# Outage Probability of Decode-and-Forward Relaying Systems with Efficient Partial Relay Selection in Nakagami Fading Channels 

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Recently, efficient partial relay selection (e-PRS) was proposed as an enhanced version of PRS. In comparing e-PRS, PRS, and the best relay selection (BRS), there is a tradeoff between complexity and performance; that is, the complexity for PRS, e-PRS, and BRS is low to high, respectively, but vice versa for performance. In this paper, we study the outage probability for e-PRS in decode-andforward (DF) relaying systems over non-identical Nakagami- $m$ fading channels, where the fading parameter $m$ is an integer. In particular, we provide closed-form expressions of the exact outage probability and asymptotic outage probability for e-PRS in DF relaying systems. Numerical results show that e-PRS achieves similar outage performance to that of BRS for a low or medium signal-to-noise ratio, a high fading parameter, a small number of relays, and a large difference between the average channel powers for the first and the second hops.

Keywords: Dual-hop relaying system, partial relay selection, outage probability, Nakagami fading channels.

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## I. Introduction

In dual-hop relaying systems with multiple relays, the best relay selection (BRS) achieves full diversity [1]. In BRS, only a single relay with the best end-to-end path between the source and the destination is selected by using the instantaneous channel information for both the first hop and the second hop for all the end-to-end paths. In [2], the outage probability and symbol error probability of BRS were presented in nonidentical Nakagami- $m$ fading channels. To reduce the complexity for BRS, partial relay selection (PRS) was proposed in [3], which is based on only the instantaneous first hop channel information for selecting a single relay. In [4], the outage probability for amplify-and-forward (AF) and decode-and-forward (DF) relaying systems with PRS was presented in identical Nakagami- $m$ fading channels. In [5] and [6], the outage performance and diversity for PRS in AF relaying systems were studied in non-identical Rayleigh and Nakagami$m$ fading channels, respectively.
To improve the outage performance of PRS, efficient PRS (e-PRS), which uses the average channel powers for the first and the second hops and the instantaneous channel information for either the first or the second hop, was proposed in [7]. Also, in [7], the outage performance of e-PRS in AF relaying systems was presented over non-identical Rayleigh fading channels. In e-PRS, a link with the smaller average channel power in the first and the second hops is chosen at each end-toend path, and the instantaneous channel information for the chosen links is then used for relay selection. Hence, e-PRS has a little higher complexity than PRS, but it can be considered as
a low-complexity alternative of BRS.
Although the performance of PRS has been well investigated, the performance study of e-PRS has been done only in AF relaying systems over non-identical Rayleigh fading channels. AF relaying causes noise amplification since an AF relay simply amplifies and forwards the signal received from the source. Therefore, in this paper, we focus on DF relaying to prevent the noise amplification, where a DF relay decodes the source's signal and re-encodes and forwards the decoded signal. Thus, we study the outage performance of e-PRS in DF relaying systems over non-identical Nakagami- $m$ fading channels. It is noted that a Nakagami fading model can represent line-of-sight channels, whereas a Rayleigh fading model represents only non-line-of-sight channels; also, a Rayleigh fading channel is a special case of a Nakagami- $m$ fading channel (that is, a Nakagami- $m$ fading channel with $m=1$ is equal to a Rayleigh fading channel). In this paper, we provide closed-form expressions of the exact outage probability and asymptotic outage probability for e-PRS in DF relaying systems. Also, we give a diversity order of the DF relaying system with e-PRS in Nakagami- $m$ fading channels. Numerical results are shown to verify the analytic expressions and to compare the outage performances of e-PRS, PRS, and BRS.

## II. System Model

We consider dual-hop DF relaying systems using relay selection, where $K$ relays $\left(R_{k}, k=1, \ldots, K\right)$ help with the communication between a source $(S)$ and a destination $(D)$. In this paper, we assume that the direct communication between the source and the destination is unavailable. In dual-hop DF relaying with relay selection, only one selected relay receives a signal from the source during the first time slot and decodes it. Then, only when the decoding succeeds, the selected relay re-encodes and forwards it to the destination during the second time slot.
$h_{S k}$ and $h_{k D}$ denote the Nakagami- $m$ fading channels for the $S-R_{k}$ link and the $R_{k}-D$ link, respectively. Let the average powers of $h_{S k}$ and $h_{k D}$ be denoted by $\beta_{S k}$ and $\beta_{k D}$, respectively, and the integer fading parameters of $h_{S k}$ and $h_{k D}$ be denoted by $m_{S k}$ and $m_{k D}$, respectively. In this paper, all the channels are assumed to be independent. Also, it is assumed that the transmit power at every transmitter is equal, denoted by $P$, and the noise power at every receiver is the same, denoted by $\sigma^{2}$.
Let $\rho=P / \sigma^{2}$, which is referred to as the average transmit signal-to-noise ratio (SNR). Then, the received SNRs for the $S-R_{k}$ link (that is, the first hop for relay $k$ ) and the $R_{k}-D$ link (that is, the second hop for relay $k$ ) are respectively given as $\gamma_{S k}=\rho\left|h_{S k}\right|^{2}$ and $\gamma_{k D}=\rho\left|h_{k D}\right|^{2}$. Using the received SNRs, a
relay selected by PRS is obtained as follows [3]:

$$
\begin{equation*}
k_{c}=\arg \max _{k=1, \cdots, K}\left\{\gamma_{S k}\right\} . \tag{1}
\end{equation*}
$$

On the other hand, a relay selected by e-PRS is obtained as follows [7]:

$$
\begin{equation*}
k_{e}=\arg \max _{k=1, \cdots, K}\left\{W_{k}\right\}, \tag{2}
\end{equation*}
$$

where

$$
W_{k}= \begin{cases}\gamma_{S k} & \text { for } \beta_{S k}<\beta_{k D},  \tag{3}\\ \gamma_{k D} & \text { for } \beta_{S k}>\beta_{k D}\end{cases}
$$

It is noted that e-PRS for AF relaying systems can be employed in DF relaying systems since the end-to-end SNR of DF relaying is dominated by the received SNR for the weakest hop [8], as in AF relaying

## III. Outage Probability Analysis

Let the outage probability be defined as the probability that the data rate, $\left\{\log _{2}\left(1+\Gamma_{\delta}\right)\right\} / 2$, falls below a target data rate in $\mathrm{bps} / \mathrm{Hz}$, denoted by $R$, where $\delta \in\left\{k_{c}, k_{e}\right\}$ and $\Gamma_{\delta}$ represents the end-to-end SNR for relay $\delta$ [7]. Then, the outage probability for e-PRS and PRS is expressed as

$$
\begin{equation*}
P_{\text {out }}(R)=\operatorname{Pr}\left\{\frac{1}{2} \log _{2}\left(1+\Gamma_{\delta}\right)<R\right\}=\operatorname{Pr}\left\{\Gamma_{\delta}<2^{2 R}-1\right\} . \tag{4}
\end{equation*}
$$

For the sake of simplicity, let $z=2^{2 R}-1$. Assuming that the decoding at relay $k$ succeeds when the received SNR for the $S-R_{k}$ link exceeds $z$ [9], and applying (1) and (2) into (4), the exact outage probability of DF relaying systems using the e-PRS and PRS is obtained by

$$
\begin{align*}
P_{\text {out }}^{D F}(R)= & \sum_{k=1}^{K} \operatorname{Pr}\left\{\gamma_{k D}<z, \gamma_{S k}>z, Y_{k}>Y_{1}, \cdots,\right. \\
& \left.\quad Y_{k}>Y_{k-1}, Y_{k}>Y_{k+1}, \cdots, Y_{k}>Y_{K}\right\} \\
+ & \sum_{k=1}^{K} \operatorname{Pr}\left\{\gamma_{S k}<z, Y_{k}>Y_{1}, \cdots,\right. \\
& \left.\quad Y_{k}>Y_{k-1}, Y_{k}>Y_{k+1}, \cdots, Y_{k}>Y_{K}\right\}, \tag{5}
\end{align*}
$$

where

$$
Y_{k}= \begin{cases}\gamma_{S k} & \text { for } \operatorname{PRS},  \tag{6}\\ W_{k} & \text { for e-PRS } .\end{cases}
$$

In (5), the first part reflects the outage probability when selected relay $k$ succeeds in decoding the signal received from the source, and the second part represents the outage probability when selected relay $k$ fails to decode it. It is noted that the e-PRS is the same as PRS when $\beta_{S k}<\beta_{k D}$ for all $k$.
Hereafter, the outage probability of e-PRS in DF relaying systems is derived. Using the probability density function (PDF) of the Nakagami- $m$ random variable in [10], the PDFs
of $\gamma_{S k}$ and $\gamma_{k D}$ are respectively given by

$$
\begin{equation*}
f_{\gamma_{S k}}(x)=\frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x^{m_{S k}-1} e^{-\frac{m_{S k} x}{\rho p_{S k}}} \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{\gamma_{k D}}(x)=\frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x^{m_{k D}-1} e^{-\frac{m_{k D} x}{\rho \beta_{k D}}}, \tag{8}
\end{equation*}
$$

where $\Gamma(\cdot)$ denotes the gamma function. Let the mean of random variable $W_{k}$ be denoted by $\rho \lambda_{k}$. Thus, for e-PRS, $\lambda_{k}=\beta_{S k}$ and $\eta_{k}=m_{S k}$ when $\beta_{S k}<\beta_{k D}$, but $\lambda_{k}=\beta_{k D}$ and $\eta_{k}=m_{k D}$ when $\beta_{S k}>\beta_{k D}$.

When $W_{k}=\gamma_{S k}$, using (7) and (8), the first and the second parts in (5) are respectively obtained by

$$
\begin{equation*}
=\left\{1-e^{-\frac{m_{m z}}{\rho \beta_{k D}}} \sum_{i=1}^{m_{k D}}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}-i} \frac{1}{\left(m_{k D}-i\right)!}\right\} \tag{9}
\end{equation*}
$$

$$
\times\left[e^{-\frac{m_{S k}}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right.
$$

$$
+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{l_{i}}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}
$$

$$
\times \frac{1}{\left(m_{S k}-1\right)!}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}}(-1)^{i} e^{-z\left(\frac{m_{S k}}{\rho \beta_{k}}+\sum \frac{n_{q}}{q \in L_{i}} \frac{p_{q} q_{q}}{)}\right)}
$$

$$
\begin{equation*}
\left.\times \sum_{v=1}^{w_{S k}} z^{w_{S k}-v} \frac{\left(w_{S k}-1\right)!}{\left(w_{S k}-v\right)!}\left(\frac{m_{S k}}{\rho \beta_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-v}\right] \tag{10}
\end{equation*}
$$

and

$$
\begin{aligned}
& \operatorname{Pr}\left\{\gamma_{S k}<z, \gamma_{S k}>W_{1}, \cdots, \gamma_{S k}>W_{k-1}, \gamma_{S k}>W_{k+1},\right. \\
& \left.\cdots, \gamma_{S k}>W_{K}\right\} \\
& =\int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S S} x_{k}}{\rho \beta_{S k}}}\right\}
\end{aligned}
$$

$$
\begin{aligned}
& \operatorname{Pr}\left\{\gamma_{k D}<z\right\} \operatorname{Pr}\left\{\gamma_{S k}>z, \gamma_{S k}>W_{1}, \cdots, \gamma_{S k}>W_{k-1},\right. \\
& \left.\gamma_{S k}>W_{k+1}, \cdots, \gamma_{S k}>W_{K}\right\} \\
& =\left\{1-e^{-\frac{m_{k D}}{\rho \beta_{k i}}} \sum_{i=1}^{m_{k D}}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}-i} \frac{1}{\left(m_{k D}-i\right)!}\right\} \\
& \times \int_{z}^{\infty} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S S k}}{\rho P_{S k}}}\right\} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1} e^{-\frac{n x_{i}}{\rho \lambda_{i}}}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k},
\end{aligned}
$$

$$
\begin{align*}
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1} e^{-\frac{-\eta_{i x i}}{\rho \rho_{i}}}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k}, \\
& =\left\{1-e^{-\frac{m_{S k} z}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\}  \tag{11}\\
& +\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{1}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{l_{i}}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\} \\
& \times \frac{1}{\left(m_{S k}-1\right)!}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}}(-1)^{i}\left\{1-e^{-z\left(\frac{m_{S k}}{\rho \rho_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho_{q}}\right)}\right. \\
& \left.\times \sum_{v=1}^{w_{S k}} z^{w_{S k}-v} \frac{\left(w_{S k}-1\right)!}{\left(w_{S k}-v\right)!}\left(\frac{m_{S k}}{\rho \beta_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-v}\right\}, \tag{12}
\end{align*}
$$

where $E_{k}=\{1, \ldots, k-1, k+1, \ldots, K\}$ and $L_{i}$ is all possible subsets of $E_{k}$ whose cardinality is $i$. Furthermore, $l_{u}$ for $u=1, \ldots, i$ denotes the $u$-th element in set $L_{i}$, and $w_{S k}=m_{S k}+\sum_{q \in L_{i}}\left(\eta_{q}-n_{q}\right)$. The detailed derivations to obtain (10) and (12) are shown in Appendix A.

When $W_{k}=\gamma_{k D}$, using (7) and (8), the first and the second parts in (5) are respectively obtained by

$$
\begin{align*}
& \operatorname{Pr}\left\{\gamma_{S k}>z\right\} \operatorname{Pr}\left\{\gamma_{k D}<z, \gamma_{k D}>W_{1}, \cdots, \gamma_{k D}>W_{k-1}\right. \text {, } \\
& \left.\gamma_{k D}>W_{k+1}, \cdots, \gamma_{k D}>W_{K}\right\} \\
& =\left\{e^{-\frac{m_{s k z}}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \\
& \times \int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{k b} x_{k}}{\rho \beta_{k D}}}\right\} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1} e^{-\frac{\eta_{i} x_{i}}{\rho i_{i}}}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k}, \tag{13}
\end{align*}
$$

$$
\begin{align*}
& +\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l_{1}}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{i}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}  \tag{14}\\
& \times \frac{1}{\left(m_{k D}-1\right)!}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}}(-1)^{i}\left\{1-e^{-z\left(\frac{m_{k D}}{\rho \beta_{k D}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)}\right.
\end{align*}
$$

$$
\left.\left.\times \sum_{v=1}^{w_{k D}} z^{w_{k D}-v} \frac{\left(w_{k D}-1\right)!}{\left(w_{k D}-v\right)!}\left(\frac{m_{k D}}{\rho \beta_{k D}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-v}\right\}\right]
$$

and

$$
\begin{equation*}
=\left\{1-e^{-\frac{m_{S k}}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \tag{15}
\end{equation*}
$$

$$
\times\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l_{1}}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{i j}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}\right.
$$

$$
\begin{equation*}
\left.\times \frac{\left(w_{k D}-1\right)!}{\left(m_{k D}-1\right)!}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}}(-1)^{i}\left(\frac{m_{k D}}{\rho \beta_{k D}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-w_{k D}}\right] \tag{16}
\end{equation*}
$$

where $w_{k D}=m_{k D}+\sum_{q \in L_{i}}\left(\eta_{q}-n_{q}\right)$. The detailed derivations to obtain (14) and (16) are shown in Appendix A.

Finally, inserting (10) and (12) into (5) when $W_{k}=\gamma_{S k}$, and using (14) and (16) as substitutes in (5) when $W_{k}=\gamma_{k D}$, a closed-form expression of the exact outage probability for DF relaying systems with e-PRS is obtained in non-identical Nakagami- $m$ fading channels. For DF relaying systems with PRS, the exact outage probability is easily obtained by inserting only (10) and (12) for all $k$ into (5).

## IV. Asymptotic Performance Analysis

To simplify the closed-form expression of the exact outage probability for DF relaying systems using e-PRS and to analyze its diversity order, the asymptotic outage probability is derived in this section. Using high SNR approximation, (10) can be approximated as

$$
\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}}\left(\frac{1}{m_{k D}!}\right)\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{1}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{i}=1}^{\eta_{l_{i}}} \frac{\left(w_{S k}-1\right)!}{\left(m_{S k}-1\right)!}\right.
$$

$$
\begin{aligned}
& \operatorname{Pr}\left\{\gamma_{S k}<z\right\} \operatorname{Pr}\left\{\gamma_{k D}>W_{1}, \cdots, \gamma_{k D}>W_{k-1},\right. \\
& \left.\gamma_{k D}>W_{k+1}, \cdots, \gamma_{k D}>W_{K}\right\} \\
& =\left\{1-e^{-\frac{m_{s k}}{\rho p_{k S}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \\
& \times \int_{0}^{\infty} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{m x_{k}}}{\rho \beta_{k D}}}\right\} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1} e^{-\frac{\eta_{k i}}{\rho_{i j}}}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k},
\end{aligned}
$$

$$
\begin{align*}
& \times\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}\left(\frac{m_{S k}}{\beta_{S k}}\right)^{m_{S k}}(-1)^{i} \\
& \left.\times\left(\frac{m_{S k}}{\beta_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\lambda_{q}}\right)^{-w_{S k}}\right], \tag{17}
\end{align*}
$$

where we use the following approximations:

$$
\begin{gather*}
1-e^{-\frac{m_{D D} D}{\rho \beta_{k D}} \sum_{i=1}^{m_{k D}}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}-i} \frac{1}{\left(m_{k D}-i\right)!} \stackrel{\rho \rightarrow \infty}{\approx}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}}\left(\frac{1}{m_{k D}!}\right),}  \tag{18}\\
e^{-\frac{m_{S z}}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!} \stackrel{\rho \rightarrow \infty}{\approx} 1,  \tag{19}\\
e^{-z\left(\frac{m_{S k}}{\rho \rho_{S k}+} \sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)} \stackrel{\rho \rightarrow \infty}{\approx 1,} \tag{20}
\end{gather*}
$$

and

$$
\begin{align*}
& \left.\sum_{v=1}^{w_{S k}} z^{w_{S k}-v} \frac{\left(w_{S k}-1\right)!}{\left(w_{S k}-v\right)!}\left(\frac{m_{S k}}{\rho \beta_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-v}\right] \stackrel{\stackrel{\rho \rightarrow \infty}{\approx}\left(w_{S k}-1\right)!}{\times\left(\frac{m_{S k}}{\rho \beta_{S k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)^{-w_{S k}} .} .
\end{align*}
$$

The approximations in (18) and (19) are obtained from [11]. Also, using $e^{a / \rho} \approx 1$ for real number $a$ when $\rho \rightarrow \infty$, (11) can be approximated as

$$
\begin{align*}
& \int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1}\right\} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k} \\
& =\frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}}\right\} \\
& \times \int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}} x_{k}^{m_{S k}-1}\left(\prod_{i=1, i \neq k}^{K} x_{i}^{\eta_{i}-1}\right) d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k} \\
& =\left(\frac{z}{\rho}\right)^{m_{S k}+\sum_{i=1, i \neq k}^{K}} \eta_{i}\left(m_{S k}+\sum_{i=1, i \neq k}^{K} \eta_{i}\right)^{-1} \frac{1}{\left(m_{S k}-1\right)!}\left(\frac{m_{S k}}{\beta_{S k}}\right)^{m_{S k}} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\eta_{i}!}\left(\frac{\eta_{i}}{\lambda_{i}}\right)^{\eta_{i}}\right\} . \tag{22}
\end{align*}
$$

Then, for $W_{k}=\gamma_{S k}$, the sum of (17) and (22) is approximated as (17) since the order of $1 / \rho$ in (22) is larger than that in (17).

Analogous to (22) and (17), (13) and (16) for $W_{k}=\gamma_{k D}$ are respectively approximated as

$$
\begin{align*}
& \int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}}\left\{\frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1}\right\} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}} x_{i}^{\eta_{i}-1}\right\} d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k} \\
& =\frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\Gamma\left(\eta_{i}\right)}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}}\right\} \\
& \times \int_{0}^{z} \int_{0}^{x_{k}} \cdots \int_{0}^{x_{k}} x_{k}^{m_{k D}-1}\left(\prod_{i=1, i \neq k}^{K} x_{i}^{\eta_{i}-1}\right) d x_{1} \cdots d x_{k-1} d x_{k+1} \cdots d x_{K} d x_{k} \\
& \left.=\left(\frac{z}{\rho}\right)^{m_{k D}+\sum_{i=1, i \neq k}^{K} \eta_{i}} m_{k D}^{+} \sum_{i=1, i \neq k}^{K} \eta_{i}\right)^{-1} \frac{1}{\left(m_{k D}-1\right)!}\left(\frac{m_{k D}}{\beta_{k D}}\right)^{m_{k D}} \\
& \left.\times \prod_{i=1, i \neq k}^{K}\left\{\frac{1}{\eta_{i}!} \frac{\eta_{i}}{\lambda_{i}}\right)^{\eta_{i}}\right\} \tag{23}
\end{align*}
$$

and

$$
\begin{align*}
& \left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}}\left(\frac{1}{m_{S k}!}\right)\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l_{1}}=1}^{\eta_{l_{1}}} \cdots \sum_{n_{l_{i}}=1}^{\eta_{l_{i}}} \frac{\left(w_{k D}-1\right)!}{\left(m_{k D}-1\right)!}\right. \\
& \times\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}\left(\frac{m_{k D}}{\beta_{k D}}\right)^{m_{k D}}(-1)^{i} \\
& \left.\times\left(\frac{m_{k D}}{\beta_{k D}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\lambda_{q}}\right)^{-w_{k D}}\right] . \tag{24}
\end{align*}
$$

Thus, for $W_{k}=\gamma_{k D}$, the sum of (23) and (24) is approximated as (24) as the order of $1 / \rho$ in (23) is larger than that in (24).

Finally, using (17) and (24) as substitutes in (5) and omitting the high order terms of $1 / \rho$, the asymptotic outage probability is obtained by

$$
\begin{align*}
P_{o u t}^{D F}(R) & \stackrel{\rho \rightarrow \infty}{\approx}\left(\frac{\mu^{*} z}{\rho}\right)^{\mu^{*}}\left(\frac{1}{\mu^{*}!}\right) \sum_{k \in M}\left(\frac{1}{\zeta_{k}}\right)^{\mu^{*}}\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l}=1}^{\eta_{l_{1}}}\right. \\
& \ldots \sum_{n_{l_{i}}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\} \frac{\left(w_{k}-1\right)!}{\left(\eta_{k}-1\right)!} \\
& \left.\times\left(\frac{\eta_{k}}{\lambda_{k}}\right)^{\eta_{k}}(-1)^{i}\left(\frac{\eta_{k}}{\lambda_{k}}+\sum_{q \in L_{i}} \frac{\eta_{q}}{\lambda_{q}}\right)^{-w_{k}}\right] \tag{25}
\end{align*}
$$

where $\varsigma_{k}=\beta_{k D}$ and $w_{k}=w_{S k}$ when $\beta_{S k}<\beta_{k D}$, but $\varsigma_{k}=\beta_{S k}$ and $w_{k}=w_{k D}$ when $\beta_{S k}>\beta_{k D}$. Moreover, $\mu^{*}=\min _{k=1, \ldots, K}\left\{\mu_{k}\right\}$, where $\mu_{k}=m_{k D}$ for $\beta_{S k}<\beta_{k D}$ and $\mu_{k}=m_{S k}$ for $\beta_{S k}>\beta_{k D}$, and $M$ denotes
the set of $k$ 's that satisfy $\mu_{k}=\mu^{*}$ for $k=1, \ldots, K$. Applying the definition of the diversity order in [12] to (25), it is recognized that the diversity order of e-PRS in DF relaying systems is $\mu^{*}$.

## V. Numerical Results

To present the performance gain of e-PRS over PRS, we consider the simulation cases in which the average channel power for the second hop is smaller than that for the first hop at some end-to-end paths, as shown in Table 1. In Case I, $\beta_{S 1} / \beta_{1 D}=-3 \mathrm{~dB}$ and $\beta_{S 2} / \beta_{2 D}=3 \mathrm{~dB}$, whereas in Case II, $\beta_{S 1} / \beta_{1 D}=-7 \mathrm{~dB}$ and $\beta_{S 2} / \beta_{2 D}=7 \mathrm{~dB}$. In Case III, $\beta_{S 1} / \beta_{1 D}=$ $\beta_{S 2} / \beta_{2 D}=-7 \mathrm{~dB}$ and $\beta_{S 3} / \beta_{3 D}=\beta_{S 4} / \beta_{4 D}=7 \mathrm{~dB}$. It implies that the average channel power gap between the first and the second hops for Case II is equal to that for Case III and is bigger than that for Case I.
In Figs. 1 through 3, it is assumed that the fading parameters for all links are equal, denoted by $m$; that is, $m_{S k}=m_{k D}=m$ for all $k$, to focus on the effect of the average channel powers on the outage performance. Figures 1 through 3 show the outage probability of e-PRS, PRS, and BRS in DF relaying systems for Cases I through III, respectively, when $R=1 \mathrm{bps} / \mathrm{Hz}$ and $m=1$ or $m=3$. In the figures, the results for BRS in DF relaying systems are obtained by simulations in which a single relay is selected by $\max _{k \in C}\left\{\gamma_{k D}\right\}$, where $C$ denotes the set of relays that succeed in decoding. The figures demonstrate that the analytic results for e-PRS and PRS perfectly match the simulated ones. Also, the figures illustrate that the outage performance for e-PRS becomes closer to that for BRS in the low to medium range of SNR values and is better than that for PRS as the integer fading parameter $m$ increases and the average channel power difference between the first hop and the second hop rises. In particular, e-PRS has a similar outage performance to that of BRS in the low to medium range of SNR values when the number of relays is small, as seen in Fig. 2. The reason is that the end-to-end link performance is increasingly dominated by the performance for the hop with the smaller average channel power between the first and the

Table 1. Description of three simulation cases.

| Case | Average channel power |  |
| :---: | :---: | :---: |
| Case I $(K=2)$ | $\beta_{S I}, \beta_{S 2}$ | $1.1,2.6$ |
|  | $\beta_{1 D}, \beta_{2 D}$ | $2.2,1.3$ |
| Case II $(K=2)$ | $\beta_{S 1}, \beta_{S 2}$ | $1.1,6.5$ |
|  | $\beta_{1 D}, \beta_{2 D}$ | $5.5,1.3$ |
|  | $\beta_{S 1}, \beta_{S 2}, \beta_{S 3}, \beta_{S 4}$ | $1.1,1.3,7.5,8.5$ |
|  | $\beta_{1 D}, \beta_{2 D}, \beta_{3 D}, \beta_{4 D}$ | $5.5,6.5,1.5,1.7$ |



Fig. 1. Outage probability comparison of e-PRS, PRS, and BRS for Case I when $K=2$ and $R=1 \mathrm{bps} / \mathrm{Hz}$.


Fig. 2. Outage probability of e-PRS, PRS, and BRS for Case II when $K=2$ and $R=1 \mathrm{bps} / \mathrm{Hz}$.
second hops as the fading parameter and the average channel power gap go up, and also the diversity order difference between e-PRS and BRS becomes smaller as the number of relays diminishes.
The diversity gain of e-PRS is analyzed according to the outage performances for various fading parameters, which are reflected in Fig. 4. In comparing the performance results for $m_{1 D}=1, m_{1 D}=2$, and $m_{1 D}=3$, it is observed that the diversity order of e-PRS is the same as $\mu^{*}=\min \left\{m_{1 D}, m_{2 D}, m_{S 3}, m_{S 4}\right\}$ given in (25). Also, the figure illustrates that the outage performances for $m_{1 D}=m_{2 D}=m_{S 3}=m_{S 4}$ are worse than those for $m_{1 D}<m_{2 D}=m_{S 3}=m_{S 4}$ since $M=\{1,2,3,4\}$ for $m_{1 D}=m_{2 D}=$ $m_{S 3}=m_{S 4}$, but $M=\{1\}$ for $m_{1 D}<m_{2 D}=m_{S 3}=m_{S 4}$; notably, the enlargement of set $M$ induces a decrease in the coding gain, as shown in (25).


Fig. 3. Outage probability of e-PRS, PRS, and BRS for Case III when $K=4$ and $R=1 \mathrm{bps} / \mathrm{Hz}$.


Fig. 4. Outage probability of e-PRS for Case III when $K=4$, $m_{S 1}=m_{S 2}=m_{3 D}=m_{4 D}=2$, and $R=1 \mathrm{bps} / \mathrm{Hz}$.

## VI. Conclusion

We provided closed-form expressions of the exact outage probability and asymptotic outage probability for e-PRS in dual-hop DF relaying systems over non-identical Nakagami-m fading channels, where the fading parameter $m$ is an integer. Also, we presented the remarkable result that e-PRS offers similar outage performance to that of BRS for a low or medium SNR, a high fading parameter, a small number of relays, and a large difference between the average channel powers for the first and the second hops. From this result, we expect that e-PRS with lower complexity than BRS can be useful and applicable in practice when all the links between the source and the relays as well as between the relays and the destination are in the line of sight.

Appendix A. Derivations of (10), (12), (14), and (16)
To derive (10), first we obtain the following equation by integrating (9) with respect to $x_{1}, \ldots, x_{k-1}, x_{k+1}, \ldots, x_{K}$.

$$
\begin{align*}
& \left\{1-e^{-\frac{m_{k D}}{\rho \beta_{k i k}}} \sum_{i=1}^{m_{k D}}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}-i} \frac{1}{\left(m_{k D}-i\right)!}\right\} \\
& \times \int_{z}^{\infty} \frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S k} x_{k}}{\rho \rho_{S k}}}  \tag{A.1}\\
& \times \prod_{i=1, i \neq k}^{K}\left\{1-\sum_{j=1}^{\eta_{i}} \frac{1}{\left(\eta_{i}-j\right)!}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}-j} x_{k}^{\eta_{i}-j} e^{-x_{k}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)}\right\} d x_{k} .
\end{align*}
$$

Then, we obtain the following equation by applying the multinomial expansion [13] into (A.1).

$$
\begin{align*}
& \left\{1-e^{-\frac{m_{m D} D^{2}}{\rho \beta_{k D}}} \sum_{i=1}^{m_{k D}}\left(\frac{m_{k D} z}{\rho \beta_{k D}}\right)^{m_{k D}-i} \frac{1}{\left(m_{k D}-i\right)!}\right\} \\
& \times \int_{z}^{\infty} \frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S k}+k}{\rho \beta_{S k}}}\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{1}=1}^{\eta_{l_{1}}}\right. \\
& \cdots \sum_{n_{i}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}(-1)^{i} x_{k}\left\{\sum_{q \in L_{i}}\left(\eta_{q}-n_{q}\right)\right\}  \tag{A.2}\\
& \left.\times e^{-x_{k}\left(\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho x_{q}}\right)}\right] d x_{k},
\end{align*}
$$

where $E_{k}=\{1, \ldots, k-1, k+1, \ldots, K\}, L_{i}$ is all possible subsets of $E_{k}$ whose cardinality is $i$, and $l_{u}$ for $u=1, \ldots, i$ denotes the $u$-th element in the set $L_{i}$. Finally, integrating (A.2) with respect to $x_{k}$ leads to (10).
To derive (12), analogous to the derivation of (A.1) and (A.2), we obtain the following equation by integrating (11) with respect to $x_{1}, \ldots, x_{k-1}, x_{k+1}, \ldots, x_{K}$ and employing the multinomial expansion [13].

$$
\begin{aligned}
& \int_{0}^{z} \frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S S} x_{k}}{\rho \beta_{S k}}} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{1-\sum_{j=1}^{\eta_{i}} \frac{1}{\left(\eta_{i}-j\right)!}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}-j} x_{k}^{\eta_{i}-j} e^{-x_{k}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)}\right\} d x_{k} \\
& =\int_{0}^{z} \frac{1}{\Gamma\left(m_{S k}\right)}\left(\frac{m_{S k}}{\rho \beta_{S k}}\right)^{m_{S k}} x_{k}^{m_{S k}-1} e^{-\frac{m_{S x_{k}}}{\rho \rho_{S k}}}\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l}=1}^{\eta_{l_{h}}}\right. \\
& \cdots \sum_{n_{i}=1}^{\eta_{i j}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}(-1)^{i} x_{k}\left\{\sum_{q L_{i}}\left(n_{q}-n_{q}\right)\right\}
\end{aligned}
$$

$$
\begin{equation*}
\left.\times e^{-x_{k}\left(\sum_{q \in L_{i}} \frac{n_{q}}{\rho_{q}}\right)}\right] d x_{k} \tag{A.3}
\end{equation*}
$$

Hence, integrating (A.3) with respect to $x_{k}$ yields (12).
To derive (14), analogous to the derivation of (A.1) and (A.2), we obtain the following equation by integrating (13) with respect to $x_{1}, \ldots, x_{k-1}, x_{k+1}, \ldots, x_{K}$ and using the multinomial expansion [13].

$$
\begin{align*}
& \left\{e^{-\frac{m_{S k z} z}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \\
& \times \int_{0}^{z} \frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{k D x_{k}}^{\rho}}{\rho \beta_{k D}}} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{1-\sum_{j=1}^{\eta_{i}} \frac{1}{\left(\eta_{i}-j\right)!}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}-j} x_{k} \eta_{i}-j_{e}^{-x_{k}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)}\right\} d x_{k} \\
& =\left\{e^{-\frac{m_{s k} \beta_{S k}}{\rho P_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \\
& \times \int_{0}^{z} \frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{k b}}{\rho \beta_{k D}}}\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{l_{1}}=1}^{\eta_{l_{1}}}\right. \\
& \ldots \sum_{n_{i}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}(-1)^{i} x_{k}\left\{\sum_{q \in L_{i}}\left(\eta_{q}-n_{q}\right)\right\} \\
& \left.\times e^{-x_{k}\left(\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho_{q}}\right)}\right] d x_{k} . \tag{A.4}
\end{align*}
$$

Therefore, integrating (A.4) with respect to $x_{k}$ leads to (14).
To derive (16), similar to the derivation of (A.1) and (A.2), we obtain the following equation by integrating (15) with respect to $x_{1}, \ldots, x_{k-1}, x_{k+1}, \ldots, x_{K}$ and adopting the multinomial expansion [13].

$$
\begin{aligned}
& \left\{1-e^{-\frac{m_{S k}}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\} \\
& \times \int_{0}^{\infty} \frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{k j k} x_{k}}{\rho k_{k D}}} \\
& \times \prod_{i=1, i \neq k}^{K}\left\{1-\sum_{j=1}^{\eta_{i}} \frac{1}{\left(\eta_{i}-j\right)!}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)^{\eta_{i}-j} x_{k}^{\eta_{i}-j} e^{-x_{k}\left(\frac{\eta_{i}}{\rho \lambda_{i}}\right)}\right\} d x_{k} \\
& =\left\{1-e^{-\frac{m_{S S z} z}{\rho \beta_{S k}}} \sum_{i=1}^{m_{S k}}\left(\frac{m_{S k} z}{\rho \beta_{S k}}\right)^{m_{S k}-i} \frac{1}{\left(m_{S k}-i\right)!}\right\}
\end{aligned}
$$

$$
\begin{align*}
& \times \int_{0}^{\infty} \frac{1}{\Gamma\left(m_{k D}\right)}\left(\frac{m_{k D}}{\rho \beta_{k D}}\right)^{m_{k D}} x_{k}^{m_{k D}-1} e^{-\frac{m_{k b} x_{k}}{\rho \beta_{k D}}}\left[1+\sum_{i=1}^{K-1} \sum_{L_{i} \subseteq E_{k}} \sum_{n_{1}=1}^{\eta_{l_{1}}}\right. \\
& \cdots \sum_{n_{l_{i}}=1}^{\eta_{l_{i}}}\left\{\prod_{j=1}^{i} \frac{1}{\left(\eta_{l_{j}}-n_{l_{j}}\right)!}\left(\frac{\eta_{l_{j}}}{\rho \lambda_{l_{j}}}\right)^{\eta_{l_{j}}-n_{l_{j}}}\right\}(-1)^{i} x_{k}\left\{\sum_{q \in L_{i}}\left(\eta_{q}-n_{q}\right)\right\} \\
& \times e^{-x_{k}\left(\sum_{q \in L_{i}} \frac{\eta_{q}}{\rho \lambda_{q}}\right)} \text { (A.5 } x_{k} . \tag{A.5}
\end{align*}
$$

Finally, integrating (A.5) with respect to $x_{k}$ yields (16).

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