# 거리 기반 이동성 관리를 위한 양방향 사용자 이동 모델 연구 ${ }^{+}$ 

# (A Study on a Bidirectional Random Walk Model for Distance Based Mobility Managements) 

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#### Abstract

요 약 거리 기반의 이동성 관리 방식은 무선 네트워크 연구 분야에서 주요한 주제로 고려 되었다. 적당한 이동성 모델로 수학적으로 분석하기 위한 많은 연구가 진행되었다. 특히 양방 향의 랜덤 워크 모델은 단순성의 장점으로 인하여 많이 활용되었다. 그럼에도 불구하고 정확한 수식에 의한 분석은 지금까지 아직 이루어지지 못하였다. 본 고에서 우리는 양방향 랜덤 워크 모델에 관한 정확한 수식을 제공한다. 이러한 수학적 모델은 거리 기반의 이동성 관리 방식에 관한 연구에 많은 도움이 될 것으로 기대된다.

핵심주제어 : 무선 네트워크, 이동성 관리, 수학적 분석 Abstract Distance based mobility management schemes have been considered as a major issue in the wireless network research area. Accordingly, many efforts have been made to analyze them numerically with suitable mobility models. In particular, bidirectional random walk model has been employed frequently due to its simplicity. Nevertheless, the exact equations are not presented so far. In this paper, we provide the exact equations regarding the bidirectional random walk model, which is very useful for the analysis of the distance based mobility management schemes.


Key Words : Wireless networks, mobility management, numerical analysis

## 1. Introduction

The issues on the power savings and handoffs have been extensively studied since they significantly influence the performance of wireless networks [1-6]. Particularly, mobility management

[^0]has been regarded as a key issue for the handoffs so that various user mobility management schemes have been developed to keep track of Mobile Stations (MSs) with small signaling overhead since the MSs are free to move around in wireless networks. Herein, distance based mobility management schemes have been regarded as one of key research issues for the user mobility management schemes. In the distance based mobility management schemes, MSs perform a
location update procedure whenever they move we derive a probability that an MS registers its beyond a predefined distance from where they location $n$ times conditioned that it crosses cell

<Fig. 1> An example of possible cells which an MS can visit.
updated their location before. In order to analyze the distance based mobility management schemes, it is necessary to adopt an appropriate user mobility model.
The bidirectional random walk model is one of popular user mobility models due to the fact that it is so simple that it is easily applicable to numerical analysis. Nevertheless, the previous work does not provide the exact equations. For example, the authors of [7] try to evaluate their handoff scheme, which can be classified as a distance based location update scheme. However, they miss the fact that an MS does not register its location if it returns back to the place where it registered most recently without going farther beyond the place where a new location update is required. In this paper, we provide full derivations of the equations representing the bidirectional random walk model.
This paper is organized as follows: In Section 2,
boundaries $k$ times when employing bidirectional random walk model. Additionally, we will validate the derivations via comparison with simulation results. In Section 3, we conclude our paper.

## 2. Analytical Modeling

Under the bidirectional random walk model, an MS can move in a back-and-forth manner with the same probability along a path composed of serial cells. Following the distance based location update strategy, the MS should register its location whenever it reaches a cell which is located at a predefined threshold distance $\left(=d_{t}\right)$ from where it did most recently.
For the analysis, we assume that an MS departs its initial cell, and thereafter, passes through cells spaced at equidistance. Accordingly, the cells can be
modelled by an one-dimensional coordinate system. We represent the initial cell by $c(0)$ and denote a cell located at the distance of $x$ by $c(x)$ or $d(-x)$
having departed $c(0)$ has to update its location at $c\left(d_{t}\right)$ or $d\left(-d_{t}\right)$. Once the MS performs location update at $d\left(d_{t}\right) / c\left(-d_{t}\right)$, no more location update is

$<$ Fig. $2>$ An example for $\mathrm{w}^{(\mathrm{i})}(\mathrm{r})$.
depending on its relative direction from $c(0)$. For example, if an MS crosses a cell boundary once after departing from $c(0)$, it is located at either $c(1)$ or $c(-1)$. In this case, with one more crossing, it will reach a cell out of three possible cells, i.e., $d(-2), d(0)$, and $d(2)$. In this manner, we can imagine which cell an MS will reach after it crosses cell boundaries $k$ times.

Fig. 1 shows cells at which an MS can be located after it crosses cell boundaries six times until it stops. In this figure, $S(i, k)$ on the righthand side represents a set of possible cells at which the MS can be located after the $i$ th crossing conditioned that it crosses cell boundaries $k$ times until it stops. We denote an element of $S(i, k)$ by $c(l / i, k)$, where 0 $\leq|l| \leq k$, indicating a cell at which an MS is located under the same condition as $S(i, k)$. When adopting distance based location update strategy, an MS
required without moving beyond $c\left(2 d_{t}\right) / c\left(-2 d_{t}\right)$ or $c(0)$. In other words, we can assume that an MS updates its location at either $c\left(d_{t}\right)$ or $c\left(-d_{t}\right)$, and then, it iterates its behavior by regarding either $c\left(d_{t}\right)$ or $c\left(-d_{t}\right)$ as initial cells for the next iteration.
Let $P^{(n)}{ }_{(d t ; k)}$, where $0 \leq n \leq k$, be the probability that an MS updates its location at least $n$ times under the condition that it crosses $k$ cells with a given value of $d_{t}$. For $d_{t}>k$, it never conducts location update, and hence, $P^{(n)}{ }_{(d t, t)}=0$. For $d_{t}=k$, an MS updates its location when it moves straight in one way by crossing $d_{t}$ cells. Accordingly, $P^{(n)}{ }_{(d t, k)}=2 \times(1 / 2)^{d t} P^{(n-1)}{ }_{(d t, k-d t)}$, where $P^{(0)}{ }_{(d t, k)}=1$. Since an MS can move toward a random direction, it can change its movement direction before reaching threshold distance. Therefore, for $d_{t} \leq k$, we should find all the possible combinatorial paths, which an MS can select. Since the rule is applied
recursively if an MS can move more after reaching the cells at threshold distance called threshold cells. Consequently, if we let $Q_{(d t, i)}$ be the number of all the possible paths to reach threshold cells with $d_{t}+2 i$ crosses, we can derive $P^{(n)}(d t, k)$ in a recursive fashion due to the fact that an MS begins to move at threshold cells with the remaining crosses (= $\left.k^{-}\left(d_{t}+2 i\right)\right)$ after reaching the threshold cells with $d_{t}+2 i$ crosses conditioned that it can cross $k$ times until it stops as denoted by:

$$
\begin{equation*}
P_{\left(d_{t}, k\right)}^{(n)}=\sum_{i=0}^{\left\lfloor\left(k-d_{t}\right) / 2\right\rfloor}\left(\frac{1}{2}\right)^{d_{t}+2 i} Q_{\left(d_{t}, i\right)} P_{\left(d_{t}, k-\left(d_{t}+2 i\right)\right)}^{(n-1)} \tag{1}
\end{equation*}
$$

Since there exist only two paths for the case that an MS can reach threshold cells with $d_{t}$ crosses when departing $c(0 \mid 0, k), Q_{(d t, i)}=2$ for $2 i=0$. Similarly, for $2 i \cdot d_{t}$, an MS can reach threshold cells with two paths when departing $c(0 \mid 2 i, k)$. Additionally, the MS can reach threshold cells by passing through $c(l / 2 i, k)$, where $1 \leq / l / \leq\left\lfloor\left(d_{t}-1\right) / 2\right\rfloor$ so that the number of paths toward threshold cells from $c(12 i, k)$ needs to be derived for the proper calculation of $Q_{(d t, i)}$.

Fig. 2 shows an example for the derivation of the number, which is referred to as $W^{(d t)}(r)$, where $r$ is a relative location of a cell destined by an MS as explained later by assuming $d_{t}=4, r=2$, and $k=$ 6. As indicated by thick lines, an MS moves toward $d(-4 \mid 6,6)$ via $c(-2 \mid 2,6)$. However, let us make an assumption that an MS begins to move from $d(-4 \mid 6,6)$ toward $c(-2 \mid 2,6)$, reversely. Consequently, the number which we want to obtain is identical to the number of paths under this assumption. For this reason, we denote $c(-4 \mid 6,6)$ and $c(-2 \mid 2,6)$ as $\hat{c}(0 \mid 0,4)$ and $\hat{\mathrm{c}}(2 \mid 4,4)$, respectively, where $\hat{\mathrm{c}}(\cdot)$ is identical to $c(\cdot)$. Under this assumption, an MS can cross $d_{t}$ times while it is prevented from passing through $\hat{c}$ $(I)$, where $l<0$. Therefore, it satisfies that $W^{(1)}{ }_{(r)}=$ $r$ only if $r=j-2$ and $W^{(j)}(r)$ is always zero for $r \leq 0$. In this example, $W^{(4)}{ }_{(2)}=2$. For general terms, $W^{(j)}{ }_{(r)}$
$=W^{(j-1)}{ }_{(r+1)}+W^{(j-1)}{ }_{(r-1)}$ by letting $r=d_{t}-2 \mid / /$. The recursion stops only if $r=j-2$ and its initial condition $W^{(j)}{ }_{(0)}=0$. By using this equation, we derive $Q_{(d t, i)}$ by:

$$
\begin{equation*}
Q_{\left(d_{t}, i\right)}=2\binom{2 i}{i}+2 \sum_{l=1}^{\left\lfloor\left(d_{i}-1\right) / 2\right\rfloor} w_{\left(d_{i}-2 l\right)}^{\left(d_{i}\right)}\binom{2 i}{i+l} . \tag{2}
\end{equation*}
$$

For $2 i \geq d_{t}$, an MS can reach a cell $c(l / 2 i, k)$, where $0 \leq|l| \leq\left\lfloor\left(d_{t}-1\right) / 2\right\rfloor$ without moving beyond threshold cells. In this figure, the number of paths to arrive a cell $d(/ / 2 i, k)$ is identical to twice the number of paths to reach $c(1 / 2 i-2, k)$ plus the number of paths for $c(1-2 \mid 2 i-2, k)$ and $c(1+2 \mid 2 i-2, k)$. Exceptionally, since an MS can reach $c \pm 1$ $\left.\left(d_{t}-1\right) / 2 \mid 2 \mathrm{i}, k\right)$, we derive $Q_{(d t, i)}$ by:

$$
\begin{gather*}
Q_{\left(d_{t, i)}=2 q_{(2 i, 0)}^{(2 i)}+2 \sum_{l=1}^{\lfloor(d-1) / 2\rfloor} w_{(d-2 l)}^{(d)} q_{(2 i, l)}^{(2 i)} .\right.}^{q_{(x, y)}^{(j)}=} \begin{array}{ll}
\frac{1}{2}\left(3+(-1)^{\left.d_{t}\right)} q_{(x, y)}^{(j-2)}+q_{(x, y-1)}^{(j-2)},\right. & y=\left\lfloor\left(d_{t}-1\right) / 2\right\rfloor, \\
\frac{1}{2}\left(3+(-1)^{d_{t}}\right) q_{(x, y)}^{(j-2)}+q_{(x, y+1)}^{(j-2)}, & y=-\left(\left\lfloor\left(d_{t}-1\right) / 2\right\rfloor\right), \\
2 q_{(x, y)}^{(j-2)}+q_{(x, y+1)}^{(j-2)}+q_{(x, y-1)}^{(j-2)}, & y \neq \pm\left(\left\lfloor\left(d_{t}-1\right) / 2\right\rfloor\right), \\
2 q_{(x, y)}^{(j-2)}, & y=0 \text { and } d_{t}=2 .
\end{array} \tag{3}
\end{gather*}
$$

From these equations for $Q_{(d t, i)}$, we derive the location update probability $p^{(n)}{ }_{(d t, k)}$ that an MS updates its location $n$ times under the condition that it can cross cell boundaries $k$ times with given threshold distance $\left(=d_{t}\right)$, i.e., simply called crossing condition, by:

$$
\begin{equation*}
p_{\left(d_{t}, k\right)}^{(n)}=P_{\left(d_{t}, k\right)}^{(n)}-P_{\left(d_{t}, k\right)}^{(n+1)} . \tag{5}
\end{equation*}
$$

$\Psi^{(n)}(d t k)$ represents a set, which consists of the elements representing the remaining number of cell boundaries which an MS can cross until the MS stops after it updates its location only $n$ times under the crossing condition. Its element $\psi\left(i, \psi^{\prime}\right)(>$ 0 ) is indexed by both $i$ and $\psi^{\prime}$, where $0 \leq i \leq \max (0$,
$\left\lfloor\frac{\left.\psi^{\psi^{\prime}}-\frac{d}{2}\right\rfloor}{2}\right\rfloor$ and $\psi^{\prime}(>0)$ is also an element of $\Psi^{(n)}(d t, k)$, recursively. We derive it by:

$$
\begin{equation*}
\Psi_{(d, t)}^{(n)}=\left\{\psi\left(i, \psi^{\prime}\right) \mid \psi\left(i, \psi^{\prime}\right)=\psi^{\prime}-\left(2 i+d_{t}\right), \psi^{\prime} \in \Psi_{\left(d_{t}, k\right)}^{(n-1)}\right\}, \tag{6}
\end{equation*}
$$

where the initial condition $\Psi_{(d, k)}^{(0)}=\{k\}$.
$c^{(n)}{ }_{(d t, k)}$ is a number of cell boundaries between a cell where an MS updated its location most recently and another cell where an MS stops its crossing under the crossing condition, and hence, can be used as a distance. We can derive $c^{(n)}\left({ }_{(t, t)}\right.$ by:

$$
\begin{equation*}
c_{\left(d_{t}, k\right)}^{(n)}=\sum_{\forall \psi \in \Psi_{\left(d_{t}, k\right)}^{(n)}}\left(\frac{1}{2}\right)^{k} L_{\left(d_{t}, \psi\right)}^{(n)} \sum_{i=-\psi}^{\psi} u(i) \cdot r(i, \psi), \tag{7}
\end{equation*}
$$

where $\quad L_{\left(d, \psi, \psi\left(\psi^{\prime}\right)\right)}^{(n)}=Q_{\left(d, \psi, \psi\left(i, \psi^{\prime}\right)\right)} L_{\left(d, \psi^{\prime} \psi^{\prime}\right.}^{(n-1)}$ and $L_{(d, k)}^{(0)}=1$. In this equation, $r(i, \psi)$ is given by:

$$
\begin{gather*}
u(i)= \begin{cases}|i|, & \text { for } \psi=\text { even and } i=\text { even, }, \\
|i|, & \text { for } \psi=\text { odd and } i=\text { odd }, \\
0, & \text { otherwise, }\end{cases}  \tag{8}\\
r(i, \psi)= \begin{cases}0, & |i| \geq d, \\
1, & |i|=\psi, \\
r(\psi-1, i-1)+r(\psi-1, i+1), & \text { otherwise. }\end{cases} \tag{9}
\end{gather*}
$$

We verify the equations are correctly derived by comparing with simulation results denoted by $c^{(n)}{ }_{(d, t)}$. In the comparison, we confirm that $\max \left(\left\{\left|p^{(n)}{ }_{(d t, k)}-s^{(n)}{ }_{(d t, k)}\right|\right\}\right)<0.004$, where $1 \leq d_{t}<100,1 \leq$ $k \triangleleft 100$, and $0 \leq n \triangleleft 100$. Figs. 3 (a)-(c) show the analysis results well match with the simulation results. In the figures, the $x$-axis represents the probabilities that an MS updates its location $x$ times when $\left(k, d_{t}\right)$ is given, where $k$ is the number of cell crossings and $d_{t}$ is the threshold distance.

## 3. Conclusion

We provide the detailed equation derivations for distance based location update strategy with the bidirectional random walk model. The equation is expected to be useful for various applications in order to analyze user mobility management schemes including paging and handoff.

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(c) $\mathrm{k}=60$
<Fig. 3> Location update probabilities.
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