Energy-Efficiency Power Allocation for Cognitive Radio MIMO-OFDM Systems

Jiakuo Zuo, Van Phuong Dao, Yongqiang Bao, Shiliang Fang, Li Zhao, and Cairong Zou

This paper studies energy-efficiency (EE) power allocation for cognitive radio MIMO-OFDM systems. Our aim is to minimize energy efficiency, measured by "Joule per bit" metric, while maintaining the minimal rate requirement of a secondary user under a total power constraint and mutual interference power constraints. However, since the formulated EE problem in this paper is non-convex, it is difficult to solve directly in general. To make it solvable, firstly we transform the original problem into an equivalent convex optimization problem via fractional programming. Then, the equivalent convex optimization problem is solved by a sequential quadratic programming algorithm. Finally, a new iterative energy-efficiency power allocation algorithm is presented. Numerical results show that the proposed method can obtain better EE performance than the maximizing capacity algorithm.

Keywords: Energy efficiency, power allocation, MIMO-OFDM, cognitive radio.

I. Introduction

Energy-efficient (EE) wireless communications have great ecological and economic benefits and are becoming more and more important in future wireless systems [1].

Recently, an EE technique was introduced to cognitive radio (CR) systems for resource allocation, something that has become a hot topic of late. For CR OFDM systems, [2] proposed a search method making use of a water-filling factor to solve the energy-efficiency power allocation (EEPA) problem. In [3], the EEPA problem is addressed via parametric programming, which is then followed by the presentation of an iterative algorithm. For CR MIMO systems, [4] studied the EE optimization problem of CR MIMO broadcast channels to improve system throughput for unit energy consumption. In [5], the throughput and EE optimization under quality-of-service constraints for CR MIMO systems are studied. In [6], a promising framework of spectrum sharing strategy selection, based on EE, is proposed for CR MIMO interference channels.

In this paper, we study the EEPA optimization for CR MIMO-OFDM systems. We try to minimize the energy efficiency of a secondary user (SU) and maintain the minimal rate requirements, without causing excessive interference to primary users (PUs). Since the EEPA problem is non-convex, to find the optimal solution the problem is first transformed into an equivalent convex problem via fractional programming (FP) [7]. Then, an optimal iterative algorithm based on sequential quadratic programming (SQP) [8] is presented.

II. Signal Model and Problem Statement

This paper considers the same CR network as described in [9]. In this CR network, there is a cognitive base station (CBS) and an SU coexisting with L PUs. The set of PUs is denoted by $\mathcal{L} = \{1, 2, \ldots, L\}$. Here, we consider the downlink scenario, whereby the CBS is equipped with N_t antennas and an SU has N_r antennas. Each PU is equipped with one receive antenna. The CR network adopts OFDM modulation for transmission. The available bands for an SU are divided into N subcarriers and the bandwidth for each subcarrier is Δf Hz. According to [9], the rate of the ith subcarrier is defined as

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$$c_i = \sum_{j=1}^{n_{\min}} \Delta f \log_2 \left(1 + \frac{p_{ij} a_{ij}^2}{N_0}\right), \quad (1)$$

where $n_{\min} = \min(N_t, N_r)$, N_0 is the additive white Gaussian noise (AWGN) variance, p_{ij} is the transmit power at the ith subcarrier at the jth antenna, and a_{ij} are the singular values of the $\boldsymbol{H}_i$ ($\boldsymbol{H}_i \in C^{N_r \times N_t}$ is the channel matrix between CBS and an SU in the ith subcarrier). The mutual interference power (MIP) introduced by CBS to the lth PU is defined as

$$I_l = \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} p_{ij} \left|g_{ij}^l\right|^2 \varphi_{ij}^l, \quad (2)$$

where the channel fading gains between the jth transmit antenna and the lth PU on the ith subcarrier are g_{ij}^l. The interference factor of the lth PU on the ith subcarrier is φ_{ij}^l [10].

The EE of the CBS is defined as

$$EE_{\text{CBS}} = \frac{P_c + \tau_c \left(\sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} p_{ij}\right)}{\sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} \Delta f \log_2 \left(1 + \frac{p_{ij} a_{ij}^2}{N_0}\right)}, \quad (3)$$

where τ_c denotes power-amplifier efficiency and P_c denotes the power consumption of circuits and base-station facilities.

In this paper, our objective is to minimize EE_{CBS}; therefore, the EEPA problem is formulated as the following OP1:

$$\begin{aligned}
\text{OP1} \quad & \min_{p_{ij}} \mu\left(\left\{p_{ij}\right\}\right) = EE_{\text{CBS}}, \\
\text{s.t.} \quad & C1: \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} p_{ij} \left|g_{ij}^l\right|^2 \varphi_{ij}^l \le I_{th}^l, \ \forall l, \\
& C2: \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} p_{ij} \le P_{\max}, \\
& C3: \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} \Delta f \log_2 \left(1 + \frac{p_{ij} a_{ij}^2}{N_0}\right) \ge R_{\min}, \\
& C4: p_{ij} \ge 0, \ i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, n_{\min},
\end{aligned} \quad (4)$$

where μ is the objective function value, I_{th}^l is the interference threshold of the lth PU, $P_{\max}$ is the total power budget of CBS, and $R_{\min}$ is the minimal rate requirement.

III. Optimal EEPA

It is difficult to solve OP1 directly, since it is non-convex. Here, we first transform the problem into a convex problem by using FP [7] and then propose an optimal power allocation method based on SQP [8]. We define the elements of the vectors $\boldsymbol{p} \in \Re^{(N \times n_{\min})}$, $\boldsymbol{\Upsilon}^l \in \Re^{(N \times n_{\min})}$, $\boldsymbol{a} \in \Re^{(N \times n_{\min})}$, and $\boldsymbol{c}(\boldsymbol{p}) \in \Re^{(L+2+N \times n_{\min})}$, respectively as

$$\boldsymbol{p}\left[(i-1) \times n_{\min} + j\right] = p_{ij}, \ i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, n_{\min};$$

$$\boldsymbol{\Upsilon}^l\left[(i-1) \times n_{\min} + j\right] = \left|g_{ij}^l\right|^2 \varphi_{ij}^l, \ i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, n_{\min};$$

$$\boldsymbol{a}\left[(i-1) \times n_{\min} + j\right] = a_{ij}^2, \ i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, n_{\min};$$

$$\boldsymbol{c}(\boldsymbol{p})[m] = \begin{cases} \sum_{n=1}^{N \times n_{\min}} \boldsymbol{p}[n] \boldsymbol{\Upsilon}^l[n] - I_{th}^l & m = l, \ l = 1, 2, \ldots, L; \\ \sum_{n=1}^{N \times n_{\min}} \boldsymbol{p}[n] - P_{\max} & m = L+1; \\ R_{\min} - \sum_{n=1}^{N \times n_{\min}} \Delta f \log_2 \left(1 + \frac{\boldsymbol{a}[n] \boldsymbol{p}[n]}{N_0}\right) & m = L+2; \\ -\boldsymbol{p}[n] & m = L+2+n, \ n = 1, 2, \ldots, N \times n_{\min}. \end{cases}$$

Then, OP1 can be rewritten in vector form as OP2

$$\begin{aligned} \text{OP2} \quad & \min_{\boldsymbol{p}} \mu(\boldsymbol{p}) = EE_{\text{CBS}}, \\ \text{s.t.} \quad & \boldsymbol{c}(\boldsymbol{p}) \le 0. \end{aligned} \quad (5)$$

Before solving OP2, we introduce a new problem OP3 as follows:

$$\text{OP3} \quad \min_{\boldsymbol{p} \in \wp} f(\boldsymbol{p}, \lambda), \quad (6)$$

where

$$f(\boldsymbol{p}, \lambda) = \left(P_c + \tau_c \sum_{n=1}^{N \times n_{\min}} \boldsymbol{p}[n]\right) - \lambda \sum_{n=1}^{N \times n_{\min}} \Delta f \log_2 \left(1 + \frac{\boldsymbol{a}[n] \boldsymbol{p}[n]}{N_0}\right),$$

λ is positive parameter, and $\wp$ denotes the feasible region of OP2.

Let $\theta(\lambda) = \min_{\boldsymbol{p} \in \wp} f(\boldsymbol{p}, \lambda)$ and $\boldsymbol{p}^{\lambda} = \arg\min_{\boldsymbol{p} \in \wp} f(\boldsymbol{p}, \lambda)$ denote the optimal value and solution of OP3, respectively. Then, OP2 and OP3 can be related to each other by the following lemma [7].

Lemma 1. The optimal solution, $\boldsymbol{p}^*$, achieves the optimal value $\lambda^* = \mu(\boldsymbol{p}^*)$ of OP2, if and only if, $\theta(\lambda^*) = 0$ and $\boldsymbol{p}^{\lambda^*} = \boldsymbol{p}^*$.

By lemma 1, solving OP1 is equivalent to solving problem OP3 for a given λ and then updating λ until Lemma 1 is satisfied. In the following, SQP is used to solve OP3 for a given λ. The Jacobian matrix $\boldsymbol{J}(\boldsymbol{p})$ of $\boldsymbol{c}(\boldsymbol{p})$ is defined as

$$\left[\boldsymbol{J}(\boldsymbol{p})\right]_{m,n} = \frac{\partial \boldsymbol{c}(\boldsymbol{p})[m]}{\partial \boldsymbol{p}[n]} = \begin{cases} \boldsymbol{\Upsilon}^l[n] & m = l\left(l = 1, 2, \ldots, L\right), \\ 1 & m = L+1, \\ \frac{-\Delta f}{\ln 2} \frac{\boldsymbol{a}[n]}{N_0 + \boldsymbol{a}[n] \boldsymbol{p}[n]} & m = L+2, \\ -1 & m = L+2+n, \\ 0 & m \ne L+2+n. \end{cases} \quad (7)$$

The Lagrangian function is

$$L(\boldsymbol{p},\beta)=\left(P_c+\sum_{n=1}^{N\times n_{\min}}\boldsymbol{p}[n]\right)-\lambda\sum_{n=1}^{N\times n_{\min}}\Delta f\log_2\left(1+\frac{\boldsymbol{a}[n]\boldsymbol{p}[n]}{N_0}\right)$$
$$+\sum_{l=1}^{L}\beta_l\left(\sum_{n=1}^{N\times n_{\min}}\boldsymbol{p}[n]\boldsymbol{\Upsilon}^l[n]-I_{th}^l\right)+\beta_{L+1}\left(\sum_{n=1}^{N\times n_{\min}}\boldsymbol{p}[n]-P_{\max}\right)$$
$$+\beta_{L+2}\left(R_{\min}-\sum_{n=1}^{N\times n_{\min}}\Delta f\log_2\left(1+\frac{\boldsymbol{a}[n]\boldsymbol{p}[n]}{N_0}\right)\right)$$
$$-\sum_{n=1}^{N\times n_{\min}}\beta_{L+2+n}\boldsymbol{p}[n], \quad (8)$$

where $\boldsymbol{\beta}=\left[\beta_1,\beta_2,\dots,\beta_{L+1},\beta_{L+2},\beta_{L+3},\dots,\beta_{L+2+N\times n_{\min}}\right]^T$ is the vector of Lagrangian multipliers. Next, we define

$$\boldsymbol{Q}(\boldsymbol{p},\boldsymbol{\beta})=\begin{bmatrix}\boldsymbol{H}(\boldsymbol{p},\boldsymbol{\beta}) & \boldsymbol{J}^T(\boldsymbol{p})\\ \boldsymbol{J}(\boldsymbol{p}) & 0\end{bmatrix}, \quad (9)$$

where $\boldsymbol{H}(\boldsymbol{p},\boldsymbol{\beta})$ is the Hessian matrix of $L(\boldsymbol{p},\boldsymbol{\beta})$; the elements of which are defined as

$$\left[\boldsymbol{H}(\boldsymbol{p},\boldsymbol{\beta})\right]_{mn}=\begin{cases}\dfrac{\Delta f\left(\lambda+\beta_{L+2}\right)}{\ln 2}\dfrac{\left(\boldsymbol{a}[n]\right)^2}{\left(N_0+\boldsymbol{a}[n]\boldsymbol{p}[n]\right)^2} & m=n,\\ 0 & m\neq n.\end{cases} \quad (10)$$

The SQP method iteratively improves the estimates of $(\boldsymbol{p}^t,\boldsymbol{\beta}^t)$ by finding the correction vectors $\boldsymbol{s}^t=\left[(\boldsymbol{s}_p^t)^T(\boldsymbol{s}_\beta^t)^T\right]^T$ at the tth iteration to construct the new estimates as follows:

$$\begin{bmatrix}\boldsymbol{p}^{t+1}\\ \boldsymbol{\beta}^{t+1}\end{bmatrix}=\begin{bmatrix}\boldsymbol{p}^t+\eta_p^t\boldsymbol{s}_p^t\\ \boldsymbol{\beta}^t+\eta_\beta^t\boldsymbol{s}_\beta^t\end{bmatrix}, \quad (11)$$

where η_p^t and η_β^t are non-negative step sizes. The vector $\boldsymbol{s}^t$ is obtained by solving the following problem:

$$\boldsymbol{Q}(\boldsymbol{p}^t,\boldsymbol{\beta}^t)\times\boldsymbol{s}^t=-\left[\boldsymbol{g}(\boldsymbol{p}^t)^T\boldsymbol{c}(\boldsymbol{p}^t)^T\right]^T, \quad (12)$$

where $\boldsymbol{g}(\boldsymbol{p}^t)$ is the gradient of the Lagrangian function (8); the elements of $\boldsymbol{g}(\boldsymbol{p})$ are

$$\boldsymbol{g}(\boldsymbol{p})[n]=1-\frac{\Delta f\left(\lambda+\beta_{L+2}\right)\boldsymbol{a}[n]}{\ln 2\left(N_0+\boldsymbol{a}[n]\boldsymbol{p}[n]\right)}+\sum_{l=1}^{L}\beta_l\boldsymbol{\Upsilon}^l[n]+\beta_{L+1}-\beta_{L+2+n}. \quad (13)$$

The step sizes η_p^t and η_β^t also need to be addressed according to the solution point. To measure the progress, a merit function $\phi(\boldsymbol{p})$ is used. The step size η_p^t is selected so that $\phi(\boldsymbol{p}^t+\eta_p^t\boldsymbol{s}_p^t)$ is sufficiently smaller than $\phi(\boldsymbol{p}^t)$, while η_β^t is set to be either η_p^t or 1. Here, we use the augmented Lagrangian penalty function to define the merit function

$$\phi_\omega(\boldsymbol{p},\omega)=f(\boldsymbol{p},\lambda)+\boldsymbol{c}(\boldsymbol{p})^T\boldsymbol{\chi}(\boldsymbol{p})+\frac{\omega}{2}\left\|\boldsymbol{c}(\boldsymbol{p})\right\|_2^2, \quad (14)$$

where $\|\bullet\|_2^2$ represents the second-order norm, ω is a constant to be chosen, and

$$\boldsymbol{\chi}(\boldsymbol{p})=-\left[\boldsymbol{J}(\boldsymbol{p})^T\boldsymbol{J}(\boldsymbol{p})\right]^{-1}\boldsymbol{J}(\boldsymbol{p})^T\times\nabla(f_0(\boldsymbol{p},\lambda)). \quad (15)$$

Here, $\nabla(f_0(\boldsymbol{p},\lambda))$ represents the gradient of function $f_0(\boldsymbol{p},\lambda)$. The multipliers $\boldsymbol{\chi}(\boldsymbol{p})$ are the least-squares estimates of the optimal multipliers based on the first-order necessary conditions, that is, KKT conditions. Generally, ω is selected sufficiently large so that the merit function $\phi_\omega(\boldsymbol{p},\omega)$ is bounded below. Now, we present the optimal EEPA algorithm based on FP and SQP, which is tabulated as follows:

Algorithm. EEPA.
1: **Initialization**: initial $(\boldsymbol{p}^0,\boldsymbol{\beta}^0)$, $\boldsymbol{p}_0^*=\boldsymbol{p}^0$, and $\phi_\omega(\boldsymbol{p}^0)$
2: **for** $\tau=0,1,\dots,\tau_{\max}$
3: update $\lambda_{\tau+1}=\mu(\boldsymbol{p}_\tau^*)$
4: **for** $t=0,1,\dots,t_{\max}$
5: calculate $\boldsymbol{Q}(\boldsymbol{p}^t,\boldsymbol{\beta}^t)$ and $\left[\boldsymbol{g}(\boldsymbol{p}^t)\ \boldsymbol{c}(\boldsymbol{p}^t)^T\right]^T$
6: solve (12) to get $\boldsymbol{s}^t$
7: select η_p^t such that $\phi(\boldsymbol{p}^t+\eta_p^t\boldsymbol{s}_p^t)<\phi(\boldsymbol{p}^t)$
8: update the estimates via (11)
9: **end for**
10: update $\boldsymbol{p}_{\tau+1}^*=\boldsymbol{p}^{t_{\max}}$
11: **end for**

Note: The detailed proofs of the convergence of the inner loop (SQP) and outer loop (FP) can be found in [8] and [7], respectively.

IV. Performance Simulation

In this section, some numerical results are presented to evaluate the performance of the proposed scheme. Here, we assume the existence of three PUs (L=3) and that the bandwidths occupied by them are respectively 1 MHz, 2 MHz, and 5 MHz. In addition, there are sixteen OFDM subcarriers (N= 16) for an SU, and the bandwidth for each subcarrier Δf is set to 0.3125 MHz. Without loss of generality, the channel gains are assumed to be Rayleigh fading with an average power gain of 1 dB. We then set the following parameters: N_t = N_r = 4, $R_{\min}$ = 10 bits/symbol, τ_c =1, P_c=10^{-2}W, and N_0=10^{-13}W. For simplicity, let I_{th}^l=I_{th}. All the results have been averaged over one thousand iterations.

Figure 1 shows the energy efficiency of EEPA versus parameter ω under different interference threshold I_{th} and total power budget $P_{\max}$. As shown in Fig. 1, when ω is large enough, the performance of EEPA will be stable. Since it is impossible to present all cases of the effects of ω on the performance of EEPA, we will, in accordance with the results

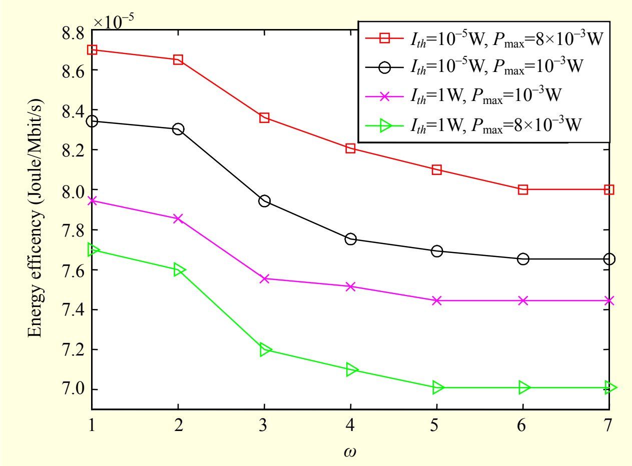

Fig. 1. Energy efficiency vs. parameter ω.

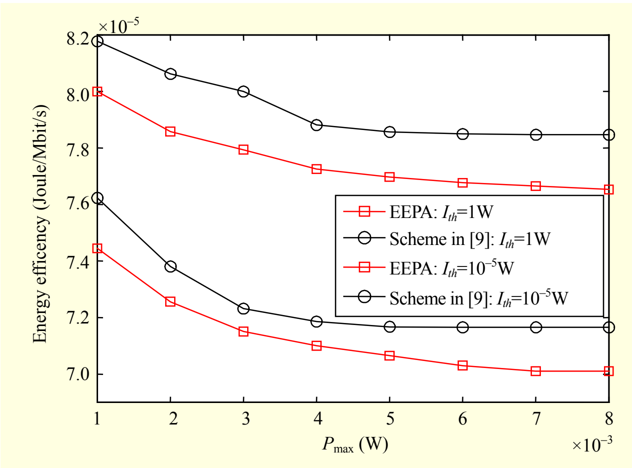

Fig. 2. Energy efficiency vs. total power budget.

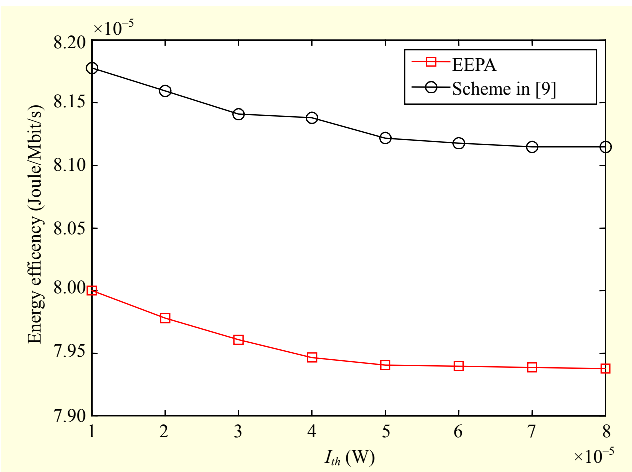

Fig. 3. Energy efficiency vs. interference threshold.

of Fig. 1, set $\omega=10^7$ in the following simulation. We compared EEPA with the capacity maximization scheme in [9]. Figure 2 depicts the energy efficiency versus P_{max}, under I_{th}. The results show that the proposed algorithm can achieve lower energy efficiency than the scheme in [9]. The energy efficiency versus I_{th} under $P_{max}=10^{-3}$W is evaluated in Fig. 3. These results depict similar properties to the results in Fig. 2.

V. Conclusion

In this letter, we investigated the EEPA problem in CR MIMO-OFDM systems. Since the objective function is non-convex, we first construct an equivalent convex problem via FP, and then a new iterative power allocation algorithm is proposed based on SQP. From the simulation results, we observed that the proposed method can improve energy efficiency compared with the conventional capacity maximization scheme.

References

[1] D. Feng et al., "A Survey of Energy-Efficient Wireless Communications," *IEEE Commun. Surveys Tutorials*, vol. 15, no. 1, Feb. 2013, pp. 167–178.

[2] J.L. Mao et al., "Energy Efficiency Optimization for OFDM-Based Cognitive Radio Systems: A Water-Filling Factor Aided Search Method," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, May 2013, pp. 2366–2375.

[3] W. Yan et al., "Optimal Energy-Efficient Power Allocation for OFDM-Based Cognitive Radio Networks," *IEEE Commun. Lett.*, vol. 16, no. 9, Sept. 2012, pp. 1420–1423.

[4] J.L. Mao et al., "Energy Efficiency Optimization for Cognitive Radio MIMO Broadcast Channels," *IEEE Commun. Lett.*, vol. 17, no. 2, Feb. 2013, pp. 337–340.

[5] S. Akin and M.C. Gursoy, "On the Throughput and Energy Efficiency of Cognitive MIMO Transmissions," *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, Sept. 2013, pp. 3245–3260.

[6] W. Zhong and J. Wang, "Energy Efficient Spectrum Sharing Strategy Selection for Cognitive MIMO Interference Channels," *IEEE Trans. Signal Process.*, vol. 61, no. 14, July 15, 2013, pp. 3705–3717.

[7] W. Dinkelbach, "On Nonlinear Fractional Programming," *Manag. Sci.*, vol. 13, no. 7, Mar. 1967, pp. 492–498.

[8] P.T. Boggs and J.W. Tolle, "Sequential Quadratic Programming," *Acta Numerica*, 1996, pp. 1–100.

[9] H.S. Shahraki and K. Mohamed-pour, "Power Allocation in Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing-Based Cognitive Radio Networks," *IET Commun.*, vol. 5, no. 3, Feb. 2011, pp. 362–370.

[10] G. Bansal, M.J. Hossain, and V.K. Bhargava, "Adaptive Power Loading for OFDM-Based Cognitive Radio Systems with Statistical Interference Constraint," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, Sept. 2011, pp. 2786–2791.