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Gain Tuning of a Fuzzy Logic Controller Superior to PD Controllers in Motor Position Control

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Abstract

Although the fuzzy logic controller is superior to the proportional integral derivative (PID) controller in motor control, the gain tuning of the fuzzy logic controller is more complicated than that of the PID controller. Using mathematical analysis of the proportional derivative (PD) and fuzzy logic controller, this study proposed a design method of a fuzzy logic controller that has the same characteristics as the PD controller in the beginning. Then a design method of a fuzzy logic controller was proposed that has superior performance to the PD controller. This fuzzy logic controller was designed by changing the envelope of the input of the of the fuzzy logic controller to nonlinear, because the fuzzy logic controller has more degree of freedom to select the control gain than the PD controller. By designing the fuzzy logic controller using the proposed method, it simplified the design of fuzzy logic controller, and it simplified the comparison of these two controllers.

Keywords: Fuzzy logic controller, PI controller, DC motor

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1. Introduction

The proportional integral derivative (PID) controller is generally used in the position or speed control of a motor. Many studies have concluded that the fuzzy logic controller is better in motor control than the PID controller [1-5]. Other studies have insisted that there is not much performance difference between fuzzy logic and PID controllers [6-9]. Moreover, although many studies insist that the fuzzy logic controller is better than the PID controller, the PID controller is used more than the fuzzy logic controller these days. This is because the gain tuning of the fuzzy logic controller−the selection of the membership function and rule table, is more complicated than that of the PID controller. While the selection of the membership function and rule table largely depend on the experience of the engineer, PID gain tuning can be often performed systematically using the Zigler-Nichols tuning method [10]. Besides, there are many publication on PID gain tuning. Therefore, this study first analyzed the similarity and the difference between proportional derivative (PD) controller and fuzzy logic controller through mathematical analysis. Afterwards, it was shown that it is always possible to design a fuzzy logic controller that has the same characteristics as the PD controller. Methods to design the fuzzy logic controller that has the same characteristics as the PD controller were proposed. Finally, design methods were proposed of the fuzzy logic controller that is superior to the PD controller by changing the membership function to nonlinear. At first, the output of the controller is shown using Matllab Simulink, that is, the system input, U, as a function of the error and the change of the error in the fuzzy logic controller. Afterwards, the comparison of the system input, U, as a function of the error and the change of the error in the PD controller is shown. By comparing the system input, U, of these two controllers, the similarity and the difference are recognized, and the fuzzy logic controller with the same characteristics as the PD controller is easily found. It is possible to obtain the fuzzy logic controller that has superior performance to the PD controller from this fuzzy logic controller by changing the gain characteristics to an S shaped nonlinear function by changing the rule table. Using this method, optimization of the PD gain is not necessary to prove the superiority of the fuzzy logic controller, because it provides a fuzzy logic controller that has the same characteristics as the PD controller. In the simulation of the fuzzy logic and PD controller in this study, the PD gain was almost optimized, but it was not important as mentioned above. The limitation of the input or the anti-windup of the integrator is also not important.

2. Fuzzy Logic Controller

In the fuzzy logic controller, the membership functions of the input and output and a rule table are used to obtain the output of the controller, system input, U. The membership function is a curve that defines how the values of a fuzzy variable in a certain region are mapped to a membership value between 0 and 1. The rule table represents a relation between the input membership function and the output membership functions. The degree of fulfillment is used to evaluate the consequent part of the rule, in the implication step of the fuzzy logic controller. In defuzzification step, the result of the implication step is converted to crisp output, and it is used as system input, U.

A membership function can have different shapes and there many membership functions such as the trapezoidal, Gaussian, two-sided Gaussian, generalized bell, sigmoid-right, sigmoid-left, difference-sigmoid, product sigmoid, polynomial-Z, polynomial-Pi and polynomial-S membership function. The triangular membership function is the most typical membership function and it has been used for the fuzzy controller in this study.

The rules of the fuzzy system are a formulation of the mapping from a given input set to an output set. If there are m membership functions for input variable e and n membership functions for input variable, Δe, then there will be $m \times n$ rules.

There are several methods of defuzzification, such as the center of area method, the height method, the mean of maxima method and the Sugeno method. In the zero-order Sugeno method, a constant membership function (singleton) is used, and the defuzzification is very simple. This study used the singleton method as an output membership function to design the fuzzy logic controller that has the same characteristic as a PD controller.

3. Comparison of the PD and Fuzzy Logic Controllers

In the position control of a motor, the PD controller is the most typically used controller to date. When we compare the fuzzy logic position controller with another controller, the general competitor is the PD controller. The output of a PD controller, the system input$U$, is proportional to an error and to the integral of this error. These gains are the P gain and D gain of the PD controller, respectively. A diagram of the PD controller is shown in Figure 1a.

The input and the output expressed in a mathematical form is shown in Eq. (1).

$$U = K_P E + K_D \Delta E \tag{1}$$

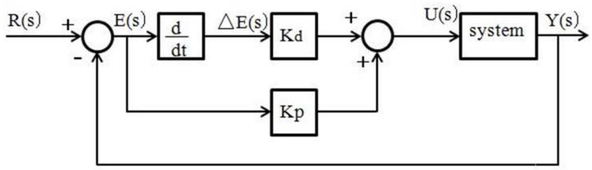
In the fuzzy logic controller, membership functions of inputs and output should be defined. Generally, the inputs to the fuzzy logic controller are the error and the change of the error for the position control of a motor. Using the membership function, fuzzy rules and defuzziffication, the output of the controller is calculated, which turns out to be the system input, U. This process is similar to that of a PD controller because the output is a function of the error and the change of the error. The error and the change of the error are expressed per unit value, between -1 and 1. To make these values between -1 and 1 in the fuzzy logic controller, K_1 is multiplied by the error and K_2 is multiplied by the change of the error.

As a result, the inputs and output can be expressed as in Eq. (2):

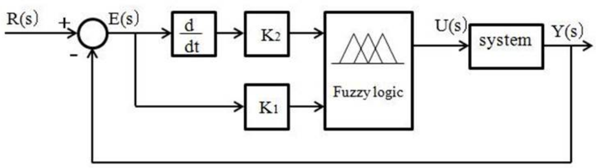
$$U = f(K_1 E,\ K_2 \Delta E) \tag{2}$$

The diagram of the fuzzy controller is shown in Figure 1b. If we assume that the relationship can be expressed in a linear form, the relationship between the inputs and output can be expressed by Eq. (3).

$$U = K_1 E + K_2 \Delta E \tag{3}$$



(a)



(b)

Figure 1. (a) Proportional derivative controller and (b) fuzzy logic controller.

Therefore, in the design of the linear fuzzy logic controller, the proportional gain, K_P, and the derivative gain, K_D, of the PD controller could be the scale factor K_1 to the error and K_2 to the change of the error of the fuzzy logic controller, respectively.

The shape of the system input, U, in a typical fuzzy logic controller with the Mamdani method is shown in Figure 2a. In this figure, the membership functions of the inputs and output in triangular form are used and the center of the area method with the minimum of the maxima is used as a defuzzification method. Many Ushapes of nonlinear forms are possible in fuzzy logic controllers, because there are many shapes of the membership function, many rule tables, and many kinds of defuzzification method. As we can see in Figure 2a, the system input, U, of the fuzzy logic controller is a nonlinear function of K_1 and K_2.

As shown in Eq. (1), and Figure 1a, the system input, U, can be found as a function of K_P and K_D in the PD controller (Figure 2b).

To design a fuzzy logic controller that has the same characteristics as PD controller, a fuzzy logic controller that has linear characteristic was deliberately composed. The system input, U, in this case is shown in Figure 2c. Here, the input membership function uses triangular form. The output membership function uses singleton form and the output value is calculated by multiplication. The defuzzification uses the center of area method. Of course, the system input, U, may take many forms, even the singleton form. Figure 2c represents one of the many possibilities. As shown in Figure 2, the fuzzy logic controller takes variable structures depending on the size of the error or the size of the change in error, which means that the proportional gain, K_P, and the derivative gain, K_D, of the PD controller are variables in the fuzzy logic controller. In other words, the fuzzy logic controller is relatively favorable to the PD controller because its gain takes variable nonlinear structures depending on its operating points. In that sense, it is possible to design a fuzzy logic controller that operates identically to the PD controller. This may be implemented by using a singleton form of the fuzzy logic controller. To extend this idea further, it was possible to design a fuzzy logic controller that operates identically to the PI or PID controller.

In other words, the singleton form fuzzy logic controller, first, makes membership functions of the error and the change of the error in triangular form, second, makes membership function of the fuzzy logic controller output in the singleton form, third, makes a rule table constantly increasing in the diagonal direction from the upper left side to the lower right side, or from the most negative to the most positive, fourth, makes defuzzification using the method of center of area of the output singleton values. Then, the system input, U, coming out of the fuzzy logic controller will be a linear function of K_1 and K_2 , comparable to that of the PD controller (Figure 2c).

In the PD controller, the input of the system, U, can only be a linear function of the proportional gain, K_P, and the derivative gain, K_D. Whereas in the fuzzy logic controller, the input of the system, U, can also be in nonlinear function of K_1, K_2.

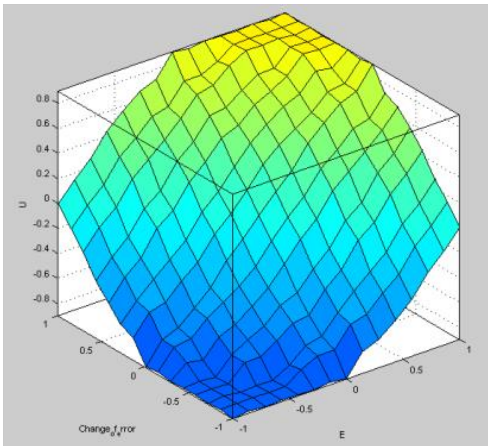
Thus, the fuzzy logic controller can take the form of PD, as one of its many nonlinear forms, but the PD controller cannot construct the nonlinear form of the fuzzy logic controller. This means that the fuzzy logic controller has many degrees of freedom that it can choose from, and thus, is more advantageous than the PD controller.

4. Design of the Fuzzy Logic Controller Corresponding to the PD Controller

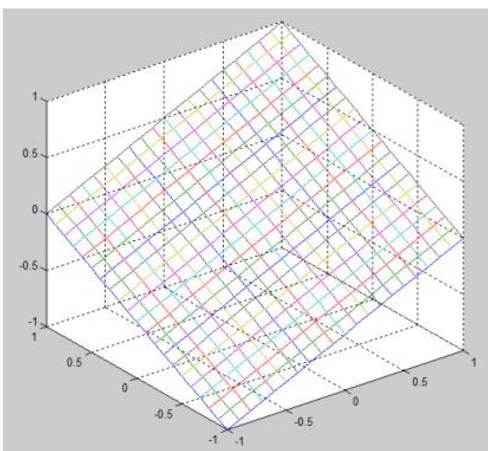
The triangular membership functions of input1 (error) and input2 (change of error) in the fuzzy logic controller (Figure 3) were used to obtain the system input, U, in Figure 2c. The membership function of the output was singleton (Figure 4).

The output membership function in the fuzzy logic controller used in this study used a singleton method, and the output membership function was distributed evenly with negative big (NB) = -1.0, negative small (NS) = -0.5, zero (ZE) = 0.0, positive small (PS) = 0.5, positive big (PB) = 1.0.

The corresponding fuzzy rule is shown in Table 1. The output was distributed evenly in the diagonal direction in the



(a)



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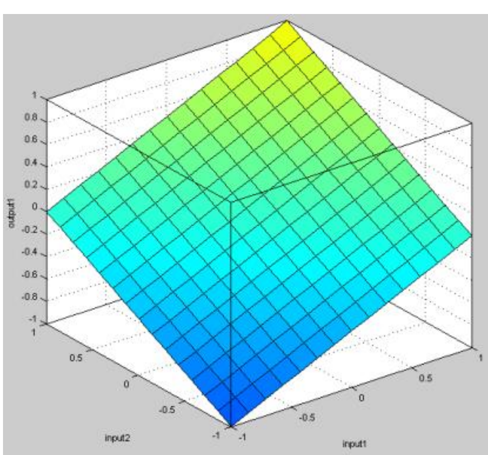


(c)

Figure 2. Envelope of the system input U in (a) a nonlinear fuzzy logic controller, (b) a PI controller, and (c) a linearized fuzzy logic controller.

rule table. Now, a fuzzy logic controller was constructed where the system input, U, was a linear function of K_1e and $K_2\Delta e$,

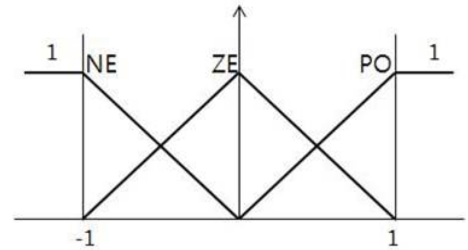


Figure 3. Input membership function.

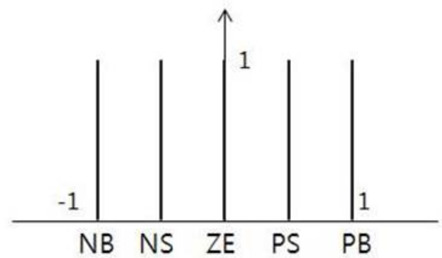


Figure 4. Output membership function.

Table 1. Fuzzy rule table

		Error		
		NE	ZE	PO
Change of error	NE	NB	NS	ZE
	ZE	NS	ZE	PS
	PO	ZE	PS	PB

NB, native big; NS, native small; ZE, zero; PS, positive small; PB, positive big.

like a PD controller. It was verified mathematically that this fuzzy logic controller was identical to a PD controller. To do so, the membership function of the input took a form as shown in Figure 3. The fuzzy rule table was in a 3×3 form (Table 1). In general cases, the singleton form of the output is defined as follows:

$$
\begin{aligned}
&PB \\
&PS = \frac{PB}{2} \\
&ZE = 0 \\
&NS = \frac{NB}{2} = -\frac{PB}{2} \\
&NB = -PB
\end{aligned} \tag{4}
$$

To scale an error and the change of error within the 0 to 1 range of membership function of the inputs, the scale factors K_1 and K_2 were multiplied by the error and the change of the error. Thus K_1e and $K_2\Delta e$ were checked mathematically in the

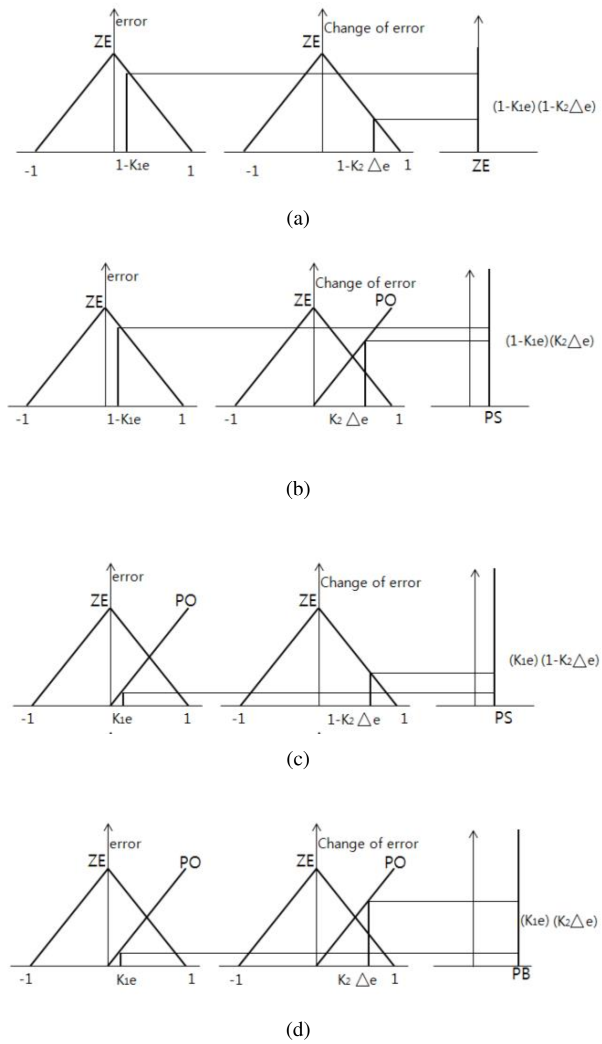


Figure 5. Calculation of the output singleton (product). (a) ZE, error; ZE, change of error. (b) ZE, error; PO, change of error. (c) PO, error; ZE, change of error; (d) PO, error; PO, change of error.

following four distinctive cases:

1) $0 \le K_1 e \le 1$, and $0 \le K_2 \Delta e \le 1$

The calculation of the output singleton (product) when $0 \le K_1 e \le 1$, and $0 \le K_2 \Delta e \le 2$ is shown in the Figure 5. Here, the output was found according to the product of the singleton value and the cases for which the output fell under the rule table are described in the following four cases. The singleton value of output, ZE, PS, PS, and PB were found through the following equations:

$$ZE : (1-k_1 e)(1-k_2 \Delta e) = a$$
$$PS : (1-k_1 e)(k_2 \Delta e) = b$$
$$PS : (k_1 e)(1-k_2 \Delta e) = c$$

$$PB : (k_1 e)(k_2 \Delta e) = d \tag{5}$$

$$U = \frac{a \cdot 0 + b \cdot PS + c \cdot PS + d \cdot PB}{a+b+c+d} \tag{6}$$

Here, the numerator and denominator can be calculated as follows:

$$\begin{aligned} a+b+c+d \quad & \text{(numerator)} \\ &= (1 - k_1 e - k_2 \Delta e + k_1 e \cdot k_2 \Delta e) \\ &\quad + (k_2 \Delta e - k_1 e \cdot k_2 \Delta e) \\ &\quad + (k_1 e - k_1 e \cdot k_2 \Delta e) \\ &\quad + k_1 e \cdot k_2 \Delta e \\ &= 1 \end{aligned} \tag{7}$$

$$\begin{aligned} a \cdot 0 + b \cdot PS + c \cdot PS + d \cdot PB \quad & \text{(denominator)} \\ &= (b+c)PS + d \cdot PB \\ &= (b+c)\frac{PB}{2} + d \cdot PB \\ &= (\frac{b+c}{2} + d)PB \\ &= (\frac{k_2 \Delta e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \\ &\quad + (\frac{k_1 e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \\ &\quad + (k_1 e + k_2 \Delta e)PB \\ &= (k_1 e + k_2 \Delta e)\frac{PB}{2} \end{aligned} \tag{8}$$

Thus,

$$U = (k_1 e + k_2 \Delta e)\frac{PB}{2} \tag{9}$$

This shows that it can take a form identical to a linear PD controller:

$$K_1 \frac{PB}{2} = K_P, \quad K_2 \frac{PB}{2} = K_D.$$

2) $0 \le K_1 e \le 1$, and $-1 \le K_2 \Delta e \le 0$

The calculation of the output singleton (product) when $0 \le K_1 e \le 1$, and $-1 \le K_2 \Delta e \le 0$ is shown in the Figure 6. Similar to case 1), singleton values of the output ZE, NS, PS, and ZE can be found as follows:

$$ZE : (1-k_1 e)(1-k_2 \Delta e) = a$$
$$NS : (1-k_1 e)(k_2 \Delta e) = b$$
$$PS : (k_1 e)(1-k_2 \Delta e) = c$$

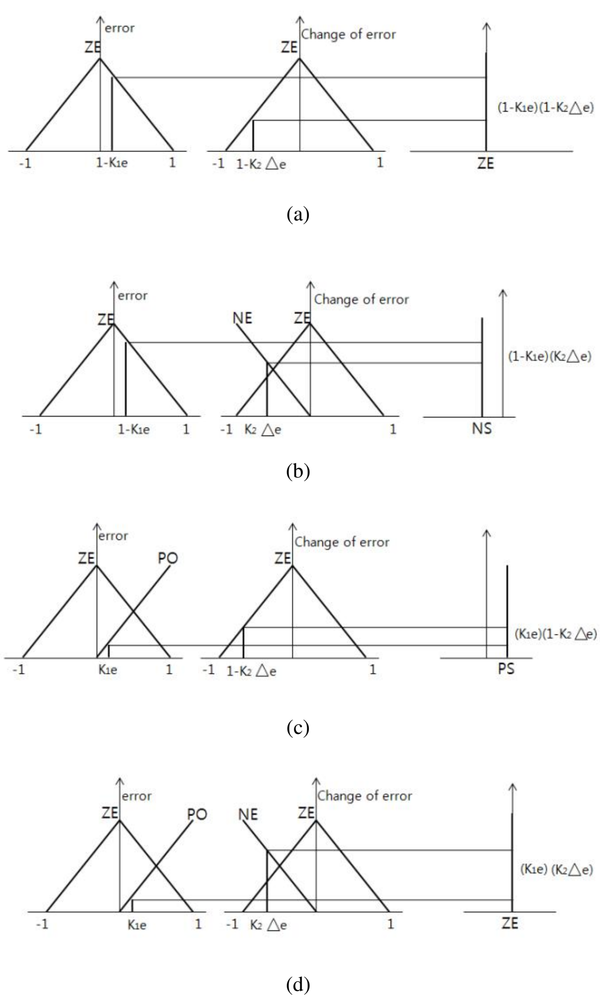


Figure 6. Calculation of the output singleton (product). (a) ZE, error; ZE, change of error. (b) ZE, error; NE, change of error. (c) PO, error; ZE, change of error. (d) PO, error; NE, change of error.

$$ZE : (k_1 e)(k_2 \Delta e) = d \tag{10}$$

$$U = \frac{a \cdot 0 + b \cdot NS + c \cdot PS + d \cdot 0}{a + b + c + d} \tag{11}$$

Here, similar to case 1), the numerator turns into 1, and the denominator can be calculated as follows:

$$\begin{aligned} a \cdot 0 + b \cdot NS + c \cdot PS + d \cdot PB \quad & \text{(denominator)} \\ &= a \cdot 0 + b \cdot (-PS) + c \cdot PS + d \cdot 0 \\ &= (-b + c)PS \\ &= (-b + c)\frac{PB}{2} \\ &= (-\frac{k_2\Delta e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \\ &\quad + (\frac{k_1 e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \end{aligned} \tag{12}$$

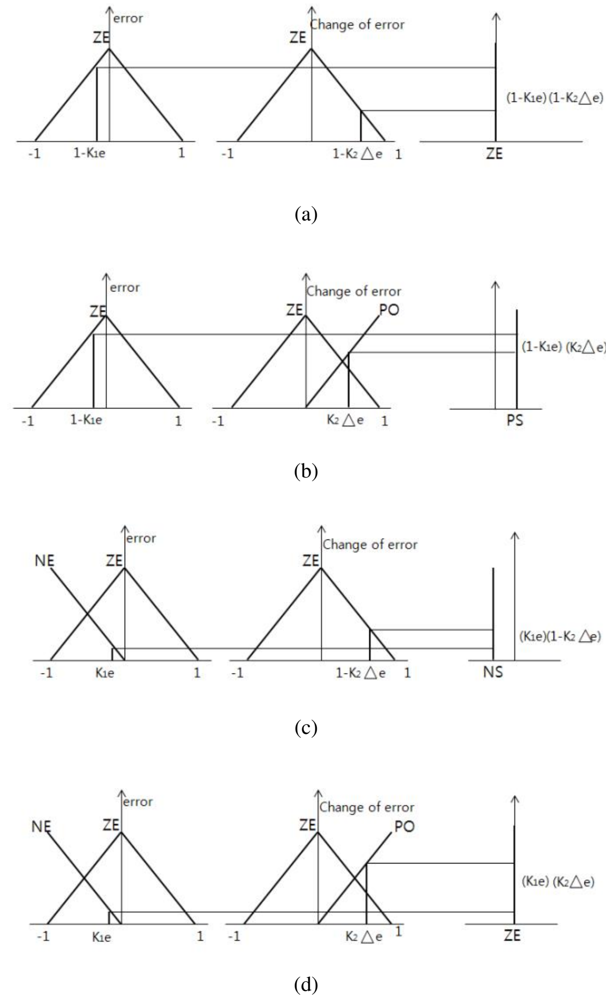


Figure 7. Calculation of the output singleton (product). (a) ZE, error; ZE, change of error. (b) ZE, error; PO, change of error. (c) NE, error; ZE, change of error. (d) NE, error; PO, change of error.

$$= (k_1 e - k_2 \Delta e)\frac{PB}{2}$$

Thus,

$$U = (k_1 e - k_2 \Delta e)\frac{PB}{2} \tag{13}$$

The reason for the sign of Eq. (13) being different from that of Eq. (9), is because Δe is changed to $-\Delta e$.

3) $-1 \leq K_1 e \leq 0$, and $0 \leq K_2 \Delta e \leq 1$

The calculation of the output singleton (product) when $-1 \leq K_1 e \leq 0$, and $0 \leq K_2 \Delta e \leq 1$ is shown in the Figure 7. Similar to case 1) and 2), singleton values of the output, ZE, PS, NS, and ZE can be found as follows:

$$ZE : (1 - k_1 e)(1 - k_2 \Delta e) = a$$

$$PS : (1 - k_1 e)(k_2 \Delta e) = b$$

$$NS : (k_1 e)(1 - k_2 \Delta e) = c$$

$$ZE : (k_1 e)(k_2 \Delta e) = d \tag{14}$$

$$U = \frac{a \cdot 0 + b \cdot PS + c \cdot NS + d \cdot 0}{a + b + c + d} \tag{15}$$

Here, similar to case 1) and 2), the numerator turns into 1, and the denominator can be calculated as follows:

$$\begin{aligned}
a \cdot 0 + b \cdot PS + c \cdot NS + d \cdot 0 \quad & \text{(denominator)} \\
&= a \cdot 0 + b \cdot PS + c \cdot (-PS) + d \cdot 0 \\
&= (b - c)PS \\
&= (b - c)\frac{PB}{2} \\
&= (\frac{k_2 \Delta e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \\
&\quad + (-\frac{k_1 e - k_1 e \cdot k_2 \Delta e}{2})\frac{PB}{2} \\
&= (-k_1 e + k_2 \Delta e)\frac{PB}{2}
\end{aligned} \tag{16}$$

Thus,

$$U = (-k_1 e + k_2 \Delta e)\frac{PB}{2} \tag{17}$$

In this case, because e turnesinto $-e$, the $k_1 e$ is changed to $-k_1 e$.

4) $-1 \leq K_1 e \leq 0$, and $-1 \leq K_2 \Delta e \leq 0$ The calculation of the output singleton (product) when $-1 \leq K_1 e \leq 0$, and $-1 \leq K_2 \Delta e \leq 0$ is shown in the Figure 8. Similar to case 1)-3), singleton values of output ZE, NS, PS, and ZE can be found as follows:

$$ZE : (1 - k_1 e)(1 - k_2 \Delta e) = a$$

$$NS : (1 - k_1 e)(k_2 \Delta e) = b$$

$$NS : (k_1 e)(1 - k_2 \Delta e) = c$$

$$NB : (k_1 e)(k_2 \Delta e) = d \tag{18}$$

$$U = \frac{a \cdot 0 + b \cdot NS + c \cdot NS + d \cdot NB}{a + b + c + d} \tag{19}$$

Here, similar to case 1)-3), the numerator turns into 1 and the denominator can be calculated as follows:

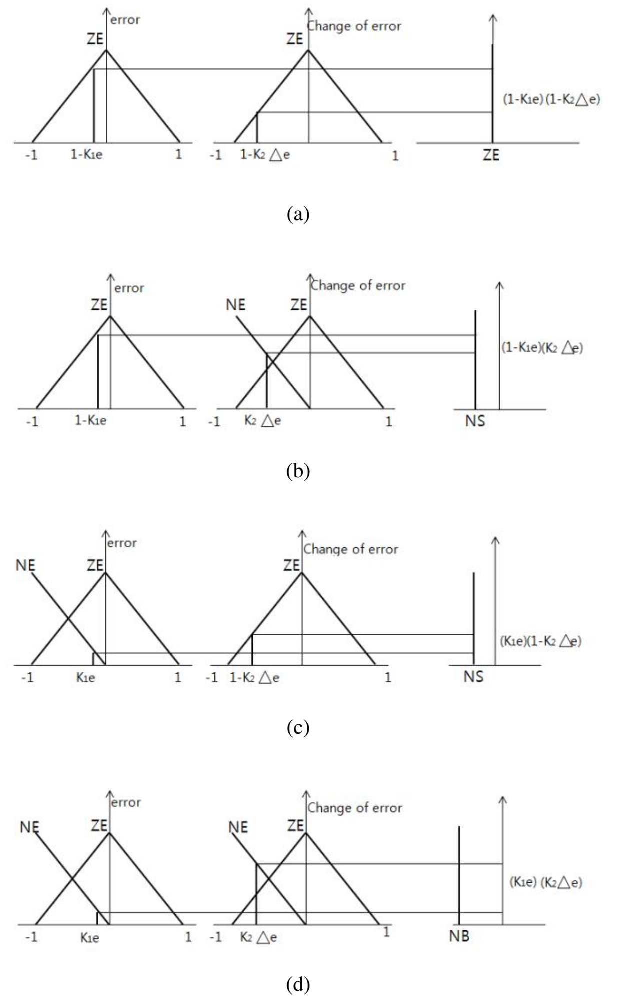


Figure 8. Calculation of the output singleton (product). (a) ZE, error; ZE, change of error. (b) ZE, error; NE, change of error. (c) NE, error; ZE, change of error. (d) NE, error; NE, change of error.

$$\begin{aligned}
a \cdot 0 + b \cdot NS + c \cdot NS + d \cdot NE \quad & \text{(denominator)} \\
&= a \cdot 0 + b \cdot (-PS) + c \cdot (-PS) + d \cdot (-PB) \\
&= (b + c)(-PS) + d \cdot (-PB) \\
&= (b + c)(-\frac{PB}{2}) + d \cdot (-PB) \\
&= (\frac{b + c}{2} + d)(-PB) \\
&= (\frac{k_2 \Delta e - k_1 e \cdot k_2 \Delta e}{2})(-\frac{PB}{2}) \\
&\quad + (\frac{k_1 e - k_1 e \cdot k_2 \Delta e}{2})(-\frac{PB}{2}) \\
&\quad + (k_1 e + k_2 \Delta e)(-PB) \\
&= (-k_1 e - k_2 \Delta e)\frac{PB}{2}.
\end{aligned} \tag{20}$$

Thus,

$$U = (-k_1 e - k_2 \Delta e)\frac{PB}{2}. \tag{21}$$

In this case, e turns into $-e$. and Δe turns into $-\Delta e$, so the sign is changed.

As we can see in Eqs. (9), (13), (17) and (21), the equations take the same form as the PI controller, Eq. (3). In other words, if$K_1 \frac{PB}{2} = K_P$, $K_2 \frac{PB}{2} = K_D$, the PD controller and the fuzzy logic controller will take an identical form. Therefore, it can be seen that a fuzzy logic controller can be construct in the same form as a PD controller. Adversely, the fuzzy logic controller may construct controllers that take nonlinear characteristics, while the PD controller may not. Thus, because of these characteristics, it is possible for fuzzy logic controllers to have superior characteristics compared to PD controllers. Afterwards, by modifying the rule table of this linear fuzzy logic controller to the nonlinear form, it was demonstrated that the fuzzy logic controller is superior to the PD controller. By showing that the fuzzy logic controller could be ab\n identical controller to the PD controller, it can be concluded that it is always possible to design a fuzzy logic controller superior to a PD controller, regardless of whether the gain selection of the PD controller has been optimized. In addition, the gain of the PD controller can be used for the design of the membership functions and the rule table. Thus, the design of the fuzzy logic controller is simplified.

Here, the membership function of the input and output of the fuzzy logic controller was constructed in 3 x 3 form, but it can be easily verified that this is also valid in n x n membership functions. That is, when the fuzzy rule table of an error and change of the error is n x n, the number of singletons for the output can be set to $(2n-1)$. Then, there will be $(n-1)$ positive values, $(n-1)$ negative values and zero. Thus, with evenly selected $(n-1)$ positive and $(n-1)$ negative values, it can be generalized to the case of n x n membership functions. This is verified in Figure 2c. In the fuzzy logic controller in Figure 2c, 5 x 5 membership functions are used. It shows that the linearized fuzzy logic controller can be designed with 5 x 5 membership functions.

5. Simulation Results

Simulations of position control of an inverted pendulum system were performed to compare the performance of the PD controller and fuzzy logic controller.

Three controllers were compared. 1) a PD controller, 2) a singleton type fuzzy logic controller that was designed to have identical characteristics to the PD controller, and 3) a singleton type fuzzy logic controller that was modified from the above singleton type fuzzy logic controller as such that the system input, U, was a nonlinear function of e and Δe (similar to the one in Figure 2a).

The fuzzy logic controller that was identical to the PD controller used the rule table shown in Table 2. As we can see in Table 2, the singleton values were evenly distributed from the upper left to lower right side. That is,

Table 2. Fuzzy rule table used in the linearized fuzzy logic controller

		Error				
		NB	NE	ZE	PO	PB
Change of error	NB	NB	NM	NS	NVS	ZE
	NE	NM	NS	NVS	ZE	PVS
	ZE	NS	NVS	ZE	PVS	PS
	PO	NVS	ZE	PVS	PS	PM
	PB	ZE	PVS	PS	PM	PB

NB, negative big; NM, negative medium; NS, negative small; NVS, native very small; ZE, zero; PVS, positive very small; PS, positive small; PM, positive medium; PB, positive big.

$$\begin{aligned} &PB \\ &PM = (3/4)PB \\ &PS = \frac{PB}{2} \\ &PVS = PB/4 \\ &ZE = 0 \\ &NVS = NB/4 = -PB/4 \\ &NS = \frac{NB}{2} = -\frac{PB}{2} \\ &NM = (3/4)NB = -(3/4)PB \\ &NB = -PB. \end{aligned} \tag{22}$$

By modifying the rule Table 2 to that in Table 3, the nonlinear fuzzy logic controller was obtained. The singleton values in Table 3 are saturated in the upper left and lower right corners. The singleton values are:

$$\begin{aligned} &PB \\ &PM = (3/4)PB \\ &PS = \frac{PB}{2} \\ &ZE = 0 \\ &NS = \frac{NB}{2} = -\frac{PB}{2} \\ &NM = (3/4)NB = -(3/4)PB \\ &NB = -PB. \end{aligned} \tag{23}$$

The fuzzy logic controller that was superior to the PD controller was obtained by changing the rule table to a nonlinear form. Therefore, the design of the fuzzy logic controller was simplified because in this technique the PD gains are applicable

Table 3. Fuzzy rule table used in the nonlinear fuzzy logic controller

		Error				
		NB	NE	ZE	PO	PB
Change of error	NB	NB	NB	NM	NS	ZE
	NE	NB	NM	NS	ZE	PS
	ZE	NM	NS	ZE	PS	PM
	PO	NS	ZE	PS	PM	PB
	PB	ZE	PS	PM	PB	PB

NB, negative big; NM, negative medium; NS, negative small; ZE, zero; PS, positive small; PM, positive medium; PB, positive big.

to the fuzzy logic controller.

In this simulation, $K_1\frac{PB}{2} = K_P = 22.5$, and $K_2\frac{PB}{2} = K_D = 4.5$ were used.

Naturally, the PD controller and the singleton fuzzy logic controller designed for identical characteristics to the PD controller, showed the same characteristics. The fuzzy logic controller with system input, U, with an S curve shaped nonlinear function showed superior performance to the linear fuzzy logic controller and to the PD controller. In the fuzzy logic controller with the nonlinear function, K_1 and K_2 were identical to that of the linear fuzzy logic controller. Only the rule table was changed to be nonlinear. In other words, the linear fuzzy logic controller that was designed to have the same characteristics as the PD controller, was merely modified to have an S shaped curve in U form for K_1 and K_2 and modified the allocation of the rule table. However, response became more advanced than that of the PD controller. That is, the reason why the fuzzy logic controller shows better characteristics than the PD controllers is because it has a nonlinear S shaped curved U. The S shaped curve means that when the error is large, it lessens the gains, and when the error is small, it increases the gains. The variableness of the gains in the fuzzy logic controller demonstrates its excellence. In addition, it shows that the fuzzy logic controller that has equal performance to the PD controller can always be implemented using the PD gains of the PD controller. It also shows that it is always possible to design fuzzy logic controllers with even more outstanding performances by using the same PD gains and modifying the rule table.

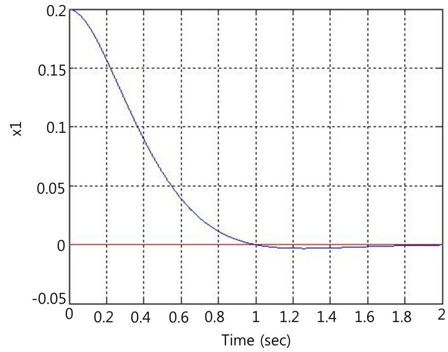
Figure 9 shows the position responses of these three controllers. It shows the responses to the position command 0, which means that the pendulum had steady state motion in the inverted position. The PD controller in Figure 9a and the linearized fuzzy logic controller in Figure 9b show identical performance and the nonlinear fuzzy logic controller shows superior performance. That is, the settling time is shortest in the nonlinear fuzzy logic controller, and the PD controller and linearized fuzzy logic controller show the same settling time. Figure 10 shows the change of the error in the position control of these three controllers. Figure 11 shows the system input, U, in these three controllers and it shows the response time. As we can see in Figures 9-11, the response was fastest in the nonlinear fuzzy controller and the PI controller and linearized fuzzy logic controller showed the same response.

To conclude, it is always possible to design a fuzzy logic controller with the same performance as a PD controller using the PD gains. It is always possible to design a fuzzy logic controller with performance superior to that of the PD controller, making the fuzzy logic controller more favorable than the PD controller.

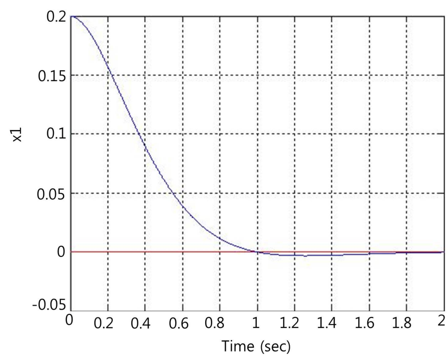
The fact that the response characteristic can be improved simply by modifying the rule table assignment to have a U with an S-shaped curve for K_1 and K_2 proves that the reason behind the fuzzy logic controller with surperior performance to that of the PD controller is the fuzzy logic controller's nonlinearity and that the PD controller's optimized gain can be used when designing a fuzzy logic controller.

6. Conclusions

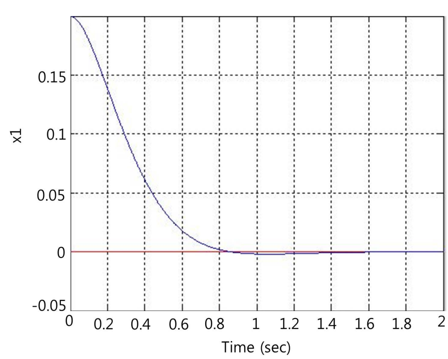
This study proposed a design method for a fuzzy logic controller that has performance characteristics identical to that of the PD controller and superior to that of the PD controller. The method was mathematically analyzed and its outcomes were confirmed through simulations. Thus, the study shows that it is possible to design a fuzzy logic controller that is superior to the PD controller using the PD gains and modifying the rule table to be nonlinear. Therefore, the problem of controller gain optimization, which arises when comparing the performance of the PD and fuzzy logic controllers, may be avoided. As described in this paper, the fuzzy logic controller with the system input U nonlinear to the error and the change of the error becomes advantageous to the linear PD controller. On the contrary, the system input, U, of the PD controller is only provided linear to the error and the change of the error. Thus, while the fuzzy logic controller may express the PD controller, the PD controller may not fully express the fuzzy logic controller. This means that there are nonlinear fuzzy logic controllers that cannot be expressed with PD controllers. Therefore, there is always the possibility of designing a fuzzy logic controller with excellent performance compared to the PD controller. In this sense, it can be said that fuzzy logic controllers always deliver equal or



(a)

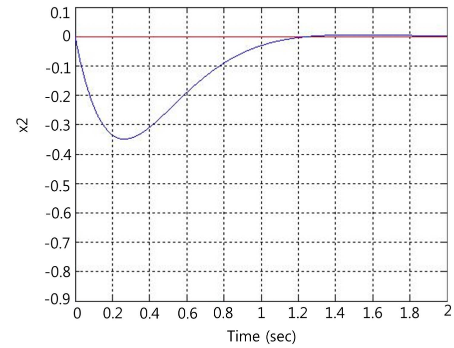


(b)

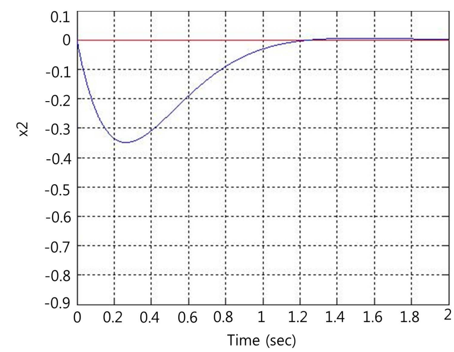


(c)

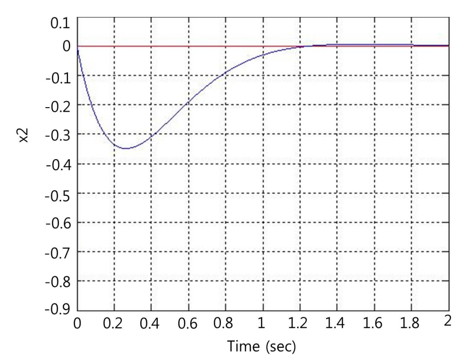
Figure 9. Position error in the position control of the inverted pendulum system for (a) proportional derivative (PD) controller, (b) linearized fuzzy logic controller (designed with the same characteristic as PD controller), (c) nonlinear fuzzy logic controller.



(a)



(b)



(c)

Figure 10. Change of the position error in the position control of the inverted pendulum system for (a) proportional derivative (PD) controller, Controller, (b) linearized fuzzy logic controller (designed with the same characteristic as PD controller), (c) nonlinear fuzzy logic controller.

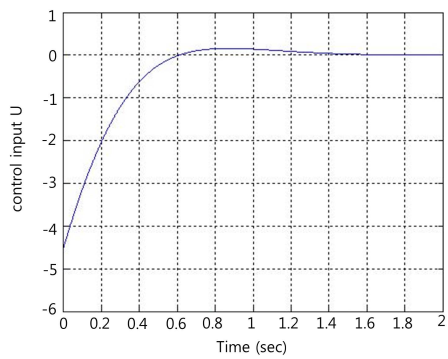
better results than PD controllers. This was also confirmed by the simulation results.

Conflict of Interest

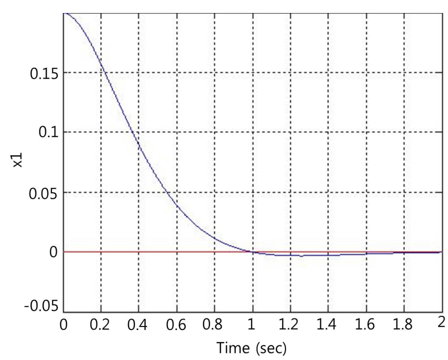
No potential conflict of interest relevant to this article was reported.

References

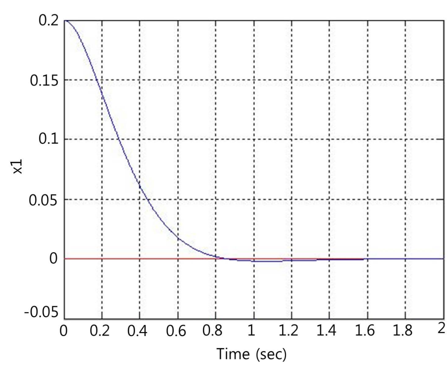
[1] B. K. Bose, "Expert system, fuzzy logic, and neural network applications in power electronics and motion control," in *Proceedings of the IEEE*, vol. 82, no. 8, pp. 1303-1323, Aug. 1994. http://dx.doi.org/10.1109/5.301690



(a)



(b)



(c)

Figure 11. The system input in the position control of the inverted pendulum system for (a) proportional derivative (PD) controller, Controller, (b) linearized fuzzy logic controller (designed with the same characteristic as PD controller), (c) nonlinear fuzzy logic controller.

[2] C. Y. Wan, S. C. Kim, and B. K. Bose, “Robust position control of induction motor using fuzzy logic control,” in *Conference Record of the IEEE Industry Applications Society Annual Meeting*, Houston, TX, October 4-9, 1992, pp. 472-481. http://dx.doi.org/10.1109/IAS.1992.244358

[3] B. Heber, L. Xu, and Y. Tang, “Fuzzy logic enhanced speed control of an indirect field-oriented induction machine drive,” *IEEE Transactions on Power Electronics*, vol. 12, no. 5, pp. 772-778, Sep. 1997. http://dx.doi.org/10.1109/63.622994

[4] M. N. Uddin, T. S. Radwan, and M. Azizur Rahman, “Performances of fuzzy-logic-based indirect vector control for induction motor drive,” *IEEE Transactions on Industry Applications*, vol. 38, no. 5, pp. 1219-1225, Sep. 2002. http://dx.doi.org/10.1109/TIA.2002.802990

[5] J. S. Yu, S. H. Kim, B. K. Lee, C. Y. Won, and J. Hur, “Fuzzy-logic-based vector control scheme for permanent-magnet synchronous motors in elevator drive applications,” *IEEE Transactions on Industrial Electronics*, vol. 54, no. 4, pp. 2190-2200, Aug. 2007. http://dx.doi.org/10.1109/TIE.2007.894692

[6] G. C. D. Sousa and B. K. Bose, “A fuzzy set theory based control of a phase-controlled converter DC machine drive,” *IEEE Transactions on Industry Applications*, vol. 30, no. 1, pp. 34-44, Jan. 1994. http://dx.doi.org/10.1109/28.273619

[7] V. Donescu, D. O. Neacsu, G. Griva, and F. Profumo, “A systematic design method for fuzzy logic speed controller for brushless DC motor drives,” in *Proceedings of the 27th Annual IEEE Power Electronics Specialists Conference*, Baveno, Italy, June 23-27, 1996, pp. 689-694. http://dx.doi.org/10.1109/PESC.1996.548656

[8] F. Mrad and G. Deeb, “Experimental comparative analysis of conventional, fuzzy logic, and adaptive fuzzy logic controllers,” in *Conference Record of the IEEE Industry Applications Conference: Thirty-Fourth IAS Annual Meeting*, Phoenix, AZ October 3-7, 1999, pp. 664-673. http://dx.doi.org/10.1109/IAS.1999.800022

[9] Z. Ibrahim and E. Levi, “A comparative analysis of fuzzy logic and PI speed control in high-performance AC drives using experimental approach,” *IEEE Transactions on Industry Applications*, vol. 38, no. 5, pp. 1210-1218, Sep. 2002. http://dx.doi.org/10.1109/TIA.2002.802993

[10] G. F. Franklin, M. L. Workman, and J. D. Powell, *Digital Control of Dynamic Systems*, 3rd ed., Half Moon Bay, CA: Ellis-Kagle Press, 2006.

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