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HYPERBOLICITY OF CHAIN TRANSITIVE SETS WITH LIMIT SHADOWING

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Abstract. In this paper we show that any chain transitive set of a diffeomorphism on a compact C^∞-manifold which is C^1-stably limit shadowable is hyperbolic. Moreover, it is proved that a locally maximal chain transitive set of a C^1-generic diffeomorphism is hyperbolic if and only if it is limit shadowable.

Transitive sets, homoclinic classes and chain components of a diffeomorphism are natural candidates to replace the hyperbolic basic sets in nonhyperbolic theory of differentiable dynamical systems, and many recent papers explored their "hyperbolic-like" properties (for more details, see [2, 6, 8, 9, 14, 15]).

In this paper we study the chain transitive sets which are limit shadowable. Let us be more precise. Let M be a compact C^∞-manifold, and let Diff(M) be the space of diffeomorphisms of M endowed with the C^1-topology. Denote by d the distance on M induced from a Riemannian metric on the tangent bundle TM. For $\delta > 0$, a sequence $\{x_n\}_{n\in\mathbb{Z}}$ in Λ is called *a δ-limit chain* if

$$\lim_{|n|\to\infty} d(f(x_n), x_{n+1}) = 0 \text{ and } d(f(x_n), x_{n+1}) < \delta$$

for any $n \in \mathbb{Z}$. For a closed f-invariant set $\Lambda \subset M$, we say that f has *the limit shadowing property* in Λ (or Λ is *limit shadowable* for f) if there exists $\delta > 0$ such that for any δ-limit chain $\xi = \{x_n\}_{n\in\mathbb{Z}}$ in Λ there exists $y \in \Lambda$ satisfying

$$d(f^n(y), x_n) \to 0 \text{ as } |n| \to \infty.$$

We also say that the δ-limit chain ξ is limit shadowed by the point y. Note that the limit shadowing property does not imply the shadowing property. For example, let f be a diffeomorphism on the unit circle $\mathbb{S}^1$ with coordinates $x \in [0, 1)$ which has only three fixed points $0, \frac{1}{3}$ and $\frac{2}{3}$ such that 0 is source, $\frac{2}{3}$ is sink and $\frac{1}{3}$ is nonhyperbolic. Then it is clear that f has the limit shadowing property in $\mathbb{S}^1$ but it does not satisfy the shadowing property.

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Definition. Let Λ be an invariant set for $f \in \mathrm{Diff(M)}$. We say that Λ admits a dominated splitting if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E \oplus F$ such that for some $C > 0$ and $0 < \lambda < 1$,

$$\|Df^n|_{E(x)}\| \cdot \|Df^{-n}|_{F(f^n(x))}\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. The set Λ is hyperbolic if the subbundle E is uniformly contracting and the subbundle F is uniformly expanding; i.e., for some $C > 0$ and $0 < \lambda < 1$,

$$\|Df^n|_{E^s(x)}\| \leq C\lambda^n \ \text{ and } \ \|Df^{-n}|_{E^u(f^n(x))}\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$.

Let $P_h(f)$ be the set of all hyperbolic periodic points of f. It is well known that for any $p \in P_h(f)$ with period k, the sets

$$W^s(p) = \{x \in M : f^{kn}(x) \to p \ \text{ as } \ n \to \infty\}, \ \text{ and}$$
$$W^u(p) = \{x \in M : f^{-kn}(x) \to p \ \text{ as } \ n \to \infty\}$$

are C^1-injectively immersed submanifolds of M. A point $x \in W^s(p) \cap W^u(p)$ is called *a homoclinic point* of f associated to p, and it is said to be *a transversal homoclinic point* of f if the above intersection is transversal at x; i.e., $x \in W^s(p) \overline{\pitchfork} W^u(p)$. The closure of the transversal homoclinic points of f associated to the orbit of p is called *the homoclinic class* of f associated to p and denoted by $H_f(p)$. Let q be another hyperbolic periodic point of f. The two points p and q are called *homoclinically related*, and write $p \sim q$ if

$$W^s(p) \overline{\pitchfork} W^u(q) \neq \emptyset \ \text{ and } \ W^u(p) \overline{\pitchfork} W^s(q) \neq \emptyset.$$

By Smale's transverse homoclinic point theorem, $H_f(p)$ coincides with the closure of the set of hyperbolic periodic points q of f such that $p \sim q$. Note that if p is a hyperbolic periodic point of f, then there is a neighborhood U of p and a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$ there exists a unique hyperbolic periodic point p_g of g in U with the same period and the same index as those of p. The point p_g is called *the continuation* of p.

Definition. We say that f has the C^1-stably limit shadowing property in Λ (or Λ is C^1-stably limit shadowable for f) if there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a compact neighborhood U of Λ such that

(1) $\Lambda = \Lambda_f(U) = \bigcap_{n\in\mathbb{Z}} f^n(U)$; i.e., Λ is locally maximal in U,
(2) Λ_g is limit shadowable for $g \in \mathcal{U}(f)$,

where $\Lambda_g = \bigcap_{n\in\mathbb{Z}} g^n(U)$ is the continuation of $\Lambda = \Lambda_f(U)$. In this case, we say that Λ is C^1-stably limit shadowable with respect to U and $\mathcal{U}(f)$.

It is known that any locally maximal hyperbolic set is C^1-stably limit shadowable (see [13]).

For given $\delta > 0$, a sequence $\{x_i\}_{i=a}^b$ $(-\infty \leq a < b \leq \infty)$, is called *a δ-pseudo orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $a \leq i \leq b-1$. A closed f-invariant set

$\Lambda \subset M$ is *chain transitive* if for any two points $x, y \in \Lambda$ and any $\delta > 0$, there is a finite δ-pseudo orbit $\{x_i\}_{i=a}^{b} \subset \Lambda$ of f with $x_a = x$ and $x_b = y$.

In this paper, we first study the hyperbolicity of a chain transitive set by making use of the limit shadowing property under C^1-open condition. More precisely, we have the following theorem.

Theorem A. *A chain transitive set Λ is C^1-stably limit shadowable if and only if it is hyperbolic.*

A subset $\mathcal{R} \subset \mathrm{Diff^r(M)}$ ($\mathrm{r} \geq 1$) is called *residual* if it contains the intersection of a countable family of open and dense subsets of $\mathrm{Diff^r(M)}$. A property "$\mathcal{P}$" is said to be C^r*-generic* if "$\mathcal{P}$" holds for all diffeomorphisms in a residual subset of $\mathrm{Diff^r(M)}$.

Recently, Abdenur and Díaz in [2] obtained a necessary and sufficient condition for a locally maximal transitive set Λ of a C^1-generic diffeomorphism f to be hyperbolic as follow: either Λ is hyperbolic, or there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a neighborhood V of Λ such that every $g \in \mathcal{U}(f)$ does not have the shadowing property on V.

As a second result of the paper, we get the following result for the limit shadowable chain transitive sets.

Theorem B. *There exists a residual subset $\mathcal{R}$ of $\mathrm{Diff}^1(M)$ such that a locally maximal chain transitive set Λ of $f \in \mathcal{R}$ is limit shadowable if and only if it is hyperbolic.*

1. Proof of Theorem A

The next lemma, known as Franks' Lemma, is a simple yet powerful result allowing us to perturb the tangent map along a finite set with an arbitrarily small support.

Lemma 1.1 (Franks' Lemma)**.** *Let $\mathcal{U}(f)$ be any given C^1-neighbor-hood of f. Then there exist $\epsilon > 0$ and a C^1-neighborhood $\mathcal{U}_0(f)$ of $\mathcal{U}(f)$ of f such that for given $g \in \mathcal{U}_0(f)$, a finite set $\{x_1, x_2, \ldots, x_N\}$, a neighborhood U of $\{x_1, x_2, \ldots, x_N\}$ and linear maps $L_i : T_{x_i}M \to T_{g(x_i)}M$ satisfying $\|L_i - D_{x_i}g\| \leq \epsilon$ for all $1 \leq i \leq N$, there exists $\tilde{g} \in \mathcal{U}(f)$ such that $\tilde{g}(x) = g(x)$ if $x \in \{x_1, x_2, \ldots, x_N\} \cup (M \setminus U)$ and $D_{x_i}\tilde{g} = L_i$ for all $1 \leq i \leq N$.*

In this section, we will prove Theorem A by using the technique developed by Mañé in [11]. To use it, we need the following lemma.

Lemma 1.2. *If f is C^1-stably limit shadowable in a chain transitive set Λ with respect to a neighborhood U of Λ and a C^1-neighborhood $\mathcal{U}(f)$ of f, then any periodic point of $g \in \mathcal{U}(f)$ in Λ_g is hyperbolic.*

Proof. Let $\epsilon > 0$ and $\mathcal{U}_0(f) \subset \mathcal{U}(f)$ are given by Lemma 1.1. Suppose that there exists a non-hyperbolic periodic point $q \in \Lambda_g$ for some $g \in \mathcal{U}_0(f)$ (since Λ is locally maximal, reducing $\mathcal{U}(f)$ if necessary, we may assume that q is contained in the interior of U). To simplify the notations, we assume that $g(q) = q$ (other case is similar). Then the use of Lemma 1.1, one may construct a diffeomorphism $g_1 \in \mathcal{U}_0(f)$ C^1-nearby g possessing either

(i) a g_1-invariant normally hyperbolic small arc $\mathcal{I}_q \subset U$ such that $g_1|_{\mathcal{I}_q}^k = id$ for some $k > 0$; or

(ii) a g_1-invariant normally hyperbolic small circle $\mathcal{C}_q \subset U$ with a small diameter and center at q such that $g_1|_{\mathcal{C}_q}$ is conjugated to an irrational rotation.

Since $\mathcal{I}_q$ and $\mathcal{C}_1$ are g_1-invariant, we see that $\mathcal{I}_q \subset \Lambda_g$ and $\mathcal{C}_q \subset \Lambda_g$. Since g_1 has the limit shadowing property on Λ_g, both $g_1^k|_{\mathcal{I}_q}$ and $g_1|_{\mathcal{C}_q}$ must have the limit shadowing property. By the C^1-stablity of limit shadowing, this is a contradiction. This completes the proof of Lemma 1.2. □

Lemma 1.3. *If f has the limit shadowing property in a chain transitive set Λ, then for any $p, q \in \Lambda \cap P_h(f)$,*

$$W^s(p) \cap W^u(q) \neq \emptyset \text{ and } W^u(p) \cap W^s(q) \neq \emptyset.$$

Proof. Let p and q be two periodic points of f in Λ. Take a constant $\delta > 0$ such that every δ-pseudo orbit in Λ is limit shadowed by a point in Λ. Since Λ is chain transitive, there is a δ-pseudo orbit $\{x_0 = p, x_1, \ldots, x_n = q\}$ in Λ. Construct a sequence

$$\xi = \{\ldots, p, p, x_0, x_1, \ldots, x_n, q, q, \ldots\}.$$

Then ξ is a δ-limit chain in Λ. Since Λ is limit shadowable for f, we can choose a point $y \in \Lambda$ such that

$$d(f^n(y), x_n) \to 0 \text{ as } |n| \to \infty.$$

For $\eta > 0$, we can choose $n_1 > 0$ sufficiently large such that

$$f^{-n}(y) \in W^u_\eta(p) \quad \text{and} \quad f^n(y) \in W^s_\eta(q)$$

for all $n \geq n_1$. Therefore

$$y \in f^n(W^u_\eta(p)) \quad \text{and} \quad y \in f^{-n}(W^s_\eta(q)).$$

Thus $y \in W^u(p) \cap W^s(q)$, and so $W^u(p) \cap W^s(q) \neq \emptyset$. Similarly we can show that $W^u(q) \cap W^s(p) \neq \emptyset$. □

Remark 1.4. Using the two lemmas above one can deduce that the index of periodic points in Λ doesn't change. In fact, if there are two periodic points p and q in Λ with different indices, then the same happens for a Kupka-Smale diffeomorphism g sufficiently close to f. On the other hand, by Lemma 1.3, stable and unstable manifolds of p_g and q_q, the continuation of p and q respectively, should intersect each other. This contradicts to g being Kupka-Smale.

By Proposition II.1 in [11] and two lemma above we have the following proposition which plays an essential rule in our proof.

Proposition 1.5. *If a chain transitive set Λ is C^1-stably limit shadowable, then there exist a neighborhood $\mathcal{U}_0(f)$ of f, a constant $0 < \lambda < 1$ and a natural number $m > 0$ such that*

(1) *for any $g \in \mathcal{U}_0(f)$, if $q \in \Lambda_g$ is a periodic point of g with period $\pi(q)$, then*

$$\prod_{i=0}^{\pi(q)-1} \|D_{g^{im}(q)} g^m|_{E^s(g^{im}(q))}\| < \lambda^{\pi(q)} \quad \text{and}$$

$$\prod_{i=0}^{\pi(q)-1} \|D_{g^{-im}(q)} g^{-m}|_{E^s(g^{im}(q))}\| < \lambda^{\pi(q)},$$

(2) *$\overline{Per(g)}$ admits a dominated splitting $E \oplus F$ with $\dim E = index(p)$.*

Before proving the hyperbolicity of Λ, we recall the Mañé's Ergodic Closing Lemma obtained in [11]. Denote by $B_\epsilon(f,x)$ an ϵ-tubular neighborhood of the f-orbit of x; that is,

$$B_\epsilon(f,x) = \{y \in M : d(f^n(x), y) < \epsilon \text{ for some } n \in \mathbb{Z}\}.$$

We say that a point $x \in M$ is *well closable* for $f \in \text{Diff(M)}$ if for any $\epsilon > 0$ there are $g \in \text{Diff(M)}$ with $d_1(f,g) < \epsilon$ and $p \in M$ such that $p \in P(g)$, $g = f$ on $M - B_\epsilon(x,f)$ and $d(f^n(x), g^n(p)) \le \epsilon$ for any $0 \le n \le \pi(p)$ and d_1 is the C^1-metric. Let Σ_f denote the set of well closable points of f. Then we can state the Closing Lemma as follows.

Lemma 1.6 (Mañé Ergodic Closing Lemma, [11])**.** *For any f-invariant probability measure μ on M, we have $\mu(\Sigma_f) = 1$.*

To complete the proof of Theorem A, it is sufficient to show the following.

Theorem 1.7. *The set $\overline{Per(f)}$ is hyperbolic.*

Proof. By Proposition 1.5(2), we know that $\overline{Per(f)}$ admits a dominated splitting $E \oplus F$ with $\dim E = index(p)$. We will show the hyperbolicity of direction E, the other one can be treated similarly. To show this, it is enough to prove that

$$\liminf_{n\to\infty} \|Df^n|_{E(x)}\| = 0$$

for all $x \in \overline{Per(f)}$. Suppose not. Then by Birkhooff's Theorem and Mañé's Ergodic Closing Lemma, we can find a point $z \in \Lambda \cap \Sigma_f$ such that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \log \|D_{f^{im}(z)} f^m|_{E(f^{im}(z))}\| \ge 0.$$

By Proposition 1.5, z can not be a periodic point of f. Let m and $0 < \lambda < 1$ be given by Proposition 1.5, and take $\lambda < \lambda_0 < 1$ and $n_0 > 0$ such that

$$\frac{1}{n} \sum_{i=0}^{n-1} \log \|D_{f^{im}(z)} f^m|_{E(f^{im}(z))}\| \ge \log \lambda_0$$

for $n > n_0$. By Mañé's Ergodic Closing Lemma, we may find $\tilde{g}$ in a connected neighborhood $\mathcal{U}_0(f)$ of f and $\tilde{z} \in \Lambda_{\tilde{g}} \cap P(\tilde{g})$ nearby z such that

$$\lambda_0^{\pi(\tilde{z})} \leq \prod_{i=0}^{\pi(\tilde{z})-1} \|D_{\tilde{g}^{im}(\tilde{z})}\tilde{g}^m|_{E(\tilde{g}^{im}(\tilde{z}))}\|. \tag{1}$$

On the other hand, by Proposition 1.5, we see that

$$\prod_{i=0}^{\pi(\tilde{z})-1} \|D_{\tilde{g}^{im}(\tilde{z})}\tilde{g}^m|_{E(\tilde{g}^{im}(\tilde{z}))}\| < \lambda^{\pi(\tilde{z})}.$$

This contradicts (1), and so E is uniformly contracting under Df. □

2. Proof of Theorem B

We begin the section by a remark assembling the behavior of a Kupka-Smale diffeomorphism with the limit shadowing.

Remark 2.1. As mentioned in the previous section, if f has limit shadowing on Λ, then any two periodic points p and q in Λ have the same index and their stable and unstable manifolds cut each other. If f is a Kupka-Smale diffeomorphism, then one can ensure the intersections are transverse. In other word, in this case $p \sim q$.

Lemma 2.2 ([10, Lemma 2.2]). *There is a residual set $\mathcal{R}_0 \subset \mathrm{Diff}(M)$ such that every $f \in \mathcal{R}_0$ satisfies the following property: For any closed f-invariant set $\Lambda \subset M$, if there are a sequence of diffeomorphisms f_n converging to f and a sequence of hyperbolic periodic orbits P_n of f_n with index k verifying $\lim_{n\to\infty} P_n = \Lambda$, then there is a sequence of hyperbolic periodic orbits Q_n of f with index k such that Λ is the Hausdorff limit of Q_n, where the index of a hyperbolic periodic orbit P is the dimension of the stable manifold of P.*

To complete the proof of Theorem B, we let $\mathcal{R} = \mathcal{R}_0 \cap \mathcal{KS}$, where $\mathcal{KS}$ denotes the set of Kupka-Smale diffeomorphisms. The following proposition is crucial in the proof of the theorem.

Proposition 2.3. *Let $f \in \mathcal{R}$, and let Λ be a limit shadowable chain transitive set of f which is locally maximal. Then there exist constants $m > 0$ and $0 < \lambda < 1$ such that for any periodic point $p \in \Lambda$,*

$$\prod_{i=0}^{\pi(p)-1} \|Df^m|_{E^s(f^{im}(p))}\| < \lambda^{\pi(p)},$$

$$\prod_{i=0}^{\pi(p)-1} \|Df^{-m}|_{E^u f^{-m}(p)}\| < \lambda^{\pi(p)}$$

and

$$\|Df^m|_{E^s(p)}\| \cdot \|Df^{-m}|_{E^u f^m(p)}\| < \lambda^2.$$

Proof. Since $f \in \mathcal{R}_0$, all periodic points in Λ have the same index and Λ is locally maximal, we can choose a C^1-neighborhood $\mathcal{U}(f)$ of f and a neighborhood U of Λ such that every $g \in \mathcal{U}(f)$ has no nonhyperbolic periodic orbit which is contained in U. In fact, if there are non-hyperbolic periodic points, then by using Franks Lemma one can produce periodic points of different index in U for diffeomorphisms sufficiently close to f. Since $f \in \mathcal{R}$, the same holds for f. We arrive at the contradiction by Remark 2.1.

By applying Lemma II.3 in [11], we get constants $K > 0$, $m_0 \in \mathbb{Z}^+$ and $0 < \lambda < 1$ such that for any periodic point $p \in \Lambda$ with $\pi(p) \geq K$,

$$\prod_{i=0}^{\pi(p)-1} \|Df^{m_0}|_{E^s(f^{im_0}(p))}\| < \lambda^{\pi(p)},$$

$$\prod_{i=0}^{\pi(p)-1} \|Df^{-m_0}|_{E^u f^{-m_0}(p)}\| < \lambda^{\pi(p)}$$

and

$$\|Df^{m_0}|_{E^s(p)}\| \cdot \|Df^{-m_0}|_{E^u(f^m(p))}\| < \lambda^2.$$

Let Λ_0 be the set of all periodic points in Λ whose periods are less than K. Since every periodic point of f is hyperbolic, there are only a finite number of periodic points in Λ_0, and so Λ_0 is hyperbolic for f. Let k be a positive integer such that

$$\|Df^{km_0}|_{E^s(x)}\| < \lambda \text{ and } \|Df^{-km_0}|_{E^u(x)}\| < \lambda$$

for all $x \in \Lambda_0$. If we let $m = km_0$, then we know that m and λ are the required constants satisfying Proposition 2.3. □

For any periodic point p of a diffeomorphism f, we can see that μ_p given by

$$\mu_p = \frac{1}{\pi(p)} \sum_{i=0}^{\pi(p)-1} \delta_{f^i(p)}$$

is a f-invariant ergodic probability measure concentrated on M. Finally we will use the following proposition comes from the Mane's ergodic closing lemma in [11] which gives the measure theoretical viewpoint of the approximation by periodic orbits.

Proposition 2.4. *There exists a residual subset $\mathcal{R}_1$ of* $\text{Diff}^1(\text{M})$ *such that for any $f \in \mathcal{R}_1$ and for any f-invariant ergodic probability measure μ of f there is a sequence of hyperobolic periodic points p_n such that*

- $\mu_{p_n} \to \mu$ *in weak** *topology,*
- $\mathcal{O}(p) \to Supp(\mu)$ *in Hausdorff metric.*

Proof of Theorem B. Put $\mathcal{R}_2 = \mathcal{R} \cap \mathcal{R}_1$, and let $f \in \mathcal{R}_2$ has the limit shadowing property in Λ. Now we will prove that the dominated splitting $E \oplus F$ given by Proposition 2.3 is in fact hyperbolic. First we show that E is uniformly contracting under Df. To this end, it suffices by using Lemma I.5 in [12], to prove that

$$\int \log(\|Df^m|_{E(x)}\|)d\mu < 0$$

for every f-invariant ergodic probability measure μ. Since $f \in \mathcal{R}_1$ there exists a sequence $\mathcal{O}(p_n)$ of periodic orbits of f with $\mathcal{O}(p_n) \to \Lambda$ in the Hausdorff topology and periodic measures μ_{p_n} concentrated on the orbit of p_n converges to μ_0 in weak*-topology. By the local maximality of Λ, for sufficiently large n, the periodic orbits $\mathcal{O}(p_n)$ are contained in Λ. On the other hand, if we apply Proposition 2.3, then we have

$$\int \log(\|Df^m|_{E(x)}\|)d\mu_n < \log\lambda$$

for sufficiently large n. Since μ_n converges to μ_0 in the weak* topology, we have

$$\int \log(\|Df^m|_{E(x)}\|)d\mu_n \to \int \log(\|Df^m|_{E(x)}\|)d\mu_0$$

as $n \to \infty$. Hence we get $\int \log(\|Df^m|_{E(x)}\|)d\mu_0 < 0$. The contradiction proves that Df is contracting on E. Similarly, one can show that Df is expanding on F. □

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