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Estimation of the half-logistic distribution based on multiply Type I hybrid censored sample

Hyejung Shin[1] · Jungdae Kim[2] · Changsoo Lee[3]

[13]Department of Flight Operation, Kyungwoon University
[2]Department of Computer Information Science, Andong Science College

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Abstract

In this paper, we consider maximum likelihood estimators of the location and scale parameters for the half-logistic distribution when samples are multiply Type I hybrid censored. The scale parameter is estimated by approximate maximum likelihood estimation methods using two different Taylor series expansion types ($\hat{\sigma}_{\mathrm{I}}$, $\hat{\sigma}_{\mathrm{II}}$). We compare the estimators in the sense of the root mean square error (RMSE). The simulation procedure is repeated 10,000 times for the sample size n=20 and 40 and various censored schemes. The approximate MLE of the second type is better than that of the first type in the sense of the RMSE. Further an illustrative example with the real data is presented.

Keywords: Approximate maximum likelihood estimator, half-logistic distribution, multiply Type I hybrid censored sample, Type I hybrid censored sample.

1. Introduction and notation

Consider a half-logistic distribution with probability density function (p.d.f.)

$$g_X(x;\sigma,\mu) = \frac{2\exp\left(-\frac{x-\mu}{\sigma}\right)}{\sigma\left[1+\exp\left(-\frac{x-\mu}{\sigma}\right)\right]^2}, \quad x \ge \mu, \quad \sigma > 0, \tag{1.1}$$

and cumulative distribution function (c.d.f.)

$$G_X(x;\sigma,\mu) = \frac{1-\exp\left(-\frac{x-\mu}{\sigma}\right)}{1+\exp\left(-\frac{x-\mu}{\sigma}\right)}, \quad x \ge \mu, \quad \sigma > 0. \tag{1.2}$$

The estimation of the parameters of the half-logistic distribution based on censored samples has been investigated by many authors such as Balakrishnan and Puthenpura (1986), Balakrishnan and Wong (1991), Kang *et al.* (2008), Lee *et al.* (2011), and Lee *et al.* (2013).

[1] Assistent professor, Department of Flight Operation, Kyungwoon University, Gyeongbuk 730-739, Korea.
[2] Professor, Department of Computer Information Science, Andong Science College, Gyeongbuk 760-709, Korea.
[3] Corresponding author: Associate professor, Department of Flight Operation, Kyungwoon University, Gyeongbuk 730-739, Korea. E-mail: cslee@ikw.ac.kr

Balakrishnan and Puthenpura (1986) introduced the best linear unbiased estimators of location and scale parameters of the half-logistic distribution. Balakrishnan and Wong (1991) obtained approximate maximum likelihood estimates (MLEs) for the location and scale parameters of the half-logistic distribution with Type II right censored sample. Kang *et al*. (2008) derived the approximate MLEs and MLE of the scale parameter in a half-logistic distribution based on progressively Type II censored samples. Recently, Lee *et al*. (2011) derived the approximate MLEs of the scale parameter in a half-logistic distribution based on doubly generalized Type II hybrid censored samples. Lee *et al*. (2013) derived the Bayes estimates of the shape parameter in a exponentiated half-logistic distribution based on Type I hybrid censored samples.

Epstein (1954) introduced a Type I hybrid censoring scheme in which the test is terminated at a random time $T^* = min\{X_{r:n}, T\}$, where $X_{r:n}$ denotes the r-th ordered failure time when the sample size is n, $r \in \{1, 2, \cdots, n\}$ and $T \in (0, \infty)$ are pre-fixed. However, there are many situation in life testing experiments in which units are lost or removed from experimentation before failure. The loss may occur carelessly or unconsciously. For example, carelessly loss may occur in the case of accidental breakage of an experimental unit. So, Lee *et al*. (2014) introduced a multiply Type I hybrid censoring scheme.

This study is to consider the MLE of the location and scale parameter when the data are multiply Type I hybrid censored samples. However, MLE cannot be obtained in a closed form. We use the approximate MLEs as an approximate estimator of scale parameter. The rest of the paper is organized as follows. In section 2, we consider the multiply Type I hybrid censoring scheme. In section 3, we derive some approximate MLEs of the σ for the half-logistic distribution under the multiply Type I hybrid censored samples. The σ is estimated by approximation method using two different Taylor series expansion types.

2. Multiply Type I hybrid censoring

Under the Type I hybrid censoring scheme, suppose the experimenter fails to observe the middle l observations. Now, let

$$\text{Case I}: \{X_{a_1:n} < X_{a_2:n} < \cdots < X_{a_{s-1}:n} < X_{a_s:n} < T\} \text{ if } X_{a_s:n} < T, \tag{2.1}$$

$$\begin{aligned}\text{Case II}: &\{X_{a_1:n} < X_{a_2:n} < \cdots < X_{a_{d-1}:n} < X_{a_d:n} < T < X_{a_d+1:n} < \cdots < X_{a_s:n}\} \\ &\text{if } d < s, \text{ and } X_{a_d:n} < T < X_{a_{d+1}:n}.\end{aligned} \tag{2.2}$$

Note that, $X_{a_i:n}$ denote the a_ith failure. $a_s = r + \sum_{i=1}^{s-1} l_i$ if $r + \sum_{i=1}^{s-1} l_i < n$, and $a_s = n$ if $r + \sum_{i=1}^{s-1} l_i \geq n$. $l_i = a_{i+1} - a_i + 1$, where $i = 1, 2, \cdots, s-1$, and $l = \sum_{i=1}^{s-1} l_i$. In the case II, $X_{a_d:n} < T < X_{a_{d+1:n}}$ means that the a_dth failure took place before T, and no failure took place between $X_{a_d:n}$ and T. A schematic representation of the multiply Type I hybrid censoring scheme is presented in Figure 2.1.

Let a_1=1, the likelihood function based on equations (2.1) and (2.2) are given by
Case I

$$L \propto \prod_{i=1}^{s} g\left(x_{a_i:n}\right)\left[1 - G\left(x_{a_s:n}\right)\right]^{n-a_s} \prod_{i=2}^{s} \left[G\left(x_{a_i:n}\right) - G\left(x_{a_{i-1}:n}\right)\right]^{l_{i-1}},$$

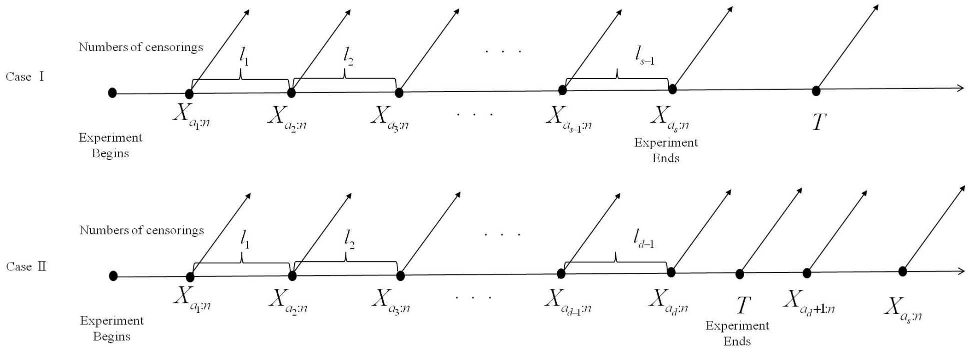

Figure 2.1 The multiply Type I hybrid censoring schemes

Case II

$$L \propto \prod_{i=1}^{d} g\left(x_{a_i:n}\right)\left[1-G\left(T\right)\right]^{n-a_d} \prod_{i=2}^{d} \left[G\left(x_{a_i:n}\right)-G\left(x_{a_{i-1}:n}\right)\right]^{l_{i-1}}.$$

Therefore, cases I and II can be combined, and can be written as

$$L \propto \prod_{i=1}^{u} g\left(x_{a_i:n}\right)\left[1-G\left(D\right)\right]^{n-a_u} \prod_{i=2}^{u} \left[G\left(x_{a_i:n}\right)-G\left(x_{a_{i-1}:n}\right)\right]^{l_{i-1}}, \tag{2.3}$$

where u denotes the number of failures; and $D = x_{a_s:n}$ if $x_{a_s:n} < T$, and $D = T$ if $x_{a_s:n} > T$.

From the equation (2.3), the likelihood function is a monotonically increasing function of μ. Thus the MLE of μ is $\hat{\mu} = X_{a_1:n}$.

Let $Z_{a_i:n} = (X_{a_i:n} - \mu)/\sigma$ and then the variables $Z_{a_i:n}$ have a standard half-logistic distribution with p.d.f. $f\left(z_{a_i:n}\right)$ and c.d.f. $F\left(z_{a_i:n}\right)$;

$$\begin{aligned} f(z) &= \frac{2\exp(-z)}{\left[1+\exp(-z)\right]^2}, \quad 0 \le z < \infty, \\ F(z) &= \frac{1-\exp(-z)}{1+\exp(-z)}, \quad 0 \le z < \infty. \end{aligned}$$

The $f(z)$, $f'(z)$, and $F(z)$ are satisfied

$$\begin{aligned} f'(z) &= -F(z)f(z), \\ f(z) &= [1-F(z)][1+F(z)]/2. \end{aligned}$$

Then, we may rewrite the above likelihood function as

$$L \propto \sigma^{-u} \prod_{i=1}^{u} f\left(z_{a_i:n}\right)\left[1-F\left(D\right)\right]^{n-a_u} \prod_{i=2}^{u} \left[F\left(z_{a_i:n}\right)-F\left(z_{a_{i-1}:n}\right)\right]^{l_{i-1}}. \tag{2.4}$$

Upon substituting for the functions $F(\cdot)$ and $f(\cdot)$, we obtain from equation (2.4) that

$$\begin{aligned}\ln L \quad &\propto \quad -u\ln\sigma + \sum_{i=1}^{u} \ln f\left(z_{a_i:n}\right) + (n-a_u)\ln\left[1-F\left(D\right)\right] \\ &\quad + \sum_{i=2}^{u} l_{i-1}\ln\left[F\left(z_{a_i:n}\right) - F\left(z_{a_{i-1}:n}\right)\right]. \qquad (2.5)\end{aligned}$$

On differentiating the log-likelihood function with respect to σ of equation (2.5) and equation to zero, we obtain the estimating equation as

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma} &= -\frac{1}{2\sigma}\left[2u - 2\sum_{i=1}^{u} F(z_{a_i:n})z_{a_i:n} - (n-a_u)(1+F(D))z_D \right. \\ &\quad \left. +2\sum_{i=2}^{u} l_{i-1}\left\{\frac{f\left(z_{a_i:n}\right)z_{a_i:n} - f\left(z_{a_{i-1}:n}\right)z_{a_{i-1}:n}}{F\left(z_{a_i:n}\right) - F\left(z_{a_{i-1}:n}\right)}\right\}\right] \\ &= 0. \qquad (2.6)\end{aligned}$$

We can find the MLE $\hat{\sigma}$ of σ that maximize the log-likelihood function in equation (2.5) by solving equation (2.6). Since equation (2.6) is very complicated, the equation does not admit an explicit solution for σ. So we use the bisection method to obtain the numerical solution of (2.6)

3. Approximate maximum likelihood estimators

Since equation (2.6) cannot be solved explicitly, it will be desirable to consider an approximation to the likelihood equations which provide us with explicit estimators for σ. See for example the work of Lee *et al.* (2014), Kwon *et al.* (2014), Lee *et al.* (2013), and Lee *et al.* (2012).

Let $\xi_{a_i:n} = F^{-1}\left(p_{a_i:n}\right) = -\ln[(1-p_{a_i:n})/(1+p_{a_i:n})]$,
where

$$p_{a_i:n} = \frac{a_i}{n+1}, \quad q_{a_i:n} = 1 - p_{a_i:n}, \quad i = 1, \cdots, u.$$

First, we expand the functions $F\left(z_{a_i:n}\right)$, $\frac{f\left(z_{a_i:n}\right)}{\left\{F\left(z_{a_i:n}\right)-F\left(z_{a_{i-1}:n}\right)\right\}}$, and $\frac{f\left(z_{a_{i-1}:n}\right)}{\left\{F\left(z_{a_i:n}\right)-F\left(z_{a_{i-1}:n}\right)\right\}}$ in Taylor series around the points $\xi_{a_i:n}$.

Then, we can approxmate the functions by

$$F\left(z_{a_i:n}\right) \simeq \alpha_{1i} + \beta_{1i}z_{a_i:n}, \qquad (3.1)$$

$$\frac{f\left(z_{a_i:n}\right)}{F\left(z_{a_i:n}\right) - F\left(z_{a_{i-1}:n}\right)} \simeq \gamma_{1i} + \delta_{1i}z_{a_i:n} + \chi_{1i}z_{a_{i-1}:n}, \qquad (3.2)$$

$$\frac{f\left(z_{a_{i-1}:n}\right)}{F\left(z_{a_i:n}\right) - F\left(z_{a_{i-1}:n}\right)} \simeq \gamma_{2i} + \delta_{2i}z_{a_i:n} + \chi_{2i}z_{a_{i-1}:n}, \qquad (3.3)$$

where

$$\begin{aligned}
\alpha_{1i} &= p_{a_i:n} - f(\xi_{a_i:n})\xi_{a_i:n} \\
\beta_{1i} &= f(\xi_{a_i:n}), \\
\gamma_{1i} &= \frac{f(\xi_{a_i:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\left[1 + p_{a_i:n}\xi_{a_i:n} + \frac{\xi_{a_i:n}f(\xi_{a_i:n}) - \xi_{a_{i-1}:n}f(\xi_{a_{i-1}:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\right], \\
\delta_{1i} &= -\frac{f(\xi_{a_i:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\left[p_{a_i:n} + \frac{f(\xi_{a_i:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\right], \\
\chi_{1i} &= \frac{f(\xi_{a_i:n})f(\xi_{a_{i-1}:n})}{(p_{a_i:n}-p_{a_{i-1}:n})^2}, \\
\gamma_{2i} &= \frac{f(\xi_{a_{i-1}:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\left[1 + p_{a_{i-1}:n}\xi_{a_{i-1}:n} + \frac{\xi_{a_i:n}f(\xi_{a_i:n}) - \xi_{a_{i-1}:n}f(\xi_{a_{i-1}:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\right], \\
\delta_{2i} &= -\frac{f(\xi_{a_i:n})f(\xi_{a_{i-1}:n})}{(p_{a_i:n}-p_{a_{i-1}:n})^2} = -\chi_{1i}, \\
\chi_{2i} &= -\frac{f(\xi_{a_{i-1}:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\left[p_{a_{i-1}:n} + \frac{f(\xi_{a_{i-1}:n})}{p_{a_i:n}-p_{a_{i-1}:n}}\right].
\end{aligned}$$

By substituting the equations (3.1), (3.2) and (3.3) into (2.6), we may approximate the equation in (2.6) by

$$\begin{aligned}
\frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^*}{\partial \sigma} = -\frac{1}{2\sigma}\Bigg[&2u - 2\sum_{i=1}^{u}(\alpha_{1i}+\beta_{1i}z_{a_i:n})z_{a_i:n} - (n-a_u)\left(1+\alpha_{1u}+\beta_{1u}z_D\right)z_D \\
&+2\sum_{i=2}^{u} l_{i-1}\{(\gamma_{1i}+\delta_{1i}z_{a_i:n}+\chi_{1i}z_{a_{i-1}:n})z_{a_i:n} - \left(\gamma_{2i}+\delta_{2i}z_{a_i:n}+\chi_{2i}z_{a_{i-1}:n}\right)z_{a_{i-1}:n}\}\Bigg] \\
&= 0. \qquad (3.4)
\end{aligned}$$

Equation (3.4) is a quadratic equation in σ, with its roots given by

$$\hat{\sigma}_{\mathrm{I}} = \frac{-B_1 + \sqrt{B_1^2 - 8uC_1}}{4u}, \qquad (3.5)$$

where

$$\begin{aligned}
B_1 &= -(n-a_u)(1+\alpha_{1u})x_D - 2\sum_{i=1}^{u}\alpha_{1i}x_{a_i:n} + 2\sum_{i=2}^{u} l_{i-1}\left(\gamma_{1i}x_{a_i:n} - \gamma_{2i}x_{a_{i-1}:n}\right) \\
&\quad + \left[(n-a_u)(1+\alpha_{1u}) + 2\sum_{i=1}^{u}\alpha_{1i} - 2\sum_{i=2}^{u} l_{i-1}\left(\gamma_{1i}-\gamma_{2i}\right)\right]\hat{\mu}, \\
C_1 &= -(n-a_u)\beta_{1u}(x_D-\hat{\mu})^2 - 2\sum_{i=1}^{u}\beta_{1i}(x_{a_i:n}-\hat{\mu})^2 \\
&\quad +2\sum_{i=2}^{u} l_{i-1}\left\{\delta_{1i}(x_{a_i:n}-\hat{\mu})^2 + 2\chi_{1i}(x_{a_i:n}-\hat{\mu})(x_{a_{i-1}:n}-\hat{\mu}) - \gamma_{2i}(x_{a_{i-1}:n}-\hat{\mu})^2\right\}.
\end{aligned}$$

Second, we expand the functions $F\left(z_{a_i:n}\right) z_{a_i:n}$ and $\frac{f\left(z_{a_i:n}\right)z_{a_i:n}-f\left(z_{a_{i-1}:n}\right)z_{a_{i-1}:n}}{F\left(z_{a_i:n}\right)-F\left(z_{a_{i-1}:n}\right)}$ in Taylor series around the points $\xi_{a_i:n}$.

Then, we can approximate the functions by the equations,

$$\begin{aligned} F\left(z_{a_i:n}\right) z_{a_i:n} &\simeq \alpha_{2i} + \beta_{2i} z_{a_i:n}, && (3.6) \\ \frac{f\left(z_{a_i:n}\right) z_{a_i:n} - f\left(z_{z_{i-1}:n}\right) z_{a_{i-1}:n}}{F\left(z_{a_i:n}\right) - F\left(z_{a_{i-1}:n}\right)} &\simeq \gamma_{3i} + \delta_{3i} z_{a_i:n} + \chi_{3i} z_{a_{i-1}:n}, && (3.7) \end{aligned}$$

where

$$\begin{aligned} \alpha_{2i} &= -f(\xi_{a_i:n})\xi^2_{a_i:n}, \\ \beta_{2i} &= p_(a_i : n) + f(\xi_{a_i:n})\xi_{a_i:n}, \\ \gamma_{3i} &= \frac{\xi^2_{a_i:n} f(\xi_{a_i:n}) p_{a_i:n} - \xi^2_{a_{i-1}:n} f(\xi_{a_{i-1}:n}) p_{a_{i-1}:n}}{p_{a_i:n} - p_{a_{i-1}:n}} + \left[\frac{\xi_{a_i:n} f(\xi_{a_i:n}) - \xi_{a_{i-1}:n} f(\xi_{a_{i-1}:n})}{p_{a_i:n} - p_{a_{i-1}:n}}\right]^2 \\ \delta_{3i} &= \frac{f(\xi_{a_i:n})}{p_{a_i:n} - p_{a_{i-1}:n}} \left[1 - p_{a_i:n}\xi_{a_i:n} - \frac{\xi_{a_i:n} f(\xi_{a_i:n}) - \xi_{a_{i-1}:n} f(\xi_{a_{i-1}:n})}{p_{a_i:n} - p_{a_{i-1}:n}}\right], \\ \chi_{3i} &= -\frac{f(\xi_{a_{i-1}:n})}{p_{a_i:n} - p_{a_{i-1}:n}} \left[1 - p_{a_{i-1}:n}\xi_{a_{i-1}:n} - \frac{\xi_{a_i:n} f(\xi_{a_i:n}) - \xi_{a_{i-1}:n} f(\xi_{a_{i-1}:n})}{p_{a_i:n} - p_{a_{i-1}:n}}\right]. \end{aligned}$$

By substituting (3.6) and (3.7) into (2.6), we may approximate the equations in (2.6) by

$$\begin{aligned} \frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^{**}}{\partial \sigma} &= -\frac{1}{2\sigma}\Bigg[2u - 2\sum_{i=1}^{u}(\alpha_{2i} + \beta_{2i} z_{a_i:n}) - (n - a_u)(z_D + \alpha_{2u} + \beta_{2u} z_D) \\ &\quad + 2\sum_{i=2}^{u} l_{i-1}(\gamma_{3i} + \delta_{3i} z_{a_i:n} + \chi_{3i} z_{a_{i-1}:n})\Bigg] \\ &= 0. \end{aligned} \tag{3.8}$$

We can derive approximate MLE as follows;

$$\hat{\sigma}_{\mathrm{II}} = \frac{B_2 + C_2\hat{\mu}}{A_2}, \tag{3.9}$$

where

$$\begin{aligned} A_2 &= 2u - (n - a_u)\alpha_{2u} - 2\sum_{i=1}^{u}\alpha_{2i} + 2\sum_{i=2}^{u} l_{i-1}\gamma_{3i}, \\ B_2 &= (n - a_u)(1 + \beta_{2u})x_D + 2\sum_{i=1}^{u}\beta_{2i} x_{a_i:n} - 2\sum_{i=2}^{u} l_{i-1}(\delta_{3i} x_{a_i:n} + \chi_{3i} x_{a_{i-1}:n}), \\ C_2 &= -(n - a_u)(1 + \beta_{2u}) - 2\sum_{i=1}^{u}\beta_{2i} + \sum_{i=2}^{u} l_{i-1}(\delta_{3i} + \chi_{3i}). \end{aligned}$$

4. Illustrative example and simulated results

In this section, we present examples to illustrate the methods and assess the performance of estimators discussed in the previous sections.

4.1. Real data

Lawless (1982) represent failure times, in minutes, for a specific type of electrical insulation that was subjected to a continuously incresing voltage stress. The data are as follows:

12.3	21.8	24.4	-	-	46.9	70.7	75.3	95.5	98.1	138.6	151.9

For this data set, Kang *et al*. (2010) indicated that the two-parameter half logistic distribution provides a satisfactory fit. In this example, we assume that the underlying distribution of this data is the half-logistic distribution based on the multiply Type I hybrid censoring scheme (i.e.,$n = 12$, $T = 100$, $r = 11$, and $a_i = 1 \sim 3$, $6 \sim 12$). From equations (3.5), and (3.9), the approximate MLEs $\hat{\mu} = 12.3$, $\hat{\sigma} = 42.64085$, $\hat{\sigma}_{\text{I}} = 42.16391$, and $\hat{\sigma}_{\text{II}} = 40.07475$ are obtained.

4.2. Simulation results

To compare the performance of the approximate MLEs of σ, we simulated the RMSE and bias of approximate MLEs, by employing the Monte Carlo simulation method. The multiply Type I hybrid censored samples for sample size $n = 20$ and 40, and various censoring schemes from the standard half-logistic distribution are generated. Using this data, the RMSEs of all approximate MLEs are calculated. We mainly compare the performances of the approximate MLEs of σ, in terms of their RMSEs for various censoring schemes. The simulation results are presented in Table 4.1.

From Table 4.1, the following general observations can be made. For all approximate MLEs and MLE, the RMSEs of all estimators decrease as sample size n increases. For fixed sample size n, the RMSE increase generally as the pre-fixed number r decreases. For fixed sample size n and pre-fixed number r, the RMSE decrease generally as the number of pre-fixed time T increases. From Table 4.1, we observed that the $\hat{\sigma}_{\text{I}}$ and $\hat{\sigma}_{\text{II}}$ are generally more efficient than the MLE $\hat{\sigma}$. The estimator $\hat{\sigma}_{\text{II}}$ is generally more efficient than the $\hat{\sigma}_{\text{I}}$.

5. Conclusions

In this paper, we consider the estimation for the half-logistic distribution based on multiply Type I hybrid censored sample. We provide the MLE of the μ, approximate MLEs of the σ, and it can be obtained explicitly. We compare the performance of the proposed estimators by Monte Carlo simulations, $\hat{\sigma}_{\text{II}}$ work quite well.

Table 4.1 The RMSEs, and biases of the μ and σ

n	T	a_i	r	RMSE (Bias) $\hat{\mu}$	$\hat{\sigma}$	$\hat{\sigma}_{\text{I}}$	$\hat{\sigma}_{\text{II}}$
20	1.5		17	.2619 (.2619)	.4114 (.0930)	.2605 (-.0145)	.2515 (-.0300)
		1∼4, 6∼20	15		.4140 (.0907)	.2631 (-.0163)	.2542 (-.0317)
			13		.4197 (.0822)	.2712 (-.0240)	.2630 (-.0394)
			17		.6126 (.1944)	.3757 (.0717)	.3457 (-.0125)
		1∼4, 8∼20	15		.6126 (.1943)	.3757 (.0717)	.3458 (-.0125)
			13		.6142 (.1921)	.3774 (.0699)	.3477 (-.0142)
			17		.9521 (.3514)	.9176 (.2566)	.8644 (.0302)
		1∼2, 8∼20	15		.9521 (.3514)	.9176 (.2566)	.8644 (.0302)
			13		.9521 (.3513)	.9176 (.2566)	.8644 (.0302)
			17		.9913 (.4015)	.9839 (.2372)	.9295 (.1082)
		1∼2, 5∼6, 10∼20	15		.9913 (.4015)	.9839 (.2372)	.9295 (.1082)
			13		.9913 (.4015)	.9839 (.2372)	.9295 (.1082)
	1.8		17		.3711 (.0780)	.2339 (-.0183)	.2292 (-.0283)
		1∼4, 6∼20	15		.3769 (.0715)	.2403 (-.0235)	.2359 (-.0334)
			13		.3826 (.0556)	.2515 (-.0382)	.2488 (-.0488)
			17		.4193 (.1474)	.2654 (.0458)	.2285 (-.0238)
		1∼4, 8∼20	15		.4197 (.1469)	.2658 (.0455)	.2290 (-.0240)
			13		.4242 (.1405)	.2706 (.0404)	.2354 (-.0290)
			17		.5091 (.2450)	.3588 (.1622)	.2261 (-.0267)
		1∼2, 8∼20	15		.5091 (.2450)	.3588 (.1622)	.2261 (-.0267)
			13		.5095 (.2446)	.3590 (.1618)	.2266 (-.0270)
			17		.5895 (.2412)	.3810 (.1121)	.2990 (.0104)
		1∼2, 5∼6, 10∼20	15		.5895 (.2412)	.3810 (.1121)	.2990 (.0104)
			13		.5898 (.2407)	.3813 (.1118)	.2994 (.0101)
40	1.5		35	.0085 (.0085)	.2591 (.0423)	.1723 (-.0107)	.1723 (-.0151)
		1∼10, 12∼40	30		.2596 (.0420)	.1728 (-.0109)	.1729 (-.0154)
			28		.2618 (.0403)	.1751 (-.0124)	.1752 (-.0168)
			35		.2779 (.0779)	.1856 (.0252)	.1726 (-.0111)
		1∼10, 14∼40	30		.2779 (.0779)	.1856 (.0252)	.1726 (-.0111)
			28		.2783 (.0776)	.1860 (.0250)	.1731 (-.0113)
			35		.3071 (.1207)	.2149 (.0750)	.1726 (-.0108)
		1∼8, 14∼40	30		.3071 (.1207)	.2149 (.0750)	.1726 (-.0108)
			28		.3071 (.1206)	.2149 (.0750)	.1726 (-.0109)
			35		.3056 (.1196)	.2127 (.0724)	.1780 (.0064)
		1∼7, 10∼11, 15∼40	30		.3056 (.1196)	.2127 (.0724)	.1780 (.0064)
			28		.3056 (.1196)	.2128 (.0724)	.1781 (.0064)
	1.8		35		.2393 (.0353)	.1586 (-.0123)	.1590 (-.0151)
		1∼10, 12∼40	30		.2421 (.0324)	.1616 (-.0147)	.1621 (-.0174)
			28		.2463 (.0258)	.1666 (-.0202)	.1673 (-.0231)
		0	35		.2538 (.0661)	.1678 (.0184)	.1598 (-.0113)
		1∼10, 14∼4	30		.2544 (.0656)	.1684 (.0180)	.1605 (-.0117)
			28		.2560 (.0633)	.1701 (.0161)	.1628 (-.0137)
			35		.2765 (.1028)	.1890 (.0605)	.1600 (-.0109)
		1∼8, 14∼40	30		.2766 (.1027)	.1891 (.0604)	.1602 (-.0109)
			28		.2770 (.1023)	.1894 (.0601)	.1608 (-.0113)
			35		.2754 (.1019)	.1875 (.0583)	.1637 (.0041)
		1∼7, 10∼11, 15∼40	30		.2755 (.1019)	.1875 (.0583)	.1638 (.0040)
			28		.2759 (.1015)	.1879 (.0579)	.1643 (.0037)

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