Commun. Korean Math. Soc. **29** (2014), No. 4, pp. 539–547
http://dx.doi.org/10.4134/CKMS.2014.29.4.539

NON-EXISTENCE OF LIGHTLIKE SUBMANIFOLDS OF INDEFINITE KAEHLER MANIFOLDS ADMITTING NON-METRIC π-CONNECTIONS

Dae Ho Jin

Abstract. In this paper, we study two types 1-lightlike submanifolds M, so called lightlike hypersurface and half lightlike submanifold, of an indefinite Kaehler manifold $\bar{M}$ admitting non-metric π-connection. We prove that there exist no such two types 1-lightlike submanifolds of an indefinite Kaehler manifold $\bar{M}$ admitting non-metric π-connections.

1. Introduction

A linear connection $\bar{\nabla}$ on a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is called a *non-metric π-connection* if, for any vector fields X, Y and Z on $\bar{M}$, it satisfies

$$(\bar{\nabla}_X \bar{g})(Y,Z) = -\,\pi(Y)\bar{g}(X,Z) - \pi(Z)\bar{g}(X,Y), \tag{1.1}$$

where π is a 1-form, associated with a non-vanishing smooth vector field ζ on $\bar{M}$ by $\pi(X) = \bar{g}(X,\zeta)$. We say that ζ is the *characteristic vector field* of $\bar{M}$.

Two special cases are important for both the mathematical study and the applications to physics: (1) A non-metric π-connection $\bar{\nabla}$ on $\bar{M}$ is called a *semi-symmetric non-metric connection* if its torsion tensor $\bar{T}$ satisfies

$$\bar{T}(X,Y) = \pi(Y)X - \pi(X)Y.$$

The notion of semi-symmetric non-metric connections on a Riemannian manifold was introduced by Ageshe and Chafle [1] and later studied by many authors. The lightlike version of Riemannian manifolds with semi-symmetric non-metric connections have been studied by some authors [15, 16, 17, 18, 23].

(2) A non-metric π-connection $\bar{\nabla}$ on $\bar{M}$ is called a *quarter-symmetric non-connection* if its torsion tensor $\bar{T}$ satisfies

$$\bar{T}(X,Y) = \pi(Y)\phi X - \pi(X)\phi Y,$$

where ϕ is a $(1,1)$-type tensor field. In particular, if $\phi X = X$, then the quarter-symmetric connection reduces to the semi-symmetric connection [8]. Thus the

Received December 16, 2013.
2010 *Mathematics Subject Classification.* Primary 53C25, 53C40, 53C50.
Key words and phrases. non-metric π-connection, lightlike hypersurface, half lightlike submanifold.

notion of the quarter-symmetric non-metric connection generalizes the notion of the semi-symmetric non-metric connection. Quarter-symmetric non-metric connection was intorduced by S. Golad [9], and then, studied by many authors [2, 3, 21, 22]. N. Pušić [20], and J. Nikić and Pušić [19] studied on quarter-symmetric metric connections on Kaehler manifold.

The theory of lightlike submanifolds is an important topic of research in differential geometry due to its application in mathematical physics, especially in the general relativity. The study of such notion was initiated by Duggal and Bejancu [4] and later studied by many authors [6, 7]. 1-lightlike submanifold is a particular case of general r-lightlike submanifold [4] such that $r = 1$. Much of its geometry will be immediately generalized in a formal way to arbitrary r-lightlike submanifolds. Moreover the theory of 1-lightlike submanifold is a simple one more than that of r-lightlike submanifold. For this reason, we study only 1-lightlike submanifolds in this paper.

Although now we have lightlike version of a large variety of Riemannian submanifolds, unfortunately, the geometry of lightlike submanifolds of indefinite Kaehler manifolds admitting non-metric π-connections has not been introduced as yet. In this paper, we study two types 1-lightlike submanifolds, named by lightlike hypersurface and half lightlike submanifold, of an indefinite Kaehler manifold $\bar{M}$ admitting non-metric π-connections. We prove that there exist no such two types 1-lightlike submanifolds of an indefinite Kaehler manifold $\bar{M}$ admitting non-metric π-connections.

2. Non-existence theorem for lightlike hypersurfaces

Let $\bar{M} = (\bar{M}, \bar{g}, J)$ be an indedinite Kaeler manifold, where $\bar{g}$ is a semi-Riemannian metric and J is an indefinite almost complex structure satisfying

$$J^2 = -I, \quad \bar{g}(JX, JY) = \bar{g}(X, Y), \quad (\bar{\nabla}_X J)Y = 0 \tag{2.1}$$

for any vector field X and Y of $\bar{M}$ ([4, 7, 10, 11, 12]).

Let (M, g) be a lightlike hypersurface of $\bar{M}$. It is well known that the normal bundle $TM^{\perp}$ of M is a subbundle of the tangent bundle TM, of rank 1. A complementary vector bundle $S(TM)$ of $TM^{\perp}$ in TM is non-degenerate distribution on M, which is called a *screen distribution* on M, such that

$$TM = TM^{\perp} \oplus_{orth} S(TM),$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M, by $\Gamma(E)$ the $F(M)$ module of smooth sections of any vector bundle E over M and by $(\cdots)_i$ the i-th equation of $(\cdots)$. We use same notations for any others. It is well-known [4] that, for any null section ξ of $TM^{\perp}$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique lightlike vector bundle $tr(TM)$ in $S(TM)^{\perp}$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \quad \forall\, X \in \Gamma(S(TM)).$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen distribution $S(TM)$, respectively. Then the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus tr(TM) = \{TM^{\perp} \oplus tr(TM)\} \oplus_{orth} S(TM).$$

In the sequel, let X, Y, Z and W be the vector fields on M, unless otherwise specified. Let $\bar{\nabla}$ be the quarter-symmetric non-metric connection of $\bar{M}$ and P the projection morphism of TM on $S(TM)$. Then the local Gauss and Weingartan formulas of M and $S(TM)$ are given, respectively, by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N, \tag{2.2}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N; \tag{2.3}$$

$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{2.4}$$

$$\nabla_X \xi = -A^*_\xi X - \sigma(X)\xi, \tag{2.5}$$

where ∇ and ∇^* are the induced linear connections on TM and $S(TM)$, respectively, B and C are the local second fundamental forms on TM and $S(TM)$, respectively, A_N and A^*_ξ are the shape operators on TM and $S(TM)$, respectively, and τ and σ are 1-forms on TM.

The induced connection ∇ on M is not metric and satisfies

$$(\nabla_X g)(Y,Z) = B(X,Y)\eta(Z) + B(X,Z)\eta(Y) - \pi(Y)g(X,Z) - \pi(Z)g(X,Y), \tag{2.6}$$

where η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X,N).$$

From the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we know that B is independent of the choice of the screen distribution $S(TM)$, and satisfies

$$B(X,\xi) = 0. \tag{2.7}$$

From (2.2), (2.5) and (2.7), we obtain

$$\bar{\nabla}_X \xi = -A^*_\xi X - \sigma(X)\xi. \tag{2.8}$$

In the entire discussion of this article, we shall assume that the characteristic vector field ζ of $\bar{M}$ to be unit spacelike, without loss of generality. Now we set $a = \pi(N)$ and $b = \pi(\xi)$. Then the above second fundamental forms B and C are related to their shape operators by

$$g(A^*_\xi X, Y) = B(X,Y) - bg(X,Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{2.9}$$

$$g(A_N X, PY) = C(X,PY) - ag(X,PY) - \eta(X)\pi(PY), \tag{2.10}$$
$$\bar{g}(A_N X, N) = -a\eta(X), \qquad \sigma(X) = \tau(X) - b\eta(X).$$

Let M be a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$ admitting a non-metric π-connection. For a lightlike hypersurface M of an indefinite Kaehler manifold $\bar{M}$, $S(TM)$ splits as follows [4, 10, 11]:

If ξ and N are local sections of $TM^{\perp}$ and $tr(TM)$, respectively, we have

$$\bar{g}(J\xi,\xi)=\bar{g}(J\xi,N)=\bar{g}(JN,\xi)=\bar{g}(JN,N)=0,\quad \bar{g}(J\xi,JN)=1.$$

These equations show that $J\xi$ and JN belong to $S(TM)$. Thus $J(TM^{\perp})$ and $J(tr(TM))$ are distributions on M, of rank 1 such that $TM^{\perp}\cap J(TM^{\perp})=\{0\}$ and $TM^{\perp}\cap J(tr(TM))=\{0\}$. Hence $J(TM^{\perp})\oplus J(tr(TM))$ is a vector subbundle of $S(TM)$, of rank 2. Then there exists a non-degenerate almost complex distribution D_o on M with respect to J, *i.e.*, $J(D_o)=D_o$, such that

$$TM=TM^{\perp}\oplus_{orth}\{J(TM^{\perp})\oplus J(tr(TM))\oplus_{orth}D_o\}.$$

Consider the 2-lightlike almost complex distribution D such that

$$D=\{TM^{\perp}\oplus_{orth}J(TM^{\perp})\}\oplus_{orth}D_o,\qquad TM=D\oplus J(tr(TM)).$$

Consider the local lightlike vector fields U and V such that

$$U=-JN,\qquad V=-J\xi. \tag{2.11}$$

Denote by S the projection morphism of TM on D with respect to the decomposition $(2.13)_2$. Then any vector field X on M is expressed as follow:

$$X=SX+u(X)U,$$

where u and v are 1-forms locally defined on M by

$$u(X)=g(X,V),\qquad v(X)=g(X,U). \tag{2.12}$$

Using (2.11), the action JX of any $X\in\Gamma(TM)$ by J is expressed as

$$JX=FX+u(X)N, \tag{2.13}$$

where F is a tensor field of type $(1,1)$ globally defined on M by $F=J\circ S$.

Theorem 2.1. *There exist no lightlike hypersurfaces of an indefinite Kaehler manifold admitting a non-metric π-connection.*

Proof. Applying $\bar{\nabla}_X$ to $(2.11)_2$ and using $(2.1)_3$, (2.8) and (2.13), we have

$$\nabla_X V=F(A^*_{\xi}X)-\sigma(X)V,\qquad B(X,V)=u(A^*_{\xi}X). \tag{2.14}$$

On the other hand, taking $Y=V$ to (2.9) and using (2.12), we have

$$B(X,V)=u(A^*_{\xi}X)+bu(X).$$

From the last two equations, we obtain $bu(X)=0$ for all $X\in\Gamma(TM)$. Thus we get $b=0$. This implies that the characteristic vector field ζ is tangent to M. It follow that $B(X,Y)=g(A^*_{\xi}X,Y)$ and $\tau=\sigma$. Applying $\bar{\nabla}_X$ to (2.13) and using $(2.1)_3$, (2.2), (2.3), (2.11) and (2.13), we have

$$(\nabla_X F)Y=u(Y)A_{N}X-B(X,Y)U,$$
$$(\nabla_X u)Y=-\,u(Y)\tau(X)-B(X,FY).$$

On the other hand, applying ∇_X to $u(Y)=g(Y,V)$ and using (2.6), (2.9), (2.14) and the facts that $b=0$, we have

$$(\nabla_X u)(Y)=-\,u(Y)\tau(X)-B(X,FY)-\pi(Y)u(X)-\pi(V)g(X,Y).$$

From the last two equations, we obtain

$$\pi(V)g(X,Y) + \pi(Y)u(X) = 0.$$

Taking the skew-symmetric part of the last equation, we have

$$\pi(X)u(Y) = \pi(Y)u(X). \tag{2.15}$$

Replacing Y by U to (2.15) and using (2.12), we get

$$\pi(X) = \pi(U)u(X).$$

Taking $X = V$ to this, we get $\pi(V) = 0$. As ζ is tangent to M, we have

$$u(\zeta) = g(\zeta, V) = \pi(V) = 0.$$

Taking $Y = \zeta$ to (2.15), we get $u(X) = u(\zeta)\pi(X) = 0$. It is a contradiction to $u(U) = 1$. Thus there exist no lightlike hypersurfaces of an indefinite Kaehler manifold admitting a non-metric π-connection. □

Corollary 2.2. *There exist no lightlike hypersurfaces of an indefinite Kaehler manifold admitting either a semi-symmetric non-metric connection or a quarter-symmetric non-metric connection.*

3. Non-existence theorem for half lightlike submanifolds

A submanifold (M, g) of a semi-Riemannian manifold $\bar{M}$ of codimension 2 is called a *half lightlike submanifold* if the radical distribution $Rad(TM) = TM \cap TM^\perp$ of M is a vector subbundle of the tangent bundle TM and the normal bundle $TM^\perp$ of rank 1. Then there exists complementary non-degenerate distributions $S(TM)$ and $S(TM^\perp)$ of $Rad(TM)$ in TM and $TM^\perp$, respectively, which are called the *screen* and *co-screen* distributions on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \; TM^\perp = Rad(TM) \oplus_{orth} S(TM^\perp).$$

We denote such a half lightlike submanifold by $M = (M, g, S(TM), S(TM^\perp))$. Choose $L \in \Gamma(S(TM^\perp))$ as a unit spacelike vector field, without loss of generality. Consider the orthogonal complementary distribution $S(TM)^\perp$ to $S(TM)$ in $T\bar{M}$. Certainly, $Rad(TM)$ and $S(TM^\perp)$ are vector subbundles of $S(TM)^\perp$. As the co-screen distribution $S(TM^\perp)$ is non-degenerate, we have

$$S(TM)^\perp = S(TM^\perp) \oplus_{orth} S(TM^\perp)^\perp,$$

where $S(TM^\perp)^\perp$ is the orthogonal complementary to $S(TM^\perp)$ in $S(TM)^\perp$. For any null section ξ of $Rad(TM)$, there exists a uniquely defined lightlike vector bundle $ltr(TM)$ and a null vector field N of $ltr(TM)$ satisfying

$$\bar{g}(\xi, N) = 1, \; \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0, \; \forall\, X \in \Gamma(S(TM)).$$

We call N, $ltr(TM)$ and $tr(TM) = S(TM^\perp) \oplus_{orth} ltr(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to $S(TM)$, respectively [5]. Thus $T\bar{M}$ is decomposed as

$$T\bar{M} = TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM)$$

$$= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}).$$

The local Gauss and Weingartan formulas of M and $S(TM)$ are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N + D(X,Y)L, \tag{3.1}$$

$$\bar{\nabla}_X N = -A_{_N} X + \tau(X)N + \rho(X)L, \tag{3.2}$$

$$\bar{\nabla}_X L = -A_{_L} X + \phi(X)N, \tag{3.3}$$

$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{3.4}$$

$$\nabla_X \xi = -A^*_\xi X - \sigma(X)\xi, \tag{3.5}$$

where ∇ and ∇^* are linear connections on TM and $S(TM)$, respectively, B and D are called the *local second fundamental forms* of M, C is called the *local second fundamental form* on $S(TM)$. $A_{_N}$, A^*_ξ and $A_{_L}$ are linear operators on TM and τ, ρ, ϕ and σ are 1-forms on TM. Using (1.1) and (3.1), we have

$$(\nabla_X g)(Y,Z) = B(X,Y)\eta(Z) + B(X,Z)\eta(Y) - \pi(Y)g(X,Z) - \pi(Z)g(X,Y). \tag{3.6}$$

From the facts $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X,Y) = \bar{g}(\bar{\nabla}_X Y, L)$, we know that B and D are independent of the choice of $S(TM)$ and satisfy

$$B(X,\xi) = 0, \qquad D(X,\xi) = -\phi(X). \tag{3.7}$$

From (3.1), (3.5) and (3.7), we obtain

$$\bar{\nabla}_X \xi = -A^*_\xi X - \sigma(X)\xi - \phi(X)L. \tag{3.8}$$

Now we set $b = \pi(\xi)$, $a = \pi(N)$ and $e = \pi(L)$. Then the above three local second fundamental forms are related to their shape operators by

$$g(A^*_\xi X, Y) = B(X,Y) - bg(X,Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{3.9}$$

$$g(A_{_L} X, Y) = D(X,Y) - eg(X,Y) + \phi(X)\eta(Y), \tag{3.10}$$
$$\bar{g}(A_{_L} X, N) = \rho(X) - e\eta(X),$$

$$g(A_{_N} X, PY) = C(X,PY) - ag(X,PY) - \eta(X)\pi(PY), \tag{3.11}$$
$$\bar{g}(A_{_N} X, N) = -a\eta(X), \qquad \sigma(X) = \tau(X) - b\eta(X).$$

Let M be a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$ admitting a non-metric π-connection. For a lightlike hypersurface M of an indefinite Kaehler manifold $\bar{M}$, $S(TM)$ splits as follows [4, 12, 13, 14]:

If ξ, N and L are local sections of $Rad(TM)$, $ltr(TM)$ and $S(TM^{\perp})$, respectively, then we have

$$\bar{g}(J\xi,\xi) = \bar{g}(J\xi,N) = \bar{g}(J\xi,L) = \bar{g}(JN,\xi) = \bar{g}(JN,N)$$
$$= \bar{g}(JN,L) = \bar{g}(JL,\xi) = \bar{g}(JL,N) = \bar{g}(JL,L) = 0,$$
$$\bar{g}(J\xi,JN) = \bar{g}(JL,JL) = 1.$$

From these equations, we show that $J\xi$, JN and JL belong to $S(TM)$. Thus $J(Rad(TM)), J(ltr(TM))$ and $J(S(TM^{\perp}))$ are distributions on M, of rank 1. Thus $J(Rad(TM)) \oplus J(ltr(TM)) \oplus_{orth} J(S(TM^{\perp}))$ is a vector subbundle of $S(TM)$, of rank 3. Then there exists a non-degenerate almost complex distribution H_o on M with respect to J, *i.e.*, $J(H_o) = H_o$, such that

$$S(TM) = J(Rad(TM)) \oplus J(ltr(TM)) \oplus_{orth} J(S(TM^{\perp}) \oplus_{orth} H_o.$$

Consider the 2-lightlike almost complex distribution H such that

$$H = \{Rad(TM) \oplus_{orth} J(Rad(TM))\} \oplus_{orth} H_o,$$
$$TM = H \oplus J(ltr(TM)) \oplus_{orth} J(S(TM^{\perp})).$$

Consider the null and spacelike vector fields $\{U, V\}$ and W such that

$$U = -JN, \quad V = -J\xi, \qquad W = -JL. \tag{3.12}$$

Denote by S the projection morphism of TM on H. Any vector field X on M is expressed as follows:

$$X = SX + u(X)U + w(X)W,$$

where u, v and w are 1-forms locally defined on M by

$$u(X) = g(X, V), \quad v(X) = g(X, U), \quad w(X) = g(X, W). \tag{3.13}$$

Using (3.12), the action JX of X by J is expressed as follow:

$$JX = FX + u(X)N + w(X)L, \tag{3.14}$$

where F is a tensor field of type $(1, 1)$ globally defined on M by $F = J \circ S$.

Theorem 3.1. *There exist no half lightlike submanifolds of an indefinite Kaehler manifold admitting a non-metric π-connection.*

Proof. In this proof we take $X \in \Gamma(TM)$. Applying $\bar{\nabla}_X$ to $(3.12)_2$ and using $(2.1)_3$, (3.1), (3.8), (3.12) and (3.14), we have

$$\nabla_X V = F(A^*_{\xi} X) - \sigma(X)V - \phi(X)W, \tag{3.15}$$

$$B(X, V) = u(A^*_{\xi} X), \quad D(X, V) = w(A^*_{\xi} X). \tag{3.16}$$

On the other hand, taking $Y = V$ to (3.9) and using (3.13), we have

$$B(X, V) = u(A^*_{\xi} X) + bu(X).$$

From this and $(3.16)_1$, we obtain $bu(X) = 0$. Thus we get $b = 0$. It follow that $B(X, Y) = g(A^*_{\xi} X, Y)$ and $\tau = \sigma$. Applying $\bar{\nabla}_X$ to $(3.12)_3$ and using $(2.1)_3$, (3.1), (3.3), (3.12) and (3.14), we have

$$\nabla_X W = F(A_{_L} X) + \phi(X)U, \tag{3.17}$$

$$B(X, W) = u(A_{_L} X), \quad D(X, W) = w(A_{_L} X). \tag{3.18}$$

On the other hand, taking $Y = W$ to (3.10), we have

$$D(X, W) = w(A_{_L} X) + ew(X).$$

From this and $(3.18)_2$, we obtain $ew(X) = 0$. Thus we get $e = 0$. As $b = e = 0$, the characteristic vector field ζ is tangent to M.

Applying $\bar{\nabla}_X$ to (3.14) and using (3.1) ∼ (3.3), (3.12) and (3.14), we have

$$(\nabla_X F)Y = u(Y)A_N X + w(Y)A_L X - B(X,Y)U - D(X,Y)W,$$
$$(\nabla_X u)Y = -u(Y)\tau(X) - w(Y)\phi(X) - B(X,FY),$$
$$(\nabla_X w)(Y) = -u(Y)\rho(X) - D(X,FY).$$

On the other hand, applying ∇_X to $u(Y) = g(Y,V)$ and $w(Y) = g(Y,W)$ by turns and using (3.6), (3.15), (3.17) and $(3.18)_1$, we have

$$\begin{aligned}(\nabla_X u)(Y) &= -u(Y)\tau(X) - w(Y)\phi(X) - B(X,FY)\\ &\quad - \pi(Y)u(X) - \pi(V)g(X,Y),\\ (\nabla_X w)(Y) &= -u(Y)\rho(X) - D(X,FY)\\ &\quad - \pi(Y)w(X) - \pi(W)g(X,Y).\end{aligned}$$

From the last four equations, for all X, $Y \in \Gamma(TM)$ we obtain

$$\pi(V)g(X,Y) + \pi(Y)u(X) = 0, \quad \pi(W)g(X,Y) + \pi(Y)w(X) = 0.$$

Taking the skew-symmetric part of the last two equations, we have

$$\pi(X)u(Y) = \pi(Y)u(X), \quad \pi(X)w(Y) = \pi(Y)w(X).$$

Replacing Y by U to the first and Y by W to the second, we have

$$\pi(X) = \pi(U)u(X), \quad \pi(X) = \pi(W)w(X).$$

Taking $X = U$ to the second equation, we get $\pi(U) = 0$. Thus, from the first of the last equations, we obtain $\pi(X) = 0$ for all $X \in \Gamma(TM)$. It is a contradiction as $\pi(\zeta) = 1$ and ζ is tangent to M. Thus there exist no half lightlike submanifolds of an indefinite Kaehler manifold admitting a non-metric π-connection. □

Corollary 3.2. *There exist no half lightlike submanifolds of an indefinite Kaehler manifold admitting either a semi-symmetric non-metric connection or a quarter-symmetric non-metric connection.*

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Department of Mathematics
Dongguk University
Kyongju 780-714, Korea
E-mail address: jindh@dongguk.ac.kr