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Derivation of Zeros from Externally-loaded Feed-forward Element of Filter Network

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Abstract

We present a mathematical method for calculation of transmission zero locations, determining a filtering characteristics of two-port systems. By adjusting element values based on the zero locations, the frequency-selectivity is characterized. The characteristic polynomial of ladder networks in externally-loaded feed-forward systems is considered by adopting chain matrices for subsystems. This method can be extended to other types of lumped systems with cross-coupled sections. We find out the zeros by solving characteristics polynomials of closed-form expressions in terms of Laplace impedances of elements. The pairs of complex zeros are shown to be solely from the cross-coupled portion of the system.

Keywords: *Ladder Networks; Transmission Zeros, Cross-Coupled (CC); Pi Section*

1. Introduction

Transmission zeros (TZs) from the cross-coupled (CC) systems are derived from an initially-synthesized ladder system with externally-loaded feed forward elements. By adding a feed forward CC bridge to the to the ladder systems an integer pair of TZs can be produced in a complex *s*-plane. The transfer function of passive networks of *R*'s and *C*'s with a cross-coupled section was derived to discuss complex and real zeros. We demonstrate in this paper the finite-frequency complex pairs of TZs are produced solely from the CC portion of the circuit.

2. ABCD matrices

A low-pass filter is a filter that passes low-frequency signals and reduces the amplitude of signals with frequencies higher than the cutoff frequency. The actual amount of attenuation for each frequency varies

depending on specific element values [1]. We consider the frequency transfer function of a low-pass filter. The complex zeros due to the cross-couples in the form of feed-forward are determined from the numerator polynomial of the transfer function. The ABCD matrix T of n cascaded filter networks is given by,

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \prod_{i=1}^{n} T_i , \qquad (1)$$

where T_i is the matrix of the i-th system[2, 3]. The entry A of Eq. (1) is given by the element (1,1) of chain matrix T, obtained by open-circuiting the output port, i.e., A is found by applying a voltage V_i at port 1, and measuring the open-circuit voltage V_o at port 2. The voltage transfer function $H(s)$ can thus be expressed as with $N(s)$ and $D(s)$ are the numerator and the denominator polynomials of $H(s)$ [4].

$$H(s) = \left. \frac{V_o}{V_i} \right|_{I_o=0} = \frac{1}{A} = \frac{1}{T(1,1)} = \frac{N(s)}{D(s)} , \qquad (2)$$

In Eq. (2), the entry (1, 1) is known be obtained from the transfer function. The canonical form of the numerator polynomial is defined as the characteristic polynomial of the TZs. Equating the polynomial to zero, the transmission zero characteristic equation (TZCE) is obtained, which is to be solved to find out TZs. We define positively cross-coupled (PCC) network: a network where the sign of the cross-coupling is the same as the sign of the main line coupling (i.e., inductive cross-coupling in an inductively coupled circuit *or* capacitive cross-coupling in an capacitively coupled circuit).

3. Ladder networks

In Fig. 1, a ladder network consists of cascaded asymmetrical *L*-sections (unbalanced) or *C*-sections (balanced). In low pass form the topology would consist of series inductors and shunt capacitors. Other band forms would have an equally simple topology transformed from the low pass topology. The transformed
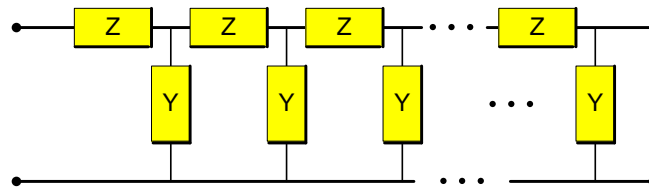

Figure 1. Ladder network

network will have shunt admittances that are dual networks of the series impedances if they were duals in the starting network, which is the case with series inductors and shunt capacitors [5]. An initially-synthesized ladder network, with shunt- connected *LC* resonators without any cross-coupling element, is a building block in the design of a cross-coupled system. Each *LC* resonator has impedances of *L* and *C* in parallel. The series- connected elements could be inductors or capacitors. There are several possibilities for adding cross-coupling elements to the circuit: skipping one resonator of tank. A string of many impedances connected between two reference voltages is an impedance string ladder network. The impedances act as voltage dividers between the referenced voltages. Each tap of the string generates a different voltage which can be compared with another voltage.

4. Pi section topology

The pi network is a specific type of attenuator circuit in electronics whereby the topology the circuit is

formed in the shape of Π. Electronic attenuators are used in order to reduce the level of a signal. Attenuators have a flat frequency response attenuating all frequencies equally in the band they are intended to operate. The attenuator has the opposite task of an amplifier. The topology of an attenuator circuit will usually follow one of the filter system. However, there is no need for more complex circuitry, as there is with filter, due to the simplicity of the frequency response required. Fig. 2 shows a prototype of pi network expressed in terms of impedances.
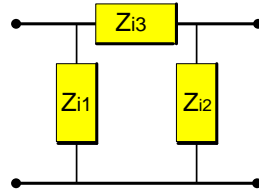

Figure 2 . Pi network

In Fig. 2, the i-th network is composed of impedances z_{i1}, z_{i2} and z_{i3}, which is a **π-network**. The impedance z_{i1} is a network of parallel connection of L_{i1} and C_{i1}, and shunt-connected. z_{i2} is a network of parallel connection of L_{i2} and C_{i2}, and shunt-connected. z_{i3} is just an impedance of single inductor, L_{i3}. The chain matrix, T_i, is given by

$$T_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \tag{3}$$

In Eq.(3), the four entries of are expressed as [6],

$$A_i = 1 + \frac{z_{i3}}{z_{i2}} \qquad\qquad B_i = Z_{i3} \tag{4}$$

$$C_i = \frac{z_{i3}}{z_{i2}} + \frac{z_{i3}}{z_{i2}} + \frac{z_{i3}}{z_{i2}} \qquad\qquad D_i = 1 + \frac{z_{i3}}{z_{i2}}$$

In these equations of Eq. (4), each impedance of the matrix entries is expressed in terms of Laplace impedances as

$$Z_{i1} = \frac{sL_{i1}}{s^2 L_{i1} C_{i1} + 1} \ , \quad Z_{i2} = \frac{sL_{i2}}{s^2 L_{i1} C_{i1} + 1}, \quad Z_{i3} = sL_{i3} \tag{5}$$

In Eq.(5), the Laplace impedances are used to obtain matrix. The i-th subsystem is a π-network used in the procedure of getting the chain matrix. The source voltage is operated with a microwave system [7].

5. Real system with one tank cross-coupled

An inductor-capacitor circuit (LC circuit) is an electrical circuit composed of inductors and capacitors. A second order LC circuit is composed of one inductor and one capacitor and is the simplest type of LC circuit. A second order LC circuit, also called a resonant circuit, tank circuit, or tuned circuit, consists of one inductor and one capacitor. The circuit can act as an electrical resonator, an electrical analogue of a tuning fork, storing energy oscillating at the circuit's resonant frequency. LC circuits are used either for generating signals at a particular frequency, or picking out a signal at a particular frequency from a more complex signal. They are key components in many electronic devices, particularly radio equipment, used in circuits such as oscillators, filters, and frequency mixers. In Fig. 3, a real system cross-coupled with one tank is shown. The transfer function can be derived from Eqs. (1)-(5). After cancellations of the common terms of poles and
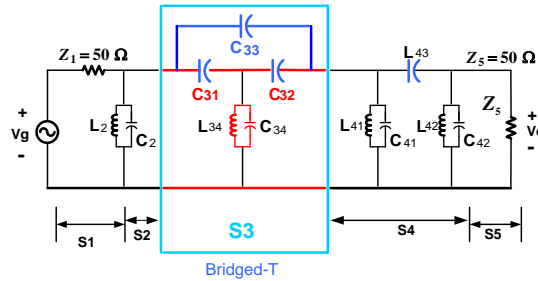
zeros in

Figure 3.   Negatively cross-coupled (NCC) filter network without skipping any resonators.

numerator and denominator polynomials, the canonical form of the numerator polynomial, i.e., TZCE in the transfer function in is obtained as a 3rd degree polynomial, namely

$$N(s) = 50\, L_2 L_{22} L_{41} L_{42}\, s \cdot [C_{31} C_{32} C_{33} C_{34} L_{34} s^2 + 1]\,.$$

Equating to zero, i.e., $N(s) = 0$, the TZCE of the network shown in Fig. 3 is expressed as a product of two functions

$$f(s) = s \cdot (a_{32} s^2 + 1) \equiv f_1(s) \cdot f_2(s) = 0\,,$$

With

$$f_1(s) = s\,.$$

This tells a single zero is always located at the origin of complex plane.

6. Conclusions

We have presented a theoretical investigation of a practical method to calculate the quadratic characteristic polynomial in externally-loaded Feed-forward Systems. It was possible for us to determine quantitatively the locations of complex transmission zeros (TZs) of positively cross-coupled systems. Solving, the location of the zeros are found out.

Acknowledgement

References

[1]   K. S. Lee, G. K. Choi, *PAPR Reduction for WPM Schemes using Filter Design Schemes*,http://dx.doi.org/10.7236, JIIBC. 2013-1-7, 2023.
[2]   A. Gerrard, J. M., Burch, *Introduction to matrix methods in optics*, Courier Dover, 1994.
[3]   M. E. Van Valkenburg, *Network Analysis* 3rd edition, Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1974.
[4]   J. S. Hong, M. J. Lancaster, *Microstrip Filters for FR/Microwave Applications,* John Wiley & Sons, Inc, 2001.
[5]   http:// www3.Alcatel-lucent.com/bstj/v.16/article/bstj16-3-303, 1997.
[6]   P.A. Rizzi *Microwave Engineering: Passive Circuits*, Prentice-Hall, Englewood Cliffs, New Jersey, 1988.
[7]   J.S. Yang, Y. H. Kim, J. Y. Kim, *Research on Spectral Interference of Microwave Systems*, JIWIT 2012-6-30, 2012.