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Usage of Dynamic Vibration Absorbers for a Beam Subjected to Moving Forces and for a System Mounted on a Moving Base

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이동하중을 받는 보와 가동 기초 위에 설치된 계에의 동흡진기의 이용

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ABSTRACT

Dynamic vibration absorbers are widely used in machinery, buildings, and structures, including bridges. Two cases of their usage are considered in this paper. One is a simply supported beam subjected to either a moving force or a sequence of moving forces, which simulates a train-bridge interaction problem. The other is a case where a primary system is mounted on a base that is not grounded and is excited by an external force. The conditions that the dynamic vibration absorbers must meet in these cases are found and compared to those for usual cases where bases of primary systems are grounded.

Keywords : Dynamic Vibration Absorber(동흡진기), Beam(보), Moving Force(이동하중), Finite Element Analysis (유한요소해석), Moving Base(가동기초)

1. Introduction

A dynamic vibration absorber (DVA) or tuned mass damper (TMD) is an efficient passive vibration suppression device. It is comprised of a mass, a spring, and a viscous or friction damper. Since invented by Frahm in 1909, DVAs have been widely used in machinery[1], buildings, and structures including bridges. The first theoretical investigation of DVA was carried out by Den Hartog[2] in 1928. He found the optimal tuning ratio and damping ratio for an auxiliary single-degree-of-freedom (SDOF) mass attached to an undamped SDOF system. Later, SDOF systems with damping were also examined and various tuning rules were proposed. To damp more t han one mode of a multiple-degree-of-freedom (MDOF) primary system, several SDOF DVAs are used. For the same purpose a MDOF DVA can be used also. Since enormous researches have been

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performed on DAVs theoretically, numerically, or experimentally, part of researches on using DVAs to suppress vibrations of bridges due to moving vehicles are listed here. A wideband multiple DVAs system was developed for reducing the multiple resonant responses of continuous truss bridges[3]. An analytical model was developed for a long span bridge, with moving vehicles under windy conditions, to which DVAs were attached[4]. Wang et al.[5] studied the applicability of passive DVAs to suppress train induced vibrations on bridges. Samani et al.[6] assessed the performances of DVAs in suppressing the vibrations of a simply supported beam subjected to an infinite sequence of regularly spaced moving loads. In their research, non-linear DVAs as well as linear ones were considered.

In this paper two cases where DVAs are used to suppress vibrations are considered. One is the suppression of vibrations of a simply supported beam subjected to either a moving force or a sequence of moving forces with uniform intervals which represents a train-bridge interaction problem. The other is the case where a primary system is mounted on a base which is not grounded and is excited by an external force. In most researches primary systems to which DVSs are attached are assumed to be mounted on a motionless ground. Usage of DVAs to a cockpit seat of a helicopter is an example of this case. A seat is mounted on the fuselage (base) of a helicopter, and the fuselage vibrates due to dynamic loads caused by a rotor system.

2. A beam subjected to a sequence of equal forces

Fig. 1 shows a simply supported beam subjected to a moving force. The dynamic responses of the beam can be obtained analytically. If a DVA is attached to a beam to suppress its dynamic responses, the dynamic responses can be hardly obtained analytically. Therefore, a finite element analysis was

Fig. 1 A simply supported beam subjected to a moving force.

performed for this case in this paper. The procedure for the analysis was programmed using MATLAB.

For the dimensions and material properties of a beam, data from an existing bridge, the second Kum-gang high speed railway bridge[7] were used: Young's modulus $E = 3.303 \times 10^{10}$ N/m^2, moment of inertia $I = 18.638$ m^4, density $\rho = 3852$ kg/m^3, cross-sectional area $A = 11.332$ m^2 and length $L = 40$ m. Though the bridge has three equal spans with $L = 40$ m, one span of the bridge was considered. The bridge was represented by a simply supported beam, and its first five natural frequencies become 23.165, 92.661, 208.487, 370.644, and 579.132 rad/s. To introduce damping into the beam, the damping ratios of the first five modes were set to 2% arbitrarily.

2.1 Finite element analysis

For a finite element analysis, the beam under consideration was divided into 10 beam elements with equal length. The mass and stiffness matrices of the beam of size 20×20 were obtained by combining element mass and stiffness matrices and considering boundary conditions. To consider damping of the beam material, Rayleigh damping was included as follows.

$$[c] = a[m] + b[k] \tag{1}$$

where $[m]$, $[k]$, and $[c]$ are the mass, stiffness, and damping matrices of the beam, respectively, and a and b are constants. For systems with this kind of damping, it can be shown that constants, a and b, are related to the natural frequencies and modal damping ratios as follows.

$$a + b\omega_r^2 = 2\zeta_r\omega_r \tag{2}$$

Using the data for the natural frequencies and damping ratios for the first two modes, the constants, *a* and *b*, can be obtained as follows.

$$b = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2} \tag{3}$$

$$a = 2\zeta_1\omega_1 - b\omega_1^2 \tag{4}$$

If more modes than two are considered, constants, *a* and *b*, in Eq. (2) can be obtained using the method of least squares. When a DVA, which is composed of a mass, a spring, and a damper, is attached to a beam, the system matrices, $[m]$, $[k]$, and $[c]$, are expanded to incorporate the mass, stiffness, and damping of the absorber and their sizes become 21×21.

A force vector in a finite element analysis represents forces applying to nodes in the directions of degree-of-freedoms. In the present beam-moving force interaction problem, a moving force is the only external force. During the time interval when a moving force is not placed on any node of a finite element model, the force is distributed between the two nodes of an element so that the distributed forces are equivalent to the original one as depicted in Fig. 2.

With the system matrices and force vector, the equation of motion of a system is expressed as

Fig. 2 A force is distributed between two nodes.

follows.

$$[m]\ddot{w} + [c]\dot{w} + [k]w = f \tag{5}$$

where w and f represent the displacement and force vectors, respectively. To solve the above equation, several numerical methods can be applied. The Newmark method[8] was employed in this paper because of the simplicity of the method. The constants, α and β, which are used in the method were selected so that the obtained solution did not diverge. These values are : $\alpha = 1/4$ and $\beta = 1/2$.

2.2 Single moving force case

A vibration absorber was attached to the beam to reduce the dynamic deflections due to moving forces. First, a single moving force case was considered. The force was set to 10,000 N. The force moved with a constant speed, v. The beam responses were calculated using a finite element analysis explained in section 2.1. To check the correctness of the programming, the deflections of the beam without an absorber due to a moving force were calculated using a finite element analysis and compared with the results obtained from an analytical analysis. Almost identical results were obtained for both approaches.

A vibration absorber was composed of a mass, a spring, and a viscous damper. An absorber was attached at the mid-span of the beam because the deflections become a maximum at that point. The absorber mass was set to 1% of the beam mass and became 17,460 kg. The stiffness of a spring was determined so that the natural frequency of the absorber was equal to the first natural frequency of the beam. The reason is that the responses due to the first mode dominate the responses of a beam as a previous research shows. At first, the damping constant was determined so that the damping ratio of the absorber became 0.1. The speed of a moving force was set to $v = 80$ m/s. The deflections at the mid-span of the beam were calculated for cases with and without the absorber, and are compared in Fig.

3. In this figure, the horizontal axis represents the non-dimensional time which is the ratio of the elapsed time to the time interval required for the moving force to pass the beam. Examining the figure, it can be found that the deflections are not reduced during the forced vibration interval while a force is moving on the beam and are reduced during the free vibration interval after a force leaves the beam.

The deflections at the mid-span of the beam were calculated for various damping ratios of a vibration absorber. For small values of the damping ratio, the amplitudes of the deflections decreased with time and increased a little bit for some interval. The value of the damping ratio which gives the smallest amplitude after a certain interval of time was found to be equal to 0.06. This damping ratio was compared with the suggestion for optimum damping of an absorber by Den Hartog[2]

$$\zeta^2 = \frac{3\mu}{8(1+\mu)^3} \tag{6}$$

where μ is the ratio of an absorber mass to a primary system mass. Remembering $\mu = 0.01$ was used for the present absorber, the above equation gives the optimum damping ratio equal to 0.06. This value is the one found above. Therefore, the suggestion for an optimum absorber by Den Hartog is applicable to designs of vibration absorbers for beams subjected to moving forces, even if the suggestion was made for single degree-of-freedom systems.

2.3 A sequence of moving forces case

Designing of an absorber for a beam subjected to a sequence of moving forces was considered. Fig. 4 shows a beam with a sequence of moving forces. The beam responses were calculated using a finite element analysis as above. The spacing between forces, d was equal to 10 m and the speed of the moving forces, v was equal to 36.9 m/s, which causes a resonance to the beam for the given spacing.

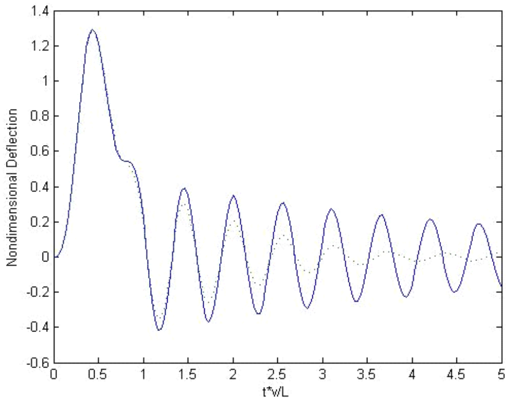

Fig. 3 Comparison of the deflections at the mid-span of the beam with (dotted line) and without (solid line) an absorber for a single force case.

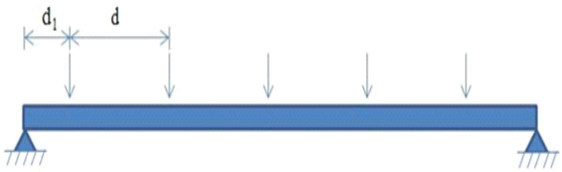

Fig. 4 A simply supported beam subjected to a sequence of moving forces.

The mass and stiffness of an absorber were designed as in a single force case. The responses of the beam with and without an absorber were calculated for various damping ratios of an absorber. From the results it was observed that the vibration amplitudes at the mid-span of the beam increased with the damping ratio of an absorber, and the amplitude was very small when the damping ratio was equal to zero. This phenomenon can be explained by reminding the facts: when a resonance due to a sequence of moving forces occurs in a beam, the mid-span of the beam vibrates with almost a single frequency, and when a SDOF system vibrates with a single frequency, its vibration amplitude can be reduced to zero by using an absorber without damping. Fig. 5 compares the deflections at the mid-span of the beam with and without an absorber when the responses reach steady states. An absorber with zero damping was used for the results in the figure. The figure shows effectiveness of using a vibration absorber for the case of a sequence of moving forces with equal spacing.

3. Case where a base is not grounded

In almost all researches on DVAs primary systems to which absorbers are attached are grounded. In other words they are mounted on fixed bases. In the preceding section the beam to which an absorber is attached is mounted on a motionless ground. However, in many actual cases primary systems are not grounded, but mounted on movable bases. If we attach absorbers to a cockpit seat of a helicopter to reduce vibrations, a cockpit seat becomes a primary system in this case, and it is not grounded but mounted on a flexible helicopter fuselage. To find out if the designing procedure of DVAs under such conditions is different from that in usual cases, primary systems which are mounted on either rigid or flexible bases will be considered.

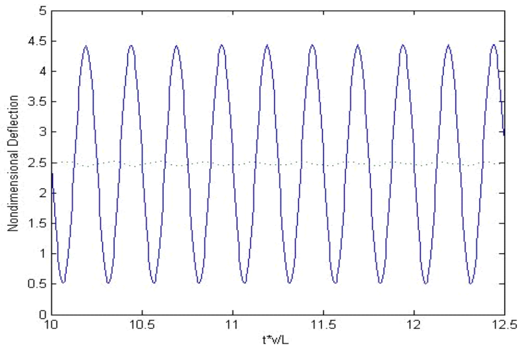

Fig. 5 Comparison of the deflections at the mid-span of the beam with (dotted line) and without (solid line) an absorber for a sequence of moving forces case

3.1 Rigid base

Consider the system shown in Fig. 6. In the figure m_1 represents a rigid base which has a stiffness k_1 against the ground, m_2 and k_2 a primary system, and m_3 and k_3 an absorber. An external force $f(t) = f_0 \sin \omega t$ applies on the base. The mass and stiffness matrices of the system are obtained as follows.

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{7}$$

$$[k] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{8}$$

The frequency response functions between forces acting on each mass and displacements of each mass can be obtained from the following equation

$$[H] = ([k] - \omega^2 [m])^{-1} \tag{9}$$

and some of them become

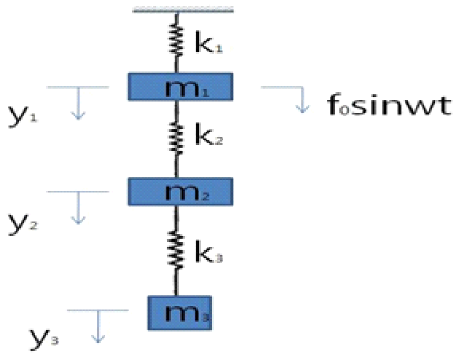

Fig. 6 A primary system and a base with a vibration absorber

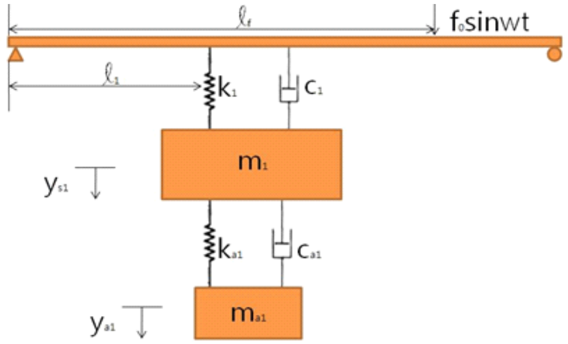

Fig. 7 A primary system mounted on a flexible base

$$H_{11}(\omega) = \frac{(k_2 + k_3 - \omega^2 m_2)(k_3 - \omega^2 m_3) - k_3^2}{|[k] - \omega^2 [m]|} \tag{10}$$

and

$$H_{21}(\omega) = \frac{k_2 (k_3 - \omega^2 m_3)}{|[k] - \omega^2 [m]|} \tag{11}$$

$H_{21}(\omega)$ represents the relationship between the force acting on mass m_1 and the displacement of mass m_2. So if $H_{21}(\omega) = 0$, then y_2 becomes zero and a satisfactory absorber is obtained. The condition for this becomes

$$\omega^2 = \frac{k_3}{m_3} \tag{12}$$

The above condition requires that the natural frequency of an absorber be the same as the exciting frequency, which is the same as the condition for usual cases where bases are grounded and primary systems are excited with a single frequency.

3.2 Flexible base

Consider the system shown in the Fig. 7. The primary system composed of m_1, k_1, and c_1 is mounted on a flexible beam, which becomes a base in this case. An external force $f(t) = f_0 \sin \omega t$ applies on the base. An absorber composed of m_{a1}, k_{a1}, and c_{a1} was attached to the primary system to reduce its vibrations. The width, thickness, and length of the beam were equal to 0.05 m, 0.05 m, and 1 m, respectively, and the material properties were: density $\rho = 7800$ kg/m^3 and Young's modulus $E = 2 \times 10^{11}$ N/m^2. The location of the point where an external force applies was $l_f = 0.8$ m, and the amplitude and frequency of the force were $f_0 = 100$ N and $\omega = 60$ rad/s. The other parameters were set to $m_1 = 3$ kg, $k_1 = 1 \times 10^5$ N/m, $l_1 = 0.3$ m, $m_{a1} = 0.3$ kg. It was assumed that the parallel spring and damper were connected to the same point for brevity. It was also assumed that the primary system and the absorber included 2% damping.

A finite element analysis was used to calculate the responses of the system. The sizes of the system matrices, $[m]$, $[k]$, and $[c]$, become 22×22 in this case. For various values of the spring constant of the absorber, k_{a1}, the displacement amplitudes of the primary system were calculated and are shown in Fig. 8. The amplitude of the primary system becomes a minimum for $k_{a1} = 1045$ N/m and changes rapidly around the value. This value of k_{a1} is close to the value obtained from Eq. (12), which is 1080 N/m. It means that the spring constant of an absorber for a primary system mounted on a flexible base can be obtained approximately using Eq. (12). Then by tuning the spring constant around this value, an optimum spring constant can be obtained.

As shown in Fig. 8, the amplitude of a primary system varies rapidly around the optimum point. To study the effect of damping of an absorber on the amplitude of a primary system, simulations similar to those performed for the results in Fig. 8 were repeated with various damping ratios of an absorber. Damping ratios 1, 2, 3, 5, and 10% were used.

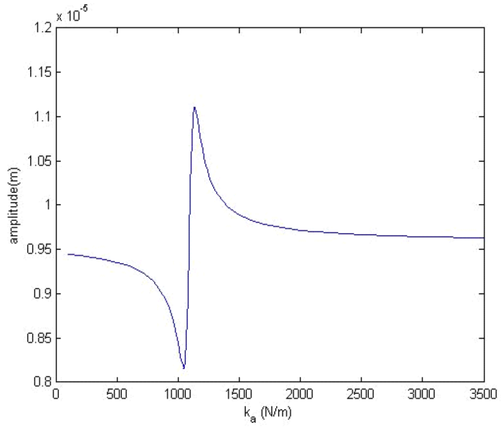

Fig. 8 Variation of the amplitude of the primary system with the spring constant of the absorber

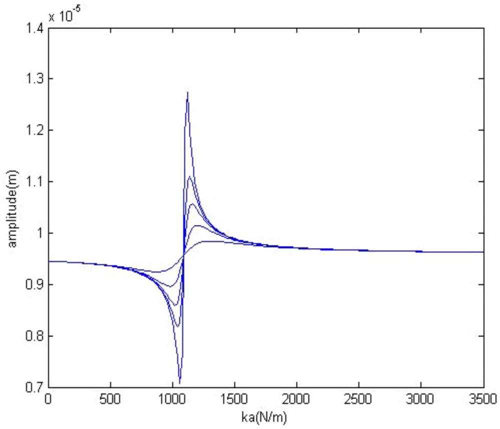

Fig. 9 Variation of the amplitude of the primary system with the spring constant of the absorber for several damping ratios

Fig. 9 shows the variation of the amplitude of the primary system with the spring constant of an absorber for each damping ratio. In the figure lines changing less rapidly correspond to increased damping ratios. Therefore, the amplitude of a primary system can be made less sensitive to variation of the natural frequency of an absorber. However, the amplitude of a primary system increases and the effectiveness of an absorber reduces consequently.

4. Conclusion

In this paper two cases where DVAs are used to suppress vibrations are considered. One is the suppression of vibrations of a simply supported beam subjected to either a moving force or a sequence of moving forces with uniform intervals. The other is the case where a primary system is mounted on a base which is not grounded and is excited by an external force.

To reduce the dynamic responses of a beam subjected to moving forces, a vibration absorber was attached at the mid-span of a beam. The responses of a beam with an absorber were calculated using a finite element analysis. For a single force case, an absorber was effective in reducing free vibrations of a beam after a force passes a beam. It was found that the suggestion by Den Hartog can be used effectively in designing absorber parameters. For a sequence of moving forces case, an absorber without damping can be used to suppress vibration amplitudes almost completely.

For primary systems mounted on bases which are not grounded, DVAs can be designed in the same way as in the usual case where bases are grounded. That is, the natural frequency of an absorber must be adjusted to the exciting frequency of a base, and an absorber for one primary system can be designed independently without considering the other primary systems. By adding damping to an absorber, the vibration amplitude of a primary system can be made less sensitive to the variation of the natural frequency of an absorber.

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