

# Optimal Amplify-and-Forward Scheme for Parallel Relay Networks with Correlated Relay Noise

Binyue Liu and Ye Yang

**This paper studies a parallel relay network where the relays employ an amplify-and-forward (AF) relaying scheme and are subjected to individual power constraints. We consider correlated effective relay noise arising from practical scenarios when the relays are exposed to common interferers. Assuming that the noise covariance and the full channel state information are available, we investigate the problem of finding the optimal AF scheme in terms of maximum end-to-end transmission rate. It is shown that the maximization problem can be equivalently transformed to a convex semi-definite program, which can be efficiently solved. Then an upper bound on the maximum achievable AF rate of this network is provided to further evaluate the performance of the optimal AF scheme. It is proved that the upper bound can be asymptotically achieved in two special regimes when the transmit power of the source node or the relays is sufficiently large. Finally, both theoretical and numerical results are given to show that, on average, noise correlation is beneficial to the transmission rate — whether the relays know the noise covariance matrix or not.**

**Keywords:** Amplify-and-forward, noise correlation, relay optimization, semi-definite relaxation, semi-definite program.

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## I. Introduction

The amplify-and-forward (AF) relaying scheme has been widely studied in the context of cooperative communications [1]–[3]. Besides its simplicity, the AF schemes can also be developed to exploit the multi-antenna gain in certain power constrained multiple-relay networks.

The problem of finding the optimal AF scheme in terms of maximum end-to-end transmission rate has been extensively studied for single-user systems in previous works [4]–[8]. The optimal AF schemes have been determined for the parallel-relay network under sum or individual relay power constraints, or both [4]–[6]. Under only the sum relay power constraint was the optimal scaling factor derived in a closed form [4]–[5]. However, the problem becomes more challenging when the relays are subject to individual power constraints. It was first solved in [6] with a semi-closed solution. Moreover, the AF relay optimization problems have also been considered for more general layered relay networks — for example, in [7] and [8]; however, only suboptimal schemes were derived.

The AF relaying scheme has also attracted great attention in the recent research community for multi-user relaying systems. Analog network coding [9] extends the AF-based one-way relaying scheme to a two-way relay channel to support communications in two directions via a two-step protocol. The problem of finding the optimal rate region of a two-way relay channel with a two-step AF protocol has been studied for different network setups [10]–[12]. The authors in [10] characterized the maximum achievable rate region for the two-way relay channel with a single multi-antenna relay, while the maximum rate region for the AF two-way relay channel with multiple single-antenna relays has been studied in [11] and [12] under both sum and individual relay power constraints.

The aforementioned schemes have all been built upon the assumption of independent relay noise. However, in wireless networks, noise correlation between nodes may occur due to common interference or noise propagation. In this aspect, it is worthwhile to mention the pioneer work [13], in which correlated relay noise is considered for a parallel  $n$ -relay network. The optimal AF scheme was obtained in closed form under a sum relay power constraint. Later, in [14], the optimal AF scheme was proposed for the same network model under a receive power constraint where relays were also exposed to correlated noise. To develop optimal AF schemes under different constraints, noise covariance and instantaneous channel coefficients are required. This important issue was first solved in [15]. The equivalent channel coefficients and the noise covariance can be jointly estimated at the destination node by an expectation-maximization algorithm-based approach. Recently, an algorithm can be concluded from [16] attempting to solve a similar problem when the relays are subject to individual power constraints. The algorithm mainly consists of a recomputation procedure and an updating procedure in each iterative step, but the description of the updating strategy is not clear, which is the key step determining the effectiveness and the complexity of the proposed algorithm. Following the algorithm from [17], let's consider the relays whose recomputed scaling factors violate the corresponding upper bound in the recomputation step. Then, let's come to the updating step. If all aforementioned relays are saturated to the maximum power simultaneously, as claimed in [16], then it is not difficult to find a counterexample to show that the algorithm cannot find the global optimal point in the sequel. However, if only one of them is saturated to the maximum power at the updating step, then the recomputation procedure may be repeated at most  $2^n - 2$  times in the worst case — that is, all the recomputed scaling factors invalidate the power constraints at every recomputation step. However, it is not proved in [16] that the worst case never occurs, which puts a limit on its implementation in practice.

In this paper, we also study the AF relay optimization problem for the parallel-relay network with correlated relay noise. Different from the existing works [13] and [14], the optimal AF scheme is developed under individual relay power constraints. The constraint is a more practical one, which makes the problem more challenging. Our main contributions can be summarized as follows.

We first formulate the problem as a semi-definite program (SDP), with the aid of several transformation tricks and the semi-definite relaxation (SDR) technique. So, the problem can be solved with only one SDP that has polynomial-time complexity. It is proved theoretically that the proposed approach can always find the global optimal solution. Then we

theoretically prove that the achievable rate of the optimal AF scheme performs close to the cut set-like bound obtained in this paper in two special regimes when either the source power or the relay power constraints are sufficiently large. Finally, we show via both analysis and simulations that noise correlation is beneficial to average transmission rate — whether the relays know the noise covariance matrix or not.

**Notation.** We use  $CN(\boldsymbol{\mu}, \mathbf{K})$  to denote an  $n$ -dimensional joint complex Gaussian distribution with means  $\boldsymbol{\mu}$  and covariance matrix  $\mathbf{K}$ . Logarithms in base 2 are denoted  $\log(\cdot)$ . Superscripts  $*$ ,  $T$ , and  $\dagger$  denote the conjugate, transpose, and conjugate transpose, respectively. The trace and rank of a matrix are denoted by  $\text{tr}(\cdot)$  and  $\text{rank}(\cdot)$ , respectively. A diagonal matrix whose diagonal elements are the components of  $\mathbf{x}$  is denoted by  $\text{diag}(\mathbf{x})$ . If  $\mathbf{X}$  is Hermitian positive semi-definite (definite), then it is denoted by  $\mathbf{X} \succeq \mathbf{0}$  ( $\succ \mathbf{0}$ ).

## II. System Model

In this section, we consider the parallel AF relay network depicted in Fig. 1. Due to channel fading, the direct path between the source and destination nodes is neglected. Thus, source  $S$  communicates to destination  $D$  with the help of multiple half-duplex single-antenna relays denoted by  $\{1, \dots, n\}$ . The channel coefficients are independently identically distributed complex Gaussian random variables with zero mean and unit variance and remain constant during one round of transmission. Therefore, the magnitude of the channel coefficients follows the Rayleigh distribution, and the phase is uniformly distributed in the interval  $[0, 2\pi]$ . We assume that the full channel state information is available at a center node.

Each relay node ( $k$ ) observes local Gaussian noise, denoted as  $w_k$ . In addition, the relays are also exposed to a set of common interferers  $\{I_1, \dots, I_m\}$  due to the broadcast nature of wireless channels. Treating the common interference as Gaussian noise, the effective Gaussian noise received at the relays is the superposition of the local Gaussian noise and the common interference and is thus, correlated. We also assume

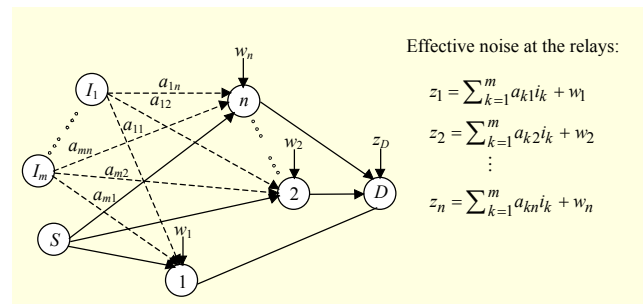


Fig. 1. Parallel AF relay channel.

that the noise covariance is available at a center node.

The data transmission takes place in two steps. During the first step, source  $S$  sends  $x_S$  with transmit power  $\mathbf{E}[|x_S|^2] = P_S$ . The  $k$ th relay receives

$$y_k = h_{S,k}x_S + z_k, \quad (1)$$

where the effective Gaussian noise  $z_k$  at relay  $k$  is drawn according to  $CN(0, \sigma_k^2)$ , which is also shown in Fig. 1, and  $h_{S,k} = |h_{S,k}| e^{j\angle h_{S,k}}$  is drawn according to  $CN(0, 1)$  and denotes the channel coefficient between source  $S$  and relay  $k$ . During the second step, the  $k$ th relay sends

$$x_k = \alpha_k^* y_k, \quad (2)$$

where  $\alpha_k = |\alpha_k| e^{j\angle \alpha_k}$  is the complex scaling factor selected by relay  $k$ .

We consider individual relay power constraints in this study. So, the scaling factor should be chosen such that

$$\mathbf{E}[|x_k|^2] = \mathbf{E}[|y_k|^2] |\alpha_k|^2 \leq P_{k,\max}, \quad (3)$$

where  $P_{k,\max}$  is the maximum allowable transmit power for relay  $k$ .

Then, the received signal at the destination node is

$$y_D = \mathbf{h}_2^T \mathbf{A}^* \mathbf{h}_1 x_S + \mathbf{h}_2^T \mathbf{A}^* \mathbf{z} + z_D, \quad (4)$$

where  $\mathbf{A} = \text{diag}\{\alpha_1, \dots, \alpha_n\}$  is a diagonal matrix,  $\mathbf{z} = [z_1, \dots, z_n]^T$  is the relay noise vector,  $z_D$  is the destination noise that is drawn according to  $CN(0, \sigma^2)$ , and  $\mathbf{h}_1 = [h_{S,1}, \dots, h_{S,n}]^T$  and  $\mathbf{h}_2 = [h_{1,D}, \dots, h_{n,D}]^T$  denote the channel vectors from the source node to the relays and from the relays to the destination node, respectively. We assume that  $\mathbf{E}[\mathbf{z}\mathbf{z}^H] = \mathbf{K}$ , where  $\mathbf{K} > \mathbf{0}$  is the covariance matrix of the relay noise. Although the destination node may also receive common interference, we assume that  $\mathbf{z}$  and  $z_D$  are independent. This is because these noise processes occur at two different steps. Note that the same assumption can also be observed in [13].

### III. Relay Optimization

#### 1. Optimal AF Scheme

In this subsection we first formulate the AF-relay optimization problem into an SDP using several transformation tricks and the SDR technique. Then the optimal AF scheme can be efficiently computed via solving only one SDP. Finally, we show that the SDP-based approach can always find the global optimal solution of the original problem.

From (4), it follows that the parallel AF relay network can be regarded as an equivalent point-to-point Gaussian channel. As

is well known, the source node adopts the Gaussian codebook with  $x_S$  drawn according to  $CN(0, P_S)$ . The achievable rate is given by  $R = 0.5 \log(1 + \text{SNR})$ , where the pre-log factor is due to the half-duplex assumption and  $\text{SNR}$  denotes the received signal-to-noise ratio (SNR) at the destination node. Note that  $\log(\cdot)$  is an increasing function. Therefore, to maximize the transmission rate is equivalent to maximizing the received SNR at the destination node. Combining with the individual power constraints (3), the problem is first formulated as

$$\max_{\mathbf{a}} \text{SNR}(\mathbf{a}) = \frac{\mathbf{a}^H \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^H \mathbf{H}_2^H \mathbf{a} P_S}{\mathbf{a}^H \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^H \mathbf{a} + \sigma^2}, \quad (5)$$

$$\text{s.t. } \mathbf{a}^H (|h_{S,k}|^2 P_S + \sigma_k^2 \mathbf{I}_k) \mathbf{a} \leq P_{k,\max}, k \in \{1, \dots, n\},$$

where  $\mathbf{a} = [\alpha_1, \dots, \alpha_n]^T$ ,  $\mathbf{H}_i = \text{diag}(\mathbf{h}_i)$ , for  $i = 1, 2$ , and  $\mathbf{I}_k = \text{diag}\{0, \dots, 0, 1, 0, \dots, 0\}$  is a diagonal matrix with the  $k$ th element equal to 1. It is clear that the magnitude and phase of  $\mathbf{a}$  should be designed jointly for a non-diagonal  $\mathbf{K}$ .

Since problem (5) is not a standard convex optimization problem, due to its non-concave objective function, we first recast it as

$$\max_{\mathbf{a}, w} \frac{\mathbf{a}^H \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^H \mathbf{H}_2^H \mathbf{a} P_S}{|w|^2}, \quad (6)$$

$$\text{s.t. } \mathbf{a}^H \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^H \mathbf{a} + \sigma^2 \leq |w|^2,$$

$$\mathbf{a}^H (|h_{S,k}|^2 P_S + \sigma_k^2) \mathbf{I}_k \mathbf{a} \leq P_{k,\max}, k \in \{1, \dots, n\}.$$

It is easy to check that the optimal solution of (6) is achieved when the first constraint takes equality. It follows that (5) and (6) are equivalent. Then, by letting  $v = 1/w$  and  $\boldsymbol{\beta} = \mathbf{a}/w$ , we have the following equivalent problem:

$$\begin{aligned} & \max_{\boldsymbol{\beta}, v} \boldsymbol{\beta}^H \mathbf{F} \boldsymbol{\beta}, \\ & \text{s.t. } \boldsymbol{\beta}^H \mathbf{F}_0 \boldsymbol{\beta} + \sigma^2 |v|^2 \leq 1, \\ & \boldsymbol{\beta}^H \mathbf{F}_k \boldsymbol{\beta} - P_{k,\max} |v|^2 \leq 0, k \in \{1, \dots, n\}, \end{aligned} \quad (7)$$

where

$$\mathbf{F} = \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^H \mathbf{H}_2^H P_S,$$

$$\mathbf{F}_0 = \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^H,$$

and

$$\mathbf{F}_k = (|h_{S,k}|^2 P_S + \sigma_k^2) \mathbf{I}_k,$$

for  $k \in \{1, \dots, n\}$ . Letting  $\mathbf{X} = \boldsymbol{\beta} \boldsymbol{\beta}^H$  and  $u = |v|^2$  we then have (7) equivalent to

$$\begin{aligned} & \max_{\mathbf{X}, u} \text{tr}(\mathbf{F} \mathbf{X}), \\ & \text{s.t. } \text{tr}(\mathbf{F}_0 \mathbf{X}) \leq 1 - \sigma^2 u, \\ & \text{tr}(\mathbf{F}_k \mathbf{X}) \leq P_{k,\max} u, k \in \{1, \dots, n\}, \\ & \mathbf{X} \succeq 0, \\ & \text{rank}(\mathbf{X}) = 1. \end{aligned} \quad (8)$$

The last constraint ( $\text{rank}(\mathbf{X}) = 1$ ) comes from the fact that  $\mathbf{X} = \boldsymbol{\beta}\boldsymbol{\beta}^\dagger$ , which is not convex. The above problem can be further converted into a convex SDP problem by applying the idea of SDR [17] as follows:

$$\begin{aligned} & \max_{\mathbf{X}, u} \text{tr}(\mathbf{F}\mathbf{X}), \\ \text{s.t. } & \text{tr}(\mathbf{F}_0\mathbf{X}) \leq 1 - \sigma^2 u, \\ & \text{tr}(\mathbf{F}_k\mathbf{X}) \leq P_{k,\max} u, \quad k \in \{1, \dots, n\}, \\ & \mathbf{X} \succeq 0, \end{aligned} \quad (9)$$

which is a convex optimization problem and can thus be efficiently solved by standard interior point methods. The detailed discussion of the algorithms used to solve the SDP problem can be found in [18]. The computational complexity of SDP using standard interior point methods is no worse than  $O(n^3)$ , where  $n$  is the number of relays. Generally speaking, the resulting optimal solution  $\mathbf{X}_{\text{opt}}$  may not be of rank one. In other words,  $\mathbf{X}_{\text{opt}}$  may not lead to an optimal solution of (8) due to dropping the constraint  $\text{rank}(\mathbf{X}) = 1$ . Interestingly, we show in the following proposition that the rank relaxation here is tight, in the sense that the rank-one constraint will be automatically satisfied at the optimum of (9).

**Proposition 1.** Suppose that problem (9) is feasible and that  $(\mathbf{X}_{\text{opt}}, u_{\text{opt}})$  is the optimal solution of (9). Then it can be shown that  $\text{rank}(\mathbf{X}_{\text{opt}}) = 1$ .

The proof is given in Appendix 1.

Proposition 1 shows that (9) is equivalent to (8). Applying rank-one decomposition to  $\mathbf{X}_{\text{opt}}$ , we get  $\mathbf{X}_{\text{opt}} = \boldsymbol{\beta}_{\text{opt}}\boldsymbol{\beta}_{\text{opt}}^\dagger$ . Combining with  $v_{\text{opt}} = \sqrt{u_{\text{opt}}}$ , the optimal solution of the problem in (5) is obtained and denoted by  $\boldsymbol{\alpha}_{\text{opt},c} = \boldsymbol{\beta}_{\text{opt}} / v_{\text{opt}}$ . Then, we derive the exact optimal AF scheme for this network.

## 2. Performance Evaluation and Implementation Discussion

In this subsection, we first propose an upper bound on the maximum achievable AF rate, which is used as a benchmark to evaluate the performance of the optimal AF scheme. It is shown that the optimal scheme can achieve multi-antenna gain in the parallel-relay network. Then we discuss the implementation issues of the scheme, which is also of great importance from a practical perspective.

From Fig. 1, it is clear that our relay network model consists of a broadcast section and a multiple-access section. Hence, there are two natural cuts to consider first, the ‘‘broadcast cut’’ and the ‘‘multiple-access cut.’’ We compute a cut set–like upper bound with these two cuts, which is given as follows:

$$\begin{aligned} \bar{C} = & \max_{\substack{p(x_S, \mathbf{x}): \mathbf{E}(|x_S|^2) \leq P_S, \\ \mathbf{E}(|x_k|^2) \leq P_{k,\max}, \quad k \in \{1, 2, \dots, n\}}} \min \{ 0.5I(x_S; \mathbf{y}, y_D | \mathbf{x}) \\ & , 0.5I(x_S, \mathbf{x}; y_D) \}, \end{aligned} \quad (10)$$

where  $\mathbf{x} = [x_1, \dots, x_n]$ ,  $\mathbf{y} = [y_1, \dots, y_n]$ , and the pre-factor 0.5 is due to the half-duplex assumption.

Let us consider the first upper bound as follows:

$$\begin{aligned} \bar{C}_1 = & \max_{\substack{p(x_S, \mathbf{x}): \mathbf{E}(|x_S|^2) \leq P_S, \\ \mathbf{E}(|x_k|^2) \leq P_{k,\max}, \quad k \in \{1, 2, \dots, n\}}} I(x_S; \mathbf{y}, y_D | \mathbf{x}) \\ = & \max_{p(x_S): \mathbf{E}(|x_S|^2) \leq P_S} h(\mathbf{y}) - h(\mathbf{z}) \\ = & \log \left( 1 + \mathbf{h}_1^\dagger \mathbf{K}^{-1} \mathbf{h}_1 P_S \right), \end{aligned} \quad (11)$$

which can be viewed as the capacity of a single-input multiple-output (SIMO) system modeled by  $\mathbf{y} = \mathbf{h}_1 x_S + \mathbf{z}$ , where  $\mathbf{K}$  is the noise covariance matrix at the multiple-antenna receiver. We call  $0.5\bar{C}_1$  the maximal-ratio combination (MRC) bound of the parallel AF relay network.

Then, consider the second upper bound to be

$$\begin{aligned} \bar{C}_2 = & \max_{\substack{p(x_S, \mathbf{x}): \mathbf{E}(|x_S|^2) \leq P_S, \\ \mathbf{E}(|x_k|^2) \leq P_{k,\max}, \quad k \in \{1, 2, \dots, n\}}} I(x_S, \mathbf{x}; y_D) \\ = & \max_{p(\mathbf{x}): \mathbf{E}(|x_k|^2) \leq P_{k,\max}, \quad k \in \{1, 2, \dots, n\}} h(y_D) - h(\mathbf{z}_D) \\ = & \log \left( 1 + \frac{1}{\sigma^2} \left( \sum_{k=1}^n |h_{k,D}| \sqrt{P_{k,\max}} \right)^2 \right), \end{aligned} \quad (12)$$

which can be viewed as the capacity of a multiple-input single-output (MISO) system modeled by  $y = \mathbf{h}_2^T \mathbf{x} + z_D$ , where the transmitter is subject to per-antenna power constraint. We call  $0.5\bar{C}_2$  the maximal-ratio transmission (MRT) bound of the parallel AF relay network.

Thus combining (10), (11), and (12), it follows that the achievable rate of the optimal AF scheme of this network is upper bounded by  $0.5 \min \{ \bar{C}_1, \bar{C}_2 \}$ .

Then, we establish the following proposition.

**Proposition 2.** The upper bound  $0.5 \min \{ \bar{C}_1, \bar{C}_2 \}$ , for the maximum achievable AF rate of the parallel-relay network with correlated relay noise and individual relay power constraints, can be asymptotically achieved when the source power  $P_S$  or the relay power budgets  $P_{k,\max}$  tend to infinity.

The proof is given in Appendix 2.

Finally, we discuss the implementation issues of the scheme. It is clear that the design of the optimal AF scheme for such a network requires side information, including the instantaneous channel coefficients and relay noise covariance. To the best of our knowledge, the expectation-maximization algorithm–based approach proposed in [15] is the first technique that can jointly estimate the channel coefficient and noise covariance. Furthermore, as shown in [15], the performance of the estimator is nearly optimal. Thus, by applying such a technique in our system model, the destination node is regarded as the center node that develops the optimal AF scheme based on the estimated side information and that then distributes the optimal

scaling factors to the relays via a backward channel before the transmission. The network overhead for distributing the optimal schemes from the destination node to the relays increases linearly as the number of relays increases. Nevertheless, learning noise covariance still results in network overhead. A natural question is whether it is worth doing so. To answer this, we compare the following scenarios.

**Scenario A.** The relay noise is uncorrelated. Therefore, the relays adopt the optimal AF scheme  $\mathbf{\alpha}_{\text{opt},i}$  obtained in [6], and the received SNR at the destination node is denoted by  $SNR_4 = SNR_0(\mathbf{\alpha}_{\text{opt},i})$ , where the definition of  $SNR_0(\mathbf{\alpha})$  is given in the proof of Proposition 3 in the sequel.

**Scenario B.** The relay noise is correlated but relays are unaware of the correlation. In such a scenario, the relays still adopt  $\mathbf{\alpha}_{\text{opt},i}$ , and the received SNR at the destination node is denoted by  $SNR_B = SNR(\mathbf{\alpha}_{\text{opt},i})$ .

**Scenario C.** The relay noise is correlated and relays are aware of the correlation. Therefore, the relays adopt the optimal AF scheme  $\mathbf{\alpha}_{\text{opt},c}$  obtained in subsection III. 1. The received SNR at the destination node is denoted by  $SNR_C = SNR(\mathbf{\alpha}_{\text{opt},c})$ .

We conclude the results in the following proposition.

**Proposition 3.** For any source power  $P_S$  and relay power constraints  $P_{k,\max}$ ,  $k \in \{1, \dots, n\}$ , the performance of the AF relaying scheme under correlated relay noise outperforms that under independent relay noise in terms of average rate over all the channel realizations; that is,

$$\mathbf{E}[R_A] \leq \mathbf{E}[R_B] \leq \mathbf{E}[R_C],$$

where  $R_x = 0.5 \log(1 + SNR_x)$ ,  $x \in \{A, B, C\}$ .

The proof of Proposition 3 is given in Appendix 3.

Proposition 3 first shows that learning noise covariance may improve the ergodic capacity of the network. In addition, it shows that the optimal AF scheme for the independent noise case can be viewed as a suboptimal scheme and that it has an even better average transmission rate when the relay noise is correlated. Regardless of the noise correlation, the suboptimal scheme can be implemented in a partially distributed manner, as shown in [6].

#### IV. Numerical Results

**Example A.** We first give a simple example to show that the algorithm concluded from [16] cannot always find the global optimal solution of (5). For simplicity, let's only consider a three-relay network with independent Gaussian noise.

The relay and source power constraints are given as

$$P_{1,\max} = 1, P_{2,\max} = \frac{64}{9}, P_{3,\max} = \frac{9}{4}, P_s = 1,$$

where it is assumed that the Gaussian noise has unit variance in

all cases. The channel gains are

$$h_{S,k} = 1, h_{k,D} = 1, \text{ for } k = 1, 2, 3.$$

Then, we have the upper bounds of all scaling factors as follows:

$$\alpha_{1,\max} = \sqrt{\frac{P_{1,\max}}{1 + h_{S,1}^2 P_s}} = \frac{1}{\sqrt{2}},$$

$$\alpha_{2,\max} = \sqrt{\frac{P_{2,\max}}{1 + h_{S,2}^2 P_s}} = \frac{8}{3\sqrt{2}},$$

$$\alpha_{3,\max} = \sqrt{\frac{P_{3,\max}}{1 + h_{S,3}^2 P_s}} = \frac{3}{2\sqrt{2}}.$$

On one hand, if the updating strategy is the first one indicated in the introduction section of [17], then we give the detailed procedure as follows:

a). Consider the hyperplane  $\alpha_1 = \alpha_{1,\max}$ . Compute  $\alpha_2^{\{1\}}$  and  $\alpha_3^{\{1\}}$ :

$$\alpha_2^{\{1\}} = \frac{h_{S,2}}{h_{2,D}} \frac{1 + \alpha_{1,\max}^2 h_{1,D}^2}{h_{S,1} \alpha_{1,\max} h_{1,D}} = \frac{3\sqrt{2}}{2} > \alpha_{2,\max},$$

$$\alpha_3^{\{1\}} = \frac{h_{S,3}}{h_{3,D}} \frac{1 + \alpha_{1,\max}^2 h_{1,D}^2}{h_{S,1} \alpha_{1,\max} h_{1,D}} = \frac{3\sqrt{2}}{2} > \alpha_{3,\max}.$$

It is clear both recomputed values  $\alpha_2^{\{1\}}$  and  $\alpha_3^{\{1\}}$  violate the corresponding upper bound. Then, the relay set  $\{2, 3\}$  should be added to the original set  $\{1\}$ . Then, we have

$$SNR(\mathbf{\alpha}_{\text{opt}}^{\{1\}}) = SNR(\alpha_{1,\max}, \alpha_{2,\max}, \alpha_{3,\max}) = 2.160.$$

b). Consider the hyperplane  $\alpha_2 = \alpha_{2,\max}$ . Compute  $\alpha_1^{\{2\}}$  and  $\alpha_3^{\{2\}}$ :

$$\alpha_1^{\{2\}} = \frac{h_{S,1}}{h_{1,D}} \frac{1 + \alpha_{2,\max}^2 h_{2,D}^2}{h_{S,2} \alpha_{2,\max} h_{2,D}} = \frac{41\sqrt{2}}{24} > \alpha_{1,\max},$$

$$\alpha_3^{\{2\}} = \frac{h_{S,3}}{h_{3,D}} \frac{1 + \alpha_{2,\max}^2 h_{2,D}^2}{h_{S,2} \alpha_{2,\max} h_{2,D}} = \frac{41\sqrt{2}}{24} > \alpha_{3,\max}.$$

It is clear both recomputed values  $\alpha_1^{\{2\}}$  and  $\alpha_3^{\{2\}}$  violate the corresponding upper bound. Then, the relay set  $\{1, 3\}$  should be added to the original set  $\{2\}$ . Then, we have

$$SNR(\mathbf{\alpha}_{\text{opt}}^{\{2\}}) = SNR(\alpha_{1,\max}, \alpha_{2,\max}, \alpha_{3,\max}) = 2.160.$$

c). Consider the hyperplane  $\alpha_3 = \alpha_{3,\max}$ . Compute  $\alpha_1^{\{3\}}$  and  $\alpha_2^{\{3\}}$ :

$$\alpha_1^{\{3\}} = \frac{h_{S,1}}{h_{1,D}} \frac{1 + \beta_{3,\max}^2 h_{3,D}^2}{h_{S,3} \beta_{3,\max} h_{3,D}} = \frac{17\sqrt{2}}{12} > \alpha_{1,\max},$$

$$\alpha_2^{\{3\}} = \frac{h_{S,2}}{h_{2,D}} \frac{1 + \alpha_{3,\max}^2 h_{3,D}^2}{h_{S,3} \alpha_{3,\max} h_{3,D}} = \frac{17\sqrt{2}}{12} > \alpha_{2,\max}.$$



It is clear both recomputed values  $\alpha_1^{(3)}$  and  $\alpha_2^{(2)}$  violate the corresponding upper bound. Then, the relay set  $\{1, 2\}$  should be added to the original set  $\{3\}$ . Then, we have

$$SNR(\alpha_{\text{opt}}^3) = SNR(\alpha_{1,\text{max}}, \alpha_{2,\text{max}}, \alpha_{3,\text{max}}) = 2.160.$$

Then, by Proposition 1 and Theorem 1 proposed in [16], the optimal AF scheme should be

$$\alpha_1 = \alpha_{1,\text{max}} = 0.707, \alpha_2 = \alpha_{2,\text{max}} = 1.886, \alpha_3 = \alpha_{3,\text{max}} = 1.061,$$

and the corresponding optimal SNR value should be 2.160. However, the optimal solution obtained by our SDP-based approach and the approach proposed in [6] is

$$\alpha_1 = 0.707, \alpha_2 = 1.485 < \alpha_{2,\text{max}}, \alpha_3 = 1.061.$$

The corresponding optimal SNR value is  $SNR_{\text{opt}} = 2.191$ . Thus, it shows that the algorithm in [16] cannot always find the global optimal point.

On the other hand, if considering a more sophisticated updating strategy — as in the second one indicated in the introduction section — then from the above example, we have already shown the existence of the worst case; that is, after the updating step, all the recomputed scaling factors exceed the corresponding upper bound. Thus in this case, the computational complexity is exponential in  $n$ . Thus, it will significantly limit the use of the algorithm in practice. Nevertheless, our algorithm is based on a convex SDP that has polynomial-time complexity.

**Example B.** We illustrate another example to show the theoretical results obtained in Propositions 2 and 3. To make the numerical results more reliable, the average achievable rates in all the following examples are averaged over 5,000 randomly generated samples of relay noise covariance matrices and channel vectors.

In the first case, we assume that the relay nodes have a fixed equal power constraint  $P_{k,\text{max}} = P_0 = 10$  dBW. Then, the average achievable rate  $R_x$  as a function of  $P_S$ , where  $x \in \{A, B, C\}$ , is shown in Figs. 2 and 3 for  $n = 5$  and  $n = 10$ , respectively. Clearly, with an increase in source power, the network performs as a MISO system with multiple relays regarded as the transmitter antennas of the source node. In this case, all three scenarios considered in Proposition 3 share the same MRT bound due to the neglecting of the relay noise. In both figures,  $\mathbf{E}[R_C]$  outperforms  $\mathbf{E}[R_B]$  and  $\mathbf{E}[R_A]$ , when  $P_S$  is small. Comparing the results in Figs. 2 and 3, we can conclude that the larger the number of relays, the more benefits can be obtained by learning noise covariance. However, it is shown that the average achievable rates under different scenarios are almost the same when  $P_S$  is greater than 25 dBW. It is because the received noise power is negligible

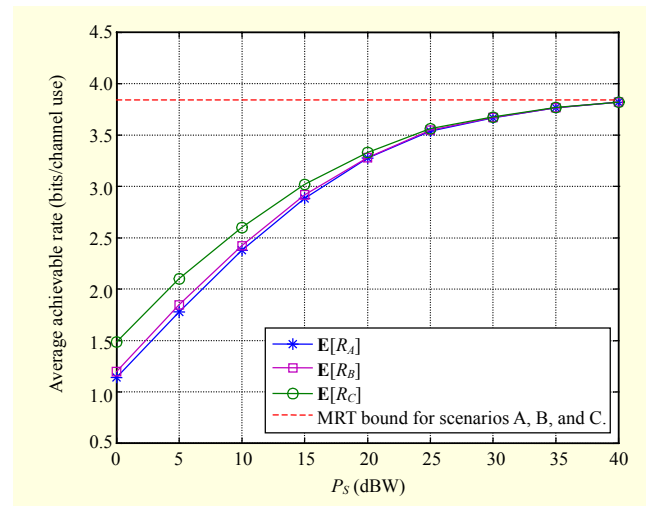


Fig. 2. Average achievable rate when  $n = 5$ .

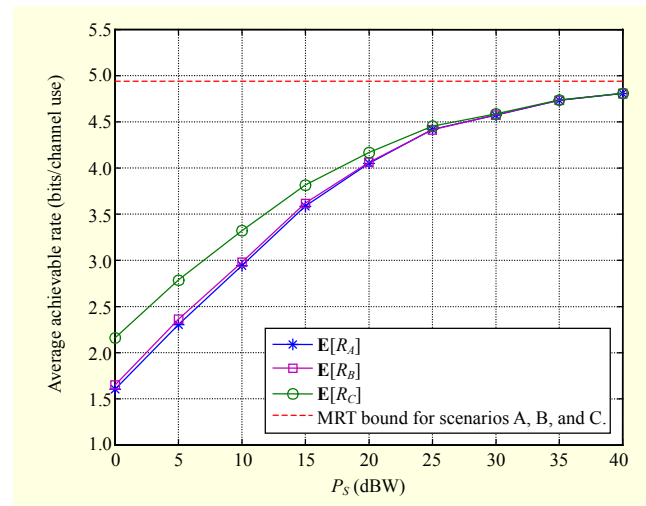


Fig. 3. Average achievable rate when  $n = 10$ .

compared with the signal power at the relays (when the transmit power of  $S$  is sufficiently large) that the correlation of relay noise is less pronounced. Figures 2 and 3 also show that relay noise correlation always benefits AF performance regardless of whether the relays know of the noise covariance matrix.

In the second case, we assume that the source node transmits with power  $P_S = 10$  dBW and each relay node has an equal power constraint  $P_{k,\text{max}} = P_0$ . Then, the average achievable rate  $P_x$  as a function of  $P_0$ , where  $x \in \{A, B, C\}$ , is shown in Figs. 4 and 5 for  $n = 5$  and  $n = 10$ , respectively. It is observed that the performance of the average achievable rate can be significantly improved by learning noise covariance, especially when  $P_0$  is greater than 10 dBW. It is also shown that the larger the number of relays, the more benefits can be obtained by learning noise correlation. The network performs as a SIMO

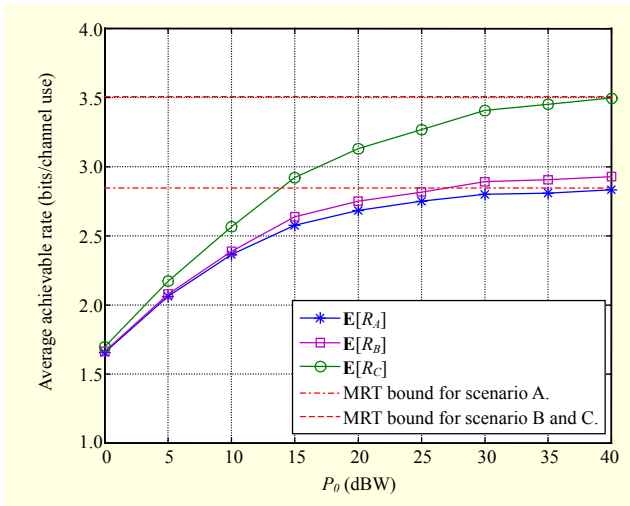


Fig. 4. Average achievable rate when  $n = 5$ .

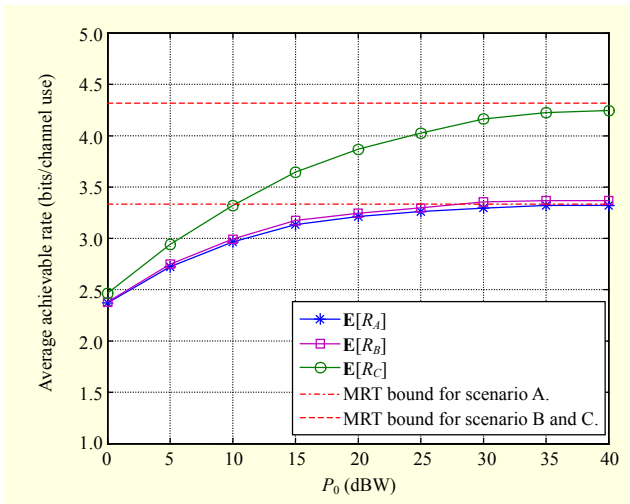


Fig. 5. Average achievable rate when  $n = 10$ .

system with the multiple relays regarded as the receiver antennas of the destination node when the relay power tends to infinity. It is shown in both figures that the optimal AF scheme can asymptotically achieve the MRC bound. It is also shown that relay noise correlation always benefits AF performance regardless of whether the relays know of the noise covariance matrix.

## V. Conclusion

We have studied the optimal AF relaying scheme for a parallel-relay channel under individual relay power constraints, where the relay noise is correlated. The performance of the maximum AF achievable rate is estimated. It is shown that noise correlation is beneficial to performance in terms of average transmission rate — even if the relays are unaware of

the correlation. The AF relay optimization problem is still open for a more general case when the destination noise is also correlated to the relay noise. This is a potential objective of our future work. Moreover, to develop an optimal AF scheme with channel coefficient or noise covariance estimation errors, or both, is also a main objective of our future work.

## Appendices

### Proof of Proposition 1.

It is readily verified that (9) satisfies Slater's constraint qualification, which implies that strong duality holds and that the primal and dual optimal solutions must satisfy the KKT conditions [19]. Let  $\mu_{0,\text{opt}}$ ,  $\mu_{k,\text{opt}}$ , for  $k \in \{1, \dots, n\}$ , and  $\mathbf{Y}_{\text{opt}}$  be the optimal dual variables associated with the constraints in (9). A part of the KKT conditions relevant to the proof are listed below

$$\mathbf{Y}_{\text{opt}} = -\mathbf{F} + \sum_{k=0}^n \mu_{k,\text{opt}} \mathbf{F}_k, \quad (13)$$

$$\mathbf{Y}_{\text{opt}} \mathbf{X}_{\text{opt}} = \mathbf{0}, \quad (14)$$

$$\mathbf{X}_{\text{opt}} \succeq \mathbf{0}, \mathbf{Y}_{\text{opt}} \succeq \mathbf{0}, \mu_{0,\text{opt}} \geq 0, \mu_{k,\text{opt}} \geq 0. \quad (15)$$

Note that  $\mu_{0,\text{opt}}$  must be positive in (15); otherwise, one can achieve a larger objective value by scaling up  $\mathbf{X}_{\text{opt}}$ , since the first constraint of (9) is inactive.

Then, by substituting (13) into (14) we have

$$\mathbf{F} \mathbf{X}_{\text{opt}} = (\mu_{0,\text{opt}} \mathbf{F}_0 + \sum_{k=1}^n \mu_{k,\text{opt}} \mathbf{F}_k) \mathbf{X}_{\text{opt}}. \quad (16)$$

According to the definitions of  $\mathbf{F}_k$  for  $k \in \{0, \dots, n\}$ , we have  $\mu_{0,\text{opt}} \mathbf{F}_0 + \sum_{k=1}^n \mu_{k,\text{opt}} \mathbf{F}_k \succ \mathbf{0}$ . Thus, it follows that

$$\begin{aligned} \text{rank}(\mathbf{X}_{\text{opt}}) &= \text{rank}((\mu_{0,\text{opt}} \mathbf{F}_0 + \sum_{k=1}^n \mu_{k,\text{opt}} \mathbf{F}_k) \mathbf{X}_{\text{opt}}) \\ &= \text{rank}(\mathbf{F} \mathbf{X}_{\text{opt}}) \leq \text{rank}(\mathbf{F}) = 1. \end{aligned} \quad (17)$$

Since  $\mathbf{X}_{\text{opt}} \neq \mathbf{0}$  in practice, it is sufficient to show that  $\text{rank}(\mathbf{X}_{\text{opt}}) = 1$ .  $\square$

### Proof of Proposition 2.

Although we cannot derive a closed-form solution of the optimal AF scheme as in [13], we propose an alternative way to show that the upper bound on the maximum AF achievable rate can be asymptotically achieved. To this end, we consider two suboptimal schemes given as follows.

Suboptimal scheme 1 is proposed as follows to deal with the correlated relay noise:

$$\mathbf{a}_{\text{sub1}} = c_0 (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger)^{-1} \mathbf{H}_2 \mathbf{h}_1, \quad (18)$$

where we denote  $\mathbf{a} = [\alpha_1, \dots, \alpha_n]^T = (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger)^{-1} \mathbf{H}_2 \mathbf{h}_1$ ,

$$\alpha_{k,\max} = \sqrt{\frac{P_{k,\max}}{P_S |h_{S,k}|^2 + \sigma_k^2}} \quad \text{for } k = 1, \dots, n, \quad \text{and}$$

$$c_0 = \min\left\{\frac{\alpha_{k,\max}}{|\alpha_k|}, k = 1, \dots, n\right\}.$$

Suboptimal scheme 2 is proposed as follows to achieve a co-phasing transmission of the relay signals:

$$\mathbf{a}_{\text{sub2}} = \left[ e^{j\angle h_1} \alpha_{1,\max}, e^{j\angle h_2} \alpha_{2,\max}, \dots, e^{j\angle h_n} \alpha_{n,\max} \right]^T, \quad (19)$$

where  $h_k = h_{S,k} h_{k,D}$ , for  $k = 1, \dots, n$ .

We conclude that destination noise can be neglected in the case of the suboptimal scheme  $\mathbf{a}_{\text{sub1}}$ , as relays have sufficiently large power constraints, that is, in (20)  $P_{k,\max} \rightarrow \infty$ ,  $k = 1, \dots, n$ .

$$\lim_{\substack{P_{k,\max} \rightarrow \infty \\ k=1,2,\dots,n}} \frac{\sigma^2}{\mathbf{a}_{\text{sub1}}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub1}}} = 0. \quad (20)$$

Then, it is easy to check that

$$\begin{aligned} \lim_{\substack{P_{k,\max} \rightarrow \infty \\ k=1,2,\dots,n}} SNR(\mathbf{a}_{\text{opt}}) &\geq \lim_{\substack{P_{k,\max} \rightarrow \infty \\ k=1,2,\dots,n}} SNR(\mathbf{a}_{\text{sub1}}) \\ &= \lim_{\substack{P_{k,\max} \rightarrow \infty \\ k=1,2,\dots,n}} f_1(\{P_{k,\max}\}) \frac{\mathbf{a}_{\text{sub1}}^\dagger \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^\dagger \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub1}} P_S}{\mathbf{a}_{\text{sub1}}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub1}}} \\ &= \mathbf{h}_1^\dagger \mathbf{K}^{-1} \mathbf{h}_1 P_S, \end{aligned} \quad (21)$$

where the last equality follows from the fact that

$$f_1(\{P_{k,\max}\}) = \left(1 + \frac{\sigma^2}{\mathbf{a}_{\text{sub1}}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub1}}}\right)^{-1} \rightarrow 1,$$

as  $P_{k,\max} \rightarrow \infty$ , for  $k = 1, \dots, n$ .

It follows that the MRC bound can be asymptotically achieved when the relay power constraints tend to infinity.

With the suboptimal scheme  $\mathbf{a}_{\text{sub2}}$ , we conclude that relay noise can be neglected as source power  $P_S \rightarrow \infty$ , because

$$\lim_{P_S \rightarrow \infty} \frac{\mathbf{a}_{\text{sub2}}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub2}}}{\sigma^2} = 0. \quad (22)$$

Therefore, we have

$$\begin{aligned} \lim_{P_S \rightarrow \infty} SNR(\mathbf{a}_{\text{opt}}) &\geq \lim_{P_S \rightarrow \infty} SNR(\mathbf{a}_{\text{sub2}}) \\ &= \lim_{P_S \rightarrow \infty} f_2(P_S) \frac{\mathbf{a}_{\text{sub2}}^\dagger \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^\dagger \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub2}} P_S}{\sigma^2} \\ &= \frac{1}{\sigma^2} \left(\sum_{k=1}^n |h_{k,D}| \sqrt{P_{k,\max}}\right)^2, \end{aligned} \quad (23)$$

where the last equality follows from the fact that

$$f_2(P_S) = \left(1 + \frac{\mathbf{a}_{\text{sub2}}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{sub2}}}{\sigma^2}\right)^{-1} \rightarrow 1,$$

as  $P_S \rightarrow \infty$ .

It follows that the MRT bound can be asymptotically achieved when the transmit power of the source node tends to

infinity.

Then, we complete the proof.  $\square$

### Proof of Proposition 3.

For fairness in comparison, we assume that the marginal distribution of the relay noise is the same for both correlated and uncorrelated scenarios; that is, if  $\mathbf{K}$  is the noise covariance matrix of correlated relay noise, then  $\mathbf{K} \circ \mathbf{I}$  is the noise covariance matrix of the uncorrelated counterpart, where  $\circ$  denotes the Hadamard product. As a result, in the case of uncorrelated relay noise, the received SNR at the destination is given by

$$SNR_0(\mathbf{a}) = \frac{\mathbf{a}^\dagger \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^\dagger \mathbf{H}_2^\dagger \mathbf{a} P_S}{\mathbf{a}^\dagger (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \circ \mathbf{I}) \mathbf{a} + \sigma^2}. \quad (24)$$

To maximize the received SNR (24) at the destination, it is better to choose scaling factors at relays such that the copies of the source signal from different relays add up in phase at the destination node. Therefore, without loss of generality, the phase of the scaling factor at relay  $k$  can be taken as  $\angle h_{S,k} + \angle h_{k,D}$ . It should be noted that selection of the phase of the scaling factor does not affect the received noise power at the destination node. Then, the system reduces to a real-valued one with all channel coefficients being positive. We denote the beamforming vector to be  $\mathbf{a}_0 = [\alpha_{0,1}, \dots, \alpha_{0,n}]^T$  in the optimal AF scheme for the real system. Then, the optimal scaling factor is given by

$$\mathbf{a}_{\text{opt},i} = \text{diag}(e^{j(\angle h_{S,1} + \angle h_{1,D})}, \dots, e^{j(\angle h_{S,n} + \angle h_{n,D})}) \mathbf{a}_0. \quad (25)$$

By substituting (25) into (24), we have

$$SNR_A = \frac{\mathbf{a}_0^T |\mathbf{H}_2 \mathbf{h}_1| |\mathbf{H}_2 \mathbf{h}_1|^\dagger \mathbf{a}_0 P_S}{\mathbf{a}_0^T (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \circ \mathbf{I}) \mathbf{a}_0 + \sigma^2}, \quad (26)$$

where  $|\mathbf{H}_2 \mathbf{h}_1| = [|h_{S,1} h_{1,D}|, \dots, |h_{S,n} h_{n,D}|]^T$ . Note that  $\mathbf{a}_0$  is independent of the phase of the channel coefficient as is  $SNR_A$ . Next, we consider  $SNR_B$ .

$$SNR_B = \frac{\mathbf{a}_{\text{opt},i}^\dagger \mathbf{H}_2 \mathbf{h}_1 \mathbf{h}_1^\dagger \mathbf{H}_2^\dagger \mathbf{a}_{\text{opt},i} P_S}{\mathbf{a}_{\text{opt},i}^\dagger \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \mathbf{a}_{\text{opt},i} + \sigma^2} = \frac{a SNR_A}{a + b}, \quad (27)$$

where  $a = \mathbf{a}_0^T (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \circ \mathbf{I}) \mathbf{a}_0 + \sigma^2$  and

$$\begin{aligned} b &= \mathbf{a}_{\text{opt},i}^\dagger (\mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger - \mathbf{H}_2 \mathbf{K} \mathbf{H}_2^\dagger \circ \mathbf{I}) \mathbf{a}_{\text{opt},i} \\ &= \sum_{i=1}^n \sum_{m \neq i}^n e^{j\phi_{i,m}} \alpha_{0,i} |h_{i,D}| \mathbf{K}(i, m) |h_{m,D}| \alpha_{0,m}, \end{aligned} \quad (28)$$

where  $\phi_{i,m} = \angle h_{S,m} - \angle h_{S,i}$ . As a result,  $b$  is a function of the phase of the channel coefficient.

Finally, from (27) we have



$$R_B = 0.5 \log \left( 1 + \frac{aSNR_A}{a+b} \right).$$

It can be shown that the second derivative of  $R_B$  with respect to  $b$  is

$$\frac{1}{2 \ln 2} \frac{aSNR_A (2(a+b) + aSNR_A)}{(a+b + aSNR_A)^2 (a+b)^2},$$

which is always positive. Therefore,  $R_B$  is convex with respect to  $b$ . Then, we have

$$\begin{aligned} \mathbf{E}[R_B] &= \mathbf{E}_m \mathbf{E}_p \left[ 0.5 \log \left( 1 + \frac{aSNR_A}{a+b} \right) \middle| |h| \right] \\ &\stackrel{(a)}{\geq} \mathbf{E}_m \left[ 0.5 \log \left( 1 + \frac{aSNR_A}{a + \mathbf{E}_p[b|h]} \right) \right] \\ &\stackrel{(b)}{=} \mathbf{E} \left[ 0.5 \log (1 + SNR_A) \right] = \mathbf{E}[R_A], \end{aligned} \quad (29)$$

where  $\mathbf{E}[\cdot]$  is the expectation over all channel coefficients,  $\mathbf{E}_m[\cdot]$  is the expectation over all channel magnitude, and  $\mathbf{E}_p[\cdot|h|]$  is the conditional expectation over all channel phase given the channel magnitude; (a) follows from Jensen's inequality, and (b) follows from the fact that  $\mathbf{E}_p[e^{j\phi_m} | |h|] = 0$  by the assumption that all channel phase is independent and uniformly distributed over the interval  $[0, 2\pi]$ .

Then, we complete the proof.  $\square$

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