International Journal of Reliability and Applications
Vol. 15, No. 1, pp. 51-64, 2014

Warranty cost anlaysis for multi-component systems with imperfect repair

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Received 6 December 2013; revised 25 April 2014; accepted 2 May 2014

Abstract. This paper develops a warranty cost model for complex systems with imperfect repair within a warranty period by addressing a practical case that the first inter-failure interval is longer than any other inter-failure intervals. The product is in its best condition before the first failure if repair is imperfect. After the imperfect repair, other inter-failure intervals which are explained by renewal processes, are stochastically smaller than the first inter-failure interval. Based on this idea, we suggest the failure-interval-failure-criterion model. In this model, we consider two random variables, X and Y where X represents failure intervals and Y represents failure criterion. We also obtain the distribution of the number of failures and conduct the warranty cost analysis. We investigate different types of warranty cost models, reliabilities and other measures for various systems including series-parallel configurations. Several numerical examples are discussed to demonstrate the applicability of the methodologies derived in the paper.

Key Words: *Failure-interval-failure-criterion model, imperfect repair, multi-component system, Quasi-renewal processes, system reliability, warranty cost*

1. INTRODUCTION

Warranty policy is considered to play an important attribute to both product manufacturers and customers (buyers). In terms of manufacturer's compensation upon their products' failures, there exists three common types of warranties; free repair/replacement warranty (FRW), pro-rata warranty (PRW) and combination warranty (CMW). Under FRW, a failed item is replaced/ repaired at no cost to the buyer if the failure occurs within the warranty period. On the other hand, under PRW, warranty services are provided at a pro-rated cost depending on the amount of usage or service time provided by the item prior to its failure (Blischke (1994), Blischke and Murthy (1996)). Combination warranty (CMW)

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contains features of both FRW and PRW; which often contains two warranty periods, a free repair and replacement period followed by a pro-rata period. In general, the warranty policy is an obligation attached to products that requires the manufacturers to provide compensation for consumers (buyers) according to the warranty terms when the warranted products fail to perform their intended functions (Wang and H. Pham (2006)). Under the warranty policy, we develop the warranty model using the renewal theory and non-renewal theory for imperfect repair.

In this paper, we aim to analyze the distribution of the number of product failures and investigate the warranty cost for the multi-component systems as well as single component system considering the effect of imperfect repair. General descriptions of different kinds of warranty policies and mathematical models can be found in Blischke and Murthy (1996). Under different warranty policies such as PRW and FRW, Bai and Pham (2006) study discounted warranty cost and Balcer and Sahin (1986) derive moments of the total replacement cost. Jung and Park (2003) consider two types of warranty policies such as renewing warranty and non-renewing warranty with warranty period and post warranty period. They derive the expressions for the expected maintenance costs for the periodic preventive maintenance during post warranty period.

Additionally, we consider the expected value and the variance of the warranty cost simultaneously. The expected warranty cost has been mainly investigated for warranty cost analysis. While the expected warranty cost is a good measure on the overall cost of warranty, it provides little information on the risk contained in a warranty program. The variance and the standard deviation provide a numerical measure of the disparity of data. These measures are useful for making comparisons between data sets that go beyond simple visual appearances. While measures of central tendency (i.e. expectation) are used to estimate 'normal' values of a dataset, measures of dispersion (i.e. variance) are important for describing the spread of the data, or its variation around a central value (Sleptchenko et al. (2002)). For example, meteorologists often use variance to help classify abnormal climatic conditions. They use variance to describe the abnormality of a data value.

There are relatively limited number of prior works on multi-component system. While many researchers have investigated simple systems, our proposed approach is to consider simple systems as well as complicated systems to conduct the warranty cost analysis in detail.

In the paper, we develop improved warranty cost models for repairable systems from the stand point of both the manufacturer and the customer. We believe the models will help warranty policy makers to make optimal decisions with the objective of downsizing manufacturers' warranty cost.

Repair service can be categorized into three classes based on the effort of repairs or the condition of repaired items; as-good-as-new repair, minimal repair and imperfect repair. As-good-as-new repair assumes that after a repair the restored system functions like new such that the failure time distribution is the same as that of a new product. Minimal repair, which is also called as-bad-as-old repair, assumes that the failure rate of a repaired system equals that of the system just before the most recent failure. Imperfect repair refers to the situation where a repair action responds to a system neither as-good-as-new nor as-bad-as-old but to a level in between. In the paper, we assume that the product is in a perfect condition upon initialization. The first inter-failure interval is stochastically greater than other inter-failure intervals which are explained by renewal models.

A few researchers consider imperfect repair for the warranty cost. Hajeeh and Jabsheh [8] study imperfect multi-component repair models. It is assumed that the performance of a system becomes inferior after each failure. So far the studies in warranty literature mostly focus on as-good-as-new repair scenario. However, we aim to conduct the study based on both imperfect repair and perfect repair. If repair is imperfect, then the inter-failure interval would be shorter. This implies that the next failure would come faster than the time it took for the previous failure.

When taking the imperfect repair into considerations, we assume that the first inter-failure interval is stochastically greater than any other inter-failure intervals. This implies that for $x>0$, $P(X_1>x)>P(X_i>x)$ where X_1 denotes the first inter-failure interval and X_i, $i=2,3,\cdots$ denotes the i^{th} inter-failure interval. After the first repair, the inter-failure intervals are modeled by renewal process.

With this idea, we develop a warranty cost model, addressing two cases: (1) Inter-failure interval as a random variable with different scales. The first inter-failure interval is stochastically greater than any other inter-failure intervals and X_1 is following distribution $F(\cdot)$ and $X_i, i=2,3,\cdots$ is following distribution $G(\cdot)$, which, for all $x>0$, $F(X)=G(aX)$, constant $a<1$ and (2) X_1 and $X_i, i=2,3,\cdots$, defined in Case (1), represent inter-failure intervals of the component and Y as a random variable of failure criterion to measure the inter-failure interval. This interval will satisfy the customer's demand and mark the limit of the imperfect repair. Using the imperfect repair assumption, we consider that the first inter-failure interval will exceed the failure criterion, that is $X_1>Y$, but the other inter-failure intervals will not be ($X_i<Y$, $i=2,3,\cdots,n$). In other words, the first inter-failure interval is longer than any other inter-failure interval. Throughout this paper, we refer to this as a *failure-interval-failure-criterion* model. Based on this model, we develop various warranty cost model, reliabilities and other measures for various systems.

The paper is organized as follows. Section 2 derives the distribution of the number of system failures for various schemes for the component level failure. Section 3 presents warranty cost analysis for the single component system and series-parallel system. Several numerical examples are given in Section 4 to illustrate the proposed cost analysis with considering Weibull distribution and finally, concluding remarks are discussed in Section 5.

Nomenclature

i.i.d. : identically and independently distributed

w : length of a warranty period

γ,θ : parameters for the distribution of the first inter-failure interval and the distribution of failure criterion *Y*, respectively

$N(t)$: number of failures by time *t*

N, N_s: number of failures and number of system failures, respectively, in the warranty period

X_i: renewal inter-failure interval between the $(i-1)^{th}$ and $(i)^{th}$ failure

S_n: arrival time of the n^{th} renewal

$f_s(\cdot)$, $F_s(\cdot)$, $R_s(\cdot)$: pdf, cdf and reliability function of system inter-failure intervals within a warranty period *w*, respectively

$f_j(\cdot)$, $F_j(\cdot)$, $R_j(\cdot)$: pdf, cdf and reliability function of component *j*'s inter-failure intervals within a warranty period *w*, respectively

$f_{ij}(\cdot)$, $F_{ij}(\cdot)$, $R_{ij}(\cdot)$: pdf, cdf and reliability function for inter-failure intervals of j^{th} component in cluster *i* in the series-parallel system

c : warranty cost per one system failure in the warranty period *w*

$Weibull(\kappa,\lambda)$: Weibull distribution with shape parameter κ and scale parameter λ

2. MODEL CONSIDERATION

2.1 Distribution of number of system failures in the warranty period

Single component system is considered and, we study the distribution of number of failures under the component level.

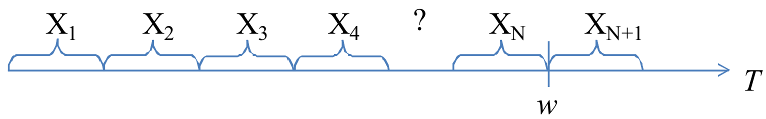

Figure 2.1. Warranty model with a warranty period *w* and *N* failures

Park and Pham (2010) obtain the distribution of the number of failures using Figure 2.1. We assume that there are *N* system failures in a warranty period. Let X_i be the i^{th} inter-arrival interval and if there are *N* failures and fixed warranty period w, the sum of inter-arrival intervals by *N* is less or equal to w and the sum of inter-arrival intervals by $N+1$ is larger than w. Therefore, the pmf of the number of component's failures is given by

$$P\left(X_1 + X_2 + \cdots + X_{N-1} + X_N \le w,\ X_1 + X_2 + \cdots + X_{N-1} + X_N + X_{N+1} > w\right)$$

In this paper, we consider $P(N=n)$ in a different way. First, we define the imperfect repair. Imperfect repair is a repair service which contributes to some noticeable improvement of the product (Park and Pham (2010)). It is a maintenance action that restores the system operating state to be somewhere between as-good-as-new and as-bad-as-old. Imperfect repairs are considered such that after each repair, the system is between the states of new and old. During the first inter-failure interval, the product is in good condition, indicating that the first inter-failure interval is stochastically greater than other inter-failure intervals which are explained by renewal models. However, after the first imperfect repair, there are no length orders for the remaining inter-failure intervals, for example, the third failure intervals may be longer than the second inter-failure interval. Using this concept, we obtain the distribution of the number of failures. If the *r.v.* N is the inter-failure index such that $X_1 \geq X_i, i = 2,3,\cdots,N$ then, $P(N)$ is given by $P(X_1 \geq X_i, i = 2,\cdots,N)$. It means that the first inter-failure interval is stochastically greater than any other inter-failure intervals. Also X_1 is following distribution $F(\cdot)$ and $X_i, i = 2,3,\cdots$ is following distribution $G(\cdot)$, which, for all $x > 0$, $F(X) = G(aX)$, constant $a < 1$. And there is another case which is failure-interval-failure-criterion model. $P(N)$ is given by $P(X_1 > Y, X_i \leq Y, i = 2,\cdots,N)$ when the first inter-failure interval is stochastically greater than another random variable failure criterion and other inter-failure interval is stochastically less than failure criterion.

2.2 When the first inter-failure interval is stochastically greater than any other inter-failure intervals

The first inter-failure interval is stochastically greater than any other intervals. Every inter-failure interval other than the first inter-failure interval is continuous *i.i.d.* random variable $X_i, i = 2,3\cdots$ in Fig. 1 where w is a warranty period. Any kind of distribution can be applied to the model. But, every inter-failure interval is assumed to follow distribution $F(\cdot)$ except that first inter-failure interval follows distribution $G(\cdot)$. Then, the random variable N is the inter-failure interval index such that $X_i \leq X_1, i = 2,3,\cdots,n$.

$$\begin{aligned} P(N = n) &= P(X_i \leq X_1, i = 2,\cdots,n) \\ &= \int_0^w P(X_i \leq x, i = 2,\cdots,n) g(x)\, dx \\ &= \int_0^w F^{n-1}(x) g(x)\, dx \\ &= \int_0^w F^{n-1}(x)\, dG(x) \end{aligned} \tag{2.1}$$

If $G(\cdot)$ follows same distribution $F(\cdot)$, Eq. (2.1) can be written as follows:

$$\begin{aligned} P(N = n) &= \int_0^w F^{n-1}(x)\, dF(x) \\ &= \frac{1}{n}\left(F^n(w) - F^n(0)\right) \end{aligned} \tag{2.2}$$

Using Eqs. (2.1) & (2.2), we can obtain the distribution of number of failure. In the next section, failure criterion is considered to measure the failure intervals and obtain the distribution of number of failures.

2.3 When the first inter-failure interval is stochastically greater than any other random variable failure criterion and other inter-failure intervals are stochastically less than failure criterion

Let X_i represent the renewal inter-failure intervals between the $(i-1)^{th}$ and $(i)^{th}$ failure. We develop a model, called a *failure-interval-failure-criterion* model. In this model, X_i represents i^{th} inter-failure interval of the component and Y represents the failure criterion to measure the inter-failure interval. Based on the feature of the imperfect repair, the first inter-failure interval exceeds the failure criterion, $X_1 > Y$, and all the other inter-failure intervals don't exceed the failure criteria, $X_i < Y$, $i = 2,3,\cdots,n$.

The first inter-failure interval is stochastically greater than any other intervals. The first failure interval is larger than *r.v.* Y, i.e. $X_1 > Y$ and other intervals are less than *r.v.* Y. The *r.v.* N is the inter-failure interval index such that $X_i \le Y$, $X_1 > Y, i = 2,\cdots,N$. Suppose that X_1, $X_i, i = 2,3,\cdots$, and Y are independent continuous random variables, we can specify the distribution of variable X_1 and $X_i, i = 2,3,\cdots$, which follow distribution $G(\cdot)$ and distribution $F_X(\cdot)$, respectively. This shows that every inter-failure interval is assumed to follow distribution $F_X(\cdot)$ except the first inter-failure interval, which follows distribution $G(\cdot)$. $F_X(x)$ is equal to $G(\gamma x)$ where constant γ is less than 1. Similarly, $F_X(x)$ is equal to $F_Y(\theta x)$ where constant θ is between $\gamma < \theta < 1$. The probability that there exists N failures, is given by

$$\begin{aligned} P(N=n) &= P(X_i \le Y, X_1 > Y, i = 2,\cdots,n) \\ &= \int_0^w P(X_i \le y, X_1 > y, i = 2,\cdots,n) f_Y(y)\,dy \\ &= \int_0^w P(X_i \le y, X_1 > y, i = 2,\cdots,n)\,dF_Y(y) \\ &= \int_0^w F_X^{\,n-1}(y)(1 - G(y))\,dF_Y(y) \end{aligned} \tag{2.3}$$

If $X_i, i = 2,3,\cdots,n$ and Y have the same distribution $F(\cdot)$, Eq. (2.3) can be written as follows:

$$\begin{aligned} P(N=n) &= \int_0^w F^{n-1}(y)(1 - G(y))\,dF(y) \\ &= \int_0^w \left(F^{n-1}(y) - F^{n-1}(y)\cdot G(y)\right)dF(y) \\ &= \left.\frac{F^n(y)}{n}\right|_0^w - \int_0^w F^{n-1}(y)\cdot G(y)\,dF(y) \\ &= \left[\frac{1}{n}F^n(w) - \frac{1}{n}F^n(0)\right] - \left[\int_0^w F^{n-1}(y)\cdot G(y)\,dF(y)\right] \end{aligned} \tag{2.4}$$

If distribution $G(\cdot)$ is same with distribution $F(\cdot)$, then $P(N=n)$ is obtained as follows.

$$P(N=n) = \frac{1}{n}\left(F^n(w) - F^n(0)\right) - \frac{1}{n+1}\left(F^{n+1}(w) - F^{n+1}(0)\right) \tag{2.5}$$

In the next section, we obtain the expected warranty cost and the variance of the warranty cost.

3. WARRANTY COST ANALYSES

In this section, we focus on the warranty cost analysis by computing the expected value of the warranty cost as well as the variance for multi-component systems such as series-parallel. Using *failure-interval-failure-criterion* model, Eq. (2.4), we conduct the warranty cost analysis. If the inter-occurrence intervals follow the regular renewal processes except the first inter-failure interval, then the expected warranty cost and the variance of warranty cost are obtained using the *failure-interval-failure-criterion* model.

$$\begin{aligned} E(N) &= \sum_{n=1}^{\infty} nP(N=n) \\ &= \sum_{n=1}^{\infty} n\left(\left[\frac{1}{n}F^n(w) - \frac{1}{n}F^n(0)\right] - \left[\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right) \end{aligned} \tag{3.1}$$

Next, we will derive the variance of the warranty system cost. First we calculate the second moment.

$$\begin{aligned} E(N^2) &= \sum_{n=1}^{\infty} n^2 P(N=n) \\ &= \sum_{n=1}^{\infty} n\left(\left[F^n(w) - F^n(0)\right] - \left[n\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right) \end{aligned} \tag{3.2}$$

Therefore, the variance of the warranty cost is given by

$$\begin{aligned} Var(N) &= \sum_{n=1}^{\infty} n\left(\left[F^n(w) - F^n(0)\right] - \left[n\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right) \\ &\quad -\left(\sum_{n=1}^{\infty} n\left(\left[\frac{1}{n}F^n(w) - \frac{1}{n}F^n(0)\right] - \left[\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right)\right)^2 \end{aligned} \tag{3.3}$$

The expected value and variance of the system warranty cost for the single component system are as follows, respectively;

$$\begin{aligned} E(C) &= cE(N) \\ &= c\sum_{n=1}^{\infty} n\left(\left[\frac{1}{n}F^n(w) - \frac{1}{n}F^n(0)\right] - \left[\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right) \end{aligned} \tag{3.4}$$

$$Var(C) = c^2 \begin{pmatrix} \sum_{n=1}^{\infty} n\left(\left[F^n(w) - F^n(0)\right] - \left[n\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right) \\ -\left(\sum_{n=1}^{\infty} n\left(\left[\frac{1}{n}F^n(w) - \frac{1}{n}F^n(0)\right] - \left[\int_0^w F^{n-1}(y)\cdot G(y)dF(y)\right]\right)\right)^2 \end{pmatrix} \tag{3.5}$$

Next, the series-parallel system is considered. The *failure-interval-failure-criterion* model is applied for the series-parallel system in Figure 3.1.

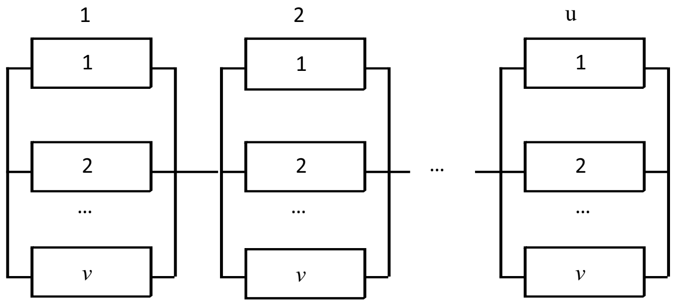

Figure 3.1. Series-parallel system with u*v components

A general series-parallel system consists of u subsystems in series with v units in parallel in each subsystem. We consider that the system has u*v components. And the cdf of inter-failure intervals of j^{th} component in cluster i is given by

$$\begin{aligned} F_{ij}(n) &= P(N \le n) \\ &= \sum_{k=1}^{n} P(N = k) \\ &= \sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w) - \frac{1}{k}F_{ij}^{k}(0)\right] - \left[\int_0^w F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right) \end{aligned} \tag{3.6}$$

where $j = 1,2,...v$ and $i = 1,2,...u$

Then, the system reliability function is given by

$$\begin{aligned} R_s(n) &= \prod_{i=1}^{u}\left[1 - \prod_{j=1}^{v}\left(1 - R_{ij}(n)\right)\right] \\ &= \prod_{i=1}^{u}\left[1 - \prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w) - \frac{1}{k}F_{ij}^{k}(0)\right] - \left[\int_0^w F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right] \end{aligned} \tag{3.7}$$

And for the cost analysis, we obtain the expectation of the cost and the variance of the cost. We obtain $f_s(n)$ when it is the series-parallel system.

$$\begin{aligned} f_s(n) &= P[N = n] \\ &= P[N \ge n] - P[N \ge n+1] \\ &= \prod_{i=1}^{u}\left[1 - \prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w) - \frac{1}{k}F_{ij}^{k}(0)\right] - \left[\int_0^w F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right] \\ &\quad -\left(\prod_{i=1}^{u}\left[1 - \prod_{j=1}^{v}\sum_{k=1}^{n+1}\left(\left[\frac{1}{k}F_{ij}^{k}(w) - \frac{1}{k}F_{ij}^{k}(0)\right] - \left[\int_0^w F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\right) \end{aligned} \tag{3.8}$$

The first moment of N_s is given by

$$E(N)=\sum_{n=1}^{\infty}nP(N=n)$$

$$=\sum_{n=1}^{\infty}n\left(\begin{array}{l}\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\\-\left(\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n+1}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\right)\end{array}\right) \tag{3.9}$$

For the variance, we calculate the second moment.

$$E(N^{2})=\sum_{n=1}^{\infty}n^{2}P(N=n)$$

$$=\sum_{n=1}^{\infty}n^{2}\left(\begin{array}{l}\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\\-\left(\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n+1}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\right)\end{array}\right) \tag{3.10}$$

The variance of warranty cost is given by

$$Var(N)=E(N^{2})-\left[E(N)\right]^{2}$$

$$=\sum_{n=1}^{\infty}n^{2}\left(\begin{array}{l}\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\\-\left(\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n+1}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\right)\end{array}\right)$$

$$-\left(\sum_{n=1}^{\infty}n\left(\begin{array}{l}\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\\-\left(\prod_{i=1}^{u}\left[1-\prod_{j=1}^{v}\sum_{k=1}^{n+1}\left(\left[\frac{1}{k}F_{ij}^{k}(w)-\frac{1}{k}F_{ij}^{k}(0)\right]-\left[\int_{0}^{w}F_{ij}^{k-1}(y)\cdot G_{ij}(y)dF_{ij}(y)\right]\right)\right]\right)\end{array}\right)\right)^{2} \tag{3.11}$$

Using the warranty cost per failure, we easily obtain the expected warranty cost and the variance of warranty cost, respectively.

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSES

In this section, numerical examples are presented to illustrate the analyses of system cost functions. We assume that the inter-failure interval of a component follows the Weibull distribution with different parameters. The reliability of the product is affected by the parameters of the Weibull distribution which is widely used in reliability engineering because other distributions such as exponential, Rayleigh, and normal are special cases of the Weibull distribution. The flexibility of the Weibull distribution also allows accurate representation of various lifetime distributions. A 2*2 series-parallel system is investigated when inter-failure intervals of each components follow the Weibull distribution.

Case 1. Single component system

We assume that the warranty cost is $c = \$2{,}000$. Suppose that an inter-failure interval random variable is said to be Weibull distributed with parameters (κ, λ) whose pdf and cdf, then

$$f(x;\kappa,\lambda) = \frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{\kappa-1} e^{\left(-\frac{x}{\lambda}\right)^{\kappa}}, x \geq 0,\ F(x;\kappa,\lambda) = 1 - e^{\left(-\frac{x}{\lambda}\right)^{\kappa}} \quad (4.1)$$

The cdf of a Weibull *r.v.* in the warranty period is given by

$$F_X(w;\kappa,\lambda) = 1 - e^{\left(-\frac{w}{\lambda}\right)^{\kappa}}, G(w;\kappa,\lambda) = 1 - e^{\left(-\frac{w}{\gamma\cdot\lambda}\right)^{\kappa}}, F_Y(w;\kappa,\lambda) = 1 - e^{\left(-\frac{w}{\theta\cdot\lambda}\right)^{\kappa}} \quad (4.2)$$

The results of the expected warranty cost, *E*(*C*), standard deviation of warranty cost, *SD*(*C*) and coefficient of variation, *CV*, are listed in Table 1. For the sensitivity analysis, we consider 20-warranty period units which start at 0.1 and finish at 2.0 for various values.

Table 4.1. *E*(*C*), *SD*(*C*) and *CV* with Weibull distribution for Case 1

	$\kappa=1,\lambda=1$			$\kappa=1,\lambda=2$			$\kappa=2,\lambda=1$			$\kappa=2,\lambda=2$		
w	E(C)	SD(C)	CV	E(C)	SD(C)	CV	E(C)	SD(C)	CV	E(C)	SD(C)	CV
0.1	209	646	3.09	22	210	9.50	105	458	4.36	5.54	105	19.01
0.2	414	913	2.20	88	419	4.76	209	646	3.09	22.12	210	9.50
0.3	616	1122	1.82	196	625	3.18	312	790	2.53	49.67	315	6.34
0.4	815	1306	1.60	345	829	2.40	414	913	2.20	88.04	419	4.76
0.5	1011	1478	1.46	531	1032	1.94	516	1022	1.98	137	522	3.81
0.6	1203	1644	1.37	751	1239	1.65	616	1122	1.82	196	625	3.18
0.7	1391	1807	1.30	1000	1458	1.46	716	1216	1.70	266	727	2.73
0.8	1576	1970	1.25	1273	1695	1.33	815	1306	1.60	345	829	2.40
0.9	1755	2130	1.21	1565	1956	1.25	913	1393	1.52	434	930	2.15
1.0	1929	2287	1.19	1864	2237	1.20	1011	1478	1.46	531	1032	1.94

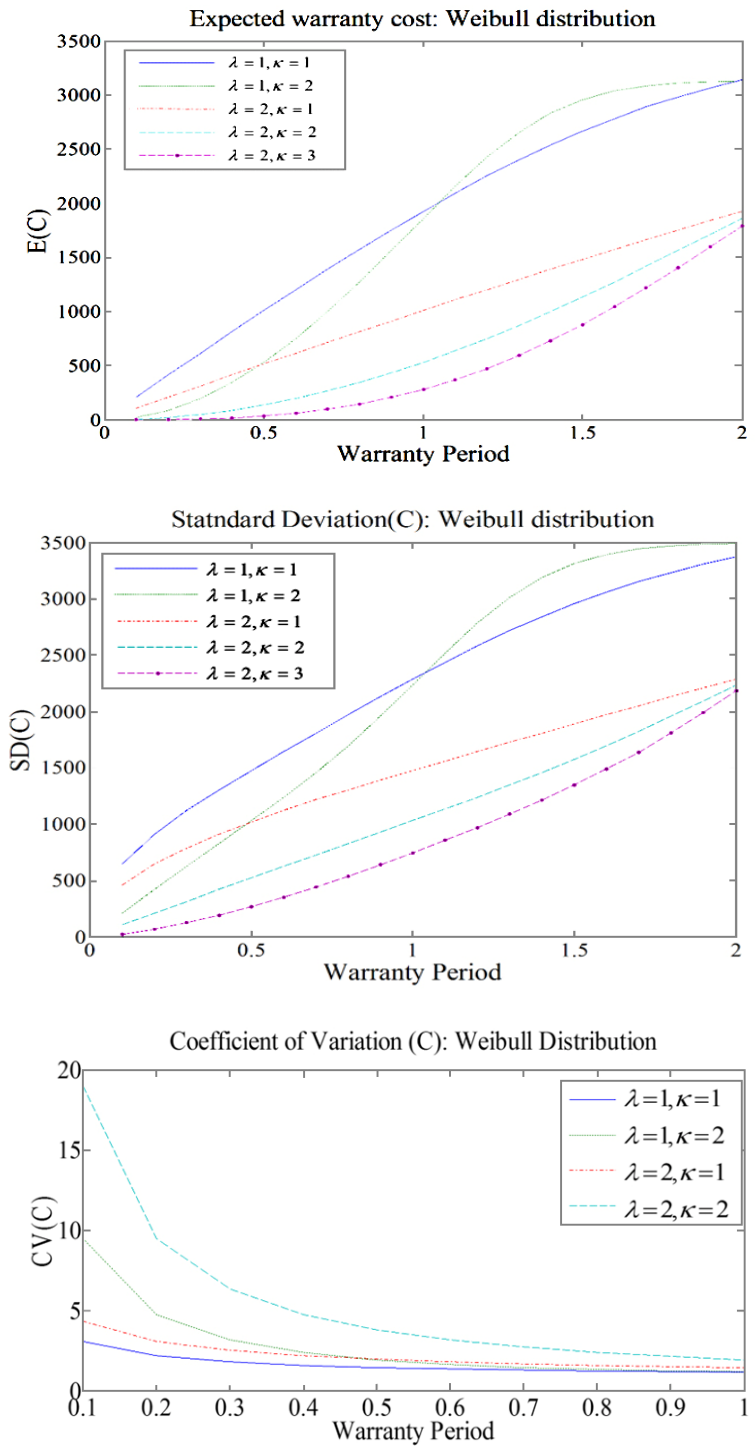

Figure 4.1. E(C) and SD(C) for single component system using Weibull distribution

We run sensitivity analyses for single-component system using Weibull distribution. Fig. 3 shows the expected warranty cost E(C), and the standard deviation of warranty cost SD(C) for single component system using Weibull distribution. The *CV* is the ratio of the standard deviation to the mean and describes the variation within the values. If the warranty period progresses and the *CV* decrease, it would imply that their variations would decrease. Under the Weibull distribution, we consider the parameters (κ, λ) = (1, 1), (1, 2), (2, 1), and (2, 2). The expected warranty cost and standard deviation of warranty cost increase and coefficient of variations decrease.

Case 2. 2*2 Series-parallel systems when each component's inter-failure intervals follow same distribution

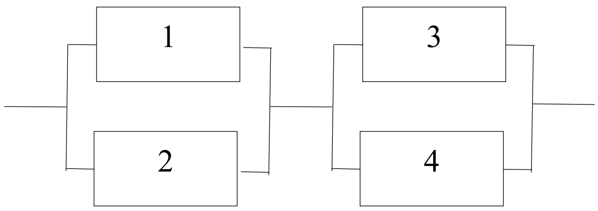

Figure 4.2. *2*2* Series-parallel systems

Consider a series-parallel system as shown in Figure 4.2. We assume that the failure time of all the components has the same distribution. To illustrate this example, we also consider Weibull distribution with different parameters. Also, the warranty costs are assumed to be $c = \$8{,}000$ because this has four components so its repair costs are more expensive than those of single component system.

Table 4.2. *E*(*C*), *SD*(*C*) and *CV* with Weibull distributions for Case 2

Weibull distribution												
	$\kappa=1, \lambda=1$			$\kappa=1, \lambda=2$			$\kappa=2, \lambda=1$			$\kappa=2, \lambda=2$		
w	E(C)	SD(C)	CV	E(C)	SD(C)	CV	E(C)	SD(C)	CV	E(C)	SD(C)	CV
0.1	15.51	376	24.23	0.02	12.92	663.5	2.08	133	64.16	0.00	2.26	N/A
0.2	108	1051	9.76	1.18	100	84.51	15.51	376	24.23	0.02	13.05	669.5
0.3	314	1889	6.02	12.46	335	26.86	48.80	687	14.08	0.22	42.28	195.4
0.4	642	2827	4.40	62.52	782	12.51	108	1051	9.76	1.18	100	84.90
0.5	1082	3818	3.53	205	1489	7.25	196	1454	7.43	4.36	195	44.64
0.6	1613	4832	3.00	510	2472	4.84	314	1889	6.02	12.46	335	26.86
0.7	2212	5845	2.64	1037	3715	3.58	463	2349	5.07	29.83	528	17.72
0.8	2856	6836	2.39	1809	5173	2.86	642	2827	4.40	62.52	782	12.51
0.9	3521	7789	2.21	2801	6778	2.42	849	3318	3.91	118	1101	9.32
1.0	4187	8690	2.08	3940	8432	2.14	1082	3818	3.53	205	1489	7.25

Similar to the first case, we are using Weibull distribution for the analysis. Then we conduct sensitivity analyses for *2*2* series-parallel systems. The results are more complicated compared to the first case. We obtain the expected warranty cost, the standard deviation of warranty cost and coefficient of variations and are shown in Table 4.2. As warranty periods are continuous, their expected warranty cost and standard deviations show pattern of increase and coefficient of variations decrease steeply for the uniform distribution. For the Weibull distribution, the same parameters are used with the first case.

5. CONCLUSION

In this paper, we assume that the repair is imperfect and that the first inter-failure interval would be longer than any other inter-failure interval. Due to the imperfect repair characteristic, except for the first inter-failure interval, other inter-failure intervals do not necessary need to have any length order or vice versa. For example, the second inter-failure interval could be shorter than the third inter-failure interval. When the first inter-failure interval is stochastically greater than another random variable failure criterion and other inter-failure interval is stochastically less than failure criterion, we suggest *failure-interval-failure-criterion* model. Using the *failure-interval-failure-criterion* model, we obtain the distribution of the number of failures and conduct the warranty cost analyses under the assumption of imperfect repair for the multi-component systems. Based on the proposed approach, we try to conduct cost analyses for the single component system and multi-component systems including series-parallel system. For the numerical examples, we show single component system and *2*2* series-parallel systems. We input different parameters using Weibull distribution and check their sensitivity analyses results. These analyses would be very helpful for various systems cost analyses in practices.

ACKNOWLEDGEMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012-002380).

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