Sensing of OFDM Signals in Cognitive Radio Systems with Time Domain Cross-Correlation

Weiyang Xu

This paper proposes an algorithm to sense orthogonal frequency-division multiplexing (OFDM) signals in cognitive radio (CR) systems. The basic idea behind this study is when a primary user is occupying a wireless channel, the covariance matrix is non-diagonal because of the time domain cross-correlation of the cyclic prefix (CP). In light of this property, a new decision metric that measures the power of the data found on two minor diagonals in the covariance matrix related to the CP is introduced. The impact of synchronization errors on the signal detection is analyzed. Besides this, a likelihood-ratio test is proposed according to the Neyman–Pearson criterion after deriving probability distribution functions of the decision metric under hypotheses of signal presence and absence. A threshold, subject to the requirement of probability of false alarm, is derived; also the probabilities of detection and false alarm are computed accordingly. Finally, numerical simulations are conducted to demonstrate the effectiveness of the proposed algorithm.

Keywords: OFDM, cyclic prefix, covariance matrix, signal detection, Neyman–Pearson criterion.

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Weiyang Xu (corresponding author, weiyangxu@cqu.edu.cn) is with College of Communication Engineering, Chongqing University, Chongqing, China.

I. Introduction

Due to its increased robustness against multipath distortion compared to single-carrier communication systems, orthogonal frequency-division multiplexing (OFDM) has been widely considered as an attractive solution to high-rate wireless data transmission [1]. On the other hand, cognitive radio (CR) has witnessed rapid development and is seen as a promising technology to solve the problem of spectrum congestion [2]. As a result, OFDM-based CR networks, which combine these two enabling techniques, have received great attention both in academia and industry, in recent decades.

For CR systems to be successfully implemented, spectrum sensing plays a key role in ensuring interference to a primary user is below a certain level [3]. Although some OFDM-based technologies, such as digital audio broadcasting and IEEE 802.16 wireless metropolitan area networks, employ data multiplexing for different primary users, single-transmission OFDM systems are also common in real applications. For such wireless systems, the CR users need to detect the primary OFDM signals to avoid interference and performance degradation [4]. However, sensing OFDM signals proves to be quite challenging due to their inherent multi-carrier characteristics [5]. Methods proposed in the literature often take advantage of the cyclic prefix (CP) [4], [6], or pilot tones embedded in OFDM symbols [7]–[8]. Lei and Chin [6] introduced a new decision metric with the aid of the CP and proposed the generalized likelihood ratio test (GLRT) [6]. However, this scheme performs badly if the signal-to-noise ratio (SNR) is low when the mobile channel is in deep fade. In

[4], both multipath and cyclic correlations are exploited to yield a novel blind spectrum sensing algorithm. Another category of detection scheme makes use of the cross-correlation incurred by pilot tones inserted in OFDM symbols, which was first introduced in [7]. Although these data-aided sensing algorithms are effective, the problem is that since pilot tones are usually pseudo-randomly coded and dedicated to the primary user's transmission, it is nontrivial or even sometimes impossible for a cognitive user to obtain this information accurately in real-world applications.

This paper focuses on the detection of OFDM signals in CR systems. The major contributions of this paper are summarized as follows: we introduce a new decision metric based on the inherent time domain cross-correlation imbedded in OFDM symbols; performance analyses concerning the distributions of the new decision metric under hypotheses of signal presence and absence are carried out, and then the sensing algorithm is proposed with the likelihood-ratio test (LRT); and the theoretical probabilities of detection and false alarm of the proposed sensing scheme are derived.

The remainder of this paper is organized as follows. Section II describes the considered OFDM-based CR system and problem formulation. A new decision metric is proposed and performance analyses are carried out in section III. Numerical results displaying the performance of the proposed sensing algorithm are presented in section IV. Finally, section V concludes this paper.

Notions. Upper- and lower-case symbols represent signals in the frequency domain and time domain, respectively. Upper- and lower-case bold characters indicate matrices and vectors, respectively. Element-wise conjugation is denoted by $\{\cdot\}^*$, and $\{\cdot\}^H$ is the transpose operation. Finally, $\mathrm{E}\{\cdot\}$ and $\mathrm{Var}\{\cdot\}$ denote the expectation and variance operations, respectively.

II. System Model and Existing Work

We consider an OFDM-based CR system with N subcarriers. CR makes it possible for cognitive users to access a spectrum hole unoccupied by a primary user; thus it is of great importance for a secondary user to be aware of the presence of the primary user's signal to avoid interference between them. Depending on whether a primary user is occupying a wireless channel or not, the detection of a primary user's OFDM signals in CR networks can be formulated as the following classical binary hypothesis testing problem:

$$\begin{cases} H_0: \; y_{m,k} = w_{m,k}, \\ H_1: \; y_{m,k} = d_{m,k} + w_{m,k}, \end{cases} \tag{1}$$

where hypotheses H_0 and H_1 indicate the absence and presence of a primary user's signals, respectively, $y_{m,k}$ represents the kth time-domain received signal of the mth OFDM symbol, $w_{m,k}$ models the additive white Gaussian noise (AWGN) with zero mean and variance σ_W^2, and $d_{m,k}$ denotes a primary user's OFDM signal contaminated by the multipath distortion; that is,

$$d_{m,k} = \frac{1}{\sqrt{N}} \sum_{l=0}^{P-1} \sum_{n=0}^{N-1} h_l X_{m,n} e^{j2\pi n(k-l-mN_T)/N}, \quad \text{for } 0 \le k \le N+G-1, \tag{2}$$

where G is the length of the CP, $N_T = N + G$ is the duration of one OFDM symbol, $X_{m,n}$ indicates the data to be transmitted in the frequency domain at the nth subcarrier of the mth OFDM symbol with zero mean and variance σ_X^2, h_l denotes the impulse response of the lth channel tap, and P is the number of taps. It is necessary that G should be larger than P to eliminate the intersymbol interference. This study wishes to determine whether the wireless channel is occupied or not by a primary user based on the received time-domain signal $y_{m,k}$ in (1). The time domain cross-correlation resided in the CP of OFDM symbols can be utilized to facilitate signal detection; the decision metric under the two hypotheses in [5] is

$$\begin{cases} H_0: \; \sum_{b=1}^{W} \dfrac{w_b w_{M+b}^*}{E\left[|y_b|^2\right]}, \\ H_1: \; \sum_{b=1}^{W} \dfrac{(d_b + w_b)(d_{M+b}^* + w_{M+b}^*)}{E\left[|d_b + w_b|^2\right]}, \end{cases} \tag{3}$$

where W denotes the length of the observation window, and d_b and w_b respectively indicate the received signal and noise samples. However, the decision metric in (3) includes all the signals in the observation window although only the signals in the CP have correlation with the originally copied part. The correlation between other signals introduces only remarkable noise and makes the detection results unreliable at low SNRs, as will be found later.

III. Proposed OFDM Spectrum Sensing Algorithm

1. Basic Idea and Proposed Spectrum Sensing Algorithm

The OFDM symbol exhibits time domain cross-correlation because of the insertion of the CP. In the transmitter end, time-domain signals are generated via exploiting the inverse discrete Fourier transform (IDFT); that is,

$$x_{m,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} e^{j2\pi nk/N}, \quad \text{for } 0 \le k \le N+G-1, \tag{4}$$

where $x_{m,k}$ represents the transformed time-domain signal in the transmitter, which is not mutually independent because of the CP. Specifically, the cross-correlation among different $x_{m,k}$ in one OFDM symbol is

$$E[x_{m,k}x_{m,k'}^*] = E\left[\frac{1}{N}\sum_{n=0}^{N-1}|X_{m,n}|^2 e^{j2\pi n(k-k')/N}\right] = \begin{cases}\sigma_X^2 & k-k'=\pm N,\\ 0 & k-k'\neq \pm N,\end{cases} \tag{5}$$

where σ_X^2 is the variance of the transmitted signal. This kind of cross-correlation, which exists between the CP and its originally copied part, is partially preserved after transmitting through the wireless channel as long as $G > P$. This is not the case for hypothesis H_0, since noise samples are mutually independent — this distinction lays the foundation of the proposed sensing algorithm.

The decision metric of the proposed scheme only considers the cross-correlation between the CP and its originally copied part as opposed to (3). To fully utilize this kind of correlation, here, the covariance matrix of the received OFDM symbol in the time domain is introduced. The covariance matrix of a certain vector **z** is defined as [9]

$$\mathrm{cov}(\mathbf{z}) = E\left[(\mathbf{z}-E[\mathbf{z}])(\mathbf{z}-E[\mathbf{z}])^H\right]. \tag{6}$$

Thus, it is more convenient to represent the classical binary hypothesis testing problem with the matrix format; that is,

$$\begin{cases} H_0: & \mathbf{y}(m)=\mathbf{w}(m),\\ H_1: & \mathbf{y}(m)=\mathbf{H}\mathbf{x}(m)+\mathbf{w}(m),\end{cases} \tag{7}$$

where

$$\mathbf{x}(m)=[x_{m,-G},\dots,x_{m,0},\dots,x_{m,N-1}]^T,$$
$$\mathbf{y}(m)=[y_{m,-G},\dots,y_{m,0},\dots,y_{m,N-1}]^T, \tag{8}$$

and $\mathbf{w}(m)=[w_{m,-G},\dots,w_{m,0},\dots,w_{m,N-1}]^T$

denote the *m*th vectors of the transmitted signal, received signal, and additive noise, respectively. The channel matrix **H** is an $N_T \times N_T$ matrix with h_0 on the principal diagonal and $h_1,\dots,h_P$ on the minor diagonals. Therefore, the covariance matrix of **y**(*m*) under the two hypotheses is

$$\begin{aligned}\mathbf{Q} &= E\left[(\mathbf{y}(m)-E[\mathbf{y}(m)])(\mathbf{y}(m)-E[\mathbf{y}(m)])^H\right]\\ &= E\left[\mathbf{y}(m)\mathbf{y}(m)^H\right]\\ &= \begin{cases} H_0: & \mathbf{I}_{N_T},\\ H_1: & \mathbf{Q}(p,q)=\dfrac{1}{N}\displaystyle\sum_{l=0}^{P-1}\sum_{n=0}^{N-1}|h_l X_{m,n}|^2 e^{j2\pi n(p-q)/N},\end{cases}\end{aligned} \tag{9}$$

where **Q**(*p*,*q*) is the (*p*,*q*)th element of matrix **Q**. The derivation of the second equation in (9) is based on the fact that **y**(*m*) has a zero mean under both hypotheses. Obviously, the covariance matrix of the received signal is diagonal because only the samples on the main diagonal are nonzero when there is no primary user occupying the wireless channel. Otherwise, it is non-diagonal.

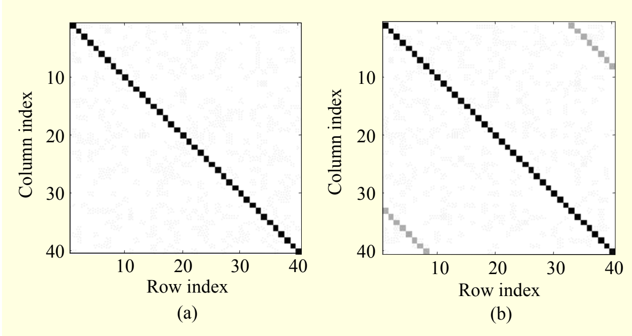

Fig. 1. Color map of the covariance matrix shows the cross-correlation between different samples in one OFDM symbol, with N = 32 and G = 8. The measurement is performed by averaging over 100 symbols. The wireless channel is based on a Rayleigh fading model: (a) in the absence of a primary user's signal and (b) in the presence of a primary user's signal.

Figure 1 draws the color map of the covariance matrix under the two hypotheses, with deeper color corresponding to more energy. Figure 1(a) shows that most energy is concentrated on the main diagonal, which demonstrates the mutual independence between the different noise samples. The small amount of cross-correlation at other places is caused by the limited number of noise samples in the observation window. Whereas in Fig. 2(b), although the main diagonal still contains a large part of the total energy, the color map contains two other energy-concentrated minor diagonals corresponding to the time domain cross-correlation incurred by the insertion of the CP. Hence, this property can be exploited to differentiate a primary user's signal from Gaussian noise by comparing the power of the entries on these two minor diagonals with a preset threshold. Thus, the proposed decision metric can be formulated accordingly as

$$\xi = \frac{\left\|\mathbf{Q}(p,q)_{|p-q|=N}\right\|_F}{\sqrt{2G}}, \tag{10}$$

where $\|\cdot\|_F$ indicates the Frobenius norm. It is noted that the metric is normalized by a factor of $\sqrt{2G}$ (since there are $2G$ entries on these two minor diagonals) and that only the entries satisfying $|p-q|=N$ are added together, ruling out signals other than those in the CP and copied part. Theoretically, the decision metric is nonzero when the wireless channel is occupied by a primary user; otherwise it is zero. Practically, a detector declares the presence of an OFDM signal if ξ is greater than a preset threshold. Otherwise, it declares there is no primary user's transmission.

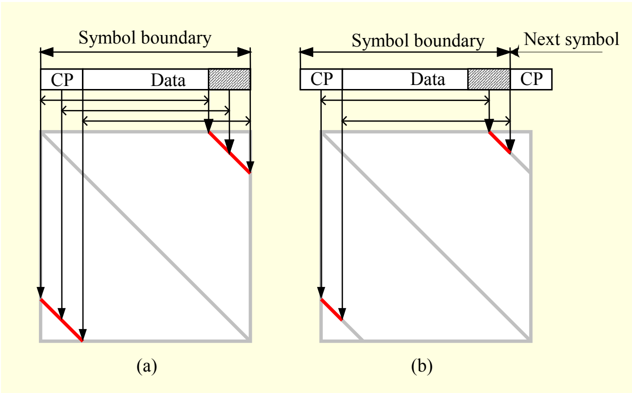

Fig. 2. Illustration of covariance matrix under two scenarios. The shaded part denotes the originally copied part of the CP. The red line on the two upper and lower minor diagonals denotes the cross-correlation: (a) timing synchronization is perfect and (b) timing synchronization is of error.

2. Impact of Synchronization Errors on Signal Detection Algorithm

In the former analysis, we assume perfect synchronization; that is, the received signal contains no baseband errors. However, the synchronization cannot always be perfect in practice, and the timing error and carrier frequency offset (CFO) inevitably exist in the received signals as a result. Hence, it is necessary to investigate the impact of these errors on the proposed signal detection algorithm.

Timing synchronization aims to find the boundaries of one symbol to perform demodulation. To understand the impact of timing error clearly, Fig. 2 draws the covariance matrix under two scenarios — namely, with and without timing error. Figure 2(a) shows that the cross-correlation is fully utilized in the case of perfect timing synchronization. Figure 2(b) shows the case where the actual timing point lags $G/2$ samples behind the ideal one. As a consequence, the cross-correlation that can be utilized is only half of that in Fig. 2(a), as clearly shown by the shortened red line. Therefore, timing uncertainty would lower the sensing accuracy of the proposed algorithm due to the reduced time domain cross-correlation in the decision metric. Fortunately, several existing timing-synchronization schemes based on the CP can be employed to compensate for the timing error efficiently [10]–[11].

Frequency offset caused by the mismatch between local oscillators and Doppler effect is another major concern in the sensing procedure. In the presence of CFO, the received time-domain signal in the presence of a primary user is

$$d_{m,k} = \frac{1}{\sqrt{N}} \sum_{l=0}^{P-1} \sum_{n=0}^{N-1} h_l X_{m,n} e^{j2\pi n(k-l-mN_T)/N + j2\pi\varepsilon k/N}, \quad \text{for } 0 \le k \le N+G-1, \tag{11}$$

where ε denotes the frequency offset normalized by the subcarrier spacing. Accordingly, the covariance matrix under the two hypotheses can be rewritten as

$$\begin{cases} H_0 : \mathbf{I}_{N_T}, \\ H_1 : \mathbf{Q}(p,q) = \dfrac{1}{N} \displaystyle\sum_{l=0}^{P-1} \sum_{n=0}^{N-1} \left| h_l X_{m,n} \right|^2 e^{j2\pi n(p-q)/N + j2\pi\varepsilon(p-q)/N}. \end{cases} \tag{12}$$

It is observed that the CFO only incurs a phase shift in each element of the covariance matrix, which can be mitigated by the norm operation and will not affect the decision metric in (10). Thus, the proposed sensing algorithm is insensitive to this kind of synchronization error.

3. Probability Distribution Functions (PDFs) of Decision Metric

The probability distribution functions of the decision metric under the two hypotheses of a primary user's signal presence and absence need to be established to gain a deeper understanding of the proposed algorithm. Specifically, the covariance matrix in real applications can be implemented via averaging over a finite number of OFDM symbols; that is,

$$\mathbf{Q} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{y}(m)\mathbf{y}(m)^H, \tag{13}$$

where M denotes the number of consecutive symbols and whose value should be subject to the requirement of sensing time — a critical parameter for sensing algorithms. It can be referred to in the Appendix that given the original signal $\mathbf{x}(m)$ is independent and identically distributed (i.i.d.) and M is sufficiently large, the decision metric under the two hypotheses H_0 and H_1 can be approximated as the sum of multiple Rayleigh and Ricean random variables, respectively. Through some manipulations and based on the central limit theorem (CLT), these two PDFs can be represented as follows:

$$\begin{aligned} p(\xi \mid H_0) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi - \mu_{H_0}\sqrt{2G}}{\sigma_{H_0}}\right)^2}, \\ p(\xi \mid H_1) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi - \mu_{H_1}\sqrt{2G}}{\sigma_{H_1}}\right)^2}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \mu_{H_0} &= \sqrt{\frac{\pi}{4M}} \sigma_W^2, \\ \sigma_{H_0}^2 &= \left(1 - \frac{\pi}{4}\right) \frac{\sigma_W^4}{M}, \\ \mu_{H_1} &= L_{1/2}\left(-\frac{MP^2\sigma_X^4\sigma_H^4}{2\sigma_W^4}\right) \sqrt{\frac{\pi}{4M}} \sigma_W^2, \end{aligned} \tag{15}$$

and $$\sigma_{H_1}^2 = \frac{\sigma_W^4}{M} + P^2\sigma_X^4\sigma_H^4 - \frac{\sigma_W^4}{4M} L_{1/2}^2\left(-\frac{MP^2\sigma_X^4\sigma_H^4}{2\sigma_W^4}\right),$$

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with $L_{1/2}(x)$ representing the Laguerre polynomial and σ_H^2 being the variance of the channel impulse response. The detailed derivation can be found in the Appendix. The PDFs in (14) are not standard Gaussian distributions. It is interesting to observe that both variances under the two hypotheses are inversely proportional to M. As a result, the variances will be significantly reduced if the observation window is large enough, which is beneficial to signal detection in deeply noisy environments.

4. LRT and Probabilities of Detection and False Alarm

With the PDFs of the decision metric under both hypotheses available, the Neyman–Pearson (NP) criterion [10], which aims to maximize the probability of detection (P_D) subject to the constraint of the probability of false alarm (P_{FA}) through properly choosing the decision threshold, is used to introduce the proposed OFDM signal sensing algorithm. Hence, the corresponding LRT can be written as [12]

$$L(\xi)=\frac{p(\xi\mid H_1)}{p(\xi\mid H_0)}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma, \tag{16}$$

where the threshold γ is constrained by the required P_{FA},

$$P_{FA}=\int_{\{\xi:L>\gamma\}}p(\xi\mid H_0)\,d\xi. \tag{17}$$

As indicated in (16), the detector of a cognitive user declares the presence of a primary transmission if $L(\xi)>\gamma$, and otherwise the absence of a primary transmission.

Substituting the PDFs of (14) into (16), we get the exact form of $L(\xi)$. Consequently, it is proved that the LRT can be expanded as

$$L(\xi)=\frac{\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\xi-\mu_{H_1}\sqrt{2G}}{\sigma_{H_1}}\right)^2\right]}{\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\xi-\mu_{H_0}\sqrt{2G}}{\sigma_{H_0}}\right)^2\right]}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma, \tag{18}$$

which is equivalent to

$$-\sigma_{H_0}^2\left(\xi-\mu_{H_1}\sqrt{2G}\right)^2+\sigma_{H_1}^2\left(\xi-\mu_{H_0}\sqrt{2G}\right)^2\underset{H_0}{\overset{H_1}{\gtrless}}2\sigma_{H_1}^2\sigma_{H_0}^2\ln\gamma. \tag{19}$$

After some simplification, we finally come to

$$L'(\xi)=\xi\underset{H_0}{\overset{H_1}{\gtrless}}\gamma', \tag{20}$$

where $\gamma'=(-c_2+\sqrt{c_2^2-4c_1c_3})/(2c_1)$ with

$$c_1=\sigma_{H_1}^2-\sigma_{H_0}^2,$$
$$c_2=2\sigma_{H_0}^2\mu_{H_1}\sqrt{2G}-2\sigma_{H_1}^2\mu_{H_0}\sqrt{2G}, \tag{21}$$

and $$c_3=2\sigma_{H_1}^2\mu_{H_0}^2G-2\sigma_{H_0}^2\mu_{H_1}^2G-2\sigma_{H_1}^2\sigma_{H_0}^2\ln\gamma.$$

Since the equivalent threshold γ' is the function of the CP duration G, it is expected that the CP duration has a close relationship with the detection performance.

According to (20), P_{FA} is obtained by

$$P_{FA}=\int_{\gamma'}^{\infty}p(\xi\mid H_0)\,d\xi. \tag{22}$$

Consequently, the equivalent threshold γ' is computed by applying the P_{FA} constraint; that is,

$$\gamma'=Q^{-1}\left(\frac{2P_{FA}}{\sqrt{\sigma_{H_0}}}\right)\sigma_{H_0}+\mu_{H_0}\sqrt{2G}, \tag{23}$$

where $Q^{-1}(\cdot)$ is the inverse complementary error function of Gaussian variables. With former analyses and discussion, the theoretical P_D can be computed as

$$P_D=Q\left(\frac{\gamma'-\mu_{H_1}\sqrt{2G}}{\sigma_{H_1}}\right), \tag{24}$$

where $Q(\cdot)$ denotes the complementary error function of Gaussian variables. Furthermore, the probability of detection can be rewritten, by substituting (23) into (24), as

$$P_D=Q\left(\frac{Q^{-1}\left(2P_{FA}/\sqrt{\sigma_{H_0}}\right)\sigma_{H_0}-(\mu_{H_1}-\mu_{H_0})\sqrt{2G}}{\sigma_{H_1}}\right). \tag{25}$$

In conclusion, we summarize the proposed OFDM sensing algorithm as comprising the following: compute the covariance matrix according to (13) from the detector's input vector. Calculate the transformed LRT in (20) in accordance to the covariance matrix. Calculate the equivalent threshold using (23). Declare the presence of a primary user's transmission if $L'(\xi)>\gamma'$, otherwise declare the absence of it.

5. Relation with Scheme in [6] and Complexity Analysis

Among the existing OFDM sensing schemes, the one that is most related to our proposed method is presented in [6], whose decision metric is (3). Motivated by this, we will next compare this estimation scheme with our proposed one.

First, the scheme in [6] sums the cross-correlation between any two samples with N samples apart. In theory, the samples, except for the CP and its copied part, are mutually independent but actually exhibit a certain amount of cross-correlation because of the limited length of the observation window. This kind of correlation is random and unpredictable, which means it should be ruled out, just as in the proposed scheme. Second,

Table 1. Complexity comparison.

Algorithms	Number of complex multiplications
Proposed scheme	$MN\log_2 N/2+2MG$
Method in [6]	$M(N+G)^2$

adding different entries of cross-correlation in the covariance matrix directly without any preprocessing, as shown in [6], may result in unreliable sensing results. This is attributed to the fact that the variation of the phases among different entries could lead to the constructive or destructive interference to the total amount of time domain cross-correlation. However, for the proposed algorithm, the norm operation prevents these entries from cancelling with each other.

The complexity of the proposed signal detection algorithm should be attached great importance because it will have an impact on the sensing time, which should be kept as minimum. The computational complexity of the proposed method is analyzed in terms of the number of complex multiplications since the complexity of complex addition is much smaller in comparison to that of the multiplication. The complexity of the proposed algorithm is mainly attributed to the computation of the covariance matrix. The results of the complexity analysis of both the proposed scheme and the scheme in [6] are listed in Table 1. It is observed that the scheme in [6] incurs higher complexity than the proposed sensing algorithm because of the inclusion of term $(N+G)^2$.

IV. Simulation Results and Discussion

In this section, numerical simulations are carried out to demonstrate the performance of the proposed sensing scheme in comparison with the existing algorithm. The considered OFDM system has $N=256$ subcarriers, a CP of length $G=64$, and a bandwidth of 5 MHz. In our simulations, a ten-ray frequency-selective fading channel model is applied with an exponential power-delay profile and is supposed to keep constant during signal detection. All source symbols are modulated using quadrature phase-shift keying. Besides, there are 50 OFDM symbols in the observation window if no other values are specified.

Figure 3 first illustrates the probability of misdetection, which is defined as $P_{MD}=1-P_D$, of both the proposed sensing method and the CP-based method in [6]. It is evident that under two conditions, namely $P_{FA}=10\%$ and $P_{FA}=1\%$, the proposed method performs better than the CP-based one. For example, the SNR gain of the proposed one over the CP-based method is about 6 dB when $P_{FA}=1\%$ at $P_{MD}=10^{-3}\%$. This is attributed

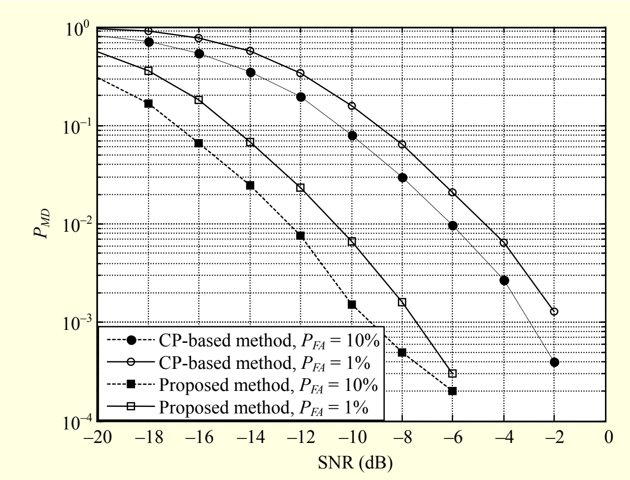

Fig. 3. Probability of misdetection of these two algorithms over wireless channel.

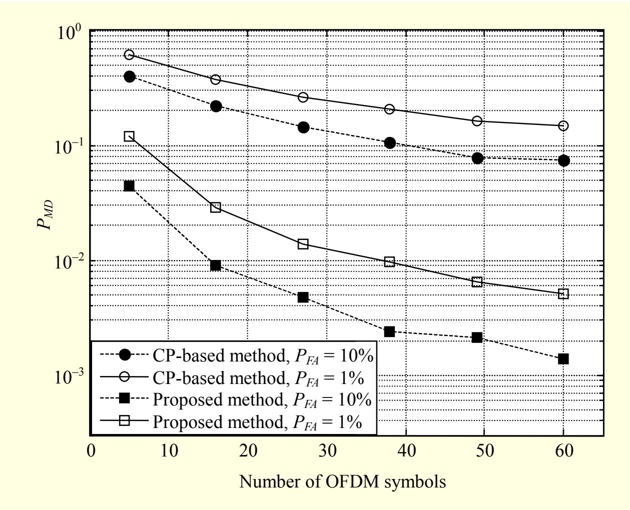

Fig. 4. Performance comparison with different numbers of OFDM symbols.

to the fact that the CP-based scheme includes not only the correlation resided in the CP but also that in other signals, which is unnecessary and adding these two types of correlation together is harmful to the correlation incurred by the CP. Furthermore, adding the correlation term of each entry in the CP directly is vulnerable to the randomness of the noise. While for the proposed method, the magnitude operation protects each correlation entry from interfering with others; thus, the performance superiority is held by the proposed method.

Figure 4 depicts the performance comparison with different numbers of OFDM symbols, here we set SNR = −10 dB. It can be observed that both schemes achieve better results with more symbols utilized, and the proposed method still enjoys superiority over the CP-based one. It is worth noting that the

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performance gain comes at the price of complexity and the requirement of sensing time should be taken into consideration, which means there is a design tradeoff between the sensing time and sensing performance when setting the length of the observation window.

Figure 5 compares the performance of the two algorithms with different CP ratios. For a fixed sensing time, it corresponds to $M = 56$ and $M = 59$ for decreased CP ratios of $G/N = 1/8$ and $G/N = 1/16$, respectively. Clearly, the probability of misdetection in both schemes rises when the CP ratio decreases, which means these two algorithms would see a limited performance if the CP length is restricted in some applications where the spectrum efficiency is of high priority.

Finally, Fig. 6 shows the receiver operating characteristic (ROC) curves of these two algorithms at three different SNRs.

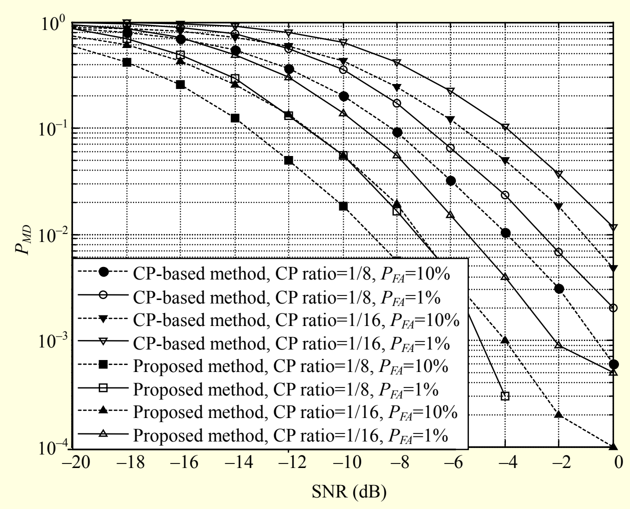

Fig. 5. Performance comparison between two algorithms with different CP ratios.

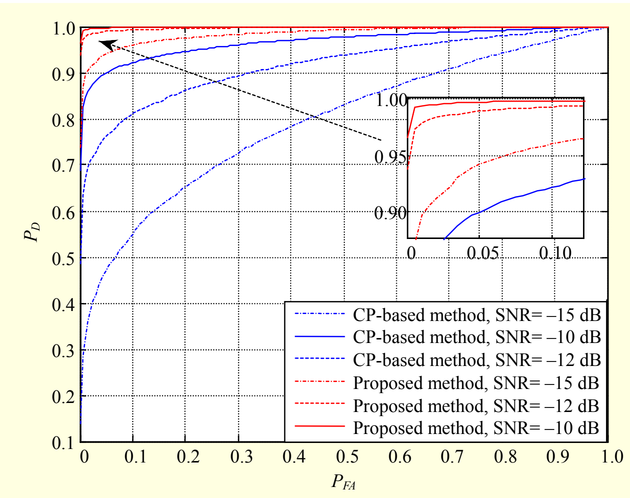

Fig. 6. ROC curves of the two schemes with different SNRs.

As can be observed, the curve of the proposed algorithm always has the steeper slope under all circumstances compared with the CP-based scheme, which is in accordance with the former results. Specifically, P_D almost approaches one when SNR is –10 dB, which means it can nearly achieve the optimum performance.

V. Conclusion

This paper proposed a sensing algorithm for OFDM signals in CR networks. A new decision metric was introduced to exploit the time domain cross-correlation resided in the CP of OFDM symbols. The proposed algorithm is robust against frequency offset but may experience performance degradation in the presence of timing error; thus, a separate scheme is required to compensate for it. We presented the LRT and derived the decision threshold and probability of detection according to the NP criterion. Numerical simulation results demonstrated that the proposed sensing algorithm can better utilize the cross-correlation incurred by the CP than the existing scheme, since it held performance superiority in terms of the probability of misdetection.

Appendix. Mathematical Derivation of (14) and (15)

If the wireless channel is not occupied by primary users, the (p,q)th element of the covariance matrix of the received time-domain signals can be computed as

$$\mathbf{Q}(p,q) = \frac{1}{M}\sum_{m=0}^{M-1} w_{m,p} w_{m,q}^{*}. \qquad (26)$$

If M is sufficiently large, $\mathbf{Q}(p,q)$ is approximated as the complex Gaussian distribution accordingly. Thus, its mean and variance can be computed directly as $\mu = 0$ and $\sigma^2 = \sigma_W^4 / M$, and the distribution of $\mathbf{Q}(p,q)$ is then $CN(0, \sigma_W^4 / M)$.

Obviously, if we let a and b denote the real and imaginary parts of $\mathbf{Q}(p,q)$ separately, it can be easily derived that they follow the distributions of $a \sim N(0, \sigma_W^4/(2M))$ and $b \sim N(0, \sigma_W^4/(2M))$. As a and b are mutually independent Gaussian random variables having zero mean and equal variance, the amplitude $\sqrt{a^2+b^2}$ is Rayleigh-distributed. Furthermore, the decision metric ξ in this situation is essentially the sum of multiple Rayleigh random variables, for which there is no closed-form expression available. Here we resort to the approximated distribution introduced in [13], and the PDF of the sum of $2G$ i.i.d. distributed Rayleigh random variables is given by

$$p(\xi \mid H_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi - \mu_{H_0}\sqrt{2G}}{\sigma_{H_0}}\right)^2} + p_E(\xi), \quad (27)$$

where μ_{H_0} and $\sigma_{H_0}^2$ are the mean and variance of $|\mathbf{Q}(p,q)|$, respectively, and $p_E(\xi)$ is a correction term that compensates for the error caused by the CLT approximation when $2G$ is relatively small. Here, this error function can be safely ignored since the length of the CP is usually large enough to keep the error within an acceptable level. As a result, we can obtain the PDF with $\mu_{H_0} = \sigma_W^2\sqrt{\pi/(4M)}$ and $\sigma_{H_0}^2 = (4-\pi)\sigma_W^4/(4M)$.

The derivation of the PDF when the channel is occupied, however, is a nontrivial task due to the complex expression of the decision metric under hypothesis H_1. In this case, the received signal seen by the detector can be represented as

$$y_{m,k} = \frac{1}{\sqrt{N}} \sum_{l=0}^{P-1} \sum_{n=0}^{N-1} h_l X_{m,n} e^{j2\pi n(k-l-mN_T)/N} + w_{m,k}. \quad (28)$$

To calculate the variance of $\mathbf{Q}(p,q)$, we assume $y_{m,k} \approx w_{m,k}$ since signal detectors usually work in harsh environments with SNR << 0 dB. Hence, the variance is simply $\sigma^2 = \sigma_W^4/M$, which is the same as that in hypothesis H_0.

As for the computation of the mean of $\mathbf{Q}(p,q)$, it can be written as follows:

$$E[\mathbf{Q}(p,q)] = \frac{1}{N} \sum_{l=0}^{P-1} \sum_{n=0}^{N-1} |h_l X_{m,n}|^2 e^{j2\pi n(p-q)/N}. \quad (29)$$

The noise term is ignored here, because of its zero mean. Based on the combination of (p,q), the elements of the covariance matrix can be categorized into two types — namely, $|p-q| = N$ or not. Here we only consider the first type of correlation, which is incurred by the CP. Based on the condition of $|p-q| = N$, (29) can be simplified as

$$\begin{aligned} E[\mathbf{Q}(p,q)] &= \frac{1}{N} \sum_{l=0}^{P-1} \sum_{n=0}^{N-1} |h_l X_{m,n}|^2 e^{j2\pi n} \\ &= \frac{P}{N} \sigma_X^2 \sigma_H^2 \sum_{n=0}^{N-1} e^{j2\pi n} \\ &= P\sigma_X^2\sigma_H^2. \end{aligned} \quad (30)$$

Again, let $\mathbf{Q}(p,q) = a + bj$. We can separately derive the distributions of real and imaginary parts as follows:

$$\begin{aligned} a &\sim N\left(P\sigma_X^2\sigma_H^2, \frac{1}{2M}\sigma_W^4\right), \\ b &\sim N\left(0, \frac{1}{2M}\sigma_W^4\right). \end{aligned} \quad (31)$$

What is different from the scenario of the absence of primary users is that the amplitude of $\mathbf{Q}(p,q)$ here is Ricean distributed. As before, since the decision metric ξ is the sum of $2G$ Rice variables, the PDF in the presence of a primary user's signal can be approximated as [13]

$$p(\xi \mid H_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi - \mu_{H_1}\sqrt{K}}{\sigma_{H_1}}\right)^2}, \quad (32)$$

where

$$\begin{aligned} \mu_{H_1} &= \sigma\sqrt{\pi/2}\, L_{1/2}(-\mu^2/2\sigma^2), \\ \sigma_{H_1}^2 &= 2\sigma^2 + \mu^2 - \frac{\pi\sigma^2}{2} L_{1/2}(-\mu^2/2\sigma^2). \end{aligned} \quad (33)$$

Substituting $\mu = P\sigma_X^2\sigma_H^2$ and $\sigma^2 = \sigma_W^4/(2M)$ into (33), the PDF in the presence of primary users is finally obtained.

References

[1] X.B. Hu, Y.M. Huang, and Z.L. Hong, "Residual Synchronization Error Elimination in OFDM Baseband Receivers," *ETRI J.*, vol. 29, no. 5, Oct. 2007, pp. 596–606.

[2] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, Feb. 2005, pp. 201–220.

[3] T. Yucek and H. Arslan, "A Survey Of Spectrum Sensing Algorithms for Cognitive Radio Applications," *IEEE Commun. Surveys Tutorials*, vol. 11, no. 1, 2009, pp. 116–130.

[4] S. Bokharaiee, H.H. Nguyen, and E. Shwedyk, "Blind Spectrum Sensing for OFDM-Based Cognitive Radio Systems," *IEEE Trans. Veh. Technol.*, vol. 60, no. 3, Mar. 2011, pp. 858–871.

[5] R. Tandra and A. Sahai, "Fundamental Limits on Detection in Low SNR under Noise Uncertainty," *IEEE Int. Conf. Wireless Netw., Commun. Mobile Comput.*, Maui, HI, USA, vol. 1, June 2005, pp. 464–469.

[6] Z. Lei and F. Chin "Sensing OFDM Systems under Frequency-Selective Fading Channels," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, May 2010, pp. 1960–1968.

[7] H.S. Chen, W. Gao, and D.G. Daut, "Spectrum Sensing for OFDM Systems Employing Pilot Tones," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, Dec. 2009, pp. 5862–5870.

[8] A. Zahedi-Ghasabeh, A. Tarighat, and B. Daneshrad, "Spectrum Sensing of OFDM Waveforms Using Embedded Pilots Subcarriers," *IEEE Int. Conf. Commun.*, Cape Town, South Africa, May 23–27, 2010, pp. 1–6.

[9] T. Roman, S. Visuri, and V. Koivunen, "Blind Frequency Synchronization in OFDM via Diagonality Criterion," *IEEE Trans. Signal Process.*, vol. 54, no. 8, Aug. 2006, pp. 3125–3135.

[10] J.J. van de Beek, M. Sandell, and P.O. Borjesson, "ML Estimation of Time and Frequency Offset in OFDM Systems," *IEEE Trans. Signal Process.*, vol. 45, no. 7, July 1997, pp. 1800–1805.

[11] T. Lv, H. Li, and J. Chen, "Joint Estimation of Symbol Timing and Carrier Frequency Offset of OFDM Signals over Fast Time-Varying Multipath Channels," *IEEE Trans. Signal Process.*, vol.

http://dx.doi.org/10.4218/etrij.14.0113.0758

53, no. 12, Dec. 2005, pp. 4526–4535.
[12] S.M. Kay, Fundamentals of Statistical Signal Processing, Detection Theory, Upper Saddle River, NJ: Prentice Hall, 1993.
[13] J.A. Lopez-Salcedo, "Simple Closed-Form Approximation to Ricean Sum Distributions," *IEEE Signal Process. Lett*., vol. 16, no. 3, Mar. 2009, pp. 153–155.

Weiyang Xu received his BS degree in electronics engineering technology from the School of Electronics and Information, Xi'an Jiao Tong University, Shannxi, China, in 2004 and his MS degree in microelectronics from the School of Electronics and Information, Xi'an Jiao Tong University, Shannxi, China, in 2007. He received his PhD degree in microelectronics from the School of Microelectronics, Fudan University, Shanghai, China, in 2010. He is now an associate professor at the College of Communication Engineering, Chongqing University, China. His research interests include design and VLSI implementation for digital wireless transceivers and signal processing algorithms involved in wireless communications.