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일반화된 삼각함수퍼지집합에 대한 정규 지수 퍼지확률

Normal and exponential fuzzy probability for generalized trigonometric fuzzy sets

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요약

일반화된 삼각함수 퍼지집합은 삼각함수 퍼지수의 일반화이다. Zadeh([7])는 확률을 이용하여 퍼지이벤트에 대한 확률을 정의하였다. 우리는 정규분포와 지수분포를 각각 이용하여 실수 ℝ 위에서 정규퍼지확률과 지수퍼지확률을 정의하고, 일반화된 삼각함수 퍼지집합에 대하여 정규퍼지확률과 지수퍼지확률을 계산하였다.

Abstract

A generalized trigonometric fuzzy set is a generalization of a trigonometric fuzzy number. Zadeh([7]) defines the probability of the fuzzy event using the probability. We define the normal and exponential fuzzy probability on ℝ using the normal and exponential distribution, respectively, and we calculate the normal and exponential fuzzy probability for generalized trigonometric fuzzy sets.

Keywords : Fuzzy event, Normal fuzzy probability, Exponential fuzzy probability

1. Introduction

We define the generalized trigonometric fuzzy set and calculate four operations of two generalized trigonometric fuzzy sets([3]). Four operations are based on the Zadeh's extension principle([6]). Zadeh defines the probability of fuzzy event as follows.

Let $(\Omega, \mathcal{F}, P)$ be a probability space, where Ω denotes the sample space, $\mathcal{F}$ the σ-algebra on Ω, and P a probability measure. A fuzzy set A on Ω is called a fuzzy event. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event A. Then the probability of the fuzzy event A is defined by Zadeh([7]) as

$$\widetilde{P}(A) = \int_\Omega \mu_A(\omega)dP(\omega), \quad \mu_A(\cdot) : \Omega \to [0,1].$$

We defined the normal fuzzy probability using the normal distribution and calculated the normal fuzzy probability for quadratic fuzzy number([4]). Then we had the explicit formula for the normal fuzzy probability for trigonometric fuzzy number([5]).

In this paper, we calculate the normal and exponential fuzzy probability for generalized trigonometric fuzzy sets.

2. Preliminaries

Let $(\Omega, \mathcal{F}, P)$ be a probability space, and X be a random variable defined on it. Let g be a real-valued Borel-measurable function on ℝ. Then $g(X)$ is also a random variable. We note that a random variable X defined on $(\Omega, \mathcal{F}, P)$ induces a measure P_X on a Borel set $B \in \boldsymbol{B}$ defined by the relation $P_X(B) = P\{X^{-1}(B)\}$. Then P_X becomes a probability measure on $\boldsymbol{B}$ and is called the probability distribution of X. It is known that if $E[g(X)]$ exists, then g is also integrable over ℝ with respect to P_X. Moreover, the

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relation

$$\int_{\Omega} g(X)dP = \int_{\mathbb{R}} g(t)dP_X(t)$$

holds.

Definition 2.1. Let the random variable X have the normal distribution given by the probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

where $\sigma^2 > 0$ and $m \in \mathbb{R}$. The induced measure P_X is called the normal distribution.

Definition 2.2. Let the random variable X have the exponential distribution given by the probability density function

$$f(x) = \lambda e^{-\lambda x}$$

where $x > 0$ and $\lambda > 0$. The induced measure P_X is called the exponential distribution.

A fuzzy set A on Ω is called a *fuzzy event*. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event A. Then the probability of the fuzzy event A is defined by Zadeh([7]) as

$$\widetilde{P}(A) = \int_{\Omega} \mu_A(\omega)dP(\omega), \quad \mu_A(\cdot) : \Omega \to [0,1].$$

Definition 2.3. The normal and exponential fuzzy probability $\widetilde{P}(A)$ of a fuzzy set A on $\mathbb{R}$ is defined by

$$\widetilde{P}(A) = \int_{\mathbb{R}} \mu_A(x)dP_X,$$

where P_X is the normal and exponential distribution, respectively.

Definition 2.4. A trigonometric fuzzy set is a fuzzy number A having membership function

$$\mu_A(x) = \begin{cases} 0, & x < \theta_1,\ \theta_3 \le x, \\ \sin(x-\theta_1), & \theta_1 \le x < \theta_3, \end{cases}$$

where $\theta_3 - \theta_1 = \pi$.

The above trigonometric fuzzy set is denoted by $A = <\theta_1, \theta_2, \theta_3>$, where $\theta_2 = \theta_1 + \frac{\pi}{2}$. For a trigonometric fuzzy number $A = <\theta_1, \theta_2, \theta_3>$, we define $\sin^{-1}(\cdot)$ as an inverse of $\sin(\cdot) : [0, \theta_2 - \theta_1] \to [0,1]$.

The operations of two fuzzy numbers (A, μ_A) and (B, μ_B) are based on the Zadeh's extension principle([5]). We consider the following four operations. For all $x \in A$ and $y \in B$

1. Addition $A(+)B$:
   $\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}$
2. Subtraction $A(-)B$:
   $\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}$
3. Multiplication $A(\cdot)B$:
   $\mu_{A(\cdot)B}(z) = \sup_{z=x\cdot y} \min\{\mu_A(x), \mu_B(y)\}$
4. Division $A(/)B$:
   $\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\}$

We studied the four operations described in introduction for two trigonometric fuzzy numbers.

Definition 2.5. ([5]) For two trigonometric fuzzy numbers $A = <c_1, c_2, c_3>$ and $B = <d_1, d_2, d_3>$, we have

1. $A(+)B = <c_1+d_1, c_2+d_2, c_3+d_3>$
2. $A(-)B = <c_1-d_3, c_2-d_2, c_3-d_1>$
3. $A(\cdot)B = <c_1 \cdot d_1, c_2 \cdot d_2, c_3 \cdot d_3>$
4. $A(/)B = <\frac{c_1}{d_3}, \frac{c_2}{d_2}, \frac{c_3}{d_1}>$

Proof. Note that

$$\mu_A(x) = \begin{cases} 0, & x < c_1,\ \frac{\pi}{k}+c_1 \le x, \\ \sin k(x-c_1), & c_1 \le x < \frac{\pi}{k}+c_1, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < d_1,\ \frac{\pi}{m}+d_1 \le x, \\ \sin m(x-d_1), & d_1 \le x < \frac{\pi}{m}+d_1. \end{cases}$$

We calculate exactly four operations using α-cuts. Let $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$ be the α-cuts of A and B, respectively. Since $\alpha = \sin k(a_1^{(\alpha)} - c_1)$ and $a_2^{(\alpha)} = \frac{\pi}{k} + 2c_1 - a_1^{(\alpha)}$, we have

$$\begin{aligned} A_\alpha &= [a_1^{(\alpha)}, a_2^{(\alpha)}] \\ &= [\frac{1}{k}\sin^{-1}\alpha + c_1, \frac{\pi}{k} + c_1 - \frac{1}{k}\sin^{-1}\alpha], \end{aligned}$$

where $c_1 \le \sin^{-1}\alpha \le c_2$. Similarly,

$$\begin{aligned} B_\alpha &= [b_1^{(\alpha)}, b_2^{(\alpha)}] \\ &= [\frac{1}{m}\sin^{-1}\alpha + d_1, \frac{\pi}{m} + d_1 - \frac{1}{m}\sin^{-1}\alpha]. \end{aligned}$$

1. Addition : By the above facts,

$$\begin{aligned} A_\alpha(+)B_\alpha &= [a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}] \\ &= [(\frac{1}{k}+\frac{1}{m})\sin^{-1}\alpha + c_1 + d_1, \\ &\quad (\frac{1}{k}+\frac{1}{m})(\pi - \sin^{-1}\alpha) + c_1 + d_1]. \end{aligned}$$

Thus $\mu_{A(+)B}(x) = 0$ on the interval $[c_1+d_1, (\frac{1}{k}+\frac{1}{m})\pi$

$+c_1+d_1]^c=[c_1+d_1, c_3+d_3]^c$ and $\mu_{A(+)B}(x)=1$ at $x=\frac{1}{2}(\frac{1}{k}+\frac{1}{m})\pi+c_1+d_1=c_2+d_2$. Hence $\mu_{A(+)B}(x)=0$ if $x<c_1+d_1, (\frac{1}{k}+\frac{1}{m})\pi+c_1+d_1 \le x$ and $\mu_{A(+)B}(x)=\sin\frac{km}{k+m}(x-c_1-d_1)$ if $c_1+d_1 \le x<(\frac{1}{k}+\frac{1}{m})\pi+c_1+d_1$. Thus $A(+)B=<c_1+d_1, c_2+d_2, c_3+d_3>$.

2. Subtraction : Since

$$\begin{aligned}A_\alpha(-)B_\alpha&=[a_1^{(\alpha)}-b_2^{(\alpha)}, a_2^{(\alpha)}-b_1^{(\alpha)}]\\&=[(\frac{1}{k}+\frac{1}{m})\sin^{-1}\alpha+c_1-d_1-\frac{\pi}{m},\\&\quad\frac{\pi}{k}+c_1-d_1-(\frac{1}{k}+\frac{1}{m})\sin^{-1}\alpha],\end{aligned}$$

we have $\mu_{A(-)B}(x)=0$ on the interval $[c_1-d_1-\frac{\pi}{m}, \frac{\pi}{k}+c_1-d_1]^c=[c_1-d_3, c_3-d_1]^c$ and $\mu_{A(-)B}(x)=1$ at $x=(\frac{1}{k}-\frac{1}{m})\frac{\pi}{2}+c_1-d_1=c_2-d_2$. Hence $\mu_{A(-)B}(x)=0$ if $x<c_1-d_1-\frac{\pi}{m}, \frac{\pi}{k}+c_1-d_1\le x$ and $\mu_{A(-)B}(x)=\sin\frac{km}{k+m}(x+\frac{\pi}{m}-c_1+d_1)$ if $c_1-d_1-\frac{\pi}{m}\le x<\frac{\pi}{k}+c_1-d_1$. Thus $A(-)B=<c_1-d_3, c_2-d_2, c_3-d_1>$.

3. Multiplication : Since

$$\begin{aligned}A_\alpha(\cdot)B_\alpha&=[a_1^{(\alpha)}\cdot b_1^{(\alpha)}, a_2^{(\alpha)}\cdot b_2^{(\alpha)}]\\&=[c_1d_1+(\frac{d_1}{k}+\frac{c_1}{m})\sin^{-1}\alpha+\frac{1}{km}(\sin^{-1}\alpha)^2,\\&\quad(\frac{d_1}{k}+\frac{c_1}{m})\pi+\frac{\pi^2}{km}+c_1d_1-(\frac{2\pi}{km}+\frac{d_1}{k}\\&\quad+\frac{c_1}{m})\sin^{-1}\alpha+\frac{1}{km}(\sin^{-1}\alpha)^2],\end{aligned}$$

we have $\mu_{A(\cdot)B}(x)=0$ on $[c_1d_1, \frac{\pi^2}{km}+(\frac{d_1}{k}+\frac{c_1}{m})\pi+c_1d_1]^c=[c_1d_1, c_3d_3]^c$ and $\mu_{A(\cdot)B}(x)=1$ at $x=\frac{\pi^2}{4km}+(\frac{d_1}{k}+\frac{c_1}{m})\frac{\pi}{2}+c_1d_1=c_2d_2$. Hence $\mu_{A(\cdot)B}(x)=0$ if $x<c_1d_1, \frac{\pi^2}{km}+(\frac{d_1}{k}+\frac{c_1}{m})\pi+c_1d_1\le x$ and $\mu_{A(\cdot)B}(x)=\sin\frac{1}{2}(-(c_1k+md_1)+km\sqrt{(\frac{d_1}{k}+\frac{c_1}{m})^2-\frac{4}{km}(c_1d_1-x)})$ if $c_1d_1\le x<\frac{\pi^2}{km}+(\frac{d_1}{k}+\frac{c_1}{m})\pi+c_1d_1-d_1$. Thus $A(\cdot)B=<c_1d_1, c_2d_2, c_3d_3>$.

4. Division : Since

$$\begin{aligned}A_\alpha(/)B_\alpha&=[\frac{a_1^{(\alpha)}}{b_2^{(\alpha)}}, \frac{a_2^{(\alpha)}}{b_1^{(\alpha)}}]\\&=[\frac{\frac{1}{k}\sin^{-1}\alpha+c_1}{-\frac{1}{m}\sin^{-1}\alpha+\frac{\pi}{m}+d_1}, \frac{-\frac{1}{k}\sin^{-1}\alpha+c_1+\frac{\pi}{k}}{\frac{1}{m}\sin^{-1}\alpha+d_1}],\end{aligned}$$

we have $\mu_{A(/)B}(x)=0$ on the interval $[\frac{c_1m}{\pi+d_1m}, \frac{\pi+c_1k}{kd_1}]^c=[\frac{c_1}{d_3}, \frac{c_3}{d_1}]^c$ and $\mu_{A(/)B}(x)=1$ at $x=\frac{m(\pi+2c_1k)}{k(\pi+2d_1m)}=\frac{c_2}{d_2}$. Hence $\mu_{A(/)B}(x)=0$ if $x<\frac{c_1m}{\pi+md_1}, \frac{\pi+c_1k}{kd_1}\le x$ and $\mu_{A(/)B}(x)=\sin\frac{km((\frac{\pi}{m}+d_1)x-c_1)}{kx+m}$ if $\frac{c_1m}{\pi+md_1}\le x<\frac{\pi+c_1k}{kd_1}-d_1$. Thus $A(/)B=<\frac{c_1}{d_3}, \frac{c_2}{d_2}, \frac{c_3}{d_1}>$.

Example 2.6. ([5]) For two trigonometric fuzzy numbers $A=<\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}>$ and $B=<\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}>$, we can calculate exactly the above four operations using α-cuts.

$$\mu_{A(+)B}(x)=\begin{cases}0, & x<\frac{7}{12}\pi, \frac{25}{12}\pi\le x,\\ \sin\frac{2}{3}(x-\frac{7}{12}\pi), & \frac{7}{12}\pi\le x<\frac{25}{12}\pi.\end{cases}$$

$$\mu_{A(-)B}(x)=\begin{cases}0, & x<-\frac{5}{12}\pi, \frac{13}{12}\pi\le x,\\ \sin\frac{2}{3}(x+\frac{5}{12}\pi), & -\frac{5}{12}\pi\le x<\frac{13}{12}\pi.\end{cases}$$

$$\mu_{A(\cdot)B}(x)=\begin{cases}0, & x<\frac{\pi^2}{12}, \pi^2\le x,\\ \sin(-\frac{5}{12}\pi+\sqrt{\frac{\pi^2}{144}+2x}), & \frac{\pi^2}{12}\le x<\pi^2.\end{cases}$$

$$\mu_{A(/)B}(x)=\begin{cases}0, & x<\frac{4}{9}, \frac{16}{3}\le x,\\ \sin\frac{9\pi x-4\pi}{6x+12}, & \frac{4}{9}\le x<\frac{16}{3}.\end{cases}$$

We define the generalized trigonometric fuzzy set. A generalized trigonometric fuzzy set is symmetric and may not have value 1.

Definition 2.7. A generalized trigonometric fuzzy set is a fuzzy set A that has a membership function

$$\mu_A(x)=\begin{cases}0, & x<c, c+\frac{\pi}{m}\le x,\\ k\sin m(x-c), & x<c+\frac{\pi}{m},\end{cases}$$

where $0<k<1$ and m, c are positive constants.

The above generalized trigonometric fuzzy set is denoted by $A=<k,m,c>$. Note that $\mu_A(\frac{\pi+2c}{2m})=k$. We define $\sin^{-1}(\cdot)$ as an inverse of $\sin(\cdot):[0,\frac{\pi}{2}]\to[0,1]$.

3. Normal and exponential fuzzy probability for generalized trigonometric fuzzy sets

We derived the explicit formula for the normal fuzzy probability for a trigonometric fuzzy number and give examples.

Theorem 3.1. ([5]) Let $A=<\theta_1,\theta_2,\theta_3>$ be a trigonometric fuzzy number and $X\sim N(m,\sigma^2)$. Then the normal fuzzy probability is

$$\begin{aligned}\widetilde{P}(A)&=\int_{\theta_1}^{\theta_3}\sin x\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}dx=-\frac{1}{4}e^{-\frac{m^2}{2\sigma^2}}\\&\times(\exp(\frac{m^2-2im\sigma^2-\sigma^4}{2\sigma^2})(\mathrm{Erf}(\frac{-m+i\sigma^2+\theta_3}{\sqrt{2}\,\sigma})\\&-\mathrm{Erf}(\frac{-m+i\sigma^2+\theta_1}{\sqrt{2}\,\sigma}))\\&+\exp(\frac{m^2+2im\sigma^2-\sigma^4}{2\sigma^2})(\mathrm{Erf}(\frac{m+i\sigma^2-\theta_3}{\sqrt{2}\,\sigma})\\&-\mathrm{Erf}(\frac{m+i\sigma^2-\theta_1}{\sqrt{2}\,\sigma}))),\end{aligned}$$

where $\mathrm{Erf}(x)=\frac{2}{i\sqrt{\pi}}\int_0^{ix}\exp(-z^2)dz.$.

Example 3.2. ([5]) In the case of the trigonometric fuzzy number $A=<0,\frac{\pi}{2},\pi>$, the normal fuzzy probability with respect to $X\sim N(3,2^2)$ is 0.3003.

$$\begin{aligned}\widetilde{P}(A)&=\int_0^{\pi}\sin x\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-3)^2}{8}}dx\\&=-\frac{1}{4}e^{-\frac{9}{8}}[e^{-\frac{7}{8}+3i}\mathrm{Erf}(\frac{-3-4i+x}{2\sqrt{2}})\\&-e^{-\frac{7}{8}-3i}\mathrm{Erf}(\frac{-3+4i+x}{2\sqrt{2}})]_0^{\pi}\\&=0.3003\end{aligned}$$

Example 3.3. ([5]) Let $X\sim N(3,2^2)$ and consider the fuzzy numbers in Example 2.6.

1. Multiplication

$$\widetilde{P}=\int_{\frac{\pi^2}{12}}^{\pi^2}\sin(-\frac{5}{12}\pi+\sqrt{\frac{\pi^2}{144}+2x})\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-3)^2}{8}}dx=0.6807$$

2. Division

$$\widetilde{P}=\int_{\frac{4}{9}}^{\frac{16}{3}}\sin(\frac{9\pi x-4\pi}{6x+12})\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-3)^2}{8}}dx=0.4245$$

In this section, we derive the explicit formula for the normal fuzzy probability for generalized trigonometric fuzzy sets and give some examples.

Theorem 3.4. Let $X\sim N(a,\sigma^2)$ and $A=<k,m,c>$ be generalized trigonometric fuzzy set. Then the normal fuzzy probability of a generalized trigonometric fuzzy set A is

$$\begin{aligned}\widetilde{P}(A)=\frac{km}{2\sqrt{2\pi}}&\sin(2\sigma(\exp(-\frac{-(a-c)^2}{2\sigma^2})\\&-\exp(-\frac{(-a+c+\frac{\pi}{m})^2}{2\sigma^2}))\\&+(a-c)\sqrt{2\pi}(\mathrm{Erf}(\frac{a-c}{\sqrt{2}\,\sigma})-\mathrm{Erf}(\frac{a-c-\frac{\pi}{m}}{\sqrt{2}\,\sigma}))).\end{aligned}$$

Example 3.5. Let $A=<\frac{2}{3},\frac{1}{2},\pi>$ be generalized trigonometric fuzzy set. Then the normal fuzzy probability of A with respect to $X\sim N(6,1^2)$ is

$$\begin{aligned}\widetilde{P}(A)&=\frac{2}{3\sqrt{2\pi}}\int_{\pi}^{3\pi}\sin\frac{1}{2}(x-\pi)\exp(-\frac{(x-6)^2}{2})dx\\&=0.5827+1.2760\times10^{-17}i\end{aligned}$$

Theorem 3.6. Let $X\sim E(\lambda)$ and $A=<k,m,c>$ be generalized trigonometric fuzzy set. Then the exponential fuzzy probability of a generalized trigonometric fuzzy set A is

$$\widetilde{P}(A)=\frac{km}{\lambda+km^2}(e^{-\lambda(c+\frac{\pi}{m})}+e^{-\lambda c}).$$

Proof. Since

$$\mu_A(x)=\begin{cases}0, & x<c,\ c+\frac{\pi}{m}\le x,\\ k\sin m(x-c), & x<c+\frac{\pi}{m},\end{cases}$$

where $0<k<1$ and m,c are positive constants, we have

$$\begin{aligned}\widetilde{P}(A) &= \int_R \mu_A(x)\,dP_X \\ &= k\lambda \int_c^{c+\frac{\pi}{m}} \sin m(x-c)\,e^{-\lambda x}dx \\ &= -k[e^{-\lambda x}\sin m(x-c)]_c^{c+\frac{\pi}{m}} \\ &\quad + km\int_c^{c+\frac{\pi}{m}} \cos m(x-c)\,e^{-\lambda x}dx \\ &= km\int_c^{c+\frac{\pi}{m}} \cos m(x-c)\,e^{-\lambda x}dx.\end{aligned}$$

Since

$$\begin{aligned}&\int_c^{c+\frac{\pi}{m}} \cos m(x-c)\,e^{-\lambda x}dx \\ &= [-\frac{1}{\lambda}e^{-\lambda x}\cos m(x-c)]_c^{c+\frac{\pi}{m}} \\ &\quad -\frac{m}{\lambda}\int_c^{c+\frac{\pi}{m}} \sin m(x-c)\,e^{-\lambda x}dx \\ &= \frac{1}{\lambda}(e^{-\lambda(c+\frac{\pi}{m})}+e^{-\lambda c}) \\ &\quad -\frac{m}{\lambda}\int_c^{c+\frac{\pi}{m}} \sin m(x-c)\,e^{-\lambda x}dx,\end{aligned}$$

we have

$$\widetilde{P}(A) = \frac{km}{\lambda}(e^{-\lambda(c+\frac{\pi}{m})}+e^{-\lambda c}) - \frac{km^2}{\lambda}\widetilde{P}(A).$$

Thus

$$\widetilde{P}(A) = \frac{km}{\lambda+km^2}(e^{-\lambda(c+\frac{\pi}{m})}+e^{-\lambda c}).$$

Example 3.7. Let $A = < \frac{2}{3}, \frac{1}{2}, 0 >$ be generalized trigonometric fuzzy set. Then the exponential fuzzy probability of A with respect to $X \sim E(2)$ is

$$\begin{aligned}\widetilde{P}(A) &= \frac{4}{3}\int_0^{2\pi} \sin\frac{x}{2}\exp(-2x)dx \\ &= 0.1569\end{aligned}$$

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