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Generalized Double Fuzzy Semi-Basically Disconnected Spaces

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Abstract

In this paper, we introduce the concept of generalized double fuzzy semi-basically disconnected space and related notions such as (r, s)-generalized fuzzy semiopen-F_σ sets, (r, s)-generalized fuzzy semiclosed-G_δ sets, generalized double fuzzy semi*-open function, generalized double fuzzy semi*-continuous function and generalized double fuzzy semi*-irresolute function. Some interesting properties and characterizations of the concepts introduced are studied.

Keywords: Double fuzzy topology, (r, s)-generalized fuzzy semiopen-F_σ, (r, s)-generalized fuzzy semiclosed-G_δ, Generalized double fuzzy semi*-irresolute function, Generalized double fuzzy semi-basically disconnected space

1. Introduction

The theory of fuzzy sets was developments by Zadeh [1], then Chang [2] used fuzzy sets to introduce the concept of a fuzzy topology. Çoker [3, 4] introduced the idea of the topology of intuitionistic fuzzy sets. Later on, Samanta and Mondal [5] succeeded to gave the definition of an intuitionistic fuzzy topological space in Kubiak-Šostak's sense. The resulting structure is given the new name "intuitionistic gradation of openness." The name "intuitionistic" didn't continue due to some doubts were thrown around the suitability of this term especially in the case of complete lattice L. These doubts were quickly ended in 2005 by García and Rodabaugh [6]. They replaced the word "intuitionistic" by "double." The notion of intuitionistic gradation of openness is given the name "double fuzzy topological spaces."

In [7–10], the notions of fuzzy (r,s)-semiopen sets, (r, s)-generalized fuzzy semiclosed sets, (r, s)-fuzzy irresolute functions and double fuzzy irresolute functions are introduced and characterized. In 2011, Sudha et al. [11] studied the notions of generalized L-fuzzy ω-basically disconnected spaces.

In this paper, motivated by the above studies, we introduce the concept of generalized double fuzzy semi-basically disconnected space and related notions of (r, s)-generalized fuzzy semiopen-F_σ, (r, s)-generalized fuzzy semiclosed-G_δ and generalized double fuzzy semi *-irresolute function. Also, we study some relationships between these new notions.

2. Preliminaries

Throughout this paper, Let X be a non-empty set, I the unit interval $[0, 1]$, $I_0 = (0, 1]$

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and $I_1 = [0, 1)$. The family of all fuzzy sets in X is denoted by I^X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $\lambda \in I^X$, $\underline{1} - \lambda$ denotes its complement. Given a function $f : I^X \longrightarrow I^Y$ and its inverse $f^{-1} : I^Y \longrightarrow I^X$ are defined by $f(\lambda)(y) = \bigvee_{f(x)=y} \lambda(x)$ and $f^{-1}(\mu)(x) = \mu(f(x))$, for each $\lambda \in I^X$, $\mu \in I^Y$ and $x \in X$, respectively. All other notations are standard notations of fuzzy set theory.

Definition 2.1. [5, 6] A double fuzzy topology (τ, τ^*) on X is a pair of maps $\tau, \tau^* : I^X \to I$, which satisfies the following properties:

(O1) $\tau(\lambda) \leq \underline{1} - \tau^*(\lambda)$ for each $\lambda \in I^X$.

(O2) $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.

(O3) $\tau(\bigvee_{i\in\Gamma} \lambda_i) \geq \bigwedge_{i\in\Gamma} \tau(\lambda_i)$ and

$$\tau^*(\bigvee_{i\in\Gamma} \lambda_i) \leq \bigvee_{i\in\Gamma} \tau^*(\lambda_i)$$

for each $\lambda_i \in I^X$, $i \in \Gamma$.

The triplet (X, τ, τ^*) is called a double fuzzy topological spaces (dfts, for short). A fuzzy set λ is called an (r, s)-fuzzy open ((r, s)-fo, for short) if $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$, λ is called an (r, s)-fuzzy closed ((r, s)-fc, for short) iff $\underline{1} - \lambda$ is an (r, s)-fo set. Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be two dfts's. A function $f : X \to Y$ is said to be a double fuzzy continuous iff $\tau_1(f^{-1}(\nu)) \geq \tau_2(\nu)$ and $\tau_1^*(f^{-1}(\nu)) \leq \tau_2^*(\nu)$ for each $\nu \in I^Y$.

Theorem 2.1. [10] Let (X, τ, τ^*) be a dfts. Then for each $r \in I_0$, $s \in I_1$, and $\lambda \in I^X$, we define an operator $C_{\tau,\tau^*} : I^X \times I_0 \times I_1 \longrightarrow I^X$ as follows:

$$C_{\tau,\tau^*}(\lambda, r, s) = \bigwedge\{\mu \in I^X \mid \lambda \leq \mu,\ \tau(\underline{1} - \mu) \geq r,\ \tau^*(\underline{1} - \mu) \leq s\}.$$

For $\lambda, \mu \in I^X$, $r, r_1, r_2 \in I_0$ and $s, s_1, s_2 \in I_1$, the operator C_{τ,τ^*} satisfies the following statements:

(C1) $C_{\tau,\tau^*}(\underline{0}, r, s) = \underline{0}$.

(C2) $\lambda \leq C_{\tau,\tau^*}(\lambda, r, s)$.

(C3) $C_{\tau,\tau^*}(\lambda, r, s) \vee C_{\tau,\tau^*}(\mu, r, s) = C_{\tau,\tau^*}(\lambda \vee \mu, r, s)$.

(C4) $C_{\tau,\tau^*}(\lambda, r_1, s_1) \leq C_{\tau,\tau^*}(\lambda, r_2, s_2)$ if $r_1 \leq r_2$ and $s_1 \geq s_2$.

(C5) $C_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s) = C_{\tau,\tau^*}(\lambda, r, s)$.

Theorem 2.2. [10] Let (X, τ, τ^*) be a dfts. Then for each $r \in I_0$, $s \in I_1$, and $\lambda \in I^X$, we define an operator $I_{\tau,\tau^*} : I^X \times I_0 \times I_1 \longrightarrow I^X$ as follows:

$$I_{\tau,\tau^*}(\lambda, r, s) = \bigvee\{\mu \in I^X \mid \mu \leq \lambda,\ \tau(\mu) \geq r,\ \tau^*(\mu) \leq s\}.$$

For $\lambda, \mu \in I^X$, $r, r_1, r_2 \in I_0$ and $s, s_1, s_2 \in I_1$, the operator I_{τ,τ^*} satisfies the following statements:

(I1) $I_{\tau,\tau^*}(\underline{1} - \lambda, r, s) = \underline{1} - C_{\tau,\tau^*}(\lambda, r, s)$.

(I2) $I_{\tau,\tau^*}(\underline{1}, r, s) = \underline{1}$.

(I3) $I_{\tau,\tau^*}(\lambda, r, s) \leq \lambda$.

(I4) $I_{\tau,\tau^*}(\lambda, r, s) \wedge I_{\tau,\tau^*}(\mu, r, s) = I_{\tau,\tau^*}(\lambda \wedge \mu, r, s)$.

(I5) $I_{\tau,\tau^*}(\lambda, r_1, s_1) \geq I_{\tau,\tau^*}(\lambda, r_2, s_2)$ if $r_1 \leq r_2$ and $s_1 \geq s_2$.

(I6) $I_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s) = I_{\tau,\tau^*}(\lambda, r, s)$.

(I7) If $I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s) = \lambda$, then $C_{\tau,\tau^*}(I_{\tau,\tau^*}(\underline{1} - \lambda, r, s), r, s) = \underline{1} - \lambda$.

Definition 2.2. Let (X, τ, τ^*) be a dfts. For each $\lambda, \mu \in I^X$, $r \in I_0$ and $s \in I_1$.

(1) A fuzzy set λ is called (r, s)-fuzzy semiopen (briefly, (r, s)-fso) [7] if $\lambda \leq C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s)$. λ is called (r, s)-fuzzy semiclosed (briefly, (r, s)-fsc) iff $\underline{1} - \lambda$ is an (r, s)-fso set.

(2) A fuzzy set λ is called (r, s)-generalized fuzzy semiclosed (briefly, (r, s)-gfsc) [8] if $C_{\tau,\tau^*}(\lambda, r, s) \leq \mu$, $\lambda \leq \mu$ and μ is an (r, s)-fso. λ is called (r, s)-generalized fuzzy semiopen (briefly, (r, s)-gfso) iff $\underline{1} - \lambda$ is (r, s)-gfsc set.

3. Generalized Double Fuzzy Semi-Basically Disconnected and Generalized Double Fuzzy Semi *-irresolute Spaces

In this section, we introduced the concepts of (r, s)-generalized fuzzy G_δ sets, generalized double fuzzy semi-basically disconnected and generalized double fuzzy semi*-irresolute spaces. Some interesting properties and characterizations of the concepts introduced are investigated.

Definition 3.1. Let (X, τ, τ^*) be a dfts. For each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$.

(1) An (r,s)-fuzzy G_δ set if $\lambda = \wedge_{i=1}^{\infty}\lambda_i$ such that λ_i is an (r,s)-fo set.

(2) An (r,s)-fuzzy F_σ set if $\lambda = \vee_{i=1}^{\infty}\lambda_i$ such that λ_i is an (r,s)-fc set.

(3) An (r,s)-generalized fuzzy semiopen-F_σ (briefly, (r,s)-gfso-F_σ) if λ is an (r,s)-gfso and (r,s)-fuzzy F_σ set.

(4) An (r,s)-generalized fuzzy semiclosed-G_δ (briefly, (r,s)-gfsc-G_δ) if λ is an (r,s)-gfsc and (r,s)-fuzzy G_δ set.

(5) An (r,s)-generalized fuzzy semiclopen-GF (briefly, (r,s)-gfsco-GF) if λ is an (r,s)-gfso-F_σ and (r,s)-gfsc-G_δ set.

(6) An (r,s)-generalized fuzzy semi*-closure of λ is defined by $GS^*C_{\tau,\tau^*}(\lambda,r,s) = \bigwedge\{\mu \in I^X \mid \lambda \le \mu$ and μ is (r,s)-gfsc-$G_\delta\}$.

(7) An (r,s)-generalized fuzzy semi*-interior of λ is defined by $GS^*I_{\tau,\tau^*}(\lambda,r,s) = \bigvee\{\mu \in I^X \mid \mu \le \lambda$ and μ is (r,s)-gfso-$F_\sigma\}$.

Remark 3.1. Let (X,τ,τ^*) be a dfts. For each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$, the following statements hold:

(1) $GS^*I_{\tau,\tau^*}(\lambda,r,s) = GS^*C_{\tau,\tau^*}(\underline{1}-\lambda,r,s)$.

(2) $GS^*C_{\tau,\tau^*}(\lambda,r,s) = GS^*I_{\tau,\tau^*}(\underline{1}-\lambda,r,s)$.

Definition 3.2. A dfts (X,τ,τ^*) is called generalized double fuzzy semi-basically disconnected if the $GS^*C(\lambda,r,s)$ is an (r,s)-gfso-F_σ, for each (r,s)-gfso-F_σ λ in I^X, $r \in I_0$ and $s \in I_1$.

Example 3.1. Let $X = \{a,b\}$. Defined fuzzy set λ_1 as follows:

$$\lambda_1(a) = 0.5,\ \lambda_1(b) = 0.5$$

Let (τ,τ^*) defined as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}\ or \underline{1}\,; \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1; \\ 0, & \text{otherwise.} \end{cases} \quad ;$$

$$\tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{0}\ or \underline{1}\,; \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1; \\ 1, & \text{otherwise.} \end{cases}$$

Since λ is an $(1/2,1/2)$-fc set and an $(1/2,1/2)$-gfso set, olso $GS^*C(\lambda,r,s)$ is an (r,s)-gfso-F_σ. Then (X,τ,τ^*) is generalized double fuzzy semi-basically disconnected space.

Definition 3.3. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. A function $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is called:

(1) generalized double fuzzy semi*-open (briefly, gdfs*-open) if $f(\lambda)$ is an (r,s)-gfso-F_σ set in I^Y, for each (r,s)-gfso-F_σ in I^X, $r \in I_0$ and $s \in I_1$.

(2) generalized double fuzzy semi*-continuous (briefly, gdfs*-c) if $f^{-1}(\lambda)$ is an (r,s)-gfsc-G_δ set in I^X, for each (r,s)-fc and (r,s)-fuzzy G_δ set λ in I^Y, $r \in I_0$ and $s \in I_1$.

(3) generalized double fuzzy semi*-irresolute (briefly, gdfs*-irr) if $f^{-1}(\lambda)$ is an (r,s)-gfso-F_σ set in I^X, for each (r,s)-gfso-F_σ in I^Y, $r \in I_0$ and $s \in I_1$.

Proposition 3.1. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. If a function $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is a gdfs*-open surjective function, then for each fuzzy set λ in I^Y, $r \in I_0$ and $s \in I_1$,

$$f^{-1}(GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)) \le GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s).$$

Proof. (1) Let λ be any fuzzy set in I^Y, $r \in I_0$ and $s \in I_1$ such that $\mu = f^{-1}(\underline{1}-\lambda)$. Then

$$GS^*I_{\tau_1,\tau_1^*}(f^{-1}(\underline{1}-\lambda),r,s) = GS^*I_{\tau_1,\tau_1^*}(\mu,r,s),$$

is an (r,s)-gfso-F_σ set in I^X. But, $GS^*I_{\tau_1,\tau_1^*}(\mu,r,s) \le \mu$, then

$$f(GS^*I_{\tau_1,\tau_1^*}(\mu,r,s)) \le f(\mu).$$

i,e.

$$GS^*I_{\tau_2,\tau_2^*}(f(GS^*I_{\tau_1,\tau_1^*}(\mu,r,s)),r,s) \le GS^*I_{\tau_2,\tau_2^*}(f(\mu),r,s).$$

Since f is gdfs*-open, $f(GS^*I_{\tau_1,\tau_1^*}(\mu,r,s))$ is an (r,s)-gfso-F_σ in I^Y, $r \in I_0$ and $s \in I_1$. Therefore,

$$\begin{aligned} f(GS^*I_{\tau_1,\tau_1^*}(\mu,r,s)) &\le GS^*I_{\tau_2,\tau_2^*}(f(\mu),r,s) \\ &= GS^*I_{\tau_2,\tau_2^*}(\underline{1}-\lambda,r,s). \end{aligned}$$

Hence,

$$\begin{aligned} GS^*I_{\tau_1,\tau_1^*}(f^{-1}(\underline{1}-\lambda),r,s) &= GS^*I_{\tau_1,\tau_1^*}(\mu,r,s) \\ &\le f^{-1}(GS^*I_{\tau_2,\tau_2^*}(\underline{1}-\lambda),r,s). \end{aligned}$$

Therefore,

$$\begin{aligned}\underline{1}-&GS^*I_{\tau_1,\tau_1^*}(f^{-1}(\underline{1}-\lambda),r,s)\\ &=\underline{1}-GS^*I_{\tau_1,\tau_1^*}(\mu,r,s)\\ &\geq \underline{1}-(f^{-1}(GS^*I_{\tau_2,\tau_2^*}(\underline{1}-\lambda,r,s))).\end{aligned}$$

Hence,

$$\begin{aligned}f^{-1}(\underline{1}-(GS^*I_{\tau_2,\tau_2^*}(\underline{1}-\lambda,r,s)))\\ \leq GS^*C_{\tau_1,\tau_1^*}(\underline{1}-f^{-1}(\underline{1}-\lambda),r,s).\end{aligned}$$

Therefore,

$$f^{-1}(GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)) \leq GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s).$$

Proposition 3.2. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. A function $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is a gdfs*-irr function if and only if $f(GS^*C_{\tau_1,\tau_1^*}(\lambda,r,s)) \leq GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s)$, for each fuzzy set λ in I^X, $r\in I_0$ and $s\in I_1$.

Proof. Suppose λ is any fuzzy set in I^X, $r\in I_0$, $s\in I_1$ and f is a gdfs*-irr function. Then, $GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s)$ is an (r,s)-gfsc-G_δ set in I^Y. But, $f^{-1}(GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s))$ is is an (r,s)-gfsc-G_δ set in I^X and

$$\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s)).$$

Hence,

$$GS^*C_{\tau_1,\tau_1^*}(\lambda,r,s) \leq f^{-1}(GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s))$$

i.e,

$$f(GS^*C_{\tau_1,\tau_1^*}(\lambda),r,s) \leq GS^*C_{\tau_2,\tau_2^*}(f(\lambda),r,s)).$$

Conversely, let λ be an (r,s)-gfsc-G_δ set in I^Y. Then,

$$GS^*C_{\tau_1,\tau_1^*}(\lambda,r,s)=\lambda.$$

Now,

$$\begin{aligned}f(GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s)) &\leq GS^*C_{\tau_2,\tau_2^*}(f(f^{-1}(\lambda),r,s))\\ &= GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s).\end{aligned}$$

This implies,

$$GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s) \leq f^{-1}(\lambda).$$

So that

$$f^{-1}(\lambda) = GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s).$$

i.e., $f^{-1}(\lambda)$ is an (r,s)-gfsc set. So, f is a gdfs*-irr function.

Theorem 3.1. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. If a function $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is a gdfs*-irr, gdfs*-open surjective function such that (X,τ_1,τ_1^*) is a generalized double fuzzy semi-basically disconnected space, then (Y,τ_2,τ_2^*) is generalized double fuzzy semi-basically disconnected space.

Proof. Suppose λ is any (r,s)-gfso-F_σ set in I^Y and f is a gdfs*-irr, then $f^{-1}(\lambda)$ is an (r,s)-gfso-F_σ set in I^X. But by hypothesis of (X,τ_1,τ_1^*) is a generalized double fuzzy semi-basically disconnected space, it follows that $GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda), r,s)$ is an (r,s)-gfso-F_σ set in I^X. Also, f is a gdfs*-open surjective function then, $f(GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s))$ is an (r,s)-gfso-F_σ set in I^Y. Now, by Proposition 3.1

$$f^{-1}(GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)) \leq GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s),$$

and hence,

$$\begin{aligned}f(f^{-1}(GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)))&\\ &= GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)\\ &\leq f(GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s))\\ &\leq GS^*C_{\tau_2,\tau_2^*}(f(f^{-1}(\lambda),r,s))\\ &= GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s),\end{aligned}$$

which implies

$$GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)) = f(GS^*C_{\tau_1,\tau_1^*}(f^{-1}(\lambda),r,s)),$$

i.e., $GS^*C_{\tau_2,\tau_2^*}(\lambda,r,s)$ is an (r,s)-gfso-F_σ set in I^Y. This implies, (Y,τ_2,τ_2^*) is generalized double fuzzy semi-basically disconnected space.

Theorem 3.2. For a dfts (X,τ,τ^*), $r\in I_0$ and $s\in I_1$, the following statements are equivalent:

(1) (X,τ,τ^*) is a generalized double fuzzy semi-basically disconnected space.

(2) For each an (r,s)-gfsc-G_δ set λ, $GS^*I_{\tau,\tau^*}(\lambda,r,s)$ is an (r,s)-gfsc-G_δ set.

(3) For each an (r,s)-gfso-F_σ set λ, $GS^*C_{\tau,\tau^*}(\lambda,r,s) \vee GS^*C_{\tau,\tau^*}(\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s),r,s)=\underline{1}$.

(4) For each (r,s)-gfo-F_σ sets λ and μ such that

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee\mu=\underline{1},$$

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\mu,r,s)=\underline{1}.$$

Proof. (1) $\Rightarrow$ (2) Suppose $\lambda\in I^X$ is an (r,s)-gfsc-G_δ set, then $\underline{1}-\lambda$ is an (r,s)-gfso-F_σ set. By (1) $GS^*C_{\tau,\tau^*}(\underline{1}-\lambda,r,s)$ is an (r,s)-gfso-F_σ set and $GS^*C_{\tau,\tau^*}(\underline{1}-\lambda,r,s)=GS^*I_{\tau,\tau^*}(\lambda,r,s)$, this implies that $GS^*I_{\tau,\tau^*}(\lambda,r,s)$ is an (r,s)-gfsc-G_δ set.

(2) $\Rightarrow$ (3) Suppose $\lambda\in I^X$ be an (r,s)-gfso-F_σ set, then

$$\begin{aligned}&GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s),r,s)\\&\quad=GS^*C_{\tau,\tau^*}(\lambda,r,s)\\&\qquad\vee GS^*C_{\tau,\tau^*}(GS^*I_{\tau,\tau^*}(\underline{1}-\lambda,r,s),r,s).\end{aligned}$$

By (2), $GS^*I_{\tau,\tau^*}(\underline{1}-\lambda,r,s)$ is an (r,s)-gfsc G_δ set. Therefore,

$$\begin{aligned}&GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s),r,s)\\&\quad=GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*I_{\tau,\tau^*}(\underline{1}-\lambda,r,s)\\&\quad=GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s)\\&\quad=\underline{1}.\end{aligned}$$

(3) $\Rightarrow$ (4) Suppose λ and μ are (r,s)-gfso-F_σ sets such that $GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee\mu=\underline{1}$.
Then by (3),

$$\begin{aligned}\underline{1}&=GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s))\\&=GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\mu,r,s).\end{aligned}$$

i.e.,

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\mu,r,s)=\underline{1}.$$

(4) $\Rightarrow$ (1) Suppose λ is an (r,s)-gfso-F_σ set. Put

$$\mu=\underline{1}-GS^*C_{\tau,\tau^*}(\lambda,r,s).$$

Then,

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee\mu=\underline{1}.$$

Therefore by (4),

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\vee GS^*C_{\tau,\tau^*}(\mu,r,s)=\underline{1}.$$

which implies, $GS^*C_{\tau,\tau^*}(\lambda,r,s)$ is an (r,s)-gfso-F_σ set, so (X,τ,τ^*) is a generalized double fuzzy semi-basically disconnected space.

Theorem 3.3. A dfts (X,τ,τ^*) is a generalized double fuzzy semi-basically disconnected iff for each (r,s)-gfso-F_σ set λ and an (r,s)-gfsc G_δ set μ such that $\lambda\le\mu$, $GS^*C_{\tau,\tau^*}(\lambda,r,s)\le GS^*I_{\tau,\tau^*}(\mu,r,s)$.

Proof. Suppose λ is an (r,s)-gfso-F_σ and $\mu=\underline{1}-\lambda$ is an (r,s)-gfsc-G_δ such that $\lambda\le\mu$. Then by (2) of Theorem 3.2 $GS^*I_{\tau,\tau^*}(\mu,r,s)$ is an (r,s)-gfsc-G_δ set. Therefore,

$$GS^*C_{\tau,\tau^*}(GS^*I_{\tau,\tau^*}(\mu,r,s),r,s)=GS^*I_{\tau,\tau^*}(\mu,r,s).$$

But λ is an (r,s)-gfso-F_σ set and $\lambda\le\mu$, $\lambda\le GS^*I_{\tau,\tau^*}(\mu,r,s)$. Therefore,

$$GS^*C_{\tau,\tau^*}(\lambda,r,s)\le GS^*I_{\tau,\tau^*}(\mu,r,s).$$

Conversely, let μ be any (r,s)-gfsc-G_δ. Then, $GS^*I_{\tau,\tau^*}(\mu,r,s)$ is an (r,s)-gfso-F_σ set in I^X such that

$$GS^*I_{\tau,\tau^*}(\mu,r,s)\le\mu.$$

Therefore,

$$GS^*C_{\tau,\tau^*}(GS^*I_{\tau,\tau^*}(\mu,r,s),r,s)\le GS^*I_{\tau,\tau^*}(\mu,r,s).$$

which implies that $GS^*I_{\tau,\tau^*}(\mu,r,s)$ is an (r,s)-gfsc-G_δ, then by (2) of Theorem 3.2, (X,τ,τ^*) is a generalized double fuzzy semi-basically disconnected space.

4. Conclusions

In this paper, we have introduced the notions of generalized double fuzzy semi-basically disconnected space and related notions such as (r,s)-generalized fuzzy semiopen-F_σ sets, (r,s)-generalized fuzzy semiclosed-G_δ sets and generalized double fuzzy semi*-irresolute function. We also studied some interesting properties and characterizations of the concepts introduced are studied and we hope these investigations will further encourage other researchers to explore the interesting connections between this area of topology and fuzzy set.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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