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Maximum Power Waveform Design for Bistatic MIMO Radar System

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** Short Paper*

Abstract: In this paper we propose a waveform design algorithm that localizes the maximum output power in the target direction. We extend existing monostatic radar optimal waveform design schemes to bistatic multiple-input multiple-output (MIMO) radar systems. The algorithm simultaneously calculates the direction of departure (DoD) and the direction of arrival (DoA) using a two-dimensional multiple signal classification (MUSIC) method, and successfully localizes the maximum transmitted power to the target locations by exploiting the calculated DoD. The simulation results confirm the performance of the proposed algorithm.

Keywords: Maximum power design, Bistatic MIMO radar, Waveform design, Beampattern

1. Introduction

The advent of multiple-input multiple-output (MIMO) radar systems has led to significant improvements in target detection, localization, and the ranging performance of radar systems. The key advantage of MIMO radar over conventional phased-array radar is that each transmitter can transmit different waveforms to optimize the overall performance. Accordingly, several waveform design techniques have been developed for monostatic MIMO radars. For example, maximum power waveform design methods used to maximize the signal power at the receiver [1] or at the target scene [2] using the signal processing characteristics of the monostatic MIMO radar have been studied.

Bistatic radar is a recent innovation in radar systems [3]. The detection of high-mobility and stealth targets has become an interesting research topic, and the demand for bistatic radar has increased. On the other hand, there have been few studies of MIMO bistatic radar systems, despite the synergetic effect of MIMO bistatic radars. Furthermore, to the best of our knowledge, no waveform design schemes have been developed for bistatic radar. One obstacle to bistatic radar waveform design is the difficulty in determining the direction of departure (DoD). Optimizing the waveform that localizes the transmitted power in the target direction requires the DoD. In this context, conventional monostatic MIMO radar has a DoD that is identical to the direction of arrival (DoA). Consequently, waveform optimization can be achieved using well known DoA estimation algorithms [4-6]. However, in the case of bistatic MIMO radar, the DoD is required, because target detection algorithms generally estimate only the DoA.

This paper describes a waveform design algorithm for bistatic radar that extends the existing power maximization waveform optimization algorithm of monostatic MIMO systems to bistatic MIMO radar systems. The algorithm utilizes the two-dimensional multiple signal characterization (2D-MUSIC) spectrum [7, 8] to determine the DoD, and applies the existing power localization waveform design rules. The remainder of this paper is organized as follows. In Section 2, we describe a model of the entire system; in Section 3, we briefly introduce the existing maximum power design method for monostatic radar systems; and in Section 4, we describe the algorithm to determine the optimal signal characteristics for target detection for bistatic MIMO radar systems based on the model discussed in Section 3. In Section 5, we discuss the potential to apply the algorithm using computer

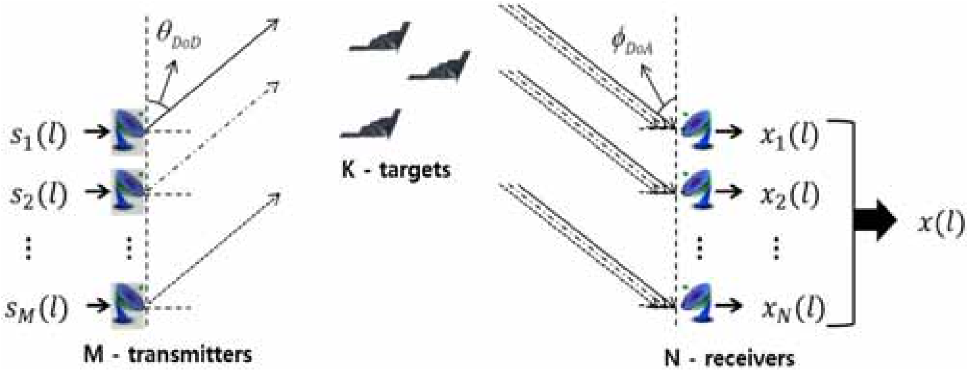

Fig. 1. Bistatic MIMO radar system model.

simulations, and the conclusions are reported in Section 6.

2. System Model

A uniform linear array (ULA) bistatic MIMO radar model which consists of M transmitter antennas and N receiver antennas was considered, as shown in Fig. 1. Let θ and φ denote the location of the target at the transmitter and receiver, respectively; the transmit steering vector $a_t(\theta)$ and receive steering vector $a_r(\varphi)$ are given by

$$\begin{aligned} a_t(\theta) &= \begin{bmatrix} 1 & e^{j2\pi\frac{d_t}{\lambda}\sin\theta} & \cdots & e^{(M-1)j2\pi\frac{d_t}{\lambda}\sin\theta} \end{bmatrix} \\ a_r(\theta) &= \begin{bmatrix} 1 & e^{j2\pi\frac{d_r}{\lambda}\sin\varphi} & \cdots & e^{(N-1)j2\pi\frac{d_r}{\lambda}\sin\varphi} \end{bmatrix} \end{aligned} \tag{1}$$

where d_t is the spacing of the transmit antennas and d_r is the spacing of the receive antennas. The distances between multiple targets were assumed to be negligible. Let $s(l) = [s_1(l) \quad s_2(l) \quad \cdots \quad s_M(l)]^T$ be the transmitted signal consisting of L discrete sample pulses, so that $l = 1,\ldots,L$. The transmitted signal at the target location θ is equal to $a_t^H(\theta)s(l)$, where $(\bullet)^T$ denotes the transpose, and $(\bullet)^H$ denotes the conjugate transpose. When the signal is transmitted, it is assumed that the transmitted power is limited to some maximum power c, i.e.,

$$\frac{1}{L}\sum_{m=1}^{M}\sum_{l=1}^{L}|s_m(l)|^2 \le c. \tag{2}$$

Let R denote the covariance matrix of $s(l)$, i.e., $R = \frac{1}{L}\sum_{l=1}^{L}s(l)s(l)^H$; Eq. (2) can then be rewritten as follows:

$$tr(R) \le c. \tag{3}$$

If there are K targets at locations $\{\varphi_1 \quad \varphi_2 \quad \cdots \quad \varphi_K\}$, the received signal at the output of the receiving array $x(l) = [x_1(l) \quad x_2(l) \quad \cdots \quad x_N(l)]^T$ has the form

$$x(l) = \sum_{k=1}^{K}\beta_k a_r(\varphi_k)a_t^H(\theta_k)s(l), \tag{4}$$

where β_k is the amplitude, which is proportional to the radar cross-sections (RCSs) of the k-th target. The matched filters $s^H(l)$ were applied to Eq. (4), so that the output signal model becomes

$$Y = \frac{1}{L}\sum_{l=1}^{L}x(l)s^H(l) + Z = \sum_{k=1}^{K}\beta_k a_r(\varphi_k)a_t^H(\theta_k)R + Z \tag{5}$$

with an additional noise matrix $Z \in C^{M\times N}$.

With the parameters defined as above, we can represent the signal power transmitted to the director located at θ, i.e.,

$$P_t(\theta) = a_t^H(\theta)Ra_t(\theta). \tag{6}$$

The spatial spectrum in Eq. (6) as a function of θ is called the transmit beam pattern [2]. The purpose of this study was to design an optimal waveform $s(l)$ for target detection by maximizing the signal power $P_t(\theta)$ accumulated at the target scenes when the total power $tr(R)$ is restricted.

3. Maximum Power Design

This section briefly introduces the maximum power design algorithms for monostatic MIMO radar; further details can be found in Refs. [2] and [9]. If the number of total targets is K, and each target is located at $\{\theta_k\}_{k=1}^{K}$, the accumulated signal power at the region of interest $\tilde{K}$ is given by

$$P_a = \sum_{k=1}^{\tilde{K}} P(\theta_k) = \sum_{k=1}^{\tilde{K}} a_t^H(\theta_k) R a_k(\theta_k) \cong tr(RB) \quad (7)$$

where

$$B = \sum_{k=1}^{\tilde{K}} a_t(\theta_k) a_t^H(\theta_k). \quad (8)$$

When the total transmitted power is limited to c, the characteristics of the desired transmit signal can be achieved by maximizing the power transmitted to the region of interest. At this point, there is usually no prior information so that the region of interest cannot be designated. In other words, the matrix B is an unknown matrix with no prior information. Therefore, considering this (worst) case, an omni-directional waveform needs to be transmitted, i.e.,

$$R = \frac{c}{M} I. \quad (9)$$

The transmitted signal should satisfy Eq. (9). Let the signal power from the m-th transmitter antenna be $s_m(l)s_m^H(l)$, then the power is distributed equally to all the transmitter antennas as $\frac{c}{M}$, and each transmitter antenna has an orthogonal signal. The aim is to determine the target location B using a target location estimation method, $\hat{B}$. Therefore the optimization problem is given by

$$\begin{aligned} R_{opt} &= \arg\max_R \{tr(RB^H)\} \\ \textit{subject to} \quad & tr(R) = c \\ & R \geq 0 \end{aligned} \quad (10)$$

To find the covariance matrix R_{opt}, we rewrite Eq. (10) as an eigenvalue decomposition of R and $\hat{B}$, i.e.,

$$tr(R\hat{B}) = tr(V \begin{bmatrix} \gamma_{\max} & & \\ & \ddots & \\ & & \gamma_{\min} \end{bmatrix} V^H U \begin{bmatrix} \lambda_{\max} & & \\ & \ddots & \\ & & \lambda_{\min} \end{bmatrix} U^H \quad (11)$$

when $V^H U = I$, i.e., when $R = U \begin{bmatrix} \lambda_{\max} & & \\ & \ddots & \\ & & \lambda_{\min} \end{bmatrix} U^H$, the right-hand side of Eq. (11) has the maximal value. From matrix theory, we have

$$tr(R\hat{B}) \leq \sum_{m=1}^{M} \gamma_m \lambda_m \leq c\lambda_{\max}. \quad (12)$$

To satisfy the upper bounds of Eq. (12), the power needs to be assigned to $\lambda_{\max}$ only. Therefore, let $U = [u_{\max} \ \cdots \ u_{\min}]$, and then R_{opt} is given by

$$R_{opt} = c u_{\max} u_{\max}^H. \quad (13)$$

For monostatic MIMO radar, the matrix $\hat{B}$ can be found easily using a one-dimensional DoA estimation algorithm, such as the amplitude and phase estimation (APES) [4], or the general likelihood ratio test (GLRT) [5] usign the Capon method [6]. In the case of bistatic MIMO radar, however, the DoD differs from the DoA. One-dimensional DoA estimations can determine only the DoA. On the other hand, the maximum power design algorithm requires the DoD. In the following section, a two-dimensional DoD and DoA joint estimation algorithm, as well as a maximum power design algorithm for bistatic MIMO radar are introduced.

4. Algorithm

Let signal $s(l)$ with the covariance matrix $R = \frac{c}{M} I$ be transmitted. Using Eq. (5), the received signal from the receiver can be written as

$$Y = \frac{c}{M} \sum_{k=1}^{K} \beta_k a_r(\varphi_k) a_t^H(\theta_k) + Z = \begin{bmatrix} Y_1 \\ \vdots \\ Y_N \end{bmatrix} \quad (14)$$

where Y_n, is the signal received by the n-th receive antenna after the matched filters, where $n = 1, \ldots, N$. To apply the two-dimensional MUSIC scheme for a joint estimation of DoD and DoA [7], we rewrite Eq. (14) in $nM \times 1$ matrix form as follows:

$$\begin{aligned} \tilde{Y} &= \begin{bmatrix} Y_1^T & \cdots & Y_N^T \end{bmatrix}^T \\ &= \frac{c}{M}[a_r(\varphi_1) \otimes a_t(\theta_1), \ldots, \\ & \quad a_r(\varphi_K) \otimes a_t(\theta_K)] b(\theta, \varphi) + Z_{M \times 1} \end{aligned} \quad (15)$$

where $b(\theta, \varphi) = [\beta_1, \beta_2, \ldots, \beta_K]^T$ denotes the amplitudes, which are proportional to the RCSs, and Z is the residual term, which includes the thermal noise, as well as any intentional or unintentional interference. The covariance of matrix $\tilde{Y}$ is calculated, and after performing an eigenvalue decomposition, $R_{\tilde{Y}} = E\{\tilde{Y}\tilde{Y}^H\}$ is given by

$$R_{\tilde{Y}} = E_s D_s E_s^H + E_n D_n E_n^H, \quad (16)$$

where D_s is a diagonal matrix, the diagonal elements of which contain the largest K eigenvalues; D_n is a diagonal matrix, the diagonal elements of which contain the smallest $MN - K$ eigenvalues; E_s is a matrix composed of the eigenvectors corresponding to the largest K eigenvalues of $R_{\tilde{Y}}$; and E_n is a matrix composed of the remaining eigenvectors. To determine the DoD, the 2D-MUSIC spectrum function was calculated using E_n from Eq. (16), i.e.,

$$f_{2dmusic}(\theta,\varphi) = \frac{1}{[a_r(\varphi)\otimes a_t(\theta)]^H E_n E_n^H [a_r(\varphi)\otimes a_t(\theta)]} . \quad (17)$$

We require DoD information only; therefore, the one-dimensional marginal spectrum is calculated from Eq. (17) as follows

$$f_{DoD}(\theta) = \int_{\varphi \in all\ possible\ range\ of\ \varphi} f_{2dmusic}(\theta,\varphi)d\varphi . \quad (18)$$

The $\tilde{K}$ peaks can be extracted and the location data $\{\hat{\theta}_1,\ldots,\hat{\theta}_{\tilde{K}}\}$ describing the peak values can be obtained by applying a one-dimensional search to Eq. (18) [7]. $\hat{B}_{2d}$ can be found for bistatic MIMO radar as follows:

$$\hat{B}_{2d} = \sum_{k=1}^{\tilde{K}} a_t(\hat{\theta}_k)a_t^H(\hat{\theta}_k) . \quad (19)$$

By substituting $\hat{B}_{2d}$ into $\hat{B}$ in Eq. (10), and using Eqs. (11) and (12), the characteristics of the transmitted signal for bistatic MIMO radar can be determined via maximum power design, i.e.,

$$R_{opt} = cu_{2d,\max}u_{2d,.ax}^H , \quad (20)$$

where $u_{2d,\max}$ is an eigenvector corresponding to the largest eigenvalues of $\hat{B}_{2d}$, and is given by the following eigenvalue decomposition:

$$\hat{B}_{2d} = \begin{bmatrix} u_{2d,\max} & \cdots & u_{2d,\min} \end{bmatrix} \begin{bmatrix} \lambda_{2d,\max} & & \\ & \ddots & \\ & & \lambda_{2d,\min} \end{bmatrix} \begin{bmatrix} u_{2d,\max} & \cdots & u_{2d,\min} \end{bmatrix}^H \quad (21)$$

A summary of the algorithm is given as follows.

1) If there is no prior information as to the target location, the transmitted signal has the covariance matrix $R = \frac{c}{M} I$.
2) Estimate the DoD using 2D-MUSIC for bistatic MIMO radar.
3) Using the estimated DoD from the 2D-MUSIC spectrum, calculate $\hat{B}_{2d} = \sum_{k=1}^{\tilde{K}} a_t(\hat{\theta}_k)a_t^H(\hat{\theta}_k)$.
4) Obtain $R_{opt} = cu_{2d,\max}u_{2d,.ax}^H$ by inserting $\hat{B}_{2d}$ into the established maximal power design.

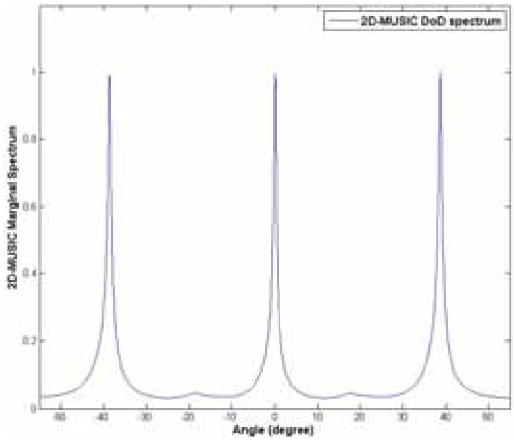

Fig. 2. 2D-MUSIC DoD marginal spatial spectrum for the initial omni-directional probing.

Through this process, the optimal signal characteristics for target detection in bistatic MIMO radar systems can be obtained using the maximum power design algorithm. Furthermore, by considering the accumulated power at the target scene, we can construct a framework of target tracking studies for moving targets.

5. Numerical Examples

Here we consider a bistatic MIMO radar with $M = 3$ transmit ULA antennas and $L = 16$ receive antennas, and with $L = 16$ transmitted signal pulse samples. We also consider a scenario whereby $K = 3$ targets are located at $(\theta_k,\varphi_k) = \{(-40^\circ,-35^\circ),(0^\circ,5^\circ),(40^\circ,45^\circ)\}$ with amplitudes $\beta_1 = \beta_2 = \beta_3 = 1$, and where zero-mean Gaussian white noise exists with a variance $\sigma^2 = -10dB$.

First, we consider that there is no prior information as to the target locations; thus the transmitted signal is as Eq. (9), i.e., $R = \frac{c}{M} I$.

In other words, we transmit an omni-directional beam pattern with signal power $c = 1$. We estimate the DoD using the received signal from the transmitted beam pattern using the 2D-MUSIC algorithm (Eq. (17)).

This estimated DoD can now be used as prior information to obtain the covariance matrix of the transmitted signal using Eq. (20). Using this covariance matrix, the transmitted beam pattern described by Eq. (6) can also be acquired. The resulting beam pattern obtained from the maximum power design is transmitted to locations θ_1, θ_2 and θ_3. The estimated location of the targets is shown in Fig. 3, where the maximal power design obtained an increase in the signal-to-noise ratio (SNR) of a factor of 2.7 compared with an omni-directional beam pattern and with the same transmitted signal power. We can therefore expect improvements in the accuracy of target detection using our algorithm. Further examples of the transmitted signal waveform are presented, as shown in Fig. 5.

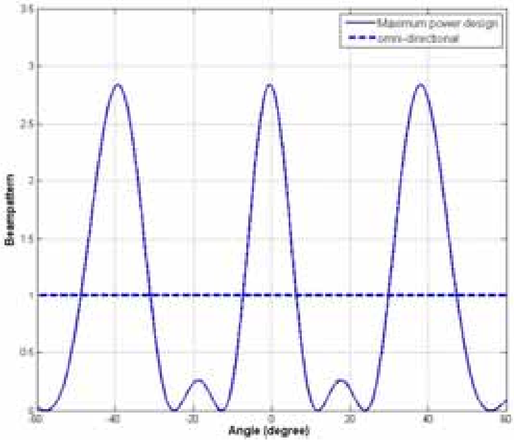

Fig. 3. Transmit beampatterns $P_t(\theta)$ formed via omni-directional probing and maximum power design for given target locations (estimated via initial omni-directional probing).

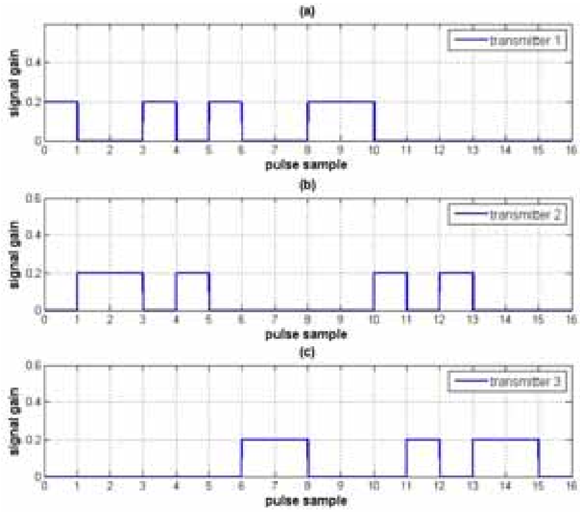

Fig. 4. Example of designed transmit waveform via omni-directional design.

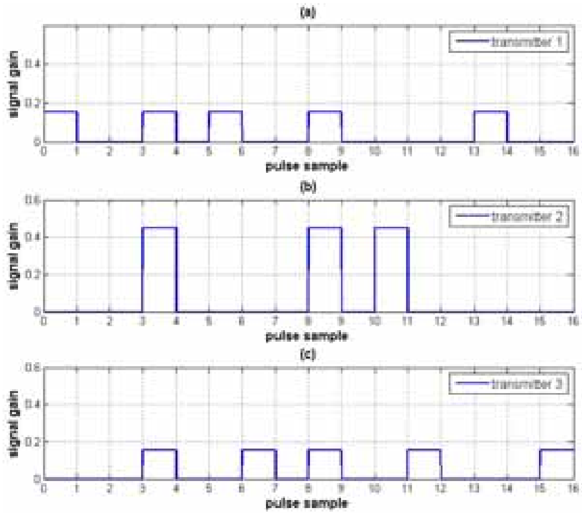

Fig. 5. Example of designed transmit waveform via maximum power desgin.

4. Conclusion

We have described a waveform design algorithm for bistatic MIMO radar. We used the 2D-MUSIC method to estimate the DoA and DoD simultaneously. Using the resulting DoD as inputs, we were able to successfully produce the power-optimized waveforms, as demonstrated by a simulated comparative analysis of the beam patterns.

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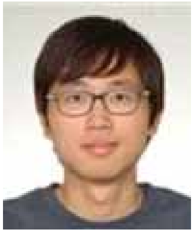

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