Robust Energy Efficiency Power Allocation for Uplink OFDM–Based Cognitive Radio Networks

Jiakuo Zuo, Van Phuong Dao, Yongqiang Bao, Shiliang Fang, Li Zhao, and Cairong Zou

This paper studies the energy efficiency power allocation for cognitive radio networks based on uplink orthogonal frequency-division multiplexing. The power allocation problem is intended to minimize the maximum energy efficiency measured by "Joule per bit" metric, under total power constraint and robust aggregate mutual interference power constraint. However, the above problem is non-convex. To make it solvable, an equivalent convex optimization problem is derived that can be solved by general fractional programming. Then, a robust energy efficiency power allocation scheme is presented. Simulation results corroborate the effectiveness of the proposed methods.

Keywords: Cognitive radio, energy efficiency, OFDM, general fractional programming.

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is considered as a promising air interface for cognitive radio (CR). Recently, energy efficiency power allocation (EEPA) for OFDM-based CR has become a hot topic [1].

J.L. Mao et al. [2] proposed a water-filling factor-aided search method to solve the EEPA problem. In [3], the EEPA problem is addressed via parametric programming and then an iterative algorithm is presented. In [4], a risk-return model is used to solve the EEPA problem. Considering the minimal throughput requirements and the proportional fairness of CR users, [5] proposed a bisection-based algorithm to solve the EERA problem. In [6], the EEPA in OFDM-based CR with cooperative relay was studied and a barrier method to solve the power allocation problem was proposed. The above studies assume that secondary users (SUs) can acquire perfect channel sate information (CSI) between SUs and primary users (PUs). However, obtaining accurate estimations of CSI between SUs and PUs is challenging. Considering the imperfect CSI, non-EEPA problems have been studied for CR systems. For example, [7] assumes that the knowledge of channel-fading statistics between SUs and PUs can be acquired. In [8] and [9], the CSI between SUs and PUs is assumed as a bounded channel uncertainty model.

In this letter, we study the robust power allocation problem for uplink OFDM–based CR networks. We try to minimize the maximum energy efficiency measured using the "Joule per bit" metric, under total power constraint and robust aggregate mutual interference power (AMIP) constraints. We first transform the robust AMIP constraints into convex constraints by introducing auxiliary variables and utilizing S-procedure [10]. Then, the EEPA problem is reformulated as a general fractional programming (GFP) problem. Finally, a new iterative robust EEPA algorithm is proposed based on the Dinkelbach-type algorithm [11].

II. Signal Model and Problem Statement

Consider an uplink CR network, there is a cognitive access point (CAP) and K SUs coexisting with L PUs. The sets of SUs and PUs are $\kappa = \{1, 2, ..., K\}$ and $L = \{1, 2, ..., L\}$, respectively. The SUs adopt an OFDM access modulation and opportunistically access the unoccupied PU bands to transmit

---

to the CAP. The available bands for SUs are located on either side of the PU bands and are divided into N subcarriers. The bandwidth for each subcarrier is Δf Hz. Let R_k denote the transmission rate of the kth SU and we have

$$R_k = \sum_{n \in \Omega_k} \Delta f \log_2 \left(1 + \frac{|H_{k,n}|^2 p_{k,n}}{\sigma_n^2} \right), \qquad (1)$$

where Ω_k is the subcarrier set allocated to the kth SU, $H_{k,n}$ is the channel-fading gains between the kth SU and the CAP on the nth subcarrier, $p_{k,n}$ is the transmitted power of the kth SU on the nth subcarrier, and σ_n^2 is the additive white Gaussian noise variance. The AMIP introduced by SUs to a PU is defined as

$$I_l = \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} |g_{k,n}^l|^2 \varphi_{k,n}^l, \quad l \in L, \qquad (2)$$

where $g_{k,n}^l$ is the channel-fading gains between the kth SU and the lth PU on the nth subcarrier. The interference factor of the lth PU on the nth subcarrier is denoted by $\varphi_{k,n}^l$ [7].

However, due to the lack of full cooperation between SUs and PUs, $g_{k,n}^l$ is challenging to estimate accurately. In particular, the robust model of $g_{k,n}^l$ is defined as

$$g_{k,n}^l = \bar{g}_{k,n}^l + \Delta_{k,n}^l, \qquad (3)$$

where $\bar{g}_{k,n}^l$ is the estimation of $g_{k,n}^l$, and $\Delta_{k,n}^l$ is the estimated error. We assume that $\Delta_{k,n}^l$ is bounded as

$$\Theta_k^l = \left\{ \Delta_{k,n}^l : \sum_{n \in \Omega_k} |\Delta_{k,n}^l|^2 \le \left(\xi_k^l \right)^2 \right\}, \quad k \in \kappa, \ l \in L, \qquad (4)$$

where ξ_k^l is the radius of the uncertainty region.

The energy efficiency (EE) of the kth SU is defined as

$$EE_k = \frac{\tau_k \left(\sum_{n \in \Omega_k} p_{k,n} \right) + P_k^c}{\sum_{n \in \Omega_k} \Delta f \log_2 \left(1 + \frac{|H_{k,n}|^2 p_{k,n}}{\sigma_n^2} \right)} = \frac{P_k^{\text{total}}}{R_k}, \qquad (5)$$

where τ_k denotes the power amplifier efficiency, and P_k^c denotes the power consumption of circuits and base-station facilities. Our objective is to minimize the maximum EE_k, therefore the EEPA problem is formulated as OP1

OP1 $\min\limits_{p_{k,n}} \max\limits_{k \in \kappa} (P_k^{\text{total}} / R_k),$

s.t $\quad C1: \sum_{n \in \Omega_k} p_{k,n} \le P_k^{\max}, \forall k \in \kappa,$

$\qquad C2: \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} |g_{k,n}^l|^2 \varphi_{k,n}^l \le I_l^{\text{th}}, \forall \Delta_{k,n}^l \in \Theta_k^l, \forall l \in L,$

$\qquad C3: p_{k,n} \ge 0, \ \forall k \in \kappa, \ \forall n \in \Omega_k,$

where $P_k^{\max}$ is the total power budget of the kth SU and I_l^{th} is the interference threshold of the lth PU.

III. Optimal Energy Efficiency Power Allocation

However, the non-convexity of the function EE_k on $p_{k,n}$ and the robust AMIP constraints in $C2$, make problem OP1 difficult to be solved directly. To make it solvable, we first transform constraint $C2$ into an equivalent simple convex constraint.

To facilitate representation, we first define column vectors as follows: $\quad g_k^l = \left\{ g_{k,n}^l \right\}_{n \in \Omega_k}, \quad \bar{g}_k^l = \left\{ \bar{g}_{k,n}^l \right\}_{n \in \Omega_k}, \quad \Delta_k^l = \left\{ \Delta_{k,n}^l \right\}_{n \in \Omega_k},$

and matrix $\quad \varphi_k^l = \text{diag}\left(\left\{ \varphi_{k,n}^l \right\}_{n \in \Omega_k} \right), \quad P_k = \text{diag}\left(\left\{ p_{k,n} \right\}_{n \in \Omega_k} \right).$

Then, we have $\quad g_k^l = \bar{g}_k^l + \Delta_k^l \quad$ and $\quad \Theta_k^l = \left\{ \Delta_{k,l} : (\Delta_k^l)^{\text{H}} \Delta_k^l \le (\xi_k^l)^2 \right\}$, where $(\cdot)^{\text{H}}$ indicates the conjugate transpose operation.

Thus, the robust AMIP constraint $C2$ in OP1 can be written in vector form as

$$\sum_{k=1}^{K} \left\{ (\Delta_k^l)^{\text{H}} P_k \varphi_k^l \Delta_k^l + 2Re\left[\left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l \Delta_k^l \right] + \left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l \bar{g}_k^l \right\} \le I_l^{\text{th}},$$

$$(\Delta_k^l)^{\text{H}} \Delta_k^l \le (\xi_k^l)^2, \ \forall l \in L. \qquad (6)$$

By introducing auxiliary variables γ_k^l with $\sum_{k=1}^{K} \gamma_k^l \le I_l^{\text{th}}$, inequality constraint (6) can be equivalently reformulated for K separable inequalities as

$$(\Delta_k^l)^{\text{H}} P_k \varphi_k^l \Delta_k^l + 2Re\left[\left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l \Delta_k^l \right] + \left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l \bar{g}_k^l \le \gamma_k^l,$$

$$(\Delta_k^l)^{\text{H}} \Delta_k^l \le (\xi_k^l)^2, \ \forall l \in L, \ \forall k \in \kappa. \qquad (7)$$

Based on S-procedure [10], there exists $\mu_k^l \ge 0$, so that (7) is equivalent to the following liner matrix inequality:

$$\begin{bmatrix} \mu_k^l I - P_k \varphi_k^l & -P_k \varphi_k^l \bar{g}_k^l \\ -\left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l & \gamma_{k,l} - \left(\bar{g}_k^l \right)^{\text{H}} P_k \varphi_k^l \bar{g}_k^l - \mu_k^l (\xi_k^l)^2 \end{bmatrix} \ge 0, \quad (8)$$

where I is the identity matrix. Observing that $\mu_k^l I - P_k \varphi_k^l$ in (8) is positive definite and invertible, which implies $\mu_k^l - \varphi_{k,n}^l p_{k,n} \ge 0$, according to Schur's Complement theory [12], we obtain an equivalent inequality for (8) as

$$\gamma_k^l - \mu_k^l (\xi_k^l)^2 - \theta_k^l \ge 0, \qquad (9)$$

where $\theta_k^l = \sum_{n \in \Omega_k} \frac{\mu_k^l |\bar{g}_{k,n}^l|^2 \varphi_{k,n}^l p_{k,n}}{\mu_k^l - \varphi_{k,n}^l p_{k,n}}$. Combining (9) with auxiliary variable γ_k^l's constraint $\sum_{k=1}^{K} \gamma_k^l \le I_l^{\text{th}}$, we have

$$\sum_{k=1}^{K}\left(\theta_k^l+\mu_k^l(\xi_k^l)^2\right)\le I_l^{\text{th}},\ \forall l\in L. \tag{10}$$

Note: since the Hessian matrix of function $\left(\theta_k^l+\mu_k^l(\xi_k^l)^2\right)$ with respect to $p_{k,n}$ and μ_k^l is positive semidefinite, this implies that the equivalent constraint (10) is convex.

Now, we have got an equivalent form (10) with $\varphi_{k,n}^l p_{k,n}\le\mu_k^l$ to replace robust AMIP constraint $C2$ in OP1. Therefore, OP1 can be rewritten as

OP2 $\displaystyle\min_{p_{k,n},\mu_k^l}\max_{k\in\kappa}(P_k^{\text{total}}/R_k),$

s.t $\quad C1:\displaystyle\sum_{n\in\Omega_k}p_{k,n}\le P_k^{\max},\ \forall k\in\kappa,$

$\quad C2:\displaystyle\sum_{k=1}^{K}\left(\theta_k^l+\mu_k^l(\xi_k^l)^2\right)\le I_l^{\text{th}},\ \forall l\in L,$

$\quad C3:p_{k,n}\ge0,\ \forall k\in\kappa,\ \forall n\in\Omega_k,$

$\quad C4:\varphi_{k,n}^l p_{k,n}\le\mu_k^l,\ \forall n\in\Omega_k,\ \forall k\in\kappa,\ \forall l\in L,$

$\quad C5:\mu_k^l\ge0,\ \forall k\in\kappa,\ \forall l\in L.$

The equivalent problem OP2 is a GFP [11] problem. Before solving it, we introduce a new optimization problem OP3

OP3 $\displaystyle\min_{p_{k,n},\mu_k^l\in\wp}\max_{k\in\kappa}(P_k^{\text{total}}-\lambda R_k),$

where $\wp$ denotes the feasible region of OP2, and λ is a positive parameter.

Let $\boldsymbol{\omega}\in\wp$ denote all the variables $\left\{p_{k,n},\mu_k^l\right\}$ in OP2, $\lambda=\displaystyle\min_{\boldsymbol{\omega}\in\wp}\max_{k\in\kappa}\left(P_k^{\text{total}}(\boldsymbol{\omega})/R_k(\boldsymbol{\omega})\right)$ is the objective value of OP2 and $F(\lambda)=\displaystyle\min_{\boldsymbol{\omega}\in\wp}\max_{k\in\kappa}\left(P_k^{\text{total}}(\boldsymbol{\omega})-\lambda R_k(\boldsymbol{\omega})\right)$ is the objective value of OP3, the following lemma can relate problem OP2 and OP3 to each other.

Lemma 1: OP2 and OP3 have the same set of optimal solutions, if and only if, $F(\lambda^*)=0$, where λ^* is the optimal objective value of OP2.

Proof: Before proving Lemma 1, we first analyze the properties of function $F(\lambda)$. (I) Since $R_k>0$, we conclude that $F(\lambda)$ is nonincreasing; (II) Function $\displaystyle\max_{k\in\kappa}\left(P_k^{\text{total}}(\boldsymbol{\omega})-\lambda R_k(\boldsymbol{\omega})\right)$ is jointly continuous in $(\boldsymbol{\omega},\lambda)$; thus, we conclude that $F(\lambda)$ is upper semicontinuous in λ. (III) Assume $F(\lambda)<0$, there exists $\boldsymbol{\omega}^\wedge\in\wp$ such that $P_k^{\text{total}}(\boldsymbol{\omega}^\wedge)-\lambda^* R_k(\boldsymbol{\omega}^\wedge)<0,\qquad\forall k\in\kappa.$ Therefore, $\lambda>\displaystyle\max_k\left(P_k^{\text{total}}(\boldsymbol{\omega}^\wedge)/R_k(\boldsymbol{\omega}^\wedge)\right)\ge\lambda^*$. If $\lambda>\lambda^*$, there exists $\boldsymbol{\omega}^\wedge\in\wp$ such that $\max\left(P_k^{\text{total}}(\boldsymbol{\omega}^\wedge)/R_k(\boldsymbol{\omega}^\wedge)\right)<\lambda$. Therefore, $P_k^{\text{total}}(\boldsymbol{\omega}^\wedge)-\lambda^* R_k(\boldsymbol{\omega}^\wedge)<0,\forall k\in\kappa$, which means $F(\lambda)<0$, and so, we conclude that $F(\lambda^*)\ge0$.

Let $\boldsymbol{\omega}^*$ denote an optimal solution of OP2, thus $\lambda^*=\max(P_k^{\text{total}}(\boldsymbol{\omega}^*)/R_k(\boldsymbol{\omega}^*))$, and $\max_k(P_k^{\text{total}}(\boldsymbol{\omega}^*)-\lambda^* R_k(\boldsymbol{\omega}^*))=0$. Since $F(\lambda^*)\ge0$, we have $F(\lambda^*)=0$. Therefore, an optimal solution $\boldsymbol{\omega}^*$ of OP2 is an optimal

solution of OP3 (where $\lambda=\lambda^*$), and we have $F(\lambda^*)=0$.

Suppose that $\boldsymbol{\omega}^*$ is an optimal solution of OP3 (where $\lambda=\lambda^*$) and $F(\lambda^*)=0$, then we have $\max_k(P_k^{\text{total}}(\boldsymbol{\omega}^*)-\lambda^* R_k(\boldsymbol{\omega}^*))=0$. Thus, $\lambda^*=\displaystyle\max_k(P_k^{\text{total}}(\boldsymbol{\omega}^*)/R_k(\boldsymbol{\omega}^*))$, which means that $\boldsymbol{\omega}^*$ is an optimal solution of OP2. $\square$

By lemma 1, it is clear that solving OP2 can be achieved by finding a solution of the equation $F(\lambda)=0$. Here we utilize the Dinkelbach-type algorithm [11] to solve this problem. At last, a new robust EEPA algorithm is present, which is termed as REEPA (Robust energy efficiency power allocation) and tabulated as follows:

Algorithm: REEPA

1: Initialization: Take $p_{k,n},\mu_k^l\in\wp$, **compute** $\quad\lambda_1=\displaystyle\max_{k\in\kappa}\frac{P_k^{\text{total}}}{R_k}$

and let $t{=}1$;

2: Determine $(p_{k,n},\mu_k^l)=\arg\displaystyle\min_{p_{k,n},\mu_k^l\in\wp}\max_{k\in\kappa}\{P_k^{\text{total}}-\lambda_k R_k\}$

3: If $F(\lambda_t)=0$

Then $(p_{k,n},\mu_k^l)$ **is an optimal solution of OP2 with value**
λ_t **and Stop**

Else Go to step 4;

4: Let $\lambda_{t+1}=\displaystyle\max_{k\in\kappa}\frac{P_k^{\text{total}}}{R_k}$

Let $t=t+1$ **, and go to step 2.**

REEPA solves at each step a subproblem OP3, and it creates a nonincreasing sequence λ_t converging from above to the optimal objective value λ^*. (The proof of convergence of the Dinkelbach-type algorithm can be found in [11]).

IV. Performance Simulations

In this section, some numerical results are presented to evaluate the performance of the proposed scheme. Assume that there are three SUs ($K{=}3$) coexisting with three PUs ($L{=}3$). The bandwidths occupied by the PUs are 1 MHz, 2 MHz, and 5 MHz, respectively. The unoccupied band is divided into twelve OFDM subcarriers ($N{=}3$), and the bandwidth for each subcarrier Δf is set to 0.3125 MHz. Assume the subcarriers have been allocated to SUs (In this paper, we only consider the power allocation problem). The channel gains are assumed to be Rayleigh-fading with an average power gain of 0 dB. Without loss of generality, the channel uncertainty is set to $(\xi_k^l)^2=\eta\cdot\displaystyle\sum_{n\in\Omega_k}\left|\bar{g}_{k,n}^l\right|^2$, with $\eta{=}0.05$. Let power amplifier efficiency $\tau_k=1$, the circuit power consumption $P_k^{\text{c}}=10^{-2}$, and the noise power $\sigma_n^2=10^{-6}\,\text{W}$. All the results have been
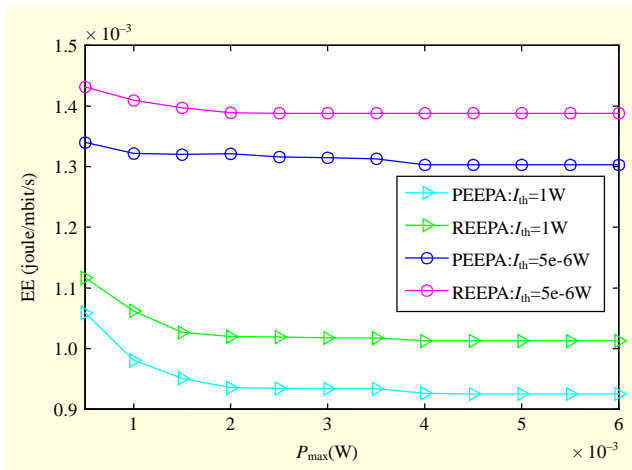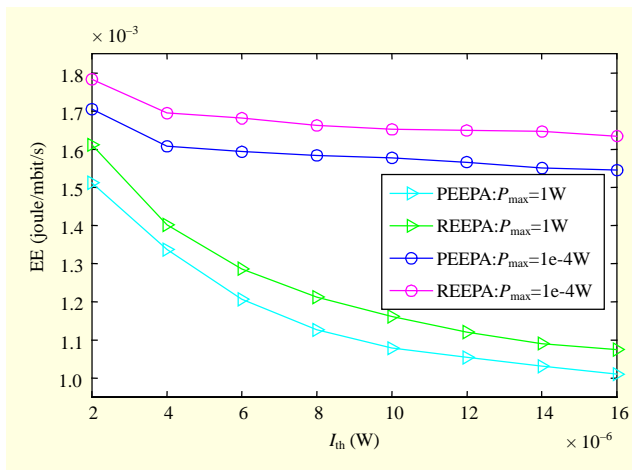
Fig. 1. EE vs. total power budget.

Fig. 2. EE vs. interference threshold.

averaged over 500 iterations.

To corroborate the effectiveness of our proposed method, we compared REEPA with PEEPA. PEEPA is the algorithm that is used to solve OP1 under perfect CSI via GFP. The procedure of PEEPA is similar to REEPA. For simplicity, we set $P_k^{max} = P_{max}$ and $I_l^{th} = I_{th}$. The EE, which we evaluated in the following experiments, is the average value of all the SUs' EE. Figure 1 depicts the EE versus the total power budget, under different interference thresholds. As shown in Fig. 1, PEEPA has a lower EE than REEPA, since PEEPA has perfect CSI, which can satisfy precisely the interference constraint. The REEPA algorithm can provide an acceptable EE, and meanwhile, never violate the aggregate mutual interference power, even with imperfect CSI. The EE versus the interference threshold, under different total power budget, is evaluated in Fig. 2. These results depict similar properties to the results in Fig. 1.

V. Conclusions

In this paper, a new, robust energy efficiency power allocation algorithm is proposed for uplink OFDM–based cognitive radio systems. Our aim is to minimize the maximum energy efficiency, under total power constraint and robust aggregate mutual interference power constraints. Simulation results corroborate the effectiveness of the proposed methods in power allocation for uplink OFDM–based cognitive systems.

References

[1] D. Feng et al., "A Survey of Energy-Efficient Wireless Communications," *IEEE J. Commun. Surveys Tutorials*, vol. 15, no. 1, 2013, pp. 167–178.

[2] J.L. Mao et al., "Energy Efficiency Optimization for OFDM-Based Cognitive Radio Systems: A Water-Filling Factor Aided Search Method," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, May 2013, pp. 2366–2375.

[3] Y. Wang et al., "Optimal Energy-Efficient Power Allocation for OFDM-Based Cognitive Radio Networks," *IEEE Commun. Lett.*, vol. 16, no. 9, Sept. 2012, pp.1420–1423.

[4] Z. Hasan et al., "Energy-Efficient Power Allocation in OFDM-Based Cognitive Radio Systems: A Risk-Return Model," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, Dec. 2009, pp. 6078–6088.

[5] M. Ge and S. Wang, "Energy-Efficient Power Allocation for Cooperative Relaying Cognitive Radio Networks," *IEEE Wireless Commun. Netw. Conf.*, Shanghai, China, Apr. 7–10, 2013, pp. 691–696.

[6] W. Shi and S. Wang, "Energy-Efficient Resource Allocation in Cognitive Radio Systems," *IEEE WCNC,* Shanghai, China, Apr. 7–10, 2013, pp. 4618–4623.

[7] G. Bansal, M.J. Hossain, and V.K. Bhargava, "Adaptive Power Loading for OFDM-Based Cogntive Radio Systems with Statistical Interference Cosntraint," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, Sept. 2011, pp. 2786–2791.

[8] J. Wang, G, Scutari, and D.P. Palomar, "Robust MIMO Cognitive Radio via Game Theory," *IEEE Trans. Signal Process.*, vol. 59, no. 3, Mar. 2011, pp. 1183–1201.

[9] Y. Zhang, E. DallAnese, and G.B. Giannakis, "Distributed Optimal Beamformers for Cogntive Radios Robust to Channel Uncertainties," *IEEE Trans. Signal Process.*, vol. 60, no. 12, Dec. 2012, pp. 6495–6508.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge University Press, 2004, p. 655.

[11] J.P. Crouzeix, J.A. Ferland, and S. Schaible, "An Algorithm for Generalized Fractional Programs," *J. Optimization Theory Appl.*, vol. 47, no. 1, 1985, pp. 35–49.

[12] R.A. Horn and C.R. Johnson, *Matrix analysis*, New York: Cambridge University Press, 1985, pp. 79–85.