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A Fuzzy Multi-Objective Linear Programming Model: A Case Study of an LPG Distribution Network

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ABSTRACT

Supply chain management is a subject that has an increasing importance due to the developments in the global markets and technology. In this paper, a fuzzy multi-objective linear programming model is developed for the supply chain of a company dealing with procurement, storage, filling, and distribution of liquefied petroleum gas (LPG) in Turkey. The model intends to determine the quantities of LPG to be procured, stored, filled to cylinders, and transported between the plants and demand centers for six planning periods. In this model, which aims to minimize both total costs (sum of procurement, storage, filling, and transportation costs) and total transportation distances, demand quantities of the main demand centers and decision maker's aspiration levels about objective functions are fuzzy. After comparing the results obtained from the model with those obtained by using different methods, it is concluded that the proposed method can be applied to real world problems practically and it may be used in this type of problems in order to generate an efficient solution.

Keywords: Supply Chain Management, Aggregate Planning, Fuzzy Multi-Objective Linear Programming, Transportation

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1. INTRODUCTION

A supply chain is composed of many subsystems and it contains much fuzziness due to its integrated structure and human factor. Along a supply chain, there exist different sources of fuzziness, such as random events, subjective aspiration levels of decision makers about objectives, lacking data or uncertainty of available data. For every plant in a supply chain, both supplies of previous plants and demands of posterior plants are ambiguous (Petrovic *et al*., 1999). Procurement of raw materials and deliveries between plants are other sources of fuzziness in the supply chains. The quantity and quality of raw and intermediate materials procured from a supplier may be different from those that are required. Most of the supply chain models developed in the literature either ignore the uncertainties that often exist in the real world problems or tend to take them into account by using probabilistic approaches (Petrovic *et al*., 1999).

Fuzzy set theory, which had a broad range of application in many different disciplines, such as operations research, management science, control theory, and artificial intelligence after Zadeh's suggestion in 1965, is an appropriate and useful tool for defining and considering uncertainties in modeling (Zadeh, 1965). Fuzzy approaches are used as an effective tool, especially in situations in which usage of standard stochastic methods is not appropriate due to the lacking data, uncertain data or even absence of any data. As a result of this, the number of researchers using fuzzy mathematical models in modeling supply chains has increased in the last two decades.

Besides those mentioned above, different and sometimes conflicting objectives must be considered together in modeling a supply chain, which causes most of the developed models to be multi-objective. For example, while minimizing production costs, storage costs must also be taken into account or distribution costs should not be minimized without considering delivery times and conditions. As subsystems composing a supply chain are rigidly interrelated, they should be considered as a whole in order to minimize the total costs along a supply chain. However, the articles that aggregate the production and distribution processes in the supply chain are few while most articles are considering them separately (Pundoor, 2005; Mokashi and Kokossis, 2003; Xie *et al.*, 2006).

2. LITERATURE REVIEW

For years, there had been a variety of work considering supply chain processes as separate processes in the literature. However, it is discerned that the supply chain performance, design and analysis have been handled as a whole for about last two decades (Beamon, 1998; Kim *et al.*, 2006; Lee and Kim, 2006). The new approach, which is based on the integration of the decisions about the different functions (procurement, production planning, inventory management, distribution, allocation, etc.) under just one optimization model, has drawn the attention of the researchers for the last two decades. Since this approach deals with many different functions, there are various studies in this field and they cannot be classified easily. After analyzing the classification techniques suggested by different resear-chers, it can be easily seen that, the criteria regarded by them in order to classify the studies are too different from each other (Ca-

Table 1. Reviewed articles

Article	Structure of supply chain	Processes included	Planning horizon	Number of objectives	Model
Chanas and Kuchta (1998)	Two-stage	Distribution	Single period	Single	Fuzzy integer linear programming
Hussein (1998)	Two-stage	Distribution	Single period	Multi	Linear programming
Liand Lai (2000)	Two-stage	Distribution	Single period	Multi	Fuzzy linear programming
Verma *et al*. (1997)	Two-stage	Distribution	Single period	Multi	Fuzzy linear/nonlinear programming
El-Wahed (2001)	Two-stage	Distribution	Single period	Multi	Fuzzy linear programming
Liang (2006)	Two-stage	Distribution	Single period	Multi	Interactive fuzzy linear programming
El-Wahed and Lee (2006)	Two-stage	Distribution	Single period	Multi	Fuzzy goal programming
Wang *et al*. (2004)	Two-stage	Distribution	Multi-period	Single	Linear programming
Shih (1997)	Two-stage	Distribution	Multi-period	Single	Mixed integer linear programming
Wang and Liang (2005)	Two-stage	Production, distribution	Multi-period	Multi	Fuzzy linear programming
Bylka (1999)	Two-stage	Production, storage, distribution	Multi-period	Single	Dynamic programming
Eksioglu *et al*. (2007)	Two-stage	Production, storage, distribution	Multi-period	Single	Dynamic programming
Ozdamar and Yazgac (1999)	Two-stage	Production, storage, distribution	Multi-period	Single	Mixed integer linear programming
Bilgen and Ozkarahan (2007)	Two-stage	Production, storage, distribution	Multi-period	Single	Mixed integer linear programming
Gen and Syarif (2005)	Two-stage	Production, storage, distribution	Multi-period	Single	Linear programming
Kanyalkar and Adil (2005)	Two-stage	Production, storage, distribution	Multi-period	Single	Linear programming
Garcia *et al*. (2004)	Two-stage	Procurement, production, distribution	Multi-period	Single.	Integer linear programming
Xie *et al*. (2006)	Serial	Production, storage	Multi-period	Single	Fuzzy linear

par *et al*., 2003; Min and Zhou, 2002; Sar-miento and Nagi, 1999; Mishi *et al*., 2009).

In order not to lose the essence of the study, the literature survey is restricted regarding some criteria. While doing this, the trend of the approach adopted moved from the general to the specific. For this aim, at first, the studies improving the mathematical programming models about integrated production-distribution planning are surveyed mostly (Altiparmak *et al*., 2006). Then, more specifically, the studies that are using the fuzzy mathematical programming mo-dels are investigated. Lastly, by taking into consideration the area of activity of the business in the case study, the studies that have an integrated approach on the production-distribution processes of petrol and petroleum products are surveyed. The literature survey is also restricted to the studies of the last decade. The papers reviewed in the literature survey by considering these priorities are given in the Table 1.

3. A FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING MODEL FOR A LPG DISTRIBUTION NETWORK

3.1 Structure of the System and the Definition of the Problem

The application of the study was carried out in a company which performs sourcing, storage, filling, and distribution of liquefied petroleum gas (LPG) in Turkey. The system under consideration consists of 6 main supply plants, 6 filling plants, and 82 main demand centers. LPG supplied from domestic refineries or foreign markets is transported to the main supply plants by pipelines or sea and land tanker fleets. After transportation, the LPG can be stored for later periods, or it can be transported to the other main supply plants and filling plants by tankers, or it can be filled to cylinders and distributed to the demand centers as bottled gas. The LPG which is transported from the main supply plants to the filling plants is filled to cylinders or stored for later periods.

In the supply chain under consideration, for the 6 planning periods, the amount of the LPG which will be supplied, stored, filled, and distributed in each period by each plant is determined. Information of between which of the main supply plants, filling plants and main demand centers transportation feasibility, as the data is taken from the company. the Together with these, the company informs that the amount of LPG supplied from domestic refineries or foreign markets by the sixth main supply plant during the six periods should not exceed 10,000 tons. In order to solve the problem, a fuzzy multi-objective linear programming model is built based on the procedure used by Liang (2006). In the model which aims to minimize both the total costs (total of the procurement, filling, storage and transportation costs) and the total transportation distances, the demands of the main demand centers and the aspiration levels of the decision maker concerning the objective functions are fuzzy.

3.2 Method

In the solution of the fuzzy multi-objective supply chain planning model constructed for the system mentioned above, the method used in the study of Wang and Liang (2004) is applied, except for the interactive decision process which is the last step of this method. In this method, the fuzzy multi-objective linear programming model can be converted to an equivalent linear programming model by using fuzzy goal programming method of Hannan (1981) and the fuzzy decision making method of Bellman and Zadeh (1970) together.

This solution procedure is formed of the following steps (Liang, 2006).

1) Original fuzzy multi-objective linear programming model is formed.
2) Considering the given minimum acceptable membership level (α), the constraints which have fuzzy right hand side values are converted to non-fuzzy constraints by using the weighted mean method.
3) For each objective function (z_g, $g = 1, 2$) the membership values ($\mu(z_g)$) concerning some objective function values are determined.
4) For every objective function, piecewise linear membership functions are sketched by using the points (z_g, $\mu(z_g)$).
5) For every membership function ($\mu(z_g)$) piecewise linear equations are constructed.
6) Original fuzzy multi-objective supply chain model is converted to an equivalent classical linear programming model after including the auxiliary variable 'L' to the model.

In the procedure, piecewise linear membership functions are used to represent fuzzy objectives and minimum operator is used to aggregate fuzzy sets. Derivation of the method mentioned is given in the Appendix 1 in detail.

3.3 Assumptions

The fuzzy multi-objective linear programming model is based on the assumptions listed below:

1) Objective functions are fuzzy.
2) Objective functions and all constraints are linear.
3) Unit costs and distances between plants are definite and constant over the planning horizon.
4) The distribution costs and the total transportation distances are directly proportional to the number of units transported.
5) Triangular fuzzy numbers are adopted to represent the estimated demand.
6) Fuzzy sets are represented by the piecewise linear membership functions.
7) The minimum operator is used to aggregate fuzzy sets.

The first assumption is about the subjective differences between the opinions, evaluations, and preferences of the decision makers in the real life. The second, the third and the fourth assumptions are essential for the model constructed to be a linear programming model. The fifth assumption states that, triangular fuzzy numbers are used when explaining the estimated demand. In many studies, triangular fuzzy numbers are used because they make the calculations easier and decrease the data need (Chen and Lee, 2004; Kikuchi, 2000; Liang, 2006; Shih, 1999). The sixth and the seventh assumptions are for the conversion of the fuzzy multi-objective problem to an equivalent linear programming structure (Liang, 2006).

3.4 Construction of the Fuzzy Multi-Objective Linear Programming Model

The parameters which are required for the model and obtained from the company are listed below:

1) The most pessimistic, possible, and optimistic quantities of demands of the main demand centers.
2) Tanker filling costs, tanker filling capacities and LPG purchase costs of the main supply centers.
3) Cylinder filling costs, cylinder filling capacities, safety inventory levels, maximum inventory levels and the storage costs of the main supply centers and filling centers.
4) The transportation distances between the plants and the demand centers.
5) The information of between which plants or from which plants to which demand centers the transportation is feasible.

The symbols below are used in the model constructed:

• *Objective Functions*

z_g — g^{th} objective function
L — total aspiration level of decision maker about fuzzy objectives

• *Indices*

i — main supply plants
j — filling plants
k — main demand centers
t — planning periods
m — all main supply plants and filling plants ($i+j$)

• *Decision Variables:*

X_{imt} — quantity of LPG to be transported from main supply plant i to plant m in period t
Y_{jkt} — quantity of LPG to be transported from filling plant j to main demand center k in period t
Z_{ikt} — quantity of LPG to be transported from main supply plant i to main demand center k in period t
I_{it} — inventory level of main supply plant i at the end of period t
N_{jt} — inventory level of filling plant j at the end of period t
Q_{it} — quantity of LPG to be procured from refineries by main supply plant i in period t
DX_{ijt} — quantity of LPG to be transported from main supply plant i to filling plant j in period t
AX_{mt} — total quantity of LPG to be transported from all main supply plants to plant m in period t
BX_{it} — total quantity of LPG to be transported from all other main supply plants to main supply plant i in period t

• *Parameters:*

c_i — purchase cost of a unit ton of LPG for the main supply plant i
p_i — filling cost of a unit ton of LPG to the tankers at the main supply plant i
r_i — filling cost of a unit ton of LPG to the LPG cylinders at the main supply plant i
dc_j — filling cost of a unit ton of LPG to the LPG cylinders at the filling plant j
h_i — inventory carrying cost of a unit ton of LPG for one period at the main supply plant i
lc_j — inventory carrying cost of a unit ton of LPG for one period at the filling plant j
ec_{im} — transportation cost of a unit ton of LPG delivered in tankers from main supply plant i to plant m
fc_{ik} — transportation cost of a unit ton of LPG delivered in LPG cylinders from main supply plant i to main demand center k
gc_{jk} — transportation cost of a unit ton of LPG delivered in LPG cylinders from filling plant j to main demand center k
AT_{imt} — 0–1 variable which takes the value of 1 if transportation from main supply plant i to plant m is possible in period t and the value of 0 if not
DM_{jkt} — 0–1 variable which takes the value of 1 if transportation from filling plant j to main demand center k is possible in period t and the value of 0 if not
IM_{ikt} — 0–1 variable which takes the value of 1 if transportation from main supply plant i to main demand center k is possible in period t and the value of 0 if not
$mesx_{im}$ — distance between the main supply plant i and the plant m
$mesy_{jk}$ — distance between the filling plant j and the main demand center k
$mesz_{ik}$ — distance between the main supply plant i and the main demand center k
G_i — maximum inventory capacity of main supply plant i
L_j — maximum inventory capacity of filling plant j
F_i — safety inventory level of main supply plant i
M_j — safety inventory level of filling plant j
S_i — tanker filling capacity of main supply plant i per one period

W_i cylinder filling capacity of main supply plant i per one period

U_j cylinder filling capacity of filling plant j per one period

d_{ge}^{+} positive deviation variables related to objective function g

d_{ge}^{-} negative deviation variables related to objective function g

The fuzzy multi-objective linear programming model constructed in order to determine the values of the decision variables for six periods (months) is as follows:

$$Min\ Z_1 \cong \sum_{i=1}^{\mathrm{i}}\left(c_i\sum_{t=1}^{T}Q_{it}\right)+\sum_{t=1}^{T}\left[\sum_{i=1}^{\mathrm{i}}\left(p_i\sum_{m=1}^{M}X_{imt}\right)+\sum_{i=1}^{\mathrm{i}}\left(r_i\sum_{k=1}^{K}Z_{ikt}\right)+\sum_{j=1}^{J}\left(dc_j\sum_{k=1}^{K}Y_{jkt}\right)\right]+\left[\sum_{i=1}^{T}\sum_{m=1}^{M}\left(ec_{im}\sum_{t=1}^{T}X_{imt}\right)+\sum_{i=1}^{\mathrm{i}}\sum_{k=1}^{K}\left(fc_{ik}\sum_{t=1}^{T}Z_{ikt}\right)+\sum_{j=1}^{J}\sum_{k=1}^{K}\left(gc_{jk}\sum_{t=1}^{T}Y_{jkt}\right)\right]+\sum_{i=1}^{\mathrm{i}}\sum_{j=1}^{J}\left(Ic_j\sum_{t=1}^{T}N_{jt}\right) \quad (1)$$

$$Min\ Z_2 \cong \sum_{i=1}^{\mathrm{i}}\sum_{m=1}^{M}\left[(mesx_{im})\sum_{t=1}^{T}X_{imt}\right]+\sum_{j=1}^{J}\sum_{k=1}^{K}\left[(mesy_{jk})\sum_{t=1}^{T}Y_{jkt}\right]+\sum_{i=1}^{\mathrm{i}}\sum_{k=1}^{K}\left[(mesz_{ik})\sum_{t=1}^{T}Z_{ikt}\right] \quad (2)$$

S.T.

$$\sum_{t=1}^{T}Q_{it}\le 10{,}000 \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (3)$$

$$\sum_{m=1}^{M}X_{imt}\le S_i \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (4)$$

$$\sum_{k=1}^{K}Z_{ikt}\le W_i \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (5)$$

$$\sum_{k=1}^{K}Y_{ikt}\le U_j \quad (j=1,2,\cdots,J)\quad (t=1,2,\cdots,T) \quad (6)$$

$$\sum_{i=1}^{\mathrm{i}}Z_{ikt}+\sum_{j=1}^{\mathrm{i}}Y_{ikt}=\tilde{D}_{kt} \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (7)$$

$$\sum_{i=1}^{\mathrm{i}}X_{imt}\le AX_m \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (8)$$

$$AX_{mt}=BX_{1t} \quad (9)$$

$$I_{it}=I_{i,t-1}+Q_{it}+BX_{it}-\sum_{m=1}^{M}X_{imt}-\sum_{k=1}^{K}Z_{ikt} \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (10)$$

$$X_{imt}=DX_{ijt} \quad (11)$$

$$N_{jt}=N_{j,t-1}+\sum_{i=1}^{\mathrm{i}}DX_{ijt}-\sum_{k=1}^{K}Y_{jkt} \quad (j=1,2,\cdots,J)\quad (t=1,2,\cdots,T) \quad (12)$$

$$Z_{imt}\le (AT_{imt})(X_{imt}) \quad (i=1,2,\cdots,\mathrm{i})\ (i=1,2,\cdots,i)\ (t=1,2,\cdots,T) \quad (13)$$

$$Y_{jkt}\le (DM_{jkt})(Y_{jkt}) \quad (j=1,2,\cdots,J)\ (k=1,2,\cdots,K)\ (t=1,2,\cdots,T) \quad (14)$$

$$Z_{ikt}\le (IM_{ikt})(Z_{ikt}) \quad (i=1,2,\cdots,\mathrm{i})\ (k=1,2,\cdots,K)\ (t=1,2,\cdots,T) \quad (15)$$

$$F_i\le I_{it}\le G_i \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (16)$$

$$M_j\le N_{jt}\le L_j \quad (i=1,2,\cdots,\mathrm{i})\quad (t=1,2,\cdots,T) \quad (17)$$

$$I_{i0}=0 \quad (18)$$

$$N_{i0}=0 \quad (19)$$

$$X_{imt},Y_{jkt},Z_{ikt},I_{it},N_{jt},Q_{it}\ge 0 \quad (20)$$

The first objective function in Eq. (1) minimizes the total costs (the total of the procurement, filling, storagee, and transportation costs) and the second objective function in Eq. (2) minimizes the total transportation distances. The symbol '$\cong$' represents the fuzziness of the aspiration levels of the decision makers related to these objective functions. The constraint (3) is related to the total amount of the LPG which the sixth main supply center can procure from the domestic refineries or foreign markets. The constraints about the periodic tanker filling capacities and the cylinder filling capacities of the main supply centers are stated in inequations (4) and (5), respectively. Restriction of the periodic cylinder filling capacities of the filling plants is stated in constraint (6). Constraint (7) is about the meeting of the demand. Constraint (10) ensure the inventory balance for the main supply centers and the filling plants respectively. The constraints (8), (11), and (12) are for the calculation of the intermediate values which are required in expressing the inventory balances and also for the assignment of some values to other new variables. The expressions (13)–(15) make the transportation amount '0' if the transportation between the related plants is not possible. Constraints (16) and (17) assure that inventory levels of the main supply plants and filling plants are between the related safety inventory level and the maximum inventory level. Eqs. (18) and (19) state that there is no initial inventory at the plants and the constraint (20) ensures that none of the decision variables take negative values.

3.5 Removing the Fuzziness in the Fuzzy Restrictions

In this study, triangular fuzzy numbers are adopted to state the fuzzy demand quantities ($\tilde{D}_{kt}$) of the demand centers and weighted mean method is adopted to convert these numbers to precise values. If the most pessimistic value (D_{kt}^{p}), the most possible value (D_{kt}^{m}), the most optimistic value (D_{k}^{o}) for $\tilde{D}_{kt}$ and the lowest acceptable membership level (α) is given, then the crisp statement of the fifth constraint will be as follows:

$$\sum_{i=1}^{I}Z_{ikt}+\sum_{j=1}^{J}Y_{jkt}=w_1D_{kt,\alpha}^{p}+w_2D_{kt,\alpha}^{m}+w_3D_{kt,\alpha}^{o} \quad (\mathrm{k}=1,2,\cdots,\mathrm{K})\ (\mathrm{t}=1,2,\cdots,\mathrm{T}) \quad (31)$$

w_1, w_2, and w_3 in Eq. (31) represent the weights of the most pessimistic, the most possible, and the most optimistic values of the fuzzy demand quantities. These are subjective values that have a total of ‘1’ and depend on the experience and the knowledge of the decision maker. In some studies in the literature, the same values of weights and ‘α’ are used to remove the fuzziness of the fuzzy constraints (Lai and Hwang, 1992; Tanaka, 1984; Wang *et al*., 2004). In this study, weights and ‘α’ value are determined as $w_1 = w_3 = 1/6$, $w_2 = 4/6$, and $\alpha = 0.5$ for the fifth constraint which is a fuzzy one. Since the most possible values for the demand quantities are more important than the extreme values, the highest weight is given to the most possible value. On the other hand, since the demand quantities take the most optimistic and pessimistic values rarely, the weight given to these values is rather low (Liang, 2006).

3.6 Construction of the Membership Functions Related to the Objective Functions

The formulation of membership functions for each objective function (z_g, g = 1, 2) on the value of the objective function a few degrees of membership ($\mu(z_g)$) is determined. Section 3.4 of the model, these values are determined for the objective function and degree of membership are given in Table 2.

The piecewise linear membership functions which are sketched for the objective functions by using the points (z_g, $\mu(z_g)$) given in Table 2 are shown in Figure 1(a) and (b).

As seen in Table 2 and Figure 1(a)–(b), the decision maker definitely does not want a higher total cost than 375,000,000 currency. The decision maker will be completely satisfied when the total cost is lower than 150,000,000 currency. The satisfaction level of the decision maker will be 80% when the total cost is 225,000,000 currency, and 50% when it is 300,000,000 currency. Similar comments can be made for the second objective function.

3.7 Construction of the Piecewise Linear Equations for the Membership Functions

Hannan’s approach (1981) is taken as a basis while constructing piecewise linear equations for the membership functions (see step 3 in Appendix 1). By using this approach, the membership functions which are sketched in Figure 1 can be expressed as follows (Tanaka, 1984).

$$\mu_{Z_1} = \begin{cases} 1 & z_1 > 150,000,000 \\ \left\{ \begin{array}{l} -1,33333\times10^{-9}\left|z_1 - 300,000,000\right| \\ -6,66667\times10^{-10}\left|z_1 - 225,000,000\right| \\ -4.66667\times10^{-9} z_1 + 1.95 \end{array} \right\} & \\ & 150,000,000 < z_1 \le 375,000,000 \\ 0 & z_1 > 375,000,000 \end{cases}$$

$$\mu_{Z_2} = \begin{cases} 1 & z_2 > 900,000,000 \\ \left\{ \begin{array}{l} -1,66667\times10^{-9}\left|z_2 - 150,000,000\right| \\ -0.5\times10^{-8}\left|z_2 - 120,000,000\right| - 0.1\times10^{-7} z_2 + 2.15 \end{array} \right\} & \\ & 900,000,000 < z_2 \le 185,000,000 \\ 0 & z_2 > 180,000,000 \end{cases}$$

Table 2. The values determined for the membership functions related to the objective functions

z_1	< 150,000,000	150,000,000	225,000,000	300,000,000	375,000,000	> 375,000,000
$\mu(z_1)$	1	1	0.8	0.5	0	0
z_2	< 90,000,000	90,000,000	120,000,000	150,000,000	180,000,000	> 180,000,000
$\mu(z_2)$	1	1	0.9	0.5	0	0

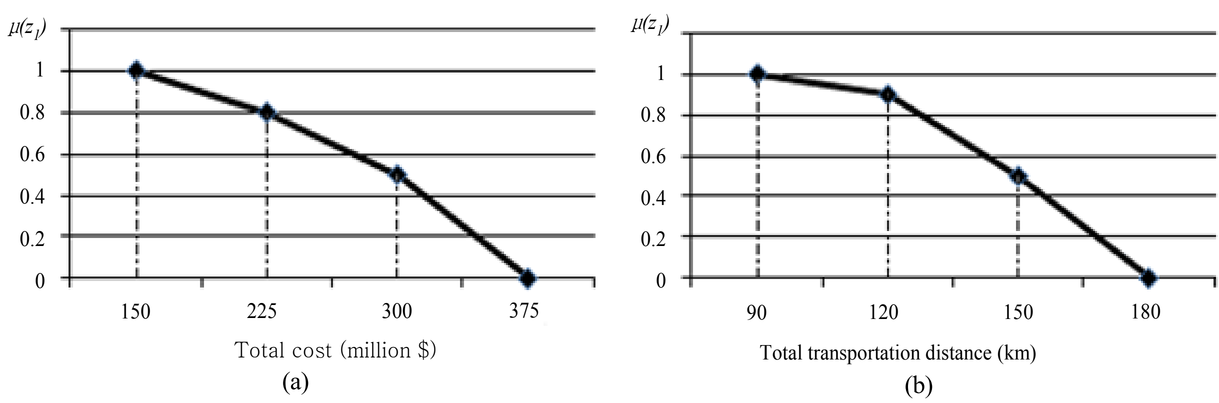

Figure 1. The graph of the piecewise linear membership function ($\mu(z_1)$) related to the first objective function (z_1).

3.8 Getting an Equivalent Linear Programming Model

The existing model is converted to an equivalent linear programming model by including the deviation variables (d_{ge}^-, d_{ge}^+) and the auxiliary variable L to the model and by using the minimum operator to aggregate the fuzzy sets. The 'L' here is defined as the total satisfaction level of the decision maker about all the fuzzy objectives (Liang, 2006). The model which is given below is equivalent to the model in Section 3.4

Max L
s.T.

$$L \le -1{,}3333\times10^{-9}(d_{11}^+ + d_{11}^-) - 6{,}66667\times10^{-10}(d_{12}^+ + d_{12}^-)$$
$$-4.66667\times10^{-9}\sum_{t=1}^{6}\left\{\left(c_i\sum_{t=1}^{6}Q_{it}\right)\sum_{t=1}^{6}\left[\sum_{i=1}^{6}\left(p_i\sum_{m=1}^{12}X_{imt}\right)+\sum_{i=1}^{6}\left(r_i\sum_{k=1}^{82}Z_{ikt}\right)+\sum_{j=1}^{6}\left(dc_i\sum_{k=1}^{82}Y_{jkt}\right)\right]\right.$$
$$+\left[\sum_{i=1}^{6}\sum_{m=1}^{12}\left(ec_{im}\sum_{t=1}^{6}X_{imt}\right)+\sum_{i=1}^{6}\sum_{k=1}^{82}\left(fc_{im}\sum_{t=1}^{6}Z_{ikt}\right)+\sum_{j=1}^{6}\sum_{k=1}^{82}\left(gc_{jk}\sum_{t=1}^{6}Y_{jkt}\right)\right]$$
$$\left.+\sum_{i=1}^{6}\left(h_i\sum_{t=1}^{6}I_{it}\right)+\sum_{j=1}^{6}\left(Ic_j\sum_{t=1}^{6}N_{jt}\right)+1.95\right\}$$

$$L \le -1{,}66667\times10^{-9}(d_{21}^+ + d_{21}^-) - 5\times10^{-9}(d_{22}^+ + d_{22}^-)$$
$$-1\times10^{-8}\left\{\sum_{i=1}^{6}\sum_{m=1}^{12}\left[(mesx_{im})\sum_{t=1}^{6}X_{imt}\right]+\sum_{j=1}^{6}\sum_{k=1}^{82}\left[(mesy_{jk})\sum_{t=1}^{6}Y_{jkt}\right]+\sum_{i=1}^{6}\sum_{k=1}^{82}\left[(mesz_{ik})\sum_{t=1}^{6}Z_{ikt}\right]+2.15\right\}$$

$$\left\{\sum_{t=1}^{6}\left(c_i\sum_{t=1}^{6}Q_{it}\right)+\sum_{t=1}^{6}\left[\sum_{i=1}^{6}\left(p_i\sum_{m=1}^{12}X_{imt}\right)+\sum_{i=1}^{6}\left(r_i\sum_{i=1}^{82}Z_{ikt}\right)+\sum_{j=1}^{6}\left(dc_j\sum_{k=1}^{82}Y_{jkt}\right)\right]\right.$$
$$+\left[\sum_{i=1}^{6}\sum_{m=1}^{12}\left(ec_{im}\sum_{t=1}^{6}X_{imt}\right)+\sum_{i=1}^{6}\sum_{k=1}^{82}\left(fc_{ik}\sum_{t=1}^{6}Z_{ikt}\right)+\sum_{j=1}^{6}\sum_{k=1}^{82}\left(gc_{jk}\sum_{t=1}^{6}Y_{jkt}\right)\right]$$
$$\left.+\sum_{i=1}^{6}\left(h_i\sum_{t=1}^{6}I_{it}\right)+\sum_{j=1}^{6}\left(IC_j\sum_{t=1}^{6}N_{jt}\right)\right\}-d_{11}^+ + d_{11}^-$$
$$=300{,}000{,}000$$

$$\left\{\sum_{i=1}^{6}\left(c_i\sum_{t=1}^{6}Q_{it}\right)+\sum_{t=1}^{6}\left[\sum_{i=1}^{6}\left(p_i\sum_{m=1}^{12}X_{imt}\right)+\sum_{i=1}^{6}\left(r_i\sum_{k=1}^{82}Z_{ikt}\right)+\sum_{j=1}^{6}\left(dc_j\sum_{k=1}^{82}Y_{jkt}\right)\right]\right.$$
$$+\left[\sum_{i=1}^{6}\sum_{m=1}^{12}\left(ec_{im}\sum_{t=1}^{6}X_{imt}\right)+\sum_{i=1}^{6}\sum_{k=1}^{82}\left(fc_{ik}\sum_{t=1}^{6}Z_{ikt}\right)+\sum_{j=1}^{6}\sum_{k=1}^{82}\left(gc_{jk}\sum_{t=1}^{6}Y_{jkt}\right)\right]$$
$$\left.+\sum_{i=1}^{6}\left(h_i\sum_{t=1}^{6}I_{it}\right)+\sum_{j=1}^{6}\left(IC_j\sum_{t=1}^{6}N_{jt}\right)\right\}-d_{12}^+ + d_{12}^-$$
$$=225{,}000{,}000$$

$$\left\{\sum_{i=1}^{6}\sum_{m=1}^{12}\left[(mesx_{im})\sum_{t=1}^{6}X_{imt}\right]+\sum_{j=1}^{6}\sum_{k=1}^{82}\left[(mesy_{jk})\sum_{t=1}^{6}Y_{jkt}\right]+\sum_{i=1}^{6}\sum_{k=1}^{82}\left[(mesz_{ik})\sum_{t=1}^{6}Z_{ikt}\right]\right\}-d_{21}^+ + d_{21}^- = 150{,}000{,}000$$

$$\left\{\sum_{i=1}^{6}\sum_{m=1}^{12}\left[(mesx_{im})\sum_{t=1}^{6}X_{imt}\right]+\sum_{j=1}^{6}\sum_{k=1}^{82}\left[(mesy_{jk})\sum_{t=1}^{6}Y_{jkt}\right]+\sum_{i=1}^{6}\sum_{k=1}^{82}\left[(mesz_{ik})\sum_{t=1}^{6}Z_{ikt}\right]\right\}-d_{22}^+ + d_{22}^- = 120{,}000{,}000$$

$$\sum_{t=1}^{6}Q_{6t}\le 10{,}000$$

$$\sum_{m=1}^{12}X_{imt}\le S_i \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$

$$\sum_{k=1}^{82}Z_{ikt}\le W_i \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$

$$\sum_{k=1}^{82}Y_{jkt}\le U_j \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$

$$\sum_{i=1}^{6}Z_{ikt}+\sum_{j=1}^{6}Y_{ikt}=w_1D_{kt}^p+w_2D_{kt}^m+w_3D_{kt}^o \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$

$$\sum_{i=1}^{6}X_{imt}\le AX_{mt} \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$

$AX_{1t}=BX_{1t}$
$AX_{2t}=BX_{2t}$
$AX_{3t}=BX_{3t}$
$AX_{4t}=BX_{4t}$
$AX_{5t}=BX_{5t}$
$AX_{6t}=BX_{6t}$

$$I_{it}=I_{i,t-1}+Q_{it}+BX_{it}-\sum_{m=1}^{12}X_{imt}-\sum_{k=1}^{82}Z_{ikt} \qquad (i=1,2,\cdots,6)\quad(t=1,2,\cdots,6)$$

$X_{i7t}=DX_{i1t}$
$X_{i8t}=DX_{i2t}$
$X_{i9t}=DX_{i3t}$
$X_{i10t}=DX_{i4t}$
$X_{i11t}=DX_{i5t}$
$X_{i12t}=DX_{i6t}$

$$N_{jt}=N_{j,t-1}+\sum_{i=1}^{6}DX_{ijt}-\sum_{k=1}^{82}Y_{jkt} \qquad (i=1,2,\cdots,6)\quad(t=1,2,\cdots,6)$$

$$X_{imt}\le (AT_{imt})(X_{imt})$$

$$(i=1,2,\cdots,6)\ (m=1,2,\cdots,12)\ (t=1,2,\cdots,6)$$
$$Y_{jkt} \le (DM_{jkt})(Y_{jkt})$$
$$(i=1,2,\cdots,6)\ (k=1,2,\cdots,82)\ (t=1,2,\cdots,6)$$
$$Z_{ikt} \le (IM_{ikt})(Z_{ikt})$$
$$(i=1,2,\cdots,6)\ (k=1,2,\cdots,82)\ (t=1,2,\cdots,6)$$
$$F_i \le I_{it} \le G_i \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$
$$M_j \le N_{jt} \le L_j \qquad (i=1,2,\cdots,6)\ (t=1,2,\cdots,6)$$
$$I_{i0}=0$$
$$N_{i0}=0$$
$$X_{imt}, Y_{jkt}, Z_{ikt}, I_{it}, N_{jt}, Q_{it},$$
$$d_{11}^{+}, d_{11}^{-}, d_{12}^{+}, d_{12}^{-}, d_{21}^{+}, d_{21}^{-}, d_{22}^{+}, d_{22}^{-} \ge 0 \qquad (32)$$

3.9 The solution of the Model and the Evaluations

The model is solved by using LINGO 8.0 package. The total cost is obtained as 168,990,400 currency and the total transportation distance is obtained as 98,236,740 km. The satisfaction level of the decision maker on this result is obtained as 94.9%.

The procedure used in this study, in which piecewise linear membership functions and minimum operator are used, is a method that generates efficient solution (Hannan, 1981). It is explained by Zimmermann (1978) why the maximization of the solution always gives efficient solution in the cases where minimum operator is used. The problem considered in the study is also solved by using the Zimmermann's method, goal programming, traditional single objective linear programming method (for each objective function, respectively) and the results are compared (see Table 3). In the case where the objective functions and demand quantities are considered fuzzily and the solution is obtained by Zimmermann's method, the total cost is acquired as 169,001,600 cu and the total transportation distance is acquired as 97,600,640 km. The satisfaction level of the decision maker on this result is obtained as 91.5%. While solving the problem by using goal programming and linear programming, demand quantities, fuzziness of which are removed by using the weighted average method, are assumed to be the precise demand quantities. In addition to the solution produced by goal programming, in which objective functions are not considered fuzzily, the solutions obtained by using the conventional linear programming method for each objective function respectively are also analyzed. If the table is investigated, it is seen that the efficient solution produced by the suggested method gives better results simultaneously for both functions in comparison with the other methods and can easily be applied to real life problems.

4. SUMMARY

The supply chains in the real life operate in uncertain environments. The fact that the relations between the elements of the supply chain depend on the operations implemented by people is one of the main reasons of uncertainty in the supply chain systems. Besides this, the integrated structure of the supply chain increases the number of real data used in the system, hence the complexity of the system substantially. The increment of the complexity of the system causes a decrease in the ability of individuals in explaining the behavior of the system certainly and meaningfully.

In the conventional mathematical programming models, even the system considered is fuzzy or has uncertainties; the decision maker should have precise and complete information. However, in the real supply chain planning problems, the decision maker has to optimize the conflicting objectives together in the fuzzy aspiration

Table 3. The results of the solution of the problem by using different models

Number of objective function	Type of objective function	Demand quantity	Objective function		L	Total cost (currency) (z_1)	Total distance (km) (z_2)
The fuzzy multi-objective linear programming	Multi	Fuzzy	Fuzzy	max L	0.9494	168,990,000	98,236,740
Zimmermann's method	Multi	Fuzzy	Fuzzy	max L	0.9155	169,001,600	97,600,640
Goal programming	Multi	Crisp	Crisp	min (d_1^-, d_2^+)		169,270,800	95,471,620
Linear programming (min z_1)	Single	Crisp	Crisp	min z_1	1	168,990,400	98,355,290
Linear programming (min z_2)	Single	Crisp	Crisp	min z_2	1	189,762,400	95,471,620

Table 4. The determination of the membership levels related to the objection function values

z_1	$> X_{10}$	X_{10}	X_{11}	X_{12}	$\cdots$	X_{1P}	$X_{1,P+1}$	$< X_{1,P+1}$
$\mu(z_1)$	0	0	q_{11}	q_{12}	$\cdots$	q_{1P}	1	1
z_2	$> X_{20}$	X_{20}	X_{21}	X_{22}	$\cdots$	X_{2P}	$X_{2,P+1}$	$< X_{2,P+1}$
$\mu(z_2)$	0	0	q_{21}	q_{22}	$\cdots$	q_{2P}	1	1

$0 \le qge \le 1.0$; $qge \le qg, e+1$ $(g=1,2)$ $(e=1,2,\cdots,P)$.

and satisfaction levels. In addition, the parameters used in the problems are usually uncertain because of the lack of data or inaccessibility of the data required in the planning period. The fuzzy set theory is proved to be a suitable and useful tool that can be used in defining the real life uncertainties and dealing with them in planning, and the fuzzy mathematical programming one of the decision-making approaches based upon the fuzzy set theory.

This study planned the procurement, production, storage, and distribution functions of a supply chain operating in a fuzzy environment, for six planning periods by using fuzzy multi-objective linear programming model. In the model that aims to minimize the total costs (procurement, filling, storage, and transportation costs) and the total transportation distance, the demand quantities concerning the main demand centers and the aspiration levels of the decision maker about the objective functions are considered fuzzily. In the solution of the model, the method proposed by Wang and Liang (2004) is used; however, the interactive decision process which is the last step of this method is not implemented. The method integrates the fuzzy goal programming method of Hannan (1981) and fuzzy decision making method of Bellman and Zadeh (1970). The problem considered in the study is solved by using the Zimmermann's method, goal programming, conventional single objective linear programming method (for each objective function, respectively) and the results are compared. The conclusion is that the proposed method can easily be applied to obtain efficient solutions to this kind of problems. Liang (2006) explained that, in general, if the decision maker wants the optimum values of the membership functions to be approximately equal or thinks that the usage of the minimum operators represents the case well, this method can be preferred to other methods. Goal programming method that provides the approach to the objectives sought to be achieved is a metod. Zimmermann's method in the case of minimal operator is always active to give solution. In this study the total distance traveled by programming advantageously solution has been reached. Thus obtained solution is applicable. In this study, a simple programming model has been proposed solution aims at generating.

In the future studies, the systems in which the parameters other than the demand quantities are fuzzy and the case that the fuzzy parameters are explained with different distributions other than the triangular distribution can be analyzed.

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Appendix 1. Derivation of the Equivalent Linear Programming Model

The equivalent linear programming model constructed for the solution of the fuzzy multi-objective linear programming model problem is derived as explained below (Liang, 2006).

Step 1: For each objective function (z_g, $g = 1, 2$) the membership values ($\mu(z_g)$) related to some objective function values are determined by the decision maker as shown Table 4.
Note: $0 \le qge \le 1.0$; $qge \le qg, e+1$ ($g = 1, 2$) ($e = 1, 2, \cdots, P$).
Step 2: For each objective function, piecewise linear membership functions are sketched by using the points (z_g, $\mu(z_g)$).
Step 3: The membership function $f_g(z_g)$ ($g = 1, 2$) is converted to the format below.

Let
$\alpha_{ge} = (|t_{g,\, e+1}| - |t_{ge}|)/2$
$\beta_g = (t_{g,\, P+1} + t_{g1})/2$
$\gamma_g = (S_{g,\, P+1} + S_{g1})/2$ be such that,

$$\mu(z_g) = \sum_{e=1}^{Pg} \alpha_{ge} \,|\, z_g - X_{ge} \,|+ \beta_g z_g + \gamma_g \qquad g = 1, 2 \tag{A.1}$$

In (A.1), if the slope of the line segment between $X_{g,\, r-1}$ and X_{gr} is t_{gr}, and if S_{gr} is the y-intercept of this line segment, then it is assumed that for every line segment $X_{g,\, r-1} \le z_g \le X_{gr}$ in the piecewise linear membership function, $f_g(z_g) = t_{gr} z_g + S_{gr}$ is satisfied.

Step 4: Let X_{ge} be the value of the objective function g at the e^{th} point. The deviation variables (d^-_{ge}, d^+_{ge}) at the e^{th} point are included to the model.

If d^+_{ge} and d^-_{ge} are defined as

$$d^+_{ge} = \begin{cases} z_g - X_{ge} & \text{if } z_g - X_{ge} \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$d^-_{ge} = \begin{cases} X_{ge} - z_g & \text{if } z_g - X_{ge} < 0 \\ 0 & \text{otherwise} \end{cases}$$

then the following equations are satisfied:

$$(d^+_{ge}) \cdot (d^-_{ge}) = 0 \tag{A.2}$$

$$z_g - X_{ge} = d^+_{ge} - d^-_{ge} \tag{A.3}$$

$$|\, z_g - X_{ge} \,| = d^+_{ge} + d^-_{ge} \tag{A.4}$$

If Eq. (A.4) is substituted in Eq. (A.1), then the equations can be expressed as follows.

$$z_g - X_{ge} = d^+_{ge} - d^-_{ge}$$
$$(d^+_{ge}) \cdot (d^-_{ge}) = 0$$
If $d^+_{ge} \ge 0$ and
$d^-_{ge} \ge 0$, then

$$\mu(z_g) = \sum_{e=1}^{Pg} \alpha_{ge} \,(d^+_{ge} + d^-_{ge}) + \beta_g z_g + \gamma_g \; g = 1, 2 \tag{A.5}$$

If all the membership functions ($\mu(z_g)$) in Eq. (A.5) are concave and piecewise linear, then Eq. (A.2) is trivially satisfied, so that this condition is not needed. In other words, if all the objectives and constraints in the original model are linear and if all the membership functions are concave and piecewise linear, then a solution to the problem can be obtained by solving a linear programming model in standard form (Hu and Fang, 1999).

Step 5: The model is converted to an equivalent linear programming model by including the auxiliary variable 'L' to the model and by using minimum operator for aggregating the fuzzy sets. The 'L' here can be defined as the total satisfaction level of the decision maker about all the fuzzy objectives (Liang, 2006).