, 28, 29, 32, 34, 36, 40, 41, 43, 45, 46, 49, 50, 51, 52, 54, 56, 62, 67, 70, 73, 75, 77, 78, 83, 85, 86, 87, 89, 91, 92, 94, 96, 100, 102, 105, 106, 108, 110, 111, 112, 114, 116, 117, 118, (17/30), (25/26), (61/64), (80/81), (59/63), (65/66), (68/69), (80/81)

used. Additionally, the impact of single contingency considerations is clearly illustrated, as expected. Furthermore, it is easy to see that, as more contingencies are considered, more PMUs are required to maintain system observability when device outages occur.

Table 3 shows the optimal PMU locations using the SOOPP model. The numbers in the table are the bus indices of the IEEE 118-bus test system. The numbers without parentheses indicate that a substation contains an installed PMU, and the PMU only monitors the corresponding bus. On the other hand, numbers divided by a slash in parentheses indicate that the PMU installed in the substation contains all of the corresponding buses in parentheses. Note that there is no instance in which two PMUs are installed in one substation because every PMU has an unlimited number of measurement channels.

In Table 4, three previous BOM approaches are chosen to compare with SOOPP for two reasons: the selected approaches can handle both lines and PMU contingency-constrained OPPs; and these BOMs provided the best results for their types of algorithms. The comparison illustrates that the proposed SOOPP method provides equivalent or better results compared to the previous methods. Note that the contingency-constrained results using the SOOPP, BILP, and ILP models are the same because of the impractical nature of the substations of the test systems. In practical power systems, one substation usually contains many more buses and branches than in the IEEE 118-bus system, for which the SOOPP model can provide much better optimal results.

4.3 Considering measurement channels limitation

As mentioned in the previous sections, the limitation of the number of PMU measurement channels in the SOOPP

Table 4. Comparison between SOOPP and other methods considering a single contingency for the IEEE 118-bus test system

Condition \ Method	RS [8]	ILP [24]	BILP [32]	SOOPP
Basic	31	28	28	27
Lines outage	63	53	53	53
Lines & channels outage	73	65	61	61

Table 5. SOOPP results considering PMU measurements limitations for the IEEE 118-bus test system under different conditions

Limitation	Optimal results shown as <No. of PMUs, No. of bus voltage channels, No. of branch current channels >					
	Without consideration of zero injections			With consideration of zero injections		
	Basic	Line outages	Lines & channels	Basic	Line outages	Lines & channels
1	<118,118,0>	<118,118,0>	<179,118,61>	<108,108,0>	<108,108,0>	<161,106,55>
2	<59,48,70>	<79,82,73>	<95,97,93>	<54,45,63>	<70,68,70>	<86,86,84>
3	<40,41,77>	<64,67,107>	<73,73,135>	<37,34,75>	<59,58,96>	<67,68,117>
4	<33,34,85>	<59,61,123>	<67,71,142>	<30,32,79>	<54,56,101>	<61,64,126>
5	<31,34,90>	<56,61,124>	<66,70,147>	<28,30,80>	<53,54,107>	<61,64,130>
10	<31,33,94>	<55,61,135>	<65,71,155>	<27,30,92>	<53,56,108>	<61,65,140>
Unlimited	<31,-,->	<55,-,->	<65,-,->	<27,-,->	<53,-,->	<61,-,->

Table 6. SOOPP results considering pmu measurement channels limitations for the IEEE 300-bus test system under different conditions

Condition Limitation	Optimal results shown as <No. of PMUs, No. of bus voltage channels, No. of branch current channels >					
	Without consideration of zero injections			With consideration of zero injections		
	Basic	Line outages	Lines & channels	Basic	Line outages	Lines & channels
5	<82,132,177>	<118,207,215>	<142,220,319>	<66,112,134>	<99,164,178>	<117,181,247>
10	<75,145,177>	<111,212,212>	<128,219,351>	<62,111,146>	<95,169,173>	<107,186,259>
UNLIMITED	<75,-,->	<111,-,->	<126,-,->	<62,-,->	<95,-,->	<106,-,->

method required much greater consideration than in the equivalent BOM implementations.

Without considering zero injections, the system requires the same or a greater number of complex voltage and current measurements than the total number of system buses to reach full system observability. With the help of zero injections, the sum of the numbers of complex measurements and zero injection buses should be equal to or greater than the total number of buses. The two assumptions above are proven in Table 5, where 118 and 108 one-channel PMUs are needed for the two basic conditions, without and with zero injections, respectively.

From Both Table 5 and Table 6, we find that fewer PMUs are needed when higher channel limitations are allowed. The relationship between the two is not linear. The rate of decrease in the number of required PMUs slows down as the limitation value increases. Furthermore, when the limitation reaches a threshold value, the number of required PMUs remains the same as that in the result of the SOOPP model without channel limitations. The importance of current measurement channels in reducing the number of required PMUs is explicit in the results provided in Tables 5 and 6. When single contingencies are considered, current measurement channels contribute even more to the robustness of system observability.

4.4 SOOPP for practical large systems

Table 7 displays the results and execution times for the

Table 7. Comparison between SOOPP results and results using [32] method for the whole zhejiang province power system

Condition Method	Without consideration of zero injections		With consideration of zero injections	
	SOOPP	[32]	SOOPP	[32]
Number of PMUs required	315	851	121	160*
Execution time(s)	0.06	0.95	22.70	-*

* The calculation could not obtain a globally optimize solution Beca use out of memory

Table 8. Contingency-constrained SOOPP results with pmu measurement channels limitations and inclusion of zero injections for the fujian province transmission system

Number of channels	4	5	Unlimited
Number of PMUs required	155	152	150

basic SOOPP model both with and without the help of zero injections for the whole Zhejiang Province power system at all voltage level. This network contains 339 substations, 2080 buses, 3007 branches, and 1516 zero injections among the buses at the transmission level. To show the advantage of SOOPP, the optimization results using OPP method proposed in [32] are also demonstrated. In practical large power systems, the SOOPP has better convergence reliability, faster calculating speed and could obtain much more economical PMU placements than conventional BOM. The computation times are quite acceptable considering that 1516 zero injections greatly increase the scale of the problem.

Finally, SOOPP considering all of the constraints is applied to the Fujian Province power system. This network contains 205 substations, 327 buses, 395 branches, and 95 zero injections among the buses. Table 8 displays the results using different channel limitations. This is the first success of OPP when considering single contingency, zero injections and channels limitations applied to a practical real-world power system.

5. Conclusions

In this paper, we proposed the first substation-oriented OPP methodology based on equivalent integer linear programming, which considers the help of zero injection buses, PMU measurement channel limitations constraints, inclusion of single contingencies of branches and PMU measurement channels both separately and simultaneously. The proposed method is applicable and easily acceptable because of its substation-oriented feature, and the method was successfully applied to the IEEE 14-bus, 118-bus and 300-bus test systems, as well as two real-world large scale power systems. The results and execution times illustrate that the proposed SOOPP method provides a globally optimal solution for practical power system OPP problems in an acceptable period of time.

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Nomenclature

$a_{i,j}$	Binary connectivity parameter between buses i and j .
K	Number of power system substations.
k, g	Indices of substations.
M	Number of network branches (including lines and windings).
m	Index of the branch.
N	Number of network buses (including buses and virtual transformer neutral buses).
i, j, n	Indices of buses.
S	Number of network zero injection buses (including buses and virtual transformer neutral buses).
s	Index of the network zero injection buses.
z_s	Bus index of the zero injection bus s .
D	Number of transformer neutral buses (including buses and virtual transformer neutral buses).
d	Index of transformer neutral buses.
C_d	Bus index of the transformer neutral bus d .
q_k	Integer variable indicating the number of PMUs installed in substation k
b_i	Binary decision variable that is equal to 1 if a PMU is installed at bus i , and 0 otherwise.
p_i	Binary decision variable that is equal to 1 if a PMU voltage measurement channel is used to measure the voltage of bus i , and 0 otherwise.
$l_{i,j}$	binary decision variable that is equal to 1 if a PMU current measurement channel on the side of bus i is used to measure the current of the branch between bus i and bus j , and 0 otherwise. Note that $i \neq j$.
x_{i,z_s}	Binary auxiliary variable that is equal to 1 if calculation of the unknown voltage of bus i is assigned to the equation resulting from the zero injection bus s , and 0 otherwise.
O_i	Observability function of bus i .
ω_k	Installation cost of each PMU in substation k .
γ_i	Installation cost of each PMU at bus i .
MCL_k	Maximum measurement channels limitation of each PMU installed in substation k .
$h_{i,j}$	Binary auxiliary variable used to linearize the formulations, where $h_{i,j}$ is equal to 1 if the observability of bus j is related to bus i and 0 otherwise. Further explanation is provided in this work.
$\zeta_{i,j}$	Binary parameter whose value is 0 if $i \neq j$, and 1 otherwise.
$a_{i,j}^m, x_{i,j}^m, O_i^m$	The parameters, variables and functions, respectively, for the condition that branch m is out of operation.
$a_{i,j}^n, x_{i,j}^n, O_i^n$	The parameters, variables and functions, respectively, for the condition that the PMU voltage measurement channel used to measure the voltage of bus n is out of operation.

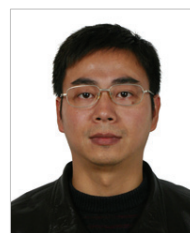
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