

Regime-dependent Characteristics of KOSPI Return

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Abstract

Stylized facts on asset return are fat-tail, asymmetry, volatility clustering and structure changes. This paper simultaneously captures these characteristics by introducing a multi-regime models: Finite mixture distribution and regime switching GARCH model. Analyzing the daily KOSPI return from 4th January 2000 to 30th June 2014, we find that a two-component mixture of t distribution is a good candidate to describe the shape of the KOSPI return from unconditional and conditional perspectives. Empirical results suggest that the equality assumption on the shape parameter of t distribution yields better discrimination of heterogeneity component in return data. We report the strong regime-dependent characteristics in volatility dynamics with high persistence and asymmetry by employing a regime switching GJR-GARCH model with t innovation model. Compared to two sub-samples, Pre-Crisis (January 2003 ~ December 2007) and Post-Crisis (January 2010 ~ June 2014), we find that the degree of persistence in the Pre-Crisis is higher than in the Post-Crisis along with a strong asymmetry in the low-volatility (high-volatility) regime during the Pre-Crisis (Post-Crisis).

Keywords: Finite mixture distribution, regime switching GJR-GARCH model, financial crisis, KOSPI.

1. Introduction

Stylized facts on asset returns are fat-tail, asymmetry, volatility clustering and structure changes. This paper simultaneously captures these characteristics by introducing multi-regime models: Finite mixture distribution and regime switching GARCH model (RS-GARCH). The former is known as a good candidate to model unconditional distribution of asset returns incorporating fat-tail and asymmetry. The latter has recently become attractive to market participants during to reflect regime-specific volatility dynamics. RS-GARCH (or Markov switching) models describe asset returns by mixtures of heteroscedastic volatility processes governed by an unobserved state variable, which is generally assumed to evolve according to a first-order Markov chain with two states that implies a high and low volatility regime.

The density of a finite mixture distributions is expressed as a linear combination of multiple component densities. In finance, a mixture distribution arises naturally when component densities are interpreted as different market regimes. In a two-component mixture model, the first component, with a relatively high mean and small variance, may be interpreted as the bull market regime, occurring with probability; however, the second regime represents a bear market with a lower expected return and a greater variance. For the detailed accounts of finite mixture model and its financial application, see Dempster *et al.* (1977) and Behr and Pötter (2009).

Schwert (1989) considers a model in which returns can have a high or low variance; consequently, switches between these states are determined by a two-state Markov process. Cai (1994) and Hamilton and Susmel (1994) propose the RS-ARCH model to account for the possible presence of structural

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breaks. Gray (1996) suggests a tractable RS-GARCH, *i.e.* path-independent RS-GARCH framework which permits direct estimation of all model parameters using quasi maximum likelihood (QML) techniques. Klaassen (2002) suggests a modification of Gray's approach by introducing the conditional expectation of the lagged conditional variance with a broader information set than Gray (1996). The advantage of the approach by Klaassen (2002) over other models are its high flexibility to capture the persistence of shocks to volatility, and the recursive expression for the multi-step ahead volatility forecasts. Haas *et al.* (2004) showed how to overcome the estimation difficulties for RS-GARCH models. A review of recent contributions on the estimation of RS-GARCH models can be found in Marcucci (2005) and Bauwens *et al.* (2012). For more financial applications to stock and exchange rate returns of RS-GARCH models, see Henry (2009) and Wilfling (2009).

This paper analyzes analyzing the daily KOSPI return series from 4th January 2000 to 30th June 2014. Since our analysis focuses on recent years where stock markets have undergone extreme shocks caused by the Global Financial Crisis (GFC) and European countries Debt Crisis (EDC), we construct three sub-periods, labeled as Pre-Crisis, Crisis and Post-Crisis. The time periods of each sub-sample consist on 2nd January 2003 ~ 28th December 2007 (Pre-Crisis), 2nd January 2008 ~ 30th December 2009 (Crisis) and 4th January 2010 ~ 30th June 2014 (Post-Crisis), respectively. To examine the distributional shape of asset return, we consider finite mixture models based on skew-elliptical distributions such as normal, Student's t , skew-normal and skew- t distributions. Empirical results show that two-component mixture of t distribution is a good candidate to describe the shape of an unconditional KOSPI. In addition, we suggest that the equality assumption on the shape parameter of t mixture distribution yields a superior discrimination of heterogeneity component.

Moving to volatility dynamics, our results show the strong persistence and asymmetry in volatility dynamics, regardless of GJR-GARCH innovations and sub-sample periods. Furthermore, we find the significant regime-dependent characteristics in volatility dynamics with high persistence and asymmetry, by examining RS-GJR-GARCH with t distribution model. The estimated shape parameter reveals the heavy tails for the conditional distribution of KOSPI returns. The estimated transition probabilities are very close to one, implying infrequent mixing between states. Compared to two sub-samples, Pre-Crisis and Post-Crisis, the degree of persistence in Pre-Crisis is higher than that in the Post-Crisis, and we find strong asymmetry in the low-volatility regime during the Pre-Crisis; an addition, this phenomenon is evident in the high-volatility regime in Post-Crisis.

The rest of this paper is organized as follows. Section 2 briefly explains the finite mixture model with normal, Student's t , skew-normal and skew- t distributions. Section 3 provides the specification of RS-GJR-GARCH(1,1) with t innovation model. Section 4 presents empirical results and discusses our findings. Section 5 concludes.

2. Finite Mixture Distributions

The general representation of K -component mixture distribution is given as

$$g(r) = \pi_i f_i(r; \theta_i) \quad (2.1)$$

where π_i denotes the mixing weight satisfying $0 \leq \pi_i \leq 1$ and $f_i(x; \theta_i)$ denotes the probability density function (pdf) of one of skew-elliptical distributions such as normal, Student's t , skew-normal and skew- t distributions with parameter vector θ_i , for $i = 1, \dots, K$.

Let r_1, \dots, r_T be observed return series, and the log-likelihood of finite mixture distribution is

given by

$$\ln L(\Theta) = \sum_{t=1}^T \log \left(\sum_{i=1}^K \pi_i f_i(r_t; \theta_i) \right) \tag{2.2}$$

where $\Theta = (\pi_1, \dots, \pi_K, \theta_1, \dots, \theta_K)$ contains all unknown parameters of finite mixture distribution. There is generally no explicit analytical solution for the log-likelihood function of equation (2.2); consequently, the computation of the maximum likelihood estimates of the parameters in finite mixture model is typically achieved through numerical iteration such as an expectation maximization (EM) algorithm by Dempster *et al.* (1977). The empirical analysis is conducted using R package *gamlss.dist*.

3. Regime Switching GJR-GARCH Model

The conditional distribution of r_t in two-state regime switching (RS) model is expressed as a mixture of two distributions, *i.e.*

$$r_t | F_{t-1} \sim \begin{cases} f(\theta_t^{S_t=1}) & w.p. p_{1,t}, \\ f(\theta_t^{S_t=2}) & w.p. p_{2,t} \end{cases} \tag{3.1}$$

where F_{t-1} denotes the observed information up to time $t - 1$, $f(\cdot)$ is the pdf with parameter vector $\theta_t^{S_t=i}$, for $i = 1, 2$, in the i -th regime and *w.p.* stands for with probability. The $p_{i,t} = \Pr(S_t = i | F_{t-1})$ is called as the ex-ante probability of being in i -th regime at time t , and $p_{1,t} = 1 - p_{2,t}$ in two-state RS model. The ex-ante probability is based solely on information already available at time $t - 1$ and forecasts the prevailing regime in the next period.

To capture the conditional fat-tailedness of asset return, we use Student's t distribution in this paper. Thus, the conditional density of r_t in state $S_t = i$ with given $\theta_t^{S_t=i}$ and F_{t-1} is given by

$$f(r_t | S_t = i, \theta_t^{S_t=i}, F_{t-1}) = \frac{\Gamma\left(\frac{\nu_{i,t}+1}{2}\right)}{\Gamma\left(\frac{\nu_{i,t}}{2}\right) \sqrt{\pi(\nu_{i,t}-2)\sigma_{i,t}^2}} \left[1 + \frac{(r_t - \mu_{i,t})^2}{(\nu_{i,t}-2)\sigma_{i,t}^2} \right]^{-\frac{\nu_{i,t}+1}{2}} \tag{3.2}$$

where $\Gamma(\cdot)$ denotes the Gamma function. The formula (3.2) means that r_t follows a Student's t distribution with conditional mean $\mu_{i,t}$, variance $\sigma_{i,t}^2$ and shape parameter $\nu_{i,t}$, in each state i . The parameters in each state are specified as

$$\mu_{i,t} = \mu, \tag{3.3}$$

$$\sigma_{i,t}^2 = w_i + \{\alpha_i^+ I_{\{\varepsilon_{t-1} \geq 0\}} + \alpha_i^- I_{\{\varepsilon_{t-1} < 0\}}\} \varepsilon_{t-1}^2 + \beta_i \sigma_{i,t-1}^2, \tag{3.4}$$

$$\nu_{i,t} = \nu_i \tag{3.5}$$

where $\varepsilon_t = r_t - \mu$ and the indicator function $I_{\{\cdot\}}$ is equal to one if the constraint holds and zero otherwise. Developed by Glosten *et al.* (1993), the time-varying variance in equation (3.4) is known as GJR-GARCH(1,1) and is a popular asymmetric GARCH models. We note that there exists asymmetry (or leverage effect) if $\alpha_i^- > \alpha_i^+$, meaning that negative innovations generate more volatility than positive innovations of equal magnitude. In addition, there exists a bull market and a bear market in stock returns if the parameters have significant differences between regimes.

The conditional variance σ_t^2 of RS-GJR-GARCH(1,1) model is probability-weighted average of conditional variances from both regimes, *i.e.*

$$\sigma_t^2 = p_{1,t}\sigma_{1,t}^2 + p_{2,t}\sigma_{2,t}^2 \quad (3.6)$$

The unconditional variance of each state is given as

$$\bar{\sigma}_i^2 = \frac{w_i}{1 - \left(\frac{\alpha_i^+ + \alpha_i^-}{2} + \beta_i\right)} \quad (3.7)$$

for $i = 1, 2$, provided that $(\alpha_i^+ + \alpha_i^-)/2 + \beta_i < 1$.

The state variable $\{S_t\}$ is assumed to evolve according to a first-order Markov chain with transition matrix

$$P = (p_{ij}) = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \quad (3.8)$$

where $p_{ij} = \Pr(S_{t+1} = j | S_t = i)$, for $i, j = 1, 2$, is the transition probability of moving from state i at time t to state j at time $t + 1$. The ergodic probability (unconditional probability) of being in state $S_t = 1$ is given by $p_1 = (1 - p_{11})/(2 - p_{11} - p_{22})$. We note that a large p_{ii} indicates that the return series has more tendency to stay in the i -th state, equivalently implying less mobility.

The log-likelihood function corresponding to RS-GJR-GARCH with the t innovation model given by

$$L(\theta) = \sum_{t=2}^T \ln f(r_t | \theta, F_{t-1}) \quad (3.9)$$

where $\theta = (w_1, w_2, \alpha_1^-, \alpha_2^-, \alpha_1^+, \alpha_2^+, \beta_1, \beta_2, \nu_1, \nu_2, p_{11}, p_{22})$ and the conditional density is derived as

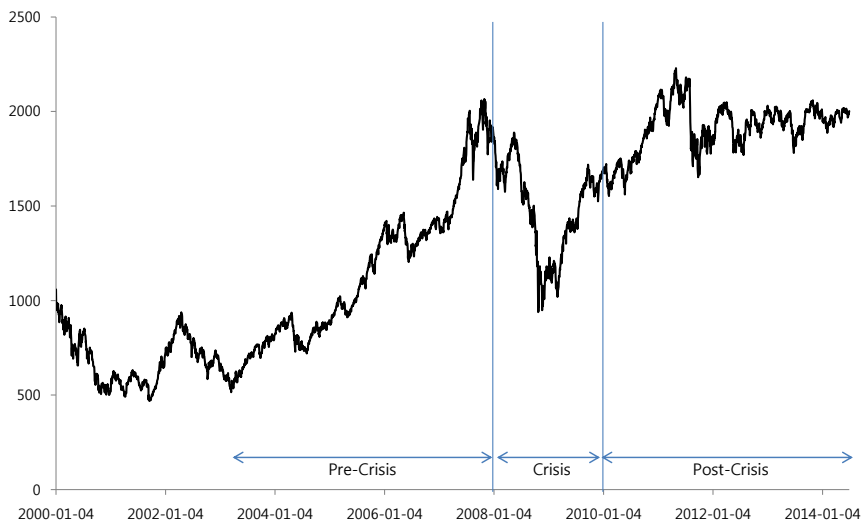
$$f(r_t | \theta, F_{t-1}) = \sum_{i=1}^2 \sum_{j=1}^2 p_{ij} \eta_{i,t-1} f(r_t | S_t = i, \theta_t^{S_t=i}, F_{t-1}) \quad (3.10)$$

where $\eta_{i,t-1} = \Pr(S_t = i | \theta_t^{S_t=i}, F_{t-1})$ is the filtered probability of the i -th state at time $t - 1$, which is obtained by an iterative algorithm, known as Hamilton filter (Hamilton, 1989). The maximum likelihood estimator $\hat{\theta}$ is obtained by maximizing equation (3.9). We set the following restrictions $\nu_i > 2$, $w_i > 0$ and $\alpha_i^-, \alpha_i^+, \beta_i \geq 0$ to obtain finite and positive conditional variances along with $(\alpha_i^+ + \alpha_i^-)/2 + \beta_i < 1$ for positive unconditional variance. We require the transition probabilities of the state variable to lie within the unit interval. The estimation of RS-GJR-GRACH models are obtained using R package *DEoptim*, developed by Ardia and Mullen (2010).

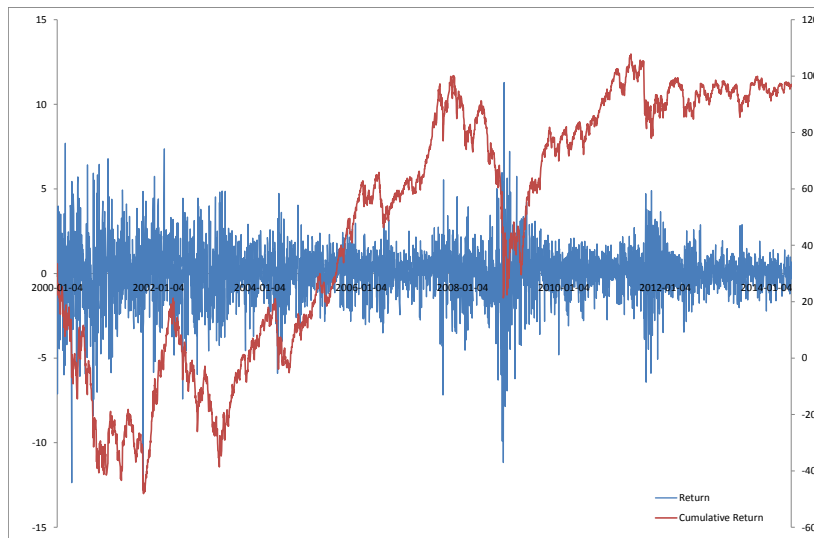
4. Empirical Analysis

4.1. Data

The empirical data consists of the daily return series of Korean stock market index (KOSPI) from the 4th January 2000 to 30th June 2014, totally 3,582 observations. The KOSPI is issued by the Korea Stock Exchange and obtained from <http://www.krx.co.kr>. The continuous compound rate of return is calculated as $r_t = 100 \times \ln(p_t/p_{t-1})$, where p_t stands for the daily closing index at time t .



(a) Level



(b) Return and Cumulative Return

Figure 1: Time plots of KOSPI and its return

Note: Daily index (levels) of the KOSPI in Panel (a), and the daily return and cumulative returns of KOSPI in panel (b) for the period from 4th January 2000 to 30th June 2014.

The panel (a) in Figure 1 depicts KOSPI at level, and the panel (b) in Figure 1 displays the log-return and cumulative return over times. The KOSPI shows a downward trend from January 2000 to December 2001, due to the aftermath of Asian Currency Crisis. We observe an outstanding upward momentum after the cessation of market turmoil, starting from the beginning of 2003 and lasting

Table 1: Summary statistics

	Full-sample	Pre-Crisis	Crisis	Post-Crisis
N	3,582	1,238	500	1,112
Mean	0.0186	0.0894	-0.024	0.0156
SD	1.6693	1.3702	2.0637	1.0993
Skewness	-0.5586	-0.3974	-0.4565	-0.4187
Kurtosis	8.4853	4.9316	8.3217	6.5738
JB	4677.03***	255.05***	607.39***	624.26***
LB(10)	13.14	15.88	4.17	22.09**
LB ² (10)	1075.94***	241.43***	326.12***	546.58***

Note: N and SD respectively denote the number of observations and standard deviation. JB and LB(10) stand for the Jarque-Bera and Ljung-Box Q statistic with order=10, respectively. The former is for normality test and the latter is for autocorrelation test. The LB² is Ljung-Box statistic using squared returns to examine autocorrelation in second order perspective. The *** and ** denotes statistically significant at 1% and 5% significance level, respectively.

almost five years until December 2007. As many studies point out, there was the extreme downward jump in the late of 2009 caused by GFC. After GFC, KOSPI has a two-year increasing momentum from 2010 ~ 2011. In October 2011, there is a short-live downside spike related to EDC, and then KOSPI stably wanders around 1900 level after 2012. Based on this result, we separate full-sample period into three sub-samples, named as Pre-Crisis, Crisis and Post-Crisis. The Pre-Crisis spans from the 2nd January 2003 ~ 28th December 2007 with 1,238 observations, and it is a representative period of upward market. The Crisis period is covered from the 2nd January 2008 ~ 30th December 2009 with 500 observations and Post-Crisis consists of the 4th January 2010 ~ 30th June 2014 with 1,112 observations. We drop out 732 daily returns from January 2000 to December 2002, to compare upward market (Pre-Crisis) with stable market (Post-Crisis) and analyze the impact on GFC with clear distinction and similar the number of observations in each sub-sample.

Table 1 provides the summary statistics of daily KOSPI return in four sample periods; full-sample, Pre-Crisis, Crisis and Post-Crisis. The mean returns are ordered as Pre-Crisis (0.0894), Post-Crisis (0.0156) and Crisis (-0.024), whereas the standard deviations are ordered as Crisis (2.0637), Pre-Crisis (1.3702) and Post-Crisis (1.0993). This result indicates that Korean stock market suffers from the negative impact by GFC with low-return and high-risk. As is typical for many index returns, we find negative skewness and strong excess kurtosis, especially the GFC period shows the extremely higher standard deviation and kurtosis. The Jarque-Bera statistics, denoted by JB in Table 1, for normality test confirms the non-normality of a KOSPI return at 1% significance level, regardless of sample periods. In addition, we test the presence of autocorrelation in return and squared return using Ljung-Box statistics, denoted by LB and LB² in Table 1, respectively. The null hypotheses of no autocorrelation are rejected in all cases of squared return series at the 1% significance level, implying strong ARCH effect.

4.2. Estimation results of finite mixture distributions

To examine the distributional shape of the KOSPI return in each sub-period, we fit single-, two- and three-component mixture distributions and provide Bayesian information criterion (BIC) of each model in Table 2. The best candidates, *i.e.* the smallest BIC, in each period among 12 candidates are reported with bold style, and the best models in each subset, grouped by the number of components, are reported with underlined style. In Pre-Crisis, skew-*t* is the best in single-component, two- and three-component mixtures of *t* distributions conclude the best candidate in mixture cases. Among them, the best model is two-component mixtures of *t* distribution, which has the smallest BIC (4220.71). In Crisis, the best candidate is single-component Student's *t* distribution, which indicates

Table 2: BIC of finite mixture distributions

No. of Component	Distribution	Pre-Crisis	Crisis	Post-Crisis
1	Normal	4299.32	2148.63	3372.30
	Student's <i>t</i>	4221.69	2045.56	<u>3239.02</u>
	Skew-normal	4288.67	2150.46	3368.32
	Skew- <i>t</i>	4214.25	2048.27	3244.36
2	Normal	4221.49	2066.29	3261.13
	Student's <i>t</i>	4220.71	<u>2059.07</u>	3257.26
	Skew-normal	4243.89	2100.49	3290.41
	Skew- <i>t</i>	4227.96	2070.77	3271.40
3	Normal	4235.34	2074.80	<u>3276.53</u>
	Student's <i>t</i>	<u>4232.43</u>	2077.65	3277.6
	Skew-normal	4265.58	2101.15	3301.89
	Skew- <i>t</i>	4253.88	2095.15	3297.74

Note: The pdf of each distribution is given as follows.

1) Normal distribution: $f_N(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(y - \mu)^2/2\sigma^2)$

2) Student's *t* distribution: $f_t(y; \mu, \sigma^2, \nu) = \frac{1}{\sigma B(1/2, \nu/2)} (1 + (y - \mu)^2/\nu\sigma^2)^{-(\nu+1)/2}$, where $B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a + b)$ denotes beta function.

3) Skew-normal distribution: $f_{SN}(y; \mu, \sigma^2, \delta) = 2f_N(z)F_N(\delta z)$, where $z = y - \mu/\sigma$ and $F_N(\cdot)$ denotes the distribution function of normal distribution.

4) Skew-*t* distribution: $f_{ST}(y; \mu, \sigma^2, \nu, \delta) = \frac{2}{\sigma} f_t(z, \nu) F_t(\sqrt{(1 + \nu)/d^{2\nu}} \delta z, 1 + \nu)$, where $F_t(\cdot)$ denotes the distribution function of Student's *t* distribution.

Table 3: Parameter estimates of two-component Student's *t* mixture distribution

Parameter	Pre-Crisis		Crisis		Post-Crisis	
	$\nu_1 = \nu_2$	$\nu_1 \neq \nu_2$	$\nu_1 = \nu_2$	$\nu_1 \neq \nu_2$	$\nu_1 = \nu_2$	$\nu_1 \neq \nu_2$
μ_1	0.3362	0.3728	0.323077	0.3399	0.1024	0.1355
σ_1	0.9394	0.7852	1.480059	1.0805	0.6281	0.4417
ν_1	4.2478	5.5490	1.100966	2.9156	8.7052	4.4542
μ_2	-0.2646	-0.9682	-0.37594	-1.0497	-0.0753	-0.1396
σ_2	1.7528	1.8933	2.612111	2.7245	1.1736	1.0108
ν_2	13.1049	5.5490	2.012594	2.9156	5.9939	4.4542
π_1	0.5861	0.7654	0.5256	0.7384	0.5529	0.5926
BIC	4195.98	4220.71	2070.23	2059.07	3270.82	3257.26

little possibility of heterogeneity in data. The best model in Post-Crisis is two-component mixture of *t* distribution with 3257.26 BIC. Our empirical results suggest that the two-component mixture of *t* distribution is a good candidate to model KOSPI returns from an unconditional perspective.

Table 3 provides the parameter estimates of two-component mixture of *t* distribution are given in. We present two results in each period, expending on the specification of shape parameters in each regime, i.e. $\nu_1 = \nu_2$ and $\nu_1 \neq \nu_2$. The estimated values of μ_1 and μ_2 are similar in both specifications in Pre-Crisis; however, the estimates of both shape parameter and mixing proportions are remarkably different. When $\nu_1 \neq \nu_2$, the estimates ν_1 and ν_2 are 4.23 and 13.10, respectively, which suggests the fat-tailed shape in first regime. The estimated mixing proportion π_1 is 0.77 (0.59) when $\nu_1 = \nu_2$ ($\nu_1 \neq \nu_2$), i.e. two states (high-low) are more evident when $\nu_1 = \nu_2$. The smaller BIC yields when $\nu_1 \neq \nu_2$. Unlike Pre-Crisis, all estimated values seem somewhat similar in Post-Crisis, except for the shape parameter estimates. The estimated ν_1 and ν_2 are 6.00 and 8.70 when $\nu_1 \neq \nu_2$, whereas the estimated shape parameter is 4.45 when $\nu_1 = \nu_2$. The mixing proportion estimates are similar; 0.59 ($\nu_1 = \nu_2$) versus 0.55 ($\nu_1 \neq \nu_2$). In addition, BIC is smaller when $\nu_1 = \nu_2$. Based on our results, we suggest that the specification of yields improve the discrimination of the heterogeneity component, whereas model fit is complicate to reach unanimous conclusion.

Table 4: Estimation results of GARCH and GJR-GARCH models

Parameter	Pre-Crisis		Crisis		Post-Crisis	
	μ	0.1650***	0.1354***	0.0762	0.0285	0.0487
w	0.0350***	0.0773***	0.0402	0.0566	0.0182**	0.0248**
α^+	0.0783***	0.0000	0.0707***	0.0000	0.0769***	0.0000
α^-	N.A.	0.1614***	N.A.	0.1356***	N.A.	0.1451***
β	0.9046***	0.8690***	0.9178***	0.9091***	0.9063***	0.9011***
ν	7.9217***	9.5571***	6.1602***	6.9528***	10.5821***	13.0305***
Persistence	0.9829	0.9497	0.9885	0.9769	0.9832	0.97365
BIC	3.3329	3.3229	3.9620	3.9439	2.7867	2.7583

Note: N.A. stands for Not Available. The *** and ** denotes the statistically significant at 1% and 5% significance level, respectively.

Table 5: Estimation results of RS-GJR-GARCH models

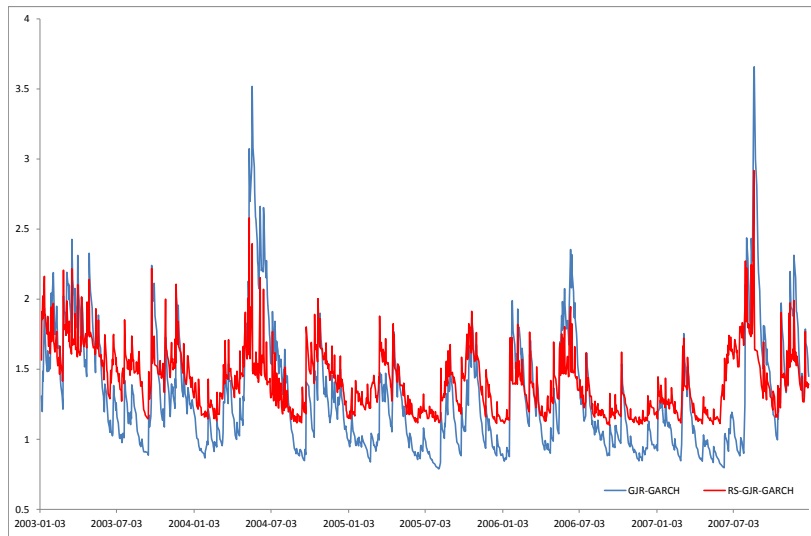
Parameter	Pre-Crisis		Post-Crisis	
	$\nu_1 = \nu_2$	$\nu_1 \neq \nu_2$	$\nu_1 = \nu_2$	$\nu_1 \neq \nu_2$
w_1	0.0500	0.0499	0.0175	0.0362
α_1^+	0.0004	0.0000	0.0020	0.0015
α_1^-	0.1489	0.1502	0.0995	0.0816
β_1	0.8848	0.8823	0.9222	0.8876
ν_1	12.0427	12.4539	9.7558	14.5908.
w_2	0.0500	0.0497	0.0358	0.0455
α_2^+	0.0272	0.0038	0.0000	0.0000
α_2^-	0.0965	0.0843	0.1953	0.1963
β_2	0.9265	0.9435	0.8790	0.8692
ν_2	9.1569	12.4539	18.5643	14.5908
p_{11}	0.9925	0.9869	0.9987	0.9990
p_{22}	0.9817	0.9696	0.9982	0.9984
Persistence1	0.9595	0.9574	0.9730	0.9292
Persistence2	0.9884	0.9876	0.9767	0.9674
BIC	3.3151	3.3125	2.7460	2.7444

4.3. RS-GJR-GARCH model results

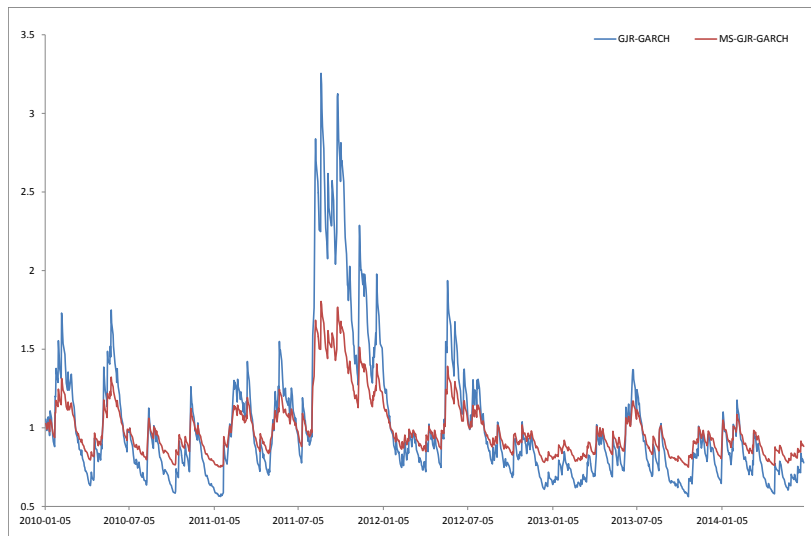
To discuss the conditional shape and asymmetric effect in volatility dynamics, we provide both the parameter estimates and BIC of the standard GARCH and GJR-GARCH with normal and Student's t innovations in Table 4. Regardless of both GARCH specifications and sampling periods, we find a strong persistence in volatility dynamics, from 0.95 to 0.99. The results suggest that the volatility is likely to remain high over several future periods once it increases. We note that there is significant asymmetric effect in stock return, since α^- , regardless of sub-samples. The estimated values of shape parameter are ordered as GFC(6.95), Pre-Crisis(9.56) and Post-Crisis(13.03), which indicates GFC has the longest tail implying the high possibility occurring extreme return or loss. The GJR-GARCH with Student's t innovation shows better in-sample fit compared to GARCH with Student's t innovation.

Table 5 presents the parameter estimates of RS-GJR-GARCH(1,1) with t innovation model. The estimated parameters show two different regimes for the conditional variance process, since the values are far apart between the regimes. Similar to single regime GJR-GARCH, we also note the presence of leverage effect in both regimes (*i.e.* for α^-), with similar levels. The estimated transition probabilities are close to one, which uncovers infrequent mixing between states. Finally, the estimated shape parameters suggest heavy tails for the conditional distribution of the log-returns.

Comparing the low and high volatility regimes in the Pre-Crisis period, it is interesting to notice that degree of volatility persistence in low-volatility regime is lower compared to the high-volatility regime, which indicates the GARCH processes in the high volatility regimes are more reactive but



(a) Pre-Crisis

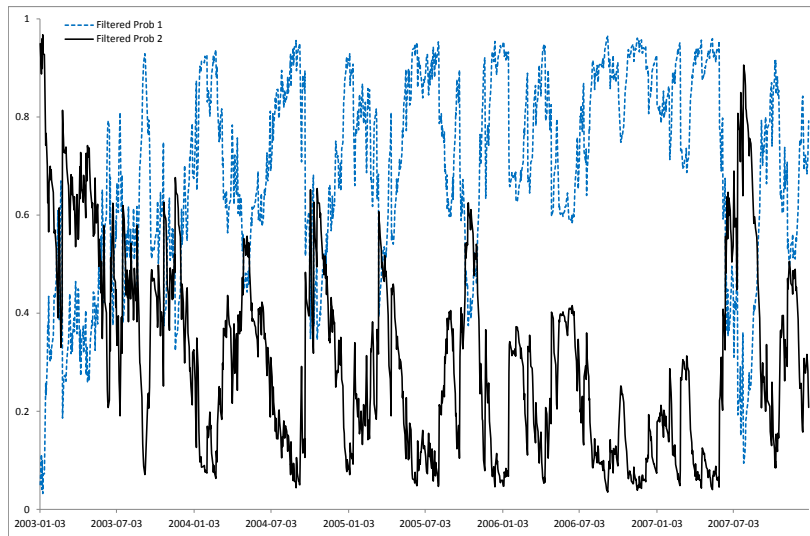


(b) Post-Crisis

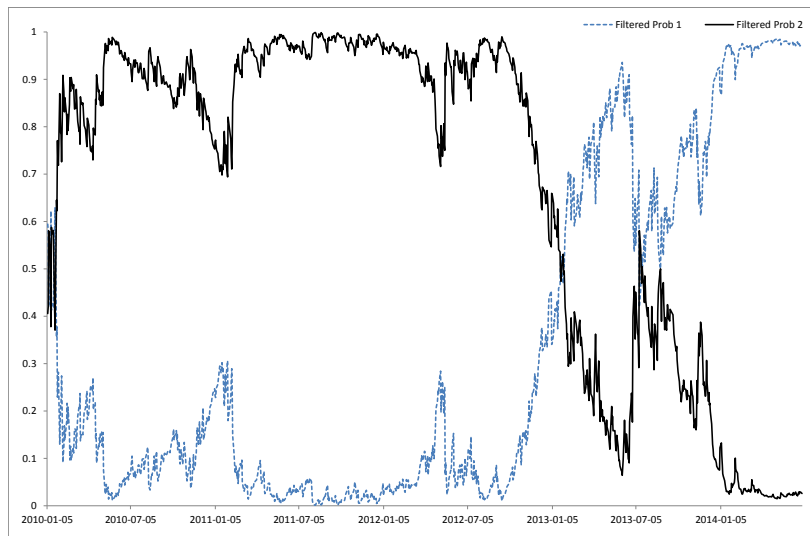
Figure 2: Conditional Volatility Dynamics

Note: The conditional volatility is obtained from the GJR-GARCH and RS-GJR-GARCH models. The y axis represents daily volatility in %.

less persistent than the low volatility regime. In addition, we see the estimated GARCH parameter in the low volatility regime is higher than that in high volatility regime, *i.e.* $\beta_1 < \beta_2$, which indicates that conditional variances of returns in the high-volatility regime exhibit higher sensitivity to recent conditional variances than in the low- volatility regime. The shape parameter estimate is 12.45 when



(a) Pre-Crisis



(b) Post-Crisis

Figure 3: Estimated filtered probability of each state

Note: Plot of estimated filtered probabilities $\eta_{i,t-1} = \Pr(S_t = i | \theta_t^{S_t=i}, F_{t-1})$ of each state during Pre- and Post-Crisis.

$\nu_1 = \nu_2$.

Moving to the Post-Crisis results, the degree of volatility persistence within low volatility regime is lower than that in the high volatility regime. Unlike Pre-Crisis, the estimated GARCH parameter in low volatility regime is lower than the high volatility regime, *i.e.* $\beta_1 > \beta_2$. The shape parameter

estimate is 14.59 when $\nu_1 = \nu_2$. The estimated transition probabilities p_{11} and p_{22} are higher than those in Pre-Crisis, which indicates the high-possibility staying at the same state in Post-Crisis.

Figure 2 depicts the time-varying volatilities obtained from GJR-GARCH(1,1) and RS-GJR-GARCH(1,1) with t innovation models. As we mentioned based on standard deviation, the volatility of the KOSPI is higher than other returns. In panel (a), we see the higher volatility levels in May 2004 and August 2007. In panel (b), the relatively higher volatility is observed in August 2011. The volatility magnitude in Post-Crisis is somewhat lower than that in Pre-Crisis.

Figure 3 displays the estimated filtered probabilities of each state in the Pre-Crisis (panel (a)) and Post-Crisis periods (panel (b)). In panel (a), the beginning of 2003 is associated with the high unconditional volatility state. Then, from January 2004 to June 2007, the returns are clearly associated with the low unconditional volatility regime. From July 2007, the model remains in the high unconditional volatility regime related to upcoming GFC. In panel (b), we see the high unconditional volatility regime from late of January 2010 to end of December 2012. After January 2013, the KOSPI returns stay in the low volatility regime to the end of our sample period.

5. Conclusion

This paper simultaneously captures unique characteristics by introducing regime dependent models. Analyzing the daily KOSPI return from 4th January 2000 to 30th June 2014, we find that two-component mixture of t distribution is a good candidate to describe the shape of the KOSPI return from both unconditional and conditional perspectives. Especially, our results suggest that the equality assumption on the shape parameter of t distribution yields better discrimination of heterogeneity component in return data. We report the strong regime-dependent characteristics in volatility dynamics with high persistence and asymmetry by employing a regime switching GJR-GARCH model with t innovation model. Compared to two sub-samples, Pre-Crisis and Post-Crisis, we find that the degree of persistence in Pre-Crisis is higher than that in Post-Crisis, and the strong asymmetry in low-volatility (high-volatility) regime during Pre-Crisis (Post-Crisis). We also report strong asymmetry in the low-volatility regime during the Pre-Crisis, whereas this phenomenon is evident in high-volatility regime in Post-Crisis.

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