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Nonlinear Time-Varying Control Based on Differential Geometry

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Abstract

This paper presents a study on nonlinear time varying systems based on differential geometry. A brief introduction about controllability and involutivity will be presented. As an example, the exact feedback linearization and the approximate feedback linearization are used in order to show some application examples.

Keywords: *Geometric Nonlinear Time-varying Control, Exact Feedback Linearization, Approximate Feedback Linearization.*

1. Introduction

The actual physical system is a time-varying parameters expressed by the time-varying nonlinear system. For example, a robot manipulator system that has load variations and follows provided trajectory, a flight vehicle system in a number of nominal routes towards the time varying disturbance, the industrial processes used in the actuator and the overall consideration of the chemical reaction process such as the both time-varying nonlinear systems.

In such system, the time-varying nonlinear differential equation is demonstrated towards the physical, chemical theories and several conditions that considered as mathematical models in coupled variation. Thus, according to the time-varying in coupled strong nonlinear and parameter, it is expressed as dynamic equation. Most cases to remove time-varying system, the theories like planning technique by ignoring complex dynamics and parameter variation of time-varying to simplify the dynamic model, and under the assumption of the system parallel state in the surrounding area, linear control in linear system that ignored the 2nd abnormal section caused by Taylor series expansion were applied. However, this kind of control method cannot be operated in accurate against the ignored section during the Taylor series expansion process and disturbances. Also, when Lagrange-Euler (LE) of the dynamic equation is expressed, a mathematical model

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towards the expressed error between the nominal model and actual model are considered linear error equation to design the linear controller. This control technique is used in linear controller design method to use in linear control technique, and to control the desired model against the error model. Nevertheless, the considered problems are the technique with the choice of a nonlinear model, arising convergence and assumption of model degree in the measurement of parameter on error model. Until the late studies on the nonlinear system, differential geometry theory of control section was introduced and this theory is strongly used to present the global and structure characteristic on nonlinear system. In fact, using differential geometry concepts and the conditions under which a non-linear feedback linearization control system that can be converted into a linear system. Which means by the state or output of the nonlinear feedback input-state or input-output linearization will be achieved. Therefore, to control the nonlinear system means the conversion to the linearization system. So, linear control theory is applied to satisfy the given performance in the linear system. And through this, it allows to consist in nonlinear system controller. In other words, while ignored nonlinear section has approximate and localized in Taylor series expansion linearization, the feedback linearization technique is global as the expressed model did not ignored the nonlinear section which makes rather more perfect linearization technique. The feedback linearization technique is developed and subdivided in various ways. But common problems with these techniques are the requirement of accurate expression on model to get necessary state variation of nonlinear system and input variation, and relatively excessive amount of calculation on inverse variation as well as the variation between the nonlinear and linear systems.

2. Analysis Necessary and Sufficient Conditions To Extended Feedback Linearization

The main objective of this paper is to provide an explicit and simple stabilizing controller for single-input single output nonlinear systems using the geometric nonlinear control for the exact feedback linearization and approximate feedback linearization. Consider a standard smooth nonlinear control system affine in the input u(t) given by:

$$\dot{x}(\mathrm{t}) = f(x(t),t) + g(x(t),t)u(t) \tag{1}$$

Where, stste x is n vector($\mathrm{x} \in \mathrm{R}^{\mathrm{n}}$), state and input character function of system - f, g - is smooth n vector field(C^{∞}), u is single input. For simplicity of expression, time t of the function of state, input is omitted.
And, let's define the following vectors.

$$\mathrm{X} = [x^T, t]^T, F = [f^T, 1]^T, G = [g^T, 0]^T \tag{2}$$

Using the equation (2) and the concept of differential geometry, the system in the form (1) is said to be input-state linearizable if there exists a region Ω in R^{n}, such that the following conditions hold :

1. The vector field $\{\mathrm{G}, \mathrm{ad}_{\mathrm{F}}\mathrm{G}, \cdots, \mathrm{ad}_{\mathrm{F}}^{\mathrm{n}-1}\mathrm{G}\}$ has rank n , is linearly independents in Ω.

This condition indicates the system controllability in which the vector fields $[\mathrm{G}, \mathrm{ad}_{\mathrm{F}}\mathrm{G}, \cdots, \mathrm{ad}_{\mathrm{F}}^{\mathrm{n}-1}\mathrm{G}]$ are equivalent to the controllability matrix for linear time-varying systems $[\mathrm{b}, \mathrm{Ab}, \mathrm{A}^2\mathrm{b}, \cdots, \mathrm{A}^{\mathrm{n}-1}\mathrm{b}]$.

2. The distribution $[\mathrm{ad}_{\mathrm{F}}^{\mathrm{i}}\mathrm{G}, \mathrm{ad}_{\mathrm{F}}^{\mathrm{j}}\mathrm{G}] = \mathrm{span}\{\mathrm{G}, \mathrm{ad}_{\mathrm{F}}\mathrm{G}, \cdots, \mathrm{ad}_{\mathrm{F}}^{\mathrm{n}-2}\mathrm{G}\}$ is involutive, for $\mathrm{i}, \mathrm{j} = 0, 1, 2 \cdots, \mathrm{n}-2$, where $[\mathrm{ad}_{\mathrm{F}}^{\mathrm{i}}\mathrm{G}, \mathrm{ad}_{\mathrm{F}}^{\mathrm{j}}\mathrm{G}]$ indicates Lie-Bracket and $\mathrm{ad}_{\mathrm{F}}^{\mathrm{i}}$ is defined as $\mathrm{ad}_{\mathrm{F}}^{\mathrm{o}}\mathrm{G} = \mathrm{G}$ and $\mathrm{ad}_{\mathrm{F}}^{\mathrm{i}}\mathrm{G} = [\mathrm{G}, \mathrm{ad}_{\mathrm{F}}^{\mathrm{i}-1}\mathrm{G}]$.

This involutivity condition enables to find out a new vector of linear state through the states feedback.

3. Accurate Extended Feedback Linearization

If the aforementioned conditions were satisfied, then it is possible to find a scalar function $\phi(\mathrm{x}, \mathrm{t})$ such that.

$$\frac{\partial \phi(\mathrm{x},\mathrm{t})}{\partial \mathrm{X}} \mathrm{ad}_{\mathrm{F}}^{\mathrm{i}}\mathrm{G} = 0, \quad \mathrm{i} = 0, 1, 2, \cdots, \mathrm{n}-2 \tag{3}$$

$$\frac{\partial \phi(x,t)}{\partial X} ad_F^{n-1} G \neq 0 \tag{4}$$

where $\phi(x,t)$ defines a state transformation **z** given by.

$$z = T(x,t) = [L_F^0\, \phi(x,t)\; L_F^1\, \phi(x,t)\,, \cdots, L_F^{n-1}\, \phi(x,t)] \tag{5}$$

The transformation T has the following characteristics :
1. Where $x = 0$ and $t = 0$, the transformation T is zero.
2. The $\phi(x,t)\; L_F^1\, \phi(x,t)\,, \cdots, L_F^{n-1}\, \phi(x,t)$ is a function of the state x and time t.
3. The $\phi(x,t)\; L_F^1\, \phi(x,t)\,, \cdots, L_F^{n-1}\, \phi(x,t)$ has a nonsingular Jacobian matrix for the state x and is one to one mapping.
4. $L_F^n \phi(x,t)$ is a function of the state x, time t and input u, inverses to the function of the input u and corresponds to one to one.
Then, the transformation $\phi(x,t)\; L_F^1\, \phi(x,t)\,, \cdots, L_F^{n-1}\, \phi(x,t)$ is the time dependent state transformation not including the input u, the $L_F^n \phi(x,t)$ is the time dependent state transformation including the input u.
An input control signal is than proposed as:

$$u = \alpha(x,t) + \beta(x,t)v = \frac{-L_F^n \phi(x,t)}{L_G L_F^{n-1} \phi(x,t)} + \frac{1}{L_G L_F^{n-1} \phi(x,t)} v \tag{6}$$

Such that the closed-loop system in its new co-ordinates is described by a linear differential equation :

$$\dot{z} = Az + bv \tag{7}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \;,\; b = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix}$$

A and **b** are in the Brunovsky canonical form. However, the generality is not lost since any representation of a linear controllable system is equivalent to the Brunovsky canonical form (4) through a state transformation.

(Example 1)
Consider a time-varying nonlinear system.

$$\dot{x_1} = x_1 + x_1^2 + u \tag{8}$$

$$\dot{x_2} = x_2 - tu \tag{9}$$

Eq. (8), (9) expressed as the Eq. (1) is as follows :

$$\dot{x}(t) = f(x(t),t) + g(x(t),t)u(t) = \begin{pmatrix} x_1 + x_1^2 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ -t \end{pmatrix} u$$

The state vector $F(x,t)$ and input vector $G(x,t)$ is as bellows :

$$F(x,t) = \begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix} \;,\; G(x,t) = \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix}$$

As it is a second-order system, the vector field for verifying the controllability will be formed by $\{G, ad_F G\}$, where,

$$G = \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix} \ , \ ad_F G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1+2x_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix} = \begin{bmatrix} -1-2x_1 \\ -1+t \\ 0 \end{bmatrix}$$

Hence, the vector field $\{G, ad_F G\}$ will be given by

$$\{G, ad_F G\} = \begin{bmatrix} 1 & -1-2x_1 \\ -t & t-1 \\ 0 & 0 \end{bmatrix} .$$

The vector fields are linearly independent; therefore, the example 1 system is controllable.
In order to calculate the involutivity of the system, it is necessary to calculate the Lie bracket of $\{G, ad_F G\}$ and verify if it may be written as a linear combination of G and $ad_f G$

$$[G, ad_F G] = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1-2x_1 \\ t-1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}.$$

The Lie bracket of $\{G, ad_F G\}$ is the column vectors of the matrix are constant. Therefore, the system is involutive, which satisfy the conditions to apply input-state linearization.
Next to calculation of the new state vector and the input variable using the differential geometry, the first component of the state vector T_1 should be obtained through the expression (3).

As the order of the system is equal to 2, equation ∂T_1 becomes $\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial t} \end{bmatrix} \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix} = 0$, resulting in $\frac{\partial T_1}{\partial x_1} - t \frac{\partial T_1}{\partial x_2} = 0$. And from (4), $\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial t} \end{bmatrix} \begin{bmatrix} -1-2x_1 \\ t-1 \\ 0 \end{bmatrix} \neq 0$, $\frac{\partial T_1}{\partial x_1}(-1-2x_1) + (t-1)\frac{\partial T_1}{\partial x_2} \neq 0$.

Therefore, in order to satisfy the above conditions, to be chosen the following equation.

$$T_1 = tx_1 + x_2 \tag{10}$$

We use (5) to compute the second component of the new state vector T_2.

$$T_2 = \dot{T}_1 = (t+1)x_1 + tx_1^2 + x_2 = L_F T_1 \tag{11}$$

$$T_3 = \dot{T}_2 = (t+2)x_1 + (3t+2)x_1^2 + 2tx_1^3 + x_2 + (1+2tx_1)u = L_F^2 T_1 + L_G L_F T_1 u \tag{12}$$

This gives the new state vector z and input v.
$z = [z_1, z_2] = [T_1, T_2]$, $v = T_3$
Therefore, we obtains the same type as equation (7).

$$\dot{z} = Az + Bv = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v \tag{13}$$

(Example 2)
Consider another time-varying nonlinear system.

$$\dot{x_1} = \sin(x_2) + \sqrt{(t+1)}x_2 \tag{14}$$

$$\dot{x_2} = x_1^4 \cos(x_2) + x_1 x_2 \sin(x_2) + u \tag{15}$$

Same method as in operation, the following result is obtained.

$$F(x,t) = \begin{bmatrix} \sin(x_2) + \sqrt{(t+1)}\, x_2 \\ x_1^4 \cos(x_2) + x_1 x_2 \sin(x_2) \\ 1 \end{bmatrix} \ , \ G(x,t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$ad_F G =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} -$$

$$\begin{bmatrix} 0 & \cos(x_2) + \sqrt{t+1} & \frac{1}{2}(t+1)^{\frac{-1}{2}} x_2 \\ 4x_1^3 \sin(x_2) + x_2 \sin(x_2) & -x_1^4 \cos(x_2) + x_1 \sin(x_2) + x_1 x_2 \cos(x_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} =$$

$$-\begin{bmatrix} \cos(x_2) + \sqrt{t+1} \\ -x_1^4 \cos(x_2) + x_1 \sin(x_2) + x_1 x_2 \cos(x_2) \\ 0 \end{bmatrix}$$

The vector fields $\{G, ad_F G\} = \begin{bmatrix} 0 & -\cos(x_2) - \sqrt{t+1} \\ 1 & x_1^4 \cos(x_2) - x_1 \sin(x_2) - x_1 x_2 \cos(x_2) \\ 0 & 0 \end{bmatrix}$ is linearly independent. Therefore, the example 2 system is controllable.

The system is involutivity, so that the distribution vector $\{G\}$ is a single vector.

$$[G, ad_F G] =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -\cos(x_2) - \sqrt{t+1} \\ x_1^4 \cos(x_2) - x_1 \sin(x_2) - x_1 x_2 \cos(x_2) \\ 0 \end{bmatrix} -$$

$$\begin{bmatrix} 0 & \sin(x_2) & -\frac{1}{2}(t+1)^{\frac{-1}{2}} \\ 4x_1^3 \cos(x_2) - \sin(x_2) - x_2 \cos(x_2) & -x_1^4 \sin(x_2) - x_1 \cos(x_2) - x_1 \cos(x_2) + x_1 x_2 \sin(x_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} \sin(x_2) \\ -x_1^4 \sin(x_2) - x_1 \cos(x_2) - x_1 \cos(x_2) + x_1 x_2 \sin(x_2) \\ 0 \end{bmatrix}.$$

Next to calculation of the new state vector T_1 ;

$$\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial t} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0, \text{ resulting in } \frac{\partial T_1}{\partial x_2} = 0.$$

And from (4),

$$\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial t} \end{bmatrix}\begin{bmatrix} -\cos(x_2) - \sqrt{t+1} \\ x_1^4 \cos(x_2) - x_1 \sin(x_2) - x_1 x_2 \cos(x_2) \\ 0 \end{bmatrix} \neq 0,$$

$$-\frac{\partial T_1}{\partial x_1}\left(\cos(x_2) + \sqrt{t+1}\right) + \frac{\partial T_1}{\partial x_2}\left(x_1^4 \cos(x_2) - x_1 \sin(x_2) - x_1 x_2 \cos(x_2)\right) \neq 0.$$

Therefore, in order to satisfy the above conditions, to be chosen the following equation.

$$T_1 = x_1 \tag{14}$$

We use (5) to compute the second component of the new state vector T_2.

$$T_2 = \dot{T}_1 = \sin(x_2) + \sqrt{(t+1)}\, x_2 = L_F T_1 \tag{15}$$

$$T_3 = \dot{T}_2 = \left(\cos(x_2) + \sqrt{t+1}\right)\left(x_1^4 \cos(x_2) + \dot{x_1} x_2 \sin(x_2) + \boldsymbol{u}\right) + \frac{1}{2}(t+1)^{\frac{-1}{2}} x_2 = \left(\cos(x_2) + \sqrt{t+1}\right)\left(x_1^4 \cos(x_2) + \dot{x}_1 x_2 \sin(x_2)\right) + \frac{1}{2}(t+1)^{\frac{-1}{2}} x_2 + \left(\cos(x_2) + \sqrt{t+1}\right)u = L_F^2 T_1 + L_G L_F T_1 u \quad (16)$$

This gives the new state vector z and input v.

$z = [\, z_1 , z_2 \,] = [T_1 , T_2 \,]$, $v = T_3$

Therefore, we obtains the same type as equation (7). Consequently, the linearized input-output system is expressed.

$$\dot{z} = Az + Bv = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v$$

$$y = Cz = [\,1 \quad 0\,]\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

(Example 3)

Consider a time-varying nonlinear system.

$\dot{x_1} = x_2 + \sin(0.9t)\, x_3$ (17)

$\dot{x_2} = x_3$ (18)

$\dot{x_3} = x_1^2 + u$ (19)

Eq. (17), (18), (19) expressed as the Eq. (1) is as follows :

$$\dot{x}(t) = f(x(t), t) + g(x(t), t)u(t) = \begin{pmatrix} x_2 + \sin(0.9t)x_3 \\ x_3 \\ x_1^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

The state vector $F(x, t)$ and input vector $G(x, t)$ is as bellows :

$$F(x, t) = \begin{bmatrix} x_2 + \sin(0.9t)x_3 \\ x_3 \\ x_1^2 \\ 1 \end{bmatrix}, \quad G(x, t) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

As it is a second-order system, the vector field for verifying the controllability will be formed by$\{G, ad_F G, ad_F^2 G\}$, where,

$$G = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$ad_F G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_2 + \sin(0.9t)x_3 \\ x_3 \\ x_1^2 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & \sin(0.9t) & 0.9\cos(0.9t)x_3 \\ 0 & 0 & 1 & 0 \\ 2x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin(0.9t) \\ -1 \\ 0 \\ 0 \end{bmatrix},$$

$$ad_F^2 G = \begin{bmatrix} 0 & 0 & 0 & -0.9\cos(0.9t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_2 + \sin(0.9t)x_3 \\ x_3 \\ x_1^2 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & \sin(0.9t) & 0.9\cos(0.9t)x_3 \\ 0 & 0 & 1 & 0 \\ 2x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -\sin(0.9t) \\ -1 \\ 0 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} -0.9\cos(0.9t) \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 2x_1\sin(0.9t) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 - 0.9\cos(0.9t) \\ 0 \\ -2x_1\sin(0.9t) \\ 0 \end{bmatrix}$$

Hence, the vector field $\{G, ad_F G, ad_F^2 G\}$ will be given by

$$\{G, ad_F G, ad_F^2 G\} = \begin{bmatrix} 0 & \sin(0.9t) & 1-0.9\cos(0.9t) \\ 0 & 1 & 0 \\ 1 & 0 & -2x_1\sin(0.9t) \\ 0 & 0 & 0 \end{bmatrix}.$$

The vector fields are linearly independent; therefore, the example 3 system is controllable.
In order to calculate the involutivity of the system, it is necessary to calculate the Lie bracket of $\{G, ad_F G\}, \{G, ad_F^2 G\}, \{ad_F G, ad_F^2 G\}$;

$$[G, ad_F G] = \begin{bmatrix} 0 & 0 & 0 & 0.9\cos(0.9t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sin(0.9t) \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[G, ad_F^2 G] = \begin{bmatrix} 0 & 0 & 0 & 0.9^2\sin(0.9t) \\ 0 & 0 & 0 & 0 \\ -2\sin(0.9t) & 0 & 0 & -1.8x_1\sin(0.9t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1-0.9\cos(0.9t) \\ 0 \\ -2x_1\sin(0.9t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[ad_F G, ad_F^2 G] =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0.9^2\sin(0.9t) \\ 0 & 0 & 0 & 0 \\ -2\sin(0.9t) & 0 & 0 & -1.8x_1\sin(0.9t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sin(0.9t) \\ 1 \\ 0 \\ 0 \end{bmatrix} -$$

$$\begin{bmatrix} 0 & 0 & 0 & 0.9^2\sin(0.9t) \\ 0 & 0 & 0 & 0 \\ -2\sin(0.9t) & 0 & 0 & -1.8x_1\sin(0.9t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1-0.9\cos(0.9t) \\ 0 \\ -2x_1\sin(0.9t) \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ -2(\sin(0.9t))^2 + (1-0.9\cos(0.9t)(-2\sin(0.9t)) \\ 0 \end{bmatrix}$$

The Lie bracket of $\{G, ad_F G\}, \{G, ad_F^2 G\}, \{ad_F G, ad_F^2 G\}$ is the column vectors of the matrix are constant. Therefore, the system is involutive, which satisfy the conditions to apply input-state linearization.
Next to calculation of the new state vector and the input variable using the differential geometry, the first component of the state vector T_1 should be obtained through the expression (3).

As the order of the system is equal to 2, equation ∂T_1 becomes $\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3} & \frac{\partial T_1}{\partial t} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 0$, resulting in

$\frac{\partial T_1}{\partial x_3} = 0$ and $\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3} & \frac{\partial T_1}{\partial t} \end{bmatrix} \begin{bmatrix} -\sin(0.9t) \\ -1 \\ 0 \\ 0 \end{bmatrix} = 0$, $-\sin(0.9t)\frac{\partial T_1}{\partial x_1} - \frac{\partial T_1}{\partial x_2} = 0$. And from (4),

$\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3} & \frac{\partial T_1}{\partial t} \end{bmatrix} \begin{bmatrix} 1-0.9\cos(0.9t) \\ 0 \\ -2x_1\sin(0.9t) \\ 0 \end{bmatrix} \neq 0$, $(1-0.9\cos(0.9t))\frac{\partial T_1}{\partial x_1} - 2x_1\sin(0.9t)\frac{\partial T_1}{\partial x_3} \neq 0$

Therefore, in order to satisfy the above conditions, to be chosen the following equation.

$$T_1 = -x_1 + \sin(0.9t)x_2 \tag{20}$$

We use (5) to compute the second component of the new state vector T_2.

$$T_2 = \dot{T}_1 = -\dot{x}_1 + \sin(0.9t)\dot{x}_2 + \cos(0.9t)x_2 = -(x_2 + \sin(0.9t)\,x_3) + \sin(0.9t)\,x_3 + \cos(0.9t)\,x_2 = (\cos(0.9t) - 1)x_2 = L_F T_1 \tag{21}$$

$$T_3 = \dot{T}_2 = (\cos(0.9t) - 1)\dot{x}_2 - 0.9\sin(0.9t)\,x_2 = -0.9\sin(0.9t)\,x_2 + (\cos(0.9t) - 1)x_3 = L_F^2 T_2 \tag{22}$$

$$\dot{T}_4 = \dot{T}_3 = (\cos(0.9t) - 1)\dot{x}_3 - 0.9\sin(0.9t)\,x_3 - 0.9\sin(0.9t)\,\dot{x}_2 - 0.9^2\cos(0.9t)\,x_2 = (\cos(0.9t) - 1)(x_1^2 + u) - 0.9\sin(0.9t)\,x_3 - 0.9\sin(0.9t)\,x_3 - 0.9^2\cos(0.9t)\,x_2 = (\cos(0.9t) - 1)x_1^2 - 0.9^2\cos(0.9t)\,x_2 - 1.8\sin(0.9t)\,x_3 + (\cos(0.9t) - 1)u = L_F^3 T_1 + L_G L_F^2 T_1 u \tag{23}$$

This gives the new state vector z and input v.
$z = [\, z_1, z_2, z_3 \,] = [T_1, T_2, T_3]$, $v = T_4$
Therefore, we obtains the same type as equation (7).

$$\dot{z} = Az + Bv = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \tag{24}$$

The linearization of a time-varying nonlinear system is investigated of through examples. Because the linearized system is time-invariant linear system, so through the feed beck the state, controller can be designed.

The system block diagram may be represented as shown in Figure 1. The control structure, according to Figure 1, has two feedbacks. The first feedback is responsible for system linearization, eliminating the existing linearity. The second feedback is the controller project based on the state feedback.

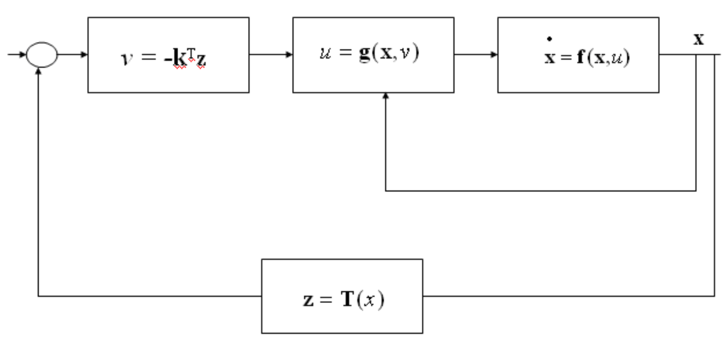

Figure 1. Control structure

4. Conclusions

In this study, the linearization for the time-varying nonlinear system were discussed. The existing time-invariant nonlinear system input-state linearization technique to extend the fixed input-state linearization technique proposed, and examples of proof of validity was confirmed through verification. Also, extended input-state linearizing technique, through a time-varying linear system to convert the time-invariant linear system was.

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