# The Stress Analysis of Planetary Gear System of Mixer Reducer for Concrete Mixer Truck

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Abstract: In general, the gears of mixer reducer for concrete mixer truck make use of the differential type planetary gear system to rotate mixer drum smoothly on the initial conditions. The planetary gear system is very important part of mixer reducer for concrete mixer truck because of strength problem. In the present study, calculating the gear specifications and analyzing the gear bending & compressive stresses of the differential planetary gear system for mixer reducer are necessary to analyze gear bending and compressive stresses confidently, for optimal design of the planetary gear system in respect to cost and reliability. As a result, analyzing actual gear bending and compressive stresses of the planetary gear system using Lewes & Hertz equation and verifying the calculated specifications of the planetary gear system, evaluate the results with the data of allowable bending and compressive stress from the Stress-No. of cycles curves of gears.

### Nomenclatures

 $T_{mi}\alpha_i$ : equivalent mean torque, (N·m)

- $N_{mi}$ : equivalent mean rotating speed, rpm
- S: actual gear bending stress, N/mm<sup>2</sup>
- P: actual gear compressive stress, N/mm<sup>2</sup>
- $S_{ab}$ : allowable gear bending stress, N/mm<sup>2</sup>
- $S_{\rm ac}$ : allowable gear compressive stress, N/mm<sup>2</sup>

#### 1. Introduction

Mixer reducer for concrete mixer truck is driven by a

hydraulic motor, as an important device to rotate the mixer drum, and to convert the required torque and rotational speed. Although the increasing initial torque resulting to the inertia moment increases of output section, the compound differential type planetary gear system applies the rotating motion that makes the mixer drum run smoothly which consists of the sun gear, the differential planetary gear and two ring gears. Gear teeth are damaged due to the lack of fatigue strength, compound planetary gears for mixer and by severe operating conditions of a concrete mixer truck that have become a problem.

The concrete mixer truck, is shown in Fig. 1, drum capacity, 6~8m<sup>3</sup> class concrete mixer truck. Fig. 2 shows schematic diagram for analytical model of mixer reducer. Table 1 shows the specifications of the mixer reducer.

Several investigations have been reported, as cited by D.E. Imwalle<sup>2)</sup> "Load equalization in planetary gear systems". D.L. Seager<sup>3)</sup> established load distribution calculation of the planetary gears. F. Cunliffe, J.D. Smith and D.B. Welbourn<sup>4)</sup> Dynamic tooth loads in

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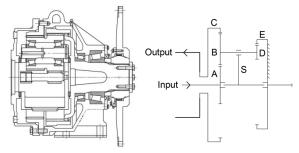
epicyclic gears for planetary gears. Castellani G., and V.P. Castelli<sup>5)</sup> also cited the gear strength analysis method.

Drum capacity	Hyd. motor max. input torque/speed	Gear ratio	Max. output torque	Installation angle of drum
6~8 m <sup>3</sup>	397 N·m /1,320rpm	132:1	52,400N∙m	15~20°

Table 1 Specifications of the mixer reducer



Fig. 1 Drum capacity,  $6 \sim 8m^3$  class concrete mixer truck and driving-related parts of the mixer drum



A : sun gear, B : NO.1 pinion gear, C : NO.1 ring gear,

D : NO.2 pinion gear, E : NO.2 ring gear, S : carrier

Fig. 2 Schematic diagram of analytical model

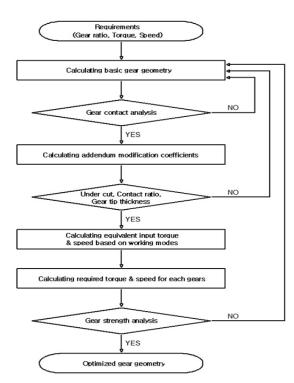


Fig. 3 Equation system solving with gear specifications calculation and stress analysis

Coy, J.J., D.P. Townsend, and E.V. Zaretsky<sup>6)</sup> further emphasized the dynamic capacity and surface pressure durability life of spur and helical gears. Oda, Satoshi and Koji Tsubokura<sup>7)</sup> similarly stressed the effect of bending endurance strength for addendum modification of spur gears and was likewise investigated. There is also an inclusion of typical bending strength calculation of planetary gears AGMA 218.01<sup>8)</sup> and Gear Handbook by Dudly, Darle W.<sup>9)</sup> that shows bending strength calculation method of planetary gears.

In this study, developing planetary gear specifications calculation program and producing detailed specifications of the planetary gear system for mixer reducer is based on Gear Handbook by Dudly, Darle W.<sup>9)</sup>

Moreover, developing also the stress analysis program of planetary gear system by Lewes<sup>1)</sup> & Hertz<sup>9)</sup> equation and analyzing the safety factor of gear bending and compressive stresses consider required life time of mixer reducer and the S/N curve presented in the Gear Handbook by Dudly, Darle W.<sup>9)</sup>

It also verified the predictive validity with respect to the developed programs. Fig. 3 shows the equation system solving with gear specifications calculation and strength analysis of the planetary gear system for mixer reducer.

# 2. Material and analytical method

## 2.1 Calculation of gear specifications

Table 2 shows the specifications for the planetary gears of mixer reducer by developed programs.

Item	Sun gear (A)	No.1 pinion gear (B)		No. 1 ring gear(C)	No. 2 pinion gear (D)	No. 2 ring gear(E)
Module	4	4		4	4	4
Pressure angle(°)	27	27		27 27		27
Helix angle(°)	0	0		0	0	0
No. gear teeth	10	35		80	31	76
Tooth modification factor	0	0		0	+0.5220	+0.5220
Pitch dia	40	140		320	124	304
Outside dia.	48	148		312	136.176	300.176
Over pin measurement	$52.501^{-0.087}_{-0.163}$ ( $\Phi$ 8)	$\begin{array}{c}151.338^{-0.104}_{-0.196}\\(\varPhi7.5)\end{array}$		$309.851^{+0.416}_{+0.257}$ $(\Phi7)$	$140.320_{-0.177}^{-0.094}$ $(\varPhi 8)$	$298.081^{+0.395}_{+0.244}_{-0.244}$ $(\varPhi7)$
Face width	56	70		72.5	71	22
Backlash	0.117 ~ 0	).220 0.		184~0.314	0.184~0.314	
Center distance	90			90	90	

Table 2 Specifications of the planetary gear system

2.2 Input equivalent torque/rotation speed analysis

The required service period of life, for a concrete mixer truck is 15 years with the vehicle operation rate of 70%, operating time is set 12 hours for a day, based on the total 28,400 hours, as shown in Table 3.

Equivalent mean torque for the average equivalent load of mixer reducer,  $T_{mi}$  is as follows:

$$T_{mi} = \left[ \frac{\sum N_i t_i T_i^n}{\sum N_i t_i} \right]^{\frac{1}{n}}$$
(1)

whereas  $T_i$  is working torque, N is rotating speed, t is working time, n is power index (n = 20.8).

Table 3 Operating mode and the required life period

Working mode	Frequency of use(%)	Working time(h)	Input			
			Torque (N • m)	Speed(rpm)	Duty cycle	Cycle ratio
Input concrete	4	1,136	189.9	1320	89971200	0.125391849
Driving	41	11,644	189.9	264	184440960	0.257053291
Normal working	12	3,408	241.7	660	134956800	0.188087774
Maximum working	1	284	284.8	132	2249280	0.003134796
Driving	38	10,792	52.4	264	170945280	0.238244514
Washing	4	1,136	52.4	1980	134956800	0.188087774
Total	100	28,400	-	-	717,520,320	1

Equivalent mean rotating speed for the average equivalent rotating speed of mixer reducer,  $N_{mi}$  is as follows:

$$N_{mi} = \left[\frac{\sum N_i t_i}{\sum t_i}\right] \tag{2}$$

whereas  $N_{mi}$  is equivalent rotating speed for the average equivalent rotating speed,  $N_i$  is rotating speed,  $t_i$  is working time.

From the equation (1) and (2), the equivalent mean torque/rotating speed was calculated 227.6N·m /421.08 rpm.

#### 2.3 Torque and number of rotation analysis

From schematic diagram in Fig. 2, the gear ratio of mixer reducer calculated by relative speed diagram method<sup>10)</sup> is as follows:

$$\Upsilon = \left\{ \frac{1 + Z_E Z_B / Z_A Z_D}{1 - Z_E Z_B / Z_D Z_C} \right\}$$
(3)

The number of rotation for each planetary gear calculated by relative speed diagram method<sup>10)</sup> is as follows:

$$N_{B} = N_{D} = \left\{ \frac{Z_{A} Z_{C} (N_{A} - N_{C})}{Z_{B} (Z_{A} + Z_{C})} \right\}$$
(4)

$$N_C = N_A / \Upsilon$$
 (5)

$$N_{S} = \left\{ \frac{Z_{A} Z_{D} N_{A}}{Z_{E} Z_{B} + Z_{A} Z_{D}} \right\}$$
(6)

From the above equations, the torque and rotation speed is shown in Table 4.

Table 4 Torque and number of rotation  $(N \cdot m/rpm)$ 

T <sub>A</sub> /N <sub>A</sub> (Torque/Number of rotation of sun gear)	227.6 / 421.08
$T_{\rm B}/N_{\rm B}$ (Torque/Number of rotation of NO.1 pinion gear)	4380.6(265.6) / 106.13
T <sub>C</sub> /N <sub>C</sub> (Torque/Number of rotation of NO.1 ring gear)	10,012.5 / 3.19
T <sub>s</sub> /N <sub>s</sub> (Torque/Number of rotation of carrier)	30,264.9 / 43.95
$T_{\rm D}/N_{\rm D}$ (Torque/Number of rotation of NO.2 pinion gear)	4,380.6 / 106.13
T <sub>E</sub> /N <sub>E</sub> (Torque/Number of rotation of NO.2 ring gear)	3,579.7 / 43.95

whereas ZA is number of teeth of sun gear(10), ZB is number of teeth of NO.1 pinion gear(35), ZC is number of teeth of NO.1 ring gear(80), ZD is number of teeth of NO.2 pinion gear(31), ZE is number of teeth of NO.2 ring gear(76)

#### 2.4 Gear bending stress analysis

The actual gear bending stress equation by Lewes<sup>1</sup> formula is as follows:

$$S = \frac{29,400\pi T}{N_a F X Z} \tag{7}$$

whereas S is actual gear bending stress(N/mm<sup>2</sup>), T is torque(N·m),  $N_a$  is length of action in the plane of rotation(mm), F is face width(mm), X is Lewes bending factor(mm), Z is number of teeth.

Allowable gear bending stress equation by Gear Handbook of Dudly, Darle W.<sup>9)</sup> including gear bending S/N curve is as follows:

$$Sab = \frac{C_1}{N_F^{\frac{1}{20.8}}}$$
 (8)

whereas *Sab* is allowable gear bending stress(N/mm<sup>2</sup>),  $N_F$  is No. of cycles,  $C_1$  is coefficient.

#### 2.5 Gear compressive stress analysis

The actual gear compressive stress  $P(N/mm^2)$  applied

to the tip of the planetary gears based on contact formula of  $\text{Hertz}^{9}$  is as follows:

$$P = 19.43 \sqrt{\frac{2\pi T \times CDSIN\alpha}{A(CDsin\Phi - A) \times F_c \times N_a \times Z}}$$
(9)

whereas  $\alpha$  is normal pressure angle,  $\Phi$  is transverse pressure angle, *T* is torque on driving gear(N·m), *F<sub>c</sub>* is active face width in contact(mm), *Z* is No. of teeth on driving gear, *CD* is operating center distance, *N<sub>a</sub>* is length of action in the plane of rotation(mm), *A* =  $\sqrt{OR^2 - BR^2}$ , *OR* is outside radius of gear, *BR* is base radius of gear.

Allowable gear compressive stress equation by Gear Handbook of Dudly, Darle W.<sup>9)</sup> including gear compressive S/N curve is as follows:

$$Sac = \frac{C_2}{N_F^{\frac{1}{6.8433}}} \tag{10}$$

whereas *Sac* is allowable gear compressive stress(N/mm<sup>2</sup>),  $N_F$  is No. of cycles,  $C_2$  is coefficient.

# 2.6 The results of gear bending and compressive stress analysis

Calculating actual gear bending and compressive stresses of planetary gear system for mixer reducer and considering allowable gear bending and compressive stresses, produce safety factors and verify the problems of gear strength for the calculated specifications of the planetary gear system for mixer reducer.

Fig. 4 shows the results of gear bending stress analysis and Fig. 5 shows the results of gear compressive stress analysis of planetary gear system for mixer reducer. Planetary gear system of mixer reducer for concrete mixer truck consisted of five gears.(A to E)

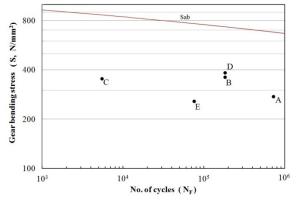


Fig. 4 The results of gear bending stress analysis

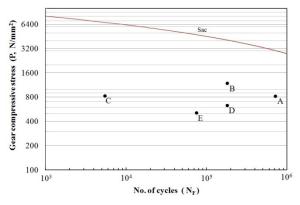


Fig. 5 The results of gear compressive stress analysis

It can be shown that actual gear bending & compressive stresses of the planetary gear system are under the allowable gear bending and compressive stresses in these S/N curves. Thus, calculation results are set safely and has been verified as valid.

# 3. Conclusion

(1) In this study, analysis actual gear bending and compressive stresses of the planetary gear system using Lewes & Hertz equation and verifying the calculated specifications of the planetary gear system, evaluate the results with the data of allowable bending and compressive stress from the Stress-No. of cycles curves of gears, based on Gear Handbook of Dudly, Darle W<sup>9</sup> for drum capacity, 6~8m<sup>3</sup>class planetary gear system of mixer reducer for concrete mixer truck.

(2) Considering the result of gear bending and compressive stress analysis of calculated specifications for planetary gear system of mixer reducer for drum capacity, 6~8m<sup>3</sup>class, concrete mixer truck, the strength of planetary gear system and the developed programs have been verified as valid.

(3) Reducer is an important component for construction machinery industry of the developed programs, calculating the specifications and analyzing the gear bending and compressive stresses of the differential planetary gear system for mixer reducer are expected to be effectively utilized.

Future research on more excellent planetary gear

system of the various reducers for construction machines is expected to be still performed.

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