

# 낮은 에러 플로어(error floor)를 사용한 효과적인 LDPC 복호 알고리즘

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An Effective Decoding Algorithm of LDPC Codes with Lowering Error Floors

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요 약

본 논문에서는, LDPC 코드의 성능을 향상시키기 위해서, LDPC 코드의 에러 플로어(error floors)를 낮추어서 복호를 수행하는 효율 좋은 알고리즘을 제안한다. 이 방법은, 바람직하지 않은 구조 때문인데, Tanner 그래프의 트래핑 세트를 줄여서 복호를 하는 방법이다. 이 알고리즘은 트래핑 세트를 줄이는 방법으로 복호의 효율성을 얻는다. 모의시험을 통해서 이 알고리즘의 개선된 성능을 확인 할 수 있었다.

ABSTRACT

In this paper, in order to improve performance of LDPC codes, we propose an effective algorithm with lowering error floor of LDPC codes. This method is done by breaking trapping sets, mostly caused by an undesirable structure. This algorithm is not need to observe all the errors, only need to break the trapping sets, to effect the effectiveness. Simulation results show that its performance can be significantly improved with this decoding algorithm.

키워드

LDPC, Trapping Set, Error Floors  
LDPC, 트래핑 세트, 에러 플로

## 1. Introduction

The low-density parity-check codes (LDPC) was error collecting code, and one of popular subjects being studied [1],[9-11]. In order to improve performance of LDPC codes, the most effective way is to lower error floor of LDPC codes. The phenomenon of error floors of LDPC code is mostly

caused by an undesirable structure, known as trapping set.

There are some methods. By using low complexity sum-product algorithm, LDPC codes can get near Shannon limit decoding performance with almost all errors detectable. Error floor is an important issue in the theory of LDPC codes and the study of algorithms. Because of the error floor,

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the error rate in high SNR region suddenly drops at a rate much slower than that in the region of moderate SNR. The phenomenon of error floor can be searched at the structures of the code's Tanner graph [2]. Small stopping sets in the Tanner graph are the error floor on binary erasure channel [3], and have similar effect over additive white Gaussian noise (AWGN) channel. More recently, trapping sets are found of error floors of many LDPC codes over AWGN channel [4]. Since trapping sets generally have complicated combinatorial properties, it is a problem to cope with them directly in code construction [5]. And some decoder-based strategies have been proposed to lower the error floor due to the trapping set [6-8]. With these methods, a prior knowledge of the dominant trapping sets in a particular code is required at the decoder, which is often difficult to obtain. About this problem some research are done[9], and there is a necessity to increase their performance.

In this paper, we propose an effective algorithm to lower error floor and increase the performance, by the method adding new check node. This algorithm is improving the iterative decoding method. First decoding is done, and if the decoding fails due to a trapping set, then selective bit-flipping (SBF) is used along with iterative decoding.

In Section II there are explanations on the existing algorithm and improved algorithm, and decoding algorithm is proposed. In Section III, Simulation results by this algorithm, which break most of the trapping sets. Section IV is conclusions. And next is references.

## II. Algorithm

### 2.1 Related two-stage iterative decoding algorithm

When being decoded by Sum-Product Algorithm, the error floors of some LDPC codes are dominated by block errors, where only a small number of check-sums are not satisfied, and these codes are called near-codewords. Near-codewords are more commonly referred to as trapping sets. A trapping set  $A(a, b)$ , called an  $(a, b)$  trapping set, is a subgraph with  $a$  variable nodes and  $b$  odd degree checks. These trapping sets are related to the most probable noise realizations that lead to decoding failure. As an example, Fig. 1 shows a  $(6, 2)$  trapping set, where the number of the variable nodes is 6, and the number of the odd-degree check nodes,  $c_1$  and  $c_7$ , is 2. If the code bits associated with the  $a$  variable nodes are all the wrong bits, then the check-sums corresponding to the  $b$  odd-degree check nodes will not be satisfied. For convenience, we refer to these nodes as unsatisfied check nodes.

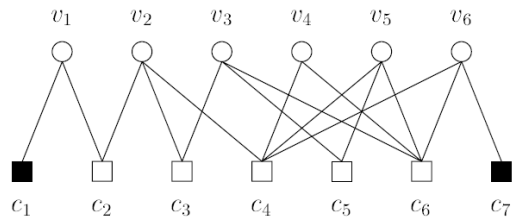


Fig. 1 Trapping set  $A(6, 2)$

The proposed decoding consists of two stages. In the first stage, the conventional iterative decoding is performed. when a decoding fails, we decide whether it is caused by a trapping set. And all the unsatisfied check nodes checked to see if there are only a relatively small number of them. Usually, because the number of unsatisfied check nodes changes whenever an iteration is done. We observe in simulations that this number does not necessarily decrease with more iterations. In fact, in many cases the number of unsatisfied check nodes still oscillate even after hundreds of

iterations. Therefore, in the decoding of each block, we keep track of the smallest number  $n_c$  of unsatisfied check nodes throughout the iterations. We also record the set  $C$  of these unsatisfied check nodes  $n_c$  and the corresponding hard decision  $\hat{X}$  of the codeword. If decoding fails and  $n_c$  is less than a predetermined threshold  $t$ , then we consider that this decoding failure is caused by a trapping set. The wrong bits in  $\hat{X}$  correspond to the set of variable nodes in the trapping set, which is denoted by  $V$ . To correct the decoding error caused by the trapping set, the decoder enters into the second stage. In the second stage, if we can locate all the variable nodes in  $V$ , the decoding is completed. Only by knowing the set of unsatisfied check nodes, it is impossible to do so in a combinatorial way. However, the set  $C$  contains some information about  $V$ . This means that each check node in  $C$  is adjacent to at least one variable node in  $V$ . To determine  $V$ , we can define a matching set  $M$  as follows, for a set of check nodes  $C$  in a Tanner graph, a matching set of  $C$  is a set of variable nodes such that each of them is adjacent to one and only one check node in  $C$ . According to the definition, a matching set of  $C$  contains  $|C|$  variable nodes, where  $|C|$  denotes the cardinality of  $C$ . Assuming that no two check nodes in  $C$  have any common neighboring variable nodes, then we can generate the set  $L$  of all the different matching sets of  $C$ . Clearly,  $|L|$  is in the order of  $d_c^{|C|}$ , where  $d_c$  is the average degree of the check nodes in  $C$ . Note that at least one of the matching sets in  $L$  is a subset of  $V$ . As an example, Fig. 1, the set  $\{v_1, v_6\}$  is such a matching set of  $\{c_1, c_7\}$ . The algorithm identifies such a matching set from  $L$  and uses it to correct all the wrong bits in  $\hat{X}$ . When each matching set  $M$  in  $L$  satisfies  $M \subseteq V$ , then the corresponding wrong bits in  $\hat{X}$  can be identified. We flip these bits by setting as the maximum possible value with opposite signs, and

perform the iterative decoding over again. If  $M \supseteq V$ , the decoding will most likely fail again. In most cases, the decoder will not get into the same trapping set, and converge to a codeword. This decoding stage is improved as below.

## 2.2 Comparison of New Algorithm with Existing algorithms

The algorithms could divide into two strategies. The first strategy is to change the construction of LDPC codes to avoid topological structure such a cycle, trapping set, stopping set.

To break the cycles, McGowan-Williamson algorithm is eliminating 4-cycle, 6-cycle, and so on. And Stopping set eliminating algorithm is executed through adding a new check node  $c_a$  so that a stopping set  $S$  can be eliminated, where if and only if one edge is used, to connect a variable node in  $S$  to  $c_a$ .

In Tanner graph covers method, for all the nodes that we considered, swapping edges at random, and the edges appearing in only one trapping set can be swapped, to eliminate all the minimal trapping sets.

The second strategy is modifying the decoding algorithm. such as two-stage iterative decoding algorithm breaks the trapping sets. First stage is a slightly modified iterative decoding. If the first stage decoding fails due to a trapping set, in second stage a selective bit-flipping along with iterative decoding is used. It can break most of the trapping set, and push the error floor down to the level limited.

But all of these algorithms need to be improved. McGowan-Williamson algorithm was breaking the cycles, but this algorithm had low efficiency. Stopping set eliminating algorithm was done by adding few check nodes, to eliminate stopping sets, but this algorithm should be keep the structure of the original code, and the degree of newly added check nodes should be consistent with that of the

original check node. Then it will be possible to introduce new stopping set.

The method of Tanner graph covers is want to eliminate all trapping sets with minimal critical number by swapping random edge, but it is difficult to eliminate all trapping set or all the stopping sets, because, no matter how a trapping set or stopping set generally has complicated with combinatorial properties, it is a problem to cope with them directly in code construction.

Two stage algorithm is combined soft decision and hard decision, because the hard decision has low complexity. It seems more practically. But this algorithm has serious disadvantage, In the stage if the unsatisfied check node number is odd times, we can not obtain a correct convergence. And only one error rate flipping is not enough to correct the error in the trapping set. In order to correct this error, we must find another bit error rate and its flipping.

Our stopping set eliminating method is add a new check, so we can combine the two method, if in the second the unsatisfied check node number is 1, we can add a new check node. With this method, it could improve greatly the effectiveness of two stage iterative decoding algorithm.

### 2.3 Improved two-stage iterative decoding algorithm

In the previous section of two-stage algorithm, the algorithm is to find the error rate of unsatisfied check node, by setting the initial value of log-likelihood ratio to reverse biggest default value and again decoding. In the process of simulation, after first decoding finish, if we find that the unsatisfied check node number is 1, then the second decoding begin, convergence will not correct, because, only with one error rate flipping, it is not enough to correct the error in the trapping set. Fig. 2 shows tanner graph not satisfying the bit flipping algorithm.

To modify this problem, a new unsatisfied check node is added, as an example Fig. 1. By adding the new check node the matching set must be satisfied. For a set of check nodes  $C$  in a Tanner graph, a matching set of  $C$  is a set of variable nodes such that each of them is adjacent to one and only one check node in  $C$ .

This improved two-stage algorithm can greatly improve the effectiveness of the two-stage iterative decoding algorithm, Specially in the high SNR.

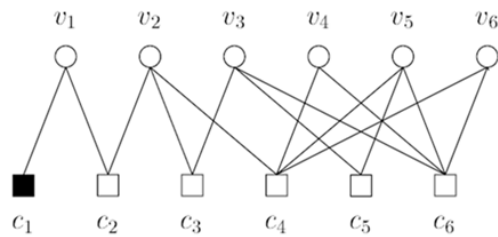


Fig. 2 Tanner graph with only one unsatisfied check node

## III. Experiment and Results

With the simulation, we can see the result that, especially in high SNR, improved two-stage method effect is good.

With MATLAB, simulation is done to seek the performance of this algorithm, under AWGN (Additive White Gaussian Noise) channel. The BER performance of LDPC codes,  $P_e = \text{bits}_{\text{error}} / \text{bits}_{\text{total}}$ , is used to represent the probability of communication error.

Mackay Neal method construction (1024, 512) is used, and LDPC code rate is 1/2. The result of the simulation is as Fig. 3. McGowan-Wilamson(MW), two-stage iterative decoding(Tanner graph cover), and Improved two-stage iterative decoding(Modified TS-BP) algorithm are compared.[10-11]

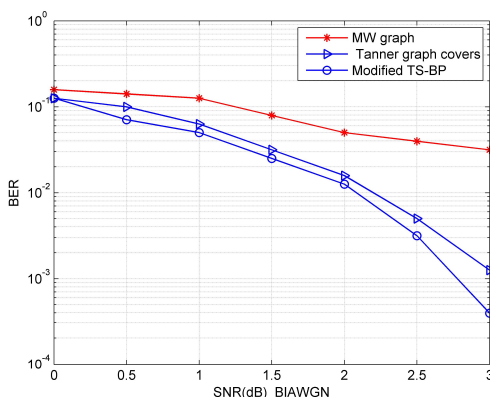


Fig. 3 Performance of algorithms decoding LDPC code under AWGN

#### IV. Conclusions

Simulation results showed that the efficiency for 60.28% was improved on high signal and noise ratio. Although on low signal-to-noise ratio, the decoding performance is not too obvious. but on the high signal-to-noise ratio, the performance is outstanding. Our algorithm has lower complexity than other algorithms which only used soft decision method, because it combines soft decision and hard decision. And theoretically this algorithm is more practical. Its decoding process can obtain stable signal, and provide more reliable and safe environment.

In this paper, we can see that the modified two stage iterative decoding algorithm is to lower the error floor, and to get more higher performance than other algorithm. Simulation result verifies that, through eliminating the trapping set to lower the error floor, the performance of LDPC codes is improved.

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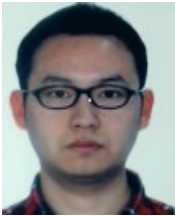
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