

Design of Distributed Beamforming for Dual-Hop Multiple-Access Relay Networks

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This paper studies a dual-hop multiple-access relay network where two independent source nodes transmit information to a common destination node with the aid of multiple single-antenna amplify-and-forward relays. Each relay node is subject to an individual power constraint. We focus on the design of distributed beamforming schemes for the relays to support the transmission rate requirements of the two sources. To this end, we first characterize the achievable rate region for this network via solving a sequence of corner point optimization problems proposed in this paper. We also develop several low-complexity suboptimal schemes in closed form. Two inner bounds of the achievable rate region are theoretically shown to be approximately optimal in two special scenarios. Finally, numerical results demonstrate the effectiveness of our proposed approaches.

Keywords: Distributed beamforming, amplify-and-forward, multiple-access relay network, achievable rate region, optimization, MARN, AF.

I. Introduction

It has been widely confirmed that amplify-and-forward (AF) relay systems increase spatial diversity and reliability of wireless communication systems — for example, [1] and [2]. From a practical standpoint, AF relaying is an interesting cooperative relaying strategy because the implementation complexity and cost of relaying — always an issue in designing cooperative networks — is minimal.

The AF-based distributed beamforming problem was first studied for one-way dual-hop relay networks with a single source-destination pair [3]–[7]. The goal is to efficiently compute the optimal beamforming vectors in terms of maximum end-to-end transmission rate under either sum or individual relay power constraints. Later, the problem was extended to layered relay networks — for example, [8] and [9]. However, only suboptimal schemes were derived.

The design of AF-based distributed beamforming has also attracted great attention in the recent research community for two-way relay systems. Unlike in the one-way case, the source nodes also perform as destinations in the case of two-way relay networks. The achievable rate region of a two-way relay network with distributed beamforming was studied in [10]. For such a network, a signal-to-noise ratio (SNR) balancing approach was investigated in [11]. The authors in [11] proposed an alternating optimization algorithm to iteratively compute a joint source power control and distributed relay beamforming scheme. The goal is to maximize the smaller SNR of the two communication links. In [12], the authors proposed a joint source power control and distributed relay beamforming scheme for a two-way relay network with multiple source-destination pairs. The aim is to minimize the relay power dissipation while the output SNR at each

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destination node is kept above a desired value.

The aforementioned schemes have all been built upon the assumption that each destination node only decodes the message from one desired source node and treats the signals from the other sources as noise. Similar to the peer-to-peer relay networks, the dual-hop multiple-access relay networks (MARNs) also play an important role in wireless relaying systems. In such networks, multiple source nodes transmit information to a common destination node simultaneously. Dual-hop MARNs can be regarded as a system model for an up-link transmission from multiple mobile terminals to a common base station or for a down-link transmission from multiple databases to a common user; where in both cases multiple AF relays are applied to help the transmission. Hence, it is worthwhile to mention the pioneer work done in characterizing the achievable rate region of a dual-hop MARN [13]. The complete achievable rate region for a dual-hop MARN was fully characterized in [13] where the AF-based distributed beamforming is employed and the relays are under a sum power constraint. The standard weighted sum-rate maximization (WSRM) problems (used in [13]) were used to solve the optimal beamforming vectors. However, this approach is not appropriate when it comes to an individual relay power constraints scenario because the WSRM problem is hard to solve — as shown in [14]. By applying such an approach, only the maximum individual rates and sum rates were obtained in some special scenarios.

In this study, we investigate the distributed beamforming problem for a dual-hop MARN where multiple AF relays, as considered in [14], are subjected to individual power constraints. The goal is to find appropriate beamforming vectors such that desired transmission rates of two independent sources can be achieved. Equivalently, we can alternatively fully characterize the complete achievable rate region of this network. Rather than formulating it into a sequence of WSRM problems, as observed in the previous works of [13] and [14], we propose a corner point (CP) optimization formulation. Then it is shown that the complete achievable rate region can be derived via solving a sequence of CP optimization problems, which yields a prohibitively high computational complexity for large networks. To deal with this issue, we focus on designing low-complexity beamforming vectors, with which two inner bounds of the complete achievable rate region can then be obtained. We show theoretically that the inner bounds are approximate to the complete achievable rate region in two special scenarios of interest. Finally, numerical results are shown to verify the effectiveness of the approaches.

Notation. The expectation operation is denoted by $\mathbf{E}[\cdot]$. A complex Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$. A diagonal matrix with x_i at the

diagonal position is denoted by $\text{diag}(\mathbf{x}) = \text{diag}(x_1, \dots, x_n)$. The Euclidean norm of a vector \mathbf{x} is denoted by $\|\mathbf{x}\|_2$. The trace of \mathbf{X} is denoted by $\text{tr}(\mathbf{X})$. A logarithm in base 2 is denoted by $\log(\cdot)$. We use $\text{cl}(\cdot)$ to denote the closure of the convex hull of a set. Let $\lambda_{\max}(\mathbf{X})$ represent the maximum eigenvalue of a matrix \mathbf{X} , and let $\text{vec}(\mathbf{X})$ denote its corresponding eigenvector.

II. System Model

We study the dual-hop MARN that is depicted in Fig. 1. Two independent sources (S_1 and S_2) simultaneously communicate to a common destination node (D) with the aid of a layer of AF relays (denoted by $\mathcal{V} = \{1, \dots, n\}$). Similar to works [13] and [14], the direct path between the sources and the destination is neglected due to large channel fading. We consider a flat-fading channel scenario; thus, all channel coefficients are assumed to be constant during a transmission. The destination node serves as a center node, which collects all channel state information using the training-based channel estimation technique proposed in [15]. Before each round of transmission, the destination node designs an appropriate beamforming vector and distributes it to the relays via feed-back channels.

In the first hop, the sources broadcast the signals to the relays simultaneously. We assume that the propagation delays from the source nodes to the relays are derived and all the relays receive signals in different frequency bands with equal bandwidth. The source node transmits the same signal on all frequency bands using different up-frequency converters. To achieve synchronization for the first hop, the source nodes control the start time of the transmission in each frequency band according to the delay information. As a result, the signal received at any given relay can be synchronized. After frequency down-conversion, the signal received at any given relay is obtained by

$$y_k = f_{1,k}x_{S_1} + f_{2,k}x_{S_2} + z_k \quad \text{for } k \in \mathcal{V}, \quad (1)$$

where x_{S_i} is the information symbol of source node S_i with

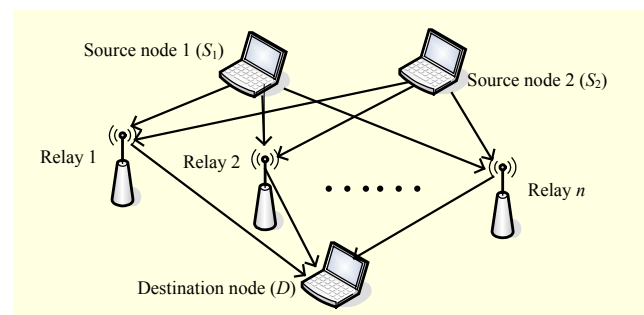


Fig. 1. Dual-hop MARN with two sources.

power $E[|x_{S_i}|^2] = P_{S_i}$, for $i=1,2$; $f_{1,k}$ and $f_{2,k}$ denote the complex-valued channel coefficients from S_1 and S_2 to relay k , respectively; and z_k denotes the complex Gaussian noise, with zero mean and unit variance, at relay k . Since the AF-based distributed beamforming scheme is employed, relay k simply amplifies and retransmits the received signal; that is,

$$x_k = \alpha_k^* y_k, \quad (2)$$

where α_k is the complex-valued scaling factor chosen at relay k and $(\cdot)^*$ represents the conjugate of a complex number.

We consider the scenario where each relay k has an individual power budget; thus, the transmit signal at relay k should satisfy the following constraint:

$$E[|x_k|^2] = P_{R,k} |\alpha_k|^2 \leq P_{k,\max}, \quad (3)$$

where $P_{R,k} = |f_{1,k}|^2 P_{S_1} + |f_{2,k}|^2 P_{S_2} + 1$ is the received signal power of relay k and $|\cdot|$ represents the absolute value of a complex number.

We use $\mathbf{a} = [\alpha_1, \dots, \alpha_n]^T$ to represent a beamforming vector and use $\Omega = \{\mathbf{a} : \mathbf{a} \text{ satisfies (3)}\}$ to denote the set of all beamforming vectors satisfying the individual relay power constraints. Given a beamforming vector \mathbf{a} , the signal received at a destination node can be expressed as

$$y_D = \mathbf{h}^T \text{diag}(\mathbf{a}^*) (\mathbf{f}_1 x_{S_1} + \mathbf{f}_2 x_{S_2} + \mathbf{z}) + z_D, \quad (4)$$

where $\mathbf{f}_1 = [f_{1,1}, \dots, f_{1,n}]^T$, $\mathbf{f}_2 = [f_{2,1}, \dots, f_{2,n}]^T$, and $\mathbf{z} = [z_1, \dots, z_n]^T$. Here, $\mathbf{h} = [h_1, \dots, h_n]^T$ denotes the channel vector from relays to the destination node and z_D is the complex Gaussian noise, with zero mean and unit variance, at the destination node. Here, we also assume that perfect synchronization is achieved. The synchronization can be achieved in a similar manner as in the first hop with the only difference being that all the relays can transmit on the same frequency band.

From (4), given a beamforming vector \mathbf{a} , the dual-hop MARN can be considered as a conventional Gaussian multiple-access channel (MAC), where $\mathbf{h}^T \text{diag}(\mathbf{a}^*) \mathbf{f}_1 x_{S_1}$ and $\mathbf{h}^T \text{diag}(\mathbf{a}^*) \mathbf{f}_2 x_{S_2}$ are the information symbols of the two equivalent sources and $\mathbf{h}^T \text{diag}(\mathbf{a}^*) \mathbf{z} + z_D$ is the equivalent Gaussian noise. To distinguish them, we denote the former as $\text{MAC}(\mathbf{a})$. The capacity region of a Gaussian MAC can be found in [16]. To achieve this region, the source nodes employ independent Gaussian codebooks, each consisting of 2^{nr} codewords. The random variables x_{S_1} and x_{S_2} , used to generate the codebooks are drawn according to $\mathcal{CN}(0, P_{S_1})$ and $\mathcal{CN}(0, P_{S_2})$, respectively. According to the capacity region of the Gaussian MAC, the achievable rate region of $\text{MAC}(\mathbf{a})$ is denoted by $\mathcal{R}(\mathbf{a})$, which is given as follows:

$$\left\{ (R_1, R_2) : R_1 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right), R_2 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right), \right. \\ \left. R_1 + R_2 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right) \right\}, \quad (5)$$

where $(\cdot)^\dagger$ represents the Hermitian transpose, $\mathbf{H} = \text{diag}(\mathbf{h})$, and $\mathcal{C}(x) = \log(1+x)$ represents the Gaussian capacity formula with SNR equal to x . For brevity of notation, we denote the union of achievable rate sets $\mathcal{R}(\mathbf{a})$'s by $\mathcal{R}(\{\mathbf{a}\}) = \bigcup_{\mathbf{a} \in \Omega} \mathcal{R}(\mathbf{a})$. By time-sharing technique, the complete achievable rate region of the network is given by $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$. To fully characterize $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$, a commonly used method is to solve a sequence of WSRM problems — as can be observed in [13] and [14]. However, it has been shown in [14] that the WSRM problem is hard to solve when it comes to individual relay power constraints. So, we shall propose an alternative approach to address this problem.

III. Characterization of Achievable Rate Region

In this section, we show that the problem of computing $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ can be formulated as a sequence of non-convex CP optimization problems. Then we solve the CP optimization problem via a semi-definite programming (SDP)-based approach.

1. Problem Formulation

We consider two special achievable rate pairs in $\mathcal{R}(\mathbf{a})$ corresponding to a specific beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$. Using the successive cancellation decoding scheme with the message of S_1 first decoded, the rate pair

$$R_1^{\text{up}}(\mathbf{a}) = \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}}{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2} + \|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right) \\ \text{and} \quad (6)$$

$$R_2^{\text{up}}(\mathbf{a}) = \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right)$$

can be achieved, which is referred to as the ‘‘upper-diagonal’’ corner point of $\mathcal{R}(\mathbf{a})$.

Reversing the decoding order, the rate pair

$$R_1^{\text{low}}(\mathbf{a}) = \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right)$$

and

$$R_2^{\text{low}}(\mathbf{a}) = \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + \|\mathbf{a}^\dagger \mathbf{H}\|_2^2 + 1} \right) \quad (7)$$

can be achieved, which is referred to as the “lower-diagonal” corner point of $\mathcal{R}(\mathbf{a})$.

According to (6), we set up a problem of maximizing the transmission rate $R_2^{\text{up}}(\mathbf{a})$ of S_2 under the individual relay power constraints (3) and the constraint that the transmission rate $R_2^{\text{up}}(\mathbf{a})$ of S_1 is no less than a desired value r_1 . Similarly, according to (7), we set up a problem of maximizing the transmission rate $R_1^{\text{low}}(\mathbf{a})$ of S_1 under the individual relay power constraints (3) and the constraint that the transmission rate $R_2^{\text{low}}(\mathbf{a})$ of S_2 is no less than a desired value r_2 . Combining the fact that $\log(\cdot)$ is an increasing function, the above-mentioned maximization problems can be unified formulated as follows, which are referred to as CP optimization problems:

$$\begin{aligned} \max_{\mathbf{a}} & \frac{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_i}}{\mathbf{a}^\dagger \mathbf{H} \mathbf{H}^\dagger \mathbf{a} + 1}, \\ \text{s.t.} & \frac{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_j}}{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_i} + \mathbf{a}^\dagger \mathbf{H} \mathbf{H}^\dagger \mathbf{a} + 1} \geq \gamma_j, \\ & \mathbf{a}^\dagger \mathbf{I}_k \mathbf{a} \leq |\alpha_{k,\max}|^2, \quad k \in \mathcal{V}, \end{aligned} \quad (8)$$

where $i, j \in \{1, 2\}$, $i \neq j$, $\gamma_j = 2^{r_j} - 1$ is the equivalent SNR constraint, $\mathbf{I}_k = \text{diag}(0, \dots, 1, \dots, 0)$ is a diagonal matrix with the k th element equal to 1, and $\alpha_{k,\max} = \sqrt{P_{k,\max}/P_{R,k}}$. We denote the optimal solution of the above problem as $\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)$ and $\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)$, for $i = 1$ and 2 , respectively. It is clear that $R_1^{\text{low}}(\mathbf{a}_{\text{opt}}^{\text{low}}(0))$ and $R_2^{\text{up}}(\mathbf{a}_{\text{opt}}^{\text{up}}(0))$ represent the maximum individual transmission rates of S_1 and S_2 respectively. Furthermore, the maximum possible value of r_j can be determined via solving the following problem:

$$\begin{aligned} \max_{\mathbf{a}} & \frac{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_j}}{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_i} + \mathbf{a}^\dagger \mathbf{H} \mathbf{H}^\dagger \mathbf{a} + 1}, \\ \text{s.t.} & \mathbf{a}^\dagger \mathbf{I}_k \mathbf{a} \leq |\alpha_{k,\max}|^2, \quad k \in \mathcal{V}. \end{aligned} \quad (9)$$

Denote the maximum objective value of the above problem by $\gamma_{j,\max}$. It follows that given any beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$, the achievable rate $R_1^{\text{up}}(\mathbf{a})$ does not exceed $r_{1,\max}$ and the achievable rate $R_2^{\text{low}}(\mathbf{a})$ does not exceed $r_{2,\max}$, where $r_{i,\max} = \log(1 + \gamma_{i,\max})$ for $i = 1, 2$.

Therefore, each value of r_j in the interval $[0, r_{j,\max}]$ leads to a CP optimization problem. Then, we can obtain a sequence of optimal solutions $\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)$ and $\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)$ by solving the corresponding CP optimization problems. For simplicity of

notation, we denote the union of the rate sets $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1))$'s and $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2))$'s as follows:

$$\begin{aligned} \mathcal{R}(\{r_1\}) &= \bigcup_{r_1 \in [0, r_{1,\max}]} \mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \\ \text{and} \\ \mathcal{R}(\{r_2\}) &= \bigcup_{r_2 \in [0, r_{2,\max}]} \mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)). \end{aligned} \quad (10)$$

It is obvious that $\text{cl}(\mathcal{R}(\{r_1\}) \cup \mathcal{R}(\{r_2\}))$ is a subset of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$. Then, we show an interesting result that the two rate sets are indeed equivalent.

Theorem 1. For any beamforming vector \mathbf{a}_0 that satisfies the individual relay power constraints in (3), the corresponding achievable rate region $\mathcal{R}(\mathbf{a}_0)$ of $\text{MAC}(\mathbf{a}_0)$ is a subset of $\text{cl}(\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \cup \mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)))$, where $r_1 = R_1^{\text{up}}(\mathbf{a}_0)$ and $r_2 = R_2^{\text{low}}(\mathbf{a}_0)$.

Proof. To prove the theorem, it is sufficient to show that the upper- and lower-diagonal corner points of $\mathcal{R}(\mathbf{a}_0)$ are contained in $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1))$ and $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2))$, respectively.

On one hand, by setting $r_1 = R_1^{\text{up}}(\mathbf{a}_0)$, \mathbf{a}_0 is apparently a feasible solution of (8) since \mathbf{a}_0 also satisfies the power constraints. So, the optimal solution $\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)$ should always exist and should satisfy the first SNR constraint of (8); that is,

$$R_1^{\text{up}}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \geq r_1 = R_1^{\text{up}}(\mathbf{a}_0). \quad (11)$$

On the other hand, $R_2^{\text{up}}(\mathbf{a})$ is maximized at $\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)$; that is,

$$R_2^{\text{up}}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \geq R_2^{\text{up}}(\mathbf{a}_0). \quad (12)$$

Therefore, it follows that the achievable rate pair $(R_1^{\text{up}}(\mathbf{a}_0), R_2^{\text{up}}(\mathbf{a}_0))$ is included in $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1))$. Following the same argument, we claim that the following relationship holds:

$$R_1^{\text{low}}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)) \geq R_1^{\text{low}}(\mathbf{a}_0) \quad (13)$$

and

$$R_2^{\text{low}}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)) \geq r_2 = R_2^{\text{low}}(\mathbf{a}_0). \quad (14)$$

As a result, it follows that the achievable rate pair $(R_1^{\text{low}}(\mathbf{a}_0), R_2^{\text{low}}(\mathbf{a}_0))$ is included in $\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2))$. By time-sharing technique, all rate pairs in $\text{cl}(\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \cup \mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)))$ are achievable and thus the rate set $\mathcal{R}(\mathbf{a}_0)$ is included in $\text{cl}(\mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{up}}(r_1)) \cup \mathcal{R}(\mathbf{a}_{\text{opt}}^{\text{low}}(r_2)))$. Then, we complete the proof. \square

From Theorem 1, it follows that both $\mathcal{R}(\{\mathbf{a}\})$ and $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ are included in $\text{cl}(\mathcal{R}(\{r_1\}) \cup \mathcal{R}(\{r_2\}))$. Then, we conclude the procedure of fully characterization of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ as follows: The interval $[0, r_{i,\max}]$ is first quantized into sufficiently fine grids. Then, for each grid point, a CP optimization problem is formulated and solved. With the obtained solutions, the complete achievable rate region

$\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ of a dual-hop AF relaying MARN under individual relay power constraints can be fully characterized.

From the above procedure, the finer the quantized grid, the more precise the complete achievable rate region is characterized; however, as a consequence, computational complexity is increased. Thus, there exists a trade-off between precision and computational complexity. For practical implementation, all the obtained schemes are stored in a look-up table in a control center. Before the transmission, the control center should search the table to decide the appropriate scheme that achieves the desirable rate pair of the sources and distributes the scheme to all the relays.

We end this subsection with a brief discussion on the design of distributed beamforming for MARNs with a general number of source nodes. Assume that there are m source nodes. With the successive cancellation decoding scheme, each rate set has $m!$ (roughly speaking, exponentially many) corner points with respect to different decoding orders. Similar to the two-source case, we shall establish an optimization problem for each corner point to fully characterize the achievable rate region. Specifically, we maximize the transmission rate of the source node for which the message is last decoded, subject to the $m-1$ rate constraints similar to that of (8) and to the individual relay power constraints. Thus, the problem formula becomes much more complicated. Besides, the search operation for each corner point becomes an $(m-1)$ dimensional one, which appears prohibitively expensive in terms of computational complexity. For simplicity of discussion, we only take the two-source case as an illustration.

2. SDP-Based Approach

Recall that the WSRM problem for a two-way relay network is solved via a bisection technique, with each step (of the bisection technique) involving a SDP when the nonreciprocal channels scenario and the individual relay power constraints are considered. Although the CP optimization problem proposed in this study is similar to the WSRM problem studied in [10], we apply several different transformation tricks to reformulate the CP problem as one SDP problem; thus, avoiding the bisection procedure. Since the optimization problems in (9) can be viewed as special cases of (8), the detailed computational procedure of (9) is skipped.

Problem (8) is then first reformulated as

$$\begin{aligned} \max_{\mathbf{a}, u} & \frac{\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_i}}{u^2}, \\ \text{s.t.} & \mathbf{a}^\dagger \mathbf{H} \mathbf{H}^\dagger \mathbf{a} + 1 = u^2, \\ & \mathbf{a}^\dagger \mathbf{A} \mathbf{a} + \gamma_j \leq \mathbf{a}^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \mathbf{a} P_{S_j}, \\ & \mathbf{a}^\dagger \mathbf{I}_k \mathbf{a} \leq |\alpha_{k, \max}|^2, \quad k \in \mathcal{V}, \end{aligned} \quad (15)$$

where $\mathbf{A} = \gamma_j (\mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger P_{S_i} + \mathbf{H} \mathbf{H}^\dagger)$ is positive definite.

Using the substitutions $v = 1/u$ and $\boldsymbol{\beta} = \mathbf{a}/u$, we have the following equivalent problem:

$$\begin{aligned} \max_{\boldsymbol{\beta}, v} & \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \boldsymbol{\beta} P_{S_i}, \\ \text{s.t.} & v^2 = 1 - \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{H}^\dagger \boldsymbol{\beta}, \\ & \boldsymbol{\beta}^\dagger \mathbf{A} \boldsymbol{\beta} + \gamma_j v^2 \leq \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \boldsymbol{\beta} P_{S_j}, \\ & \boldsymbol{\beta}^\dagger \mathbf{I}_k \boldsymbol{\beta} \leq |\alpha_{k, \max}|^2 v^2, \quad k \in \mathcal{V}. \end{aligned} \quad (16)$$

Then, substituting the first constraint into the remaining constraints of (16), we have

$$\begin{aligned} \max_{\boldsymbol{\beta}} & \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \boldsymbol{\beta} P_{S_i}, \\ \text{s.t.} & \gamma_j \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \boldsymbol{\beta} P_{S_i} + \gamma_j \leq \boldsymbol{\beta}^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \boldsymbol{\beta} P_{S_j}, \\ & \boldsymbol{\beta}^\dagger (\mathbf{I}_k + |\alpha_{k, \max}|^2 \mathbf{H} \mathbf{H}^\dagger) \boldsymbol{\beta} \leq |\alpha_{k, \max}|^2, \quad k \in \mathcal{V}. \end{aligned} \quad (17)$$

Using the substitution $\mathbf{X} = \boldsymbol{\beta} \boldsymbol{\beta}^\dagger$, (17) can be recast as

$$\begin{aligned} \max_{\mathbf{X}} & \text{tr}(P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{X}) \\ \text{s.t.} & \text{tr}[(\gamma_j P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger - P_{S_j} \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger) \mathbf{X}] + \gamma_j \leq 0, \\ & \text{tr}[(\mathbf{I}_k + |\alpha_{k, \max}|^2 \mathbf{H} \mathbf{H}^\dagger) \mathbf{X}] - |\alpha_{k, \max}|^2 \leq 0, \quad k \in \mathcal{V}, \\ & \mathbf{X} \succeq \mathbf{0}, \quad \text{rank}(\mathbf{X}) = 1. \end{aligned} \quad (18)$$

It is clear that the last rank-one constraint is non-convex. By applying the semi-definite relaxation technique [17], the above problem can be relaxed to

$$\begin{aligned} \max_{\mathbf{X}} & \text{tr}(P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{X}), \\ \text{s.t.} & \text{tr}[(\gamma_j P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger - P_{S_j} \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger) \mathbf{X}] + \gamma_j \leq 0, \\ & \text{tr}[(\mathbf{I}_k + |\alpha_{k, \max}|^2 \mathbf{H} \mathbf{H}^\dagger) \mathbf{X}] - |\alpha_{k, \max}|^2 \leq 0, \quad k \in \mathcal{V}, \\ & \mathbf{X} \succeq \mathbf{0}, \end{aligned} \quad (19)$$

which is a convex semi-definite program; thus, it can be efficiently solved via standard interior-point methods within polynomial time. Compared with the bisection search-based method proposed in [10], the proposed approach is much more efficient.

Generally speaking, the resulting optimal solution \mathbf{X}_{opt} may not be of rank one. Interestingly, we show in the following proposition that the rank relaxation is tight when the first constraint of (19) is inactive.

Proposition 1. Suppose that problem (19) is feasible and that the optimal solution \mathbf{X}_{opt} of (19) is attained with the rate constraint being inactive. Then it can be shown that $\text{rank}(\mathbf{X}_{\text{opt}}) = 1$.

The proof is given in the appendix.

Proposition 1 provides a sufficient condition to determine whether the resultant solution is of rank one. In other cases, we

shall first compute the rank of the optimal solution \mathbf{X}_{opt} . If it is of rank one, then the optimal solution of (17) can be easily obtained via rank-one decomposition of \mathbf{X}_{opt} , that is, $\mathbf{X}_{\text{opt}} = \boldsymbol{\beta}_{\text{opt}} \boldsymbol{\beta}_{\text{opt}}^\dagger$; otherwise, good rank-one approximate solutions can be derived via various techniques developed in [18]. Interestingly enough, we find that \mathbf{X}_{opt} is always of rank one in all our numerical simulations. Then, the optimal solution of (8) is given by

$$\mathbf{a}_{\text{opt}} = \boldsymbol{\beta}_{\text{opt}} / v_{\text{opt}}, \quad (20)$$

where $v_{\text{opt}} = \sqrt{1 - \boldsymbol{\beta}_{\text{opt}}^\dagger \mathbf{H} \mathbf{H}^\dagger \boldsymbol{\beta}_{\text{opt}}}$.

Nevertheless, we provide a randomization-based efficient approach to construct rank-one solutions once the rank of \mathbf{X}_{opt} is greater than one, for the sake of completion. We use \mathbf{X}_{opt} to randomly generate a set of candidate vectors $\{\boldsymbol{\beta}_l\}$. The k th element of $\boldsymbol{\beta}_l$ is chosen as $\sqrt{\mathbf{X}_{\text{opt},k}} e^{j\theta_{l,k}}$, where $\mathbf{X}_{\text{opt},k}$ denotes the k th diagonal element of \mathbf{X}_{opt} and $\theta_{l,k}$ is uniformly distributed over the interval $[0, 2\pi)$. As can be seen, $\boldsymbol{\beta}_l$ always satisfies the individual relay power constraints in (17). The best solution among all the candidates is chosen such that the violation of the first constraint of (17) is minimized; that is,

$$\tilde{\boldsymbol{\beta}}_{\text{opt}} = \arg \min_{\boldsymbol{\beta}_l} \left\{ 1 - \left(\frac{\boldsymbol{\beta}_l^\dagger \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger \boldsymbol{\beta}_l P_{S_j}}{\gamma_j} - \boldsymbol{\beta}_l^\dagger \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \boldsymbol{\beta}_l P_{S_i} \right) \right\}. \quad (21)$$

IV. Low-Complexity Suboptimal Schemes

In this section, we focus on designing low-complexity suboptimal schemes. Using these schemes, we can approximately characterize the complete achievable rate region $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ with finitely many $\mathcal{R}(\mathbf{a})$'s in two special scenarios. It is clear in this case that the implementation cost can be significantly reduced.

From (4), the received sum noise at the destination node consists of two parts. One is the relay propagation noise denoted by

$$z_R(\mathbf{a}) = \mathbf{h}^T \text{diag}(\mathbf{a}^*) \mathbf{z}. \quad (22)$$

The other is the local noise z_D introduced by the destination node. Comparing the powers of $z_R(\mathbf{a})$ and z_D , we consider the following two special scenarios.

Scenario 1. The power of z_D is relatively large compared with that of $z_R(\mathbf{a})$. In this scenario, the relays can be regarded as multiple transmit antennas of the source nodes. Intuitively, the relays shall apply co-phase transmission — with all relays using the maximum allowable powers to combat the local destination noise z_D . Thus, the beamforming vector is

chosen as

$$\mathbf{a}_{1,1} = [\alpha_{1,\max} e^{j(\angle f_{1,1} + \angle h_1)}, \dots, \alpha_{n,\max} e^{j(\angle f_{1,n} + \angle h_n)}]^T, \quad (23)$$

or

$$\mathbf{a}_{1,2} = [\alpha_{1,\max} e^{j(\angle f_{2,1} + \angle h_1)}, \dots, \alpha_{n,\max} e^{j(\angle f_{2,n} + \angle h_n)}]^T, \quad (24)$$

where $e^{j(\angle f_{i,k} + \angle h_k)} = f_{i,k} h_k / |f_{i,k}| |h_k|$ for $i=1,2$.

It is clear that $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ is an inner bound of the complete achievable rate region $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$. Our goal is to show that the low-complexity inner bound $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ is an approximate characterization of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ in this scenario.

To be rigorous, we consider the case when the power of z_D is relatively large compared with that of $z_R(\mathbf{a})$ if the condition

$$\mathbf{E} \left[|z_R(\mathbf{a}_{1,i})|^2 \right] \leq \mathbf{E} \left[|z_D|^2 \right] \text{ for } i=1,2 \quad (25)$$

is satisfied; that is, $\|\mathbf{a}_{1,i}^\dagger \mathbf{H}\|_2^2 \leq 1$ for $i=1,2$.

Due to the lack of closed-form solutions of (8), it is difficult to directly compare $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ with $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$. Hence, we first propose an outer bound of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$, which is used as a benchmark to evaluate the performance of $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$. Given any beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$, the following rate set, denoted by $\mathcal{R}_1^{\text{out}}(\mathbf{a})$, is an outer bound of $\mathcal{R}(\mathbf{a})$:

$$\left\{ (R_1, R_2) : R_1 \leq \mathcal{C}(|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}), R_2 \leq \mathcal{C}(|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}), \right. \\ \left. R_1 + R_2 \leq \mathcal{C}(|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}) \right\}. \quad (26)$$

Denoting the union of the above outer bounds by $\mathcal{R}_1^{\text{out}}(\{\mathbf{a}\}) = \cup_{\mathbf{a} \in \Omega} \mathcal{R}_1^{\text{out}}(\mathbf{a})$, it then follows that

$$\text{cl}(\mathcal{R}(\{\mathbf{a}\})) \subset \text{cl}(\mathcal{R}_1^{\text{out}}(\{\mathbf{a}\})). \quad (27)$$

From (23), (24), and (26) it follows that for any beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$, $\mathcal{R}_1^{\text{out}}(\mathbf{a})$ is included in the rate set $\bar{\mathcal{R}}_1^{\text{out}}$ given as

$$\left\{ (R_1, R_2) : R_1 \leq \mathcal{C}(|\mathbf{a}_{1,1}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}), R_2 \leq \mathcal{C}(|\mathbf{a}_{1,2}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}), \right. \\ \left. R_1 + R_2 \leq \mathcal{C}(|\mathbf{a}_{1,1}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}_{1,2}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}) \right\}, \quad (28)$$

which further implies that

$$\text{cl}(\mathcal{R}(\{\mathbf{a}\})) \subset \text{cl}(\mathcal{R}_1^{\text{out}}(\{\mathbf{a}\})) \subset \bar{\mathcal{R}}_1^{\text{out}}. \quad (29)$$

Therefore, to achieve the desired goal, it suffices to show that $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ is approximate to $\bar{\mathcal{R}}_1^{\text{out}}$ under the condition given in (25). Considering the maximum achievable individual rates of $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$, we have

$$\begin{aligned}
R_i &= \log \left(1 + \frac{|\mathbf{a}_{1,i}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i}}{\|\mathbf{a}_{1,i}^\dagger \mathbf{H}\|_2^2 + 1} \right) \\
&\stackrel{(a)}{\geq} \log \left(1 + \frac{|\mathbf{a}_{1,i}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i}}{2} \right) \\
&\geq \log \left(1 + |\mathbf{a}_{1,i}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i} \right) - 1 \quad \text{for } i=1,2;
\end{aligned} \tag{30}$$

where (a) follows from the condition (25). It follows that the maximum achievable individual rate of $\text{cl}(\cup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ is at most one bit/channel use smaller than that of the outer bound $\bar{\mathcal{R}}_1^{\text{out}}$.

Scenario 2. The power of z_D is relatively small compared with that of $z_R(\mathbf{a})$. In this scenario, the relays can be regarded as multiple-receive antennas of the destination node. It is well known that maximal-ratio combining [19] is the optimal linear combining technique that achieves the largest diversity gain in this scenario.

To be specific, for individual rates we choose the beamforming vector as the solution of

$$\begin{aligned}
\max \quad & \frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2}, \\
\text{s.t.} \quad & \mathbf{a}^\dagger \mathbf{I}_k \mathbf{a} \leq |\alpha_{k,\max}|^2, \quad k \in \mathcal{V}.
\end{aligned} \tag{31}$$

It is clear that the objective function is a Rayleigh-Ritz quotient [20]. Consequently, the optimal solution can be easily obtained in closed-form and is given as follows:

$$\mathbf{a}_{2,i} = c_i (\mathbf{H}^\dagger)^{-1} \mathbf{f}_i, \quad c_i = \min \left\{ \frac{\alpha_{k,\max} |h_k|}{|f_{i,k}|}, k \in \mathcal{V} \right\} \quad \text{for } i=1,2; \tag{32}$$

where c_i is chosen such that the relay power constraints are satisfied. The corresponding maximum objective value is $\|\mathbf{f}_i\|_2^2 P_{S_i}$. For sum rate, we choose the beamforming vector as the solution of

$$\begin{aligned}
\max \quad & \frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2}, \\
\text{s.t.} \quad & \mathbf{a}^\dagger \mathbf{I}_k \mathbf{a} \leq |\alpha_{k,\max}|^2, \quad k \in \mathcal{V}.
\end{aligned} \tag{33}$$

It is clear that the objective function is also a Rayleigh-Ritz quotient. Hence, the optimal solution can be easily obtained as

$$\mathbf{a}_{2,3} = c_3 (\mathbf{H}^\dagger)^{-1} \mathbf{f}, \quad c_3 = \min \left\{ \frac{\alpha_{k,\max} |h_k|}{|f_k|}, k \in \mathcal{V} \right\}, \tag{34}$$

where c_3 is chosen such that the relay power constraints are satisfied, $\mathbf{f} = [f_1, \dots, f_n] = \text{vec}(\mathbf{f}_1 \mathbf{f}_1^\dagger P_{S_1} + \mathbf{f}_2 \mathbf{f}_2^\dagger P_{S_2})$ and the

maximum objective value is $\lambda_{\max}(\mathbf{f}_1 \mathbf{f}_1^\dagger P_{S_1} + \mathbf{f}_2 \mathbf{f}_2^\dagger P_{S_2})$. Note that the maximum eigenvalue of a rank-two semi-definite matrix can be analytically derived, as shown in [14]; that is,

$$\lambda_{\max}(\mathbf{f}_1 \mathbf{f}_1^\dagger P_{S_1} + \mathbf{f}_2 \mathbf{f}_2^\dagger P_{S_2}) = \frac{a + \sqrt{a^2 - b}}{2}, \tag{35}$$

where

$$a = \|\mathbf{f}_1\|_2^2 P_{S_1} + \|\mathbf{f}_2\|_2^2 P_{S_2}$$

and

$$b = 4P_{S_1} P_{S_2} (\|\mathbf{f}_1\|_2^2 \|\mathbf{f}_2\|_2^2 - |\mathbf{f}_1^\dagger \mathbf{f}_2|^2).$$

Then, \mathbf{f} can be easily derived via solving a system of homogeneous linearity equations.

It is clear that $\text{cl}(\cup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is also an inner bound of the complete achievable rate region $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$. Our goal is to show that the low-complexity inner bound $\text{cl}(\cup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is an approximate characterization of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ in this scenario.

To be rigorous, we consider the case when the power of z_D is relatively small compared with that of $z_R(\mathbf{a})$ if the condition

$$\mathbf{E} \left[|z_R(\mathbf{a}_{2,i})|^2 \right] \geq \mathbf{E} [z_D^2] \quad \text{for } i=1,2,3 \tag{36}$$

is satisfied, that is, $\|\mathbf{a}_{2,i}^\dagger \mathbf{H}\|_2^2 \geq 1$ for $i=1,2,3$.

Due to the same reason as the previous scenario, we shall propose an outer bound of $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ as a benchmark to evaluate the performance of $\text{cl}(\cup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$. Given any beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$, the following rate set $\mathcal{R}_2^{\text{out}}(\mathbf{a})$ is an outer bound of $\mathcal{R}(\mathbf{a})$:

$$\begin{aligned}
& \left\{ (R_1, R_2) : R_1 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2} \right), R_2 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2} \right), \right. \\
& \left. R_1 + R_2 \leq \mathcal{C} \left(\frac{|\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}^\dagger \mathbf{H}\|_2^2} \right) \right\}. \tag{37}
\end{aligned}$$

Denoting the union of the above outer bounds by $\mathcal{R}_2^{\text{out}}(\{\mathbf{a}\}) = \cup_{\mathbf{a} \in \Omega} \mathcal{R}_2^{\text{out}}(\mathbf{a})$ it then follows that

$$\text{cl}(\mathcal{R}(\{\mathbf{a}\})) \subset \text{cl}(\mathcal{R}_2^{\text{out}}(\{\mathbf{a}\})). \tag{38}$$

Then, from (31)–(34), it follows that for any beamforming vector \mathbf{a} , $\mathbf{a} \in \Omega$, $\mathcal{R}_2^{\text{out}}(\mathbf{a})$ is included in the rate set $\bar{\mathcal{R}}_2^{\text{out}}$ given as

$$\begin{aligned}
& \left\{ (R_1, R_2) : R_1 \leq \mathcal{C}(\|\mathbf{f}_1\|_2^2 P_{S_1}), R_2 \leq \mathcal{C}(\|\mathbf{f}_2\|_2^2 P_{S_2}), \right. \\
& \left. R_1 + R_2 \leq \mathcal{C}(\lambda_{\max}(\mathbf{f}_1 \mathbf{f}_1^\dagger P_{S_1} + \mathbf{f}_2 \mathbf{f}_2^\dagger P_{S_2})) \right\}, \tag{39}
\end{aligned}$$

which further implies that

$$\text{cl}(\mathcal{R}(\{\mathbf{a}\})) \subset \text{cl}(\mathcal{R}_2^{\text{out}}(\{\mathbf{a}\})) \subset \bar{\mathcal{R}}_2^{\text{out}}. \quad (40)$$

Finally, to achieve the desired goal, it suffices to show that $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is approximate to $\bar{\mathcal{R}}_2^{\text{out}}$ under the condition given in (36). Considering the maximum achievable individual rates of $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$, we have

$$\begin{aligned} R_i &\geq \log \left(1 + \frac{|\mathbf{a}_{2,i}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i}}{\|\mathbf{a}_{2,i}^\dagger \mathbf{H}\|_2^2 + 1} \right) \\ &\stackrel{(a)}{\geq} \log \left(1 + \frac{|\mathbf{a}_{2,i}^\dagger \mathbf{H} \mathbf{f}_i|^2 P_{S_i}}{2 \|\mathbf{a}_{2,i}^\dagger \mathbf{H}\|_2^2} \right) \\ &\stackrel{(b)}{\geq} \log \left(1 + \|\mathbf{f}_i\|_2^2 P_{S_i} \right) - 1 \text{ for } i = 1, 2; \end{aligned} \quad (41)$$

where (a) follows from the condition (36) and (b) follows from (32). It follows that the maximum achievable individual rate of $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is at most half a bit/channel use smaller than that of the outer bound $\bar{\mathcal{R}}_2^{\text{out}}$. Considering the maximum achievable sum rate of the inner bound $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$, we have

$$\begin{aligned} R_1 + R_2 &\geq \log \left(1 + \frac{|\mathbf{a}_{2,3}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}_{2,3}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{\|\mathbf{a}_{2,3}^\dagger \mathbf{H}\|_2^2 + 1} \right) \\ &\stackrel{(a)}{\geq} \log \left(1 + \frac{|\mathbf{a}_{2,3}^\dagger \mathbf{H} \mathbf{f}_1|^2 P_{S_1} + |\mathbf{a}_{2,3}^\dagger \mathbf{H} \mathbf{f}_2|^2 P_{S_2}}{2 \|\mathbf{a}_{2,3}^\dagger \mathbf{H}\|_2^2} \right) \\ &\stackrel{(b)}{\geq} \log \left(1 + \lambda_{\max}(\mathbf{f}_1 \mathbf{f}_1^\dagger P_{S_1} + \mathbf{f}_2 \mathbf{f}_2^\dagger P_{S_2}) \right) - 1, \end{aligned} \quad (42)$$

where (a) follows from the condition (36) and (b) follows from (34). It follows that the maximum achievable sum rate of the inner bound $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is also at most half a bit/channel use smaller than that of the outer bound $\bar{\mathcal{R}}_2^{\text{out}}$. Combining (41) and (42), we can conclude that $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is approximate to $\bar{\mathcal{R}}_2^{\text{out}}$ and thus, to $\text{cl}(\mathcal{R}(\{\mathbf{a}\}))$ in this scenario.

V. Simulation Results

In this section, we illustrate several numerical results to show the performance of the proposed schemes. Let's consider a dual-hop MARN with three AF relays. The network setup is given as follows. The powers of the two sources P_{S_1} and P_{S_2} are both equal to 10. Each relay node has an equal individual power budget $P_{k,\max} = P_0$, for $k \in \{1, 2, 3\}$. The randomly generated channel vectors are given as follows:

$$\mathbf{f}_1 = [-0.045 + 1.283j, 0.432 + 0.221j, 0.077 + 1.276j]^T,$$

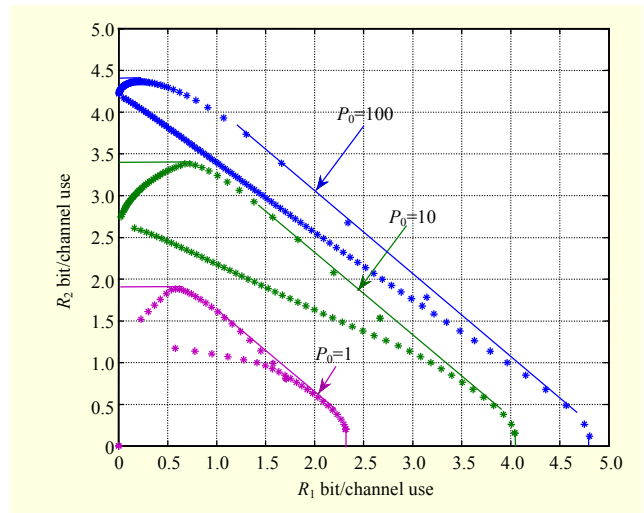


Fig. 2. Achievable rate regions vs. individual relay power budget P_0 .

$$\begin{aligned} \mathbf{f}_2 &= [-0.511 + 0.424j, 0.372 + 0.420j, -0.184 - 1.546j]^T, \\ \mathbf{h} &= [-0.938 + 1.040j, -1.019 - 0.231j, 0.284 + 0.574j]^T. \end{aligned}$$

Example 1. In the first example, three achievable rate regions of the dual-hop MARN are depicted in Fig. 2 versus different individual relay power budgets. We apply the approach proposed in section III. Each CP optimization problem (8) is solved via the SDP-based approach. Interestingly enough, we find that the solutions of the relaxed semi-definite programs are of rank one in all circumstances. Thus, we can obtain the exact optimal solution of the CP optimization problem, which leads to the full characterization of the achievable rate regions. We indicate all the optimized upper- and lower-diagonal corner points as stars in Fig. 2.

Example 2. In the second example, we show the approximate characterization of the complete achievable rate region in the two special scenarios considered in section IV. In Fig. 3, we show the inner bound $\text{cl}(\bigcup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ and the two outer bounds for $P_0=1$. It is easy to check that condition (25) is satisfied. From Fig. 3, we can see that the inner bound $\text{cl}(\bigcup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ performs close to the outer bound $\bar{\mathcal{R}}_1^{\text{out}}$, which coincides with the theoretical result that $\text{cl}(\bigcup_{i=1}^2 \mathcal{R}(\mathbf{a}_{1,i}))$ is a close approximation to the complete achievable rate region in this scenario. In Fig. 4, we show the inner bound $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ and the two outer bounds for $P_0=100$. It is easy to check that condition (36) is satisfied. From Fig. 4, we can see that the inner bound $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ performs close to the outer bound $\bar{\mathcal{R}}_2^{\text{out}}$, which coincides with the theoretical result that $\text{cl}(\bigcup_{i=1}^3 \mathcal{R}(\mathbf{a}_{2,i}))$ is a close approximation to the complete achievable rate region in this

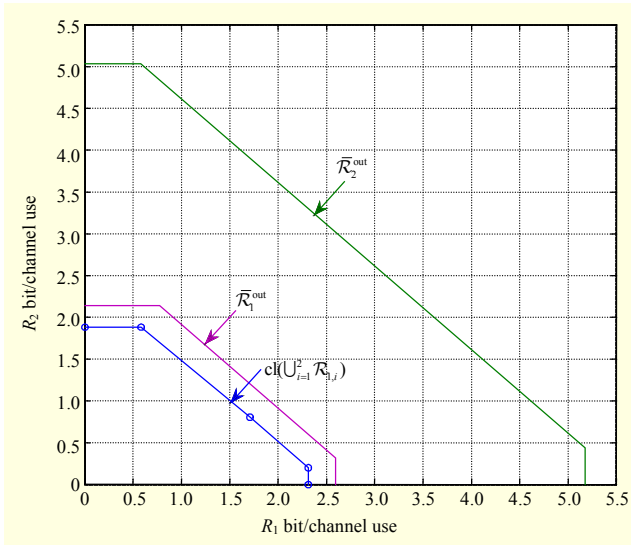


Fig. 3. Comparison of inner and outer bounds in scenario 1.

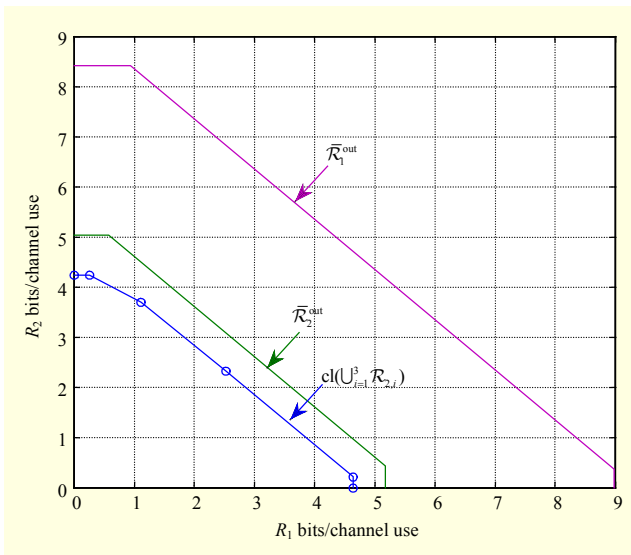


Fig. 4. Comparison of inner and outer bounds in scenario 2.

scenario.

VI. Conclusion

In this paper, we investigated the achievable rate region of a dual-hop MARN with multiple AF relays under individual power constraints. The complete achievable rate region can be fully characterized via solving a sequence of corner point optimization problems. To reduce the computational complexity, we also proposed several analytical suboptimal beamforming vectors to approximately characterize the complete achievable rate region in two special scenarios of interest. Overall, this work provides a theoretical basis for designing practical AF-based distributed relay beamforming.

Appendices

Proof of Proposition 1.

It is readily verified that (19) satisfies Slater's constraint qualification, which implies that the strong duality holds and that the primal and dual optimal solutions must satisfy the KKT conditions [21]. Let $\lambda_{0,\text{opt}}, \lambda_{k,\text{opt}}$, for $k \in \mathcal{V}$ and \mathbf{Y}_{opt} be the optimal dual variables associated with the constraints in (19). A part of the KKT conditions relevant to the proof are listed below:

$$\mathbf{Y}_{\text{opt}} = -(P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger + \lambda_{0,\text{opt}} P_{S_j} \mathbf{H} \mathbf{f}_j \mathbf{f}_j^\dagger \mathbf{H}^\dagger) + \sum_{k \in \mathcal{V}} \lambda_{k,\text{opt}} (\mathbf{I}_k + |\alpha_{k,\text{max}}|^2 \mathbf{H} \mathbf{H}^\dagger) + \lambda_{0,\text{opt}} \gamma_j P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger, \quad (43)$$

$$\mathbf{Y}_{\text{opt}} \mathbf{X}_{\text{opt}} = \mathbf{0}, \quad (44)$$

$$\mathbf{X}_{\text{opt}} \succeq \mathbf{0}, \mathbf{Y}_{\text{opt}} \succeq \mathbf{0}, \lambda_{0,\text{opt}} \geq 0, \lambda_{k,\text{opt}} \geq 0. \quad (45)$$

Note that if the first constraint is inactive, then by strong duality $\lambda_{0,\text{opt}} = 0$. Moreover, at least one $\lambda_{k,\text{opt}}$ must be positive in (45); otherwise, one can achieve a larger objective value by scaling up \mathbf{X}_{opt} . It implies that $\sum_{k \in \mathcal{V}} \lambda_{k,\text{opt}} (\mathbf{I}_k + |\alpha_{k,\text{max}}|^2 \mathbf{H} \mathbf{H}^\dagger)$ is a full-rank matrix. Then, by substituting (43) into (44), we have

$$P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{X}_{\text{opt}} = \sum_{k \in \mathcal{V}} \lambda_{k,\text{opt}} (\mathbf{I}_k + |\alpha_{k,\text{max}}|^2 \mathbf{H} \mathbf{H}^\dagger) \mathbf{X}_{\text{opt}}. \quad (46)$$

Thus it follows that:

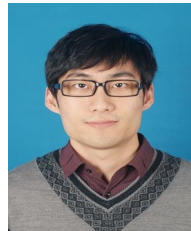
$$\begin{aligned} \text{rank}(\mathbf{X}_{\text{opt}}) &= \text{rank}\left(\sum_{k \in \mathcal{V}} \lambda_{k,\text{opt}} (\mathbf{I}_k + |\alpha_{k,\text{max}}|^2 \mathbf{H} \mathbf{H}^\dagger) \mathbf{X}_{\text{opt}}\right) \\ &= \text{rank}(P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger \mathbf{X}_{\text{opt}}) \\ &\leq \text{rank}(P_{S_i} \mathbf{H} \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}^\dagger) = 1. \end{aligned} \quad (47)$$

Since $\mathbf{X}_{\text{opt}} \neq \mathbf{0}$ in practice, it is sufficient to show that $\text{rank}(\mathbf{X}_{\text{opt}}) = 1$. \square

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