

Supply Chain Coordination in 2-Stage-Ordering-Production System with Update of Demand Information

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ABSTRACT

It is necessary for a retailer to improve responsiveness to uncertain customer demand in product sales. In order to solve this problem, this paper discusses an optimal operation for a 2-stage-ordering-production system consisting of a retailer and a manufacturer. First, based on the demand information estimated at first order time t_1 , the retailer determines the optimal initial order quantity Q_1^* , the optimal advertising cost a_1^* and the optimal retail price p_1^* of a single product at t_1 , and then the manufacturer produces Q_1^* . Next, the retailer updates the demand information at second order time t_2 . If the retailer finds that Q_1^* dissatisfies the demand indicated by the demand information updated at t_2 , the retailer determines the optimal second order quantity Q_2^* under Q_1^* and adjusts optimally the advertising cost and the retail price to a_2^* and p_2^* at t_2 . Here, decision-making approaches for two situations are made—a decentralized supply chain (DSC) whose objective is to maximize the retailer's profit and an integrated supply chain (ISC) whose objective is to maximize the whole system's profit. In the numerical analysis, the results of the optimal decisions under DSC are compared with those under ISC. In addition, supply chain coordination is discussed to adjust the unit wholesale price at each order time as Nash Bargaining solutions.

Keywords: 2-Stage-Ordering-Production System, Price-Sensitivity, Advertising Cost-Sensitivity, Update of Demand Information, Supply Chain Coordination

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1. INTRODUCTION

The past five decades have been greatly interested in inventory management issues for newsvendor-type products. An excellent review regarding this problem was shown in Khouja (1999). It is necessary for a retailer to improve responsiveness to the uncertain demand in product sales. However, the greater the uncertainty in demand fluctuations becomes, the more difficult the retailer plans procurement and inventory of product before the selling period. In this situation, multiple production modes are effective for manufactures to respond to each product order from the retailers with make-to-order policy.

Previous papers during past decades have studied supply chain management issues for newsvendor-type products with two production modes and two ordering opportunities (Donohue, 2000; Gurnani and Tang, 1999; Lau and Lau, 1997, 1998; Wang *et al.*, 2010; Weng, 2004; Zhou and Li, 2007; Zhou and Wang, 2009).

Lau and Lau (1997, 1998), Weng (2004), Zhou and Li (2007), Zhou and Wang (2009) discussed two ordering models where a manufacturer responded to each ordering of a buyer with the make-to-order policy under the same production mode.

Chen *et al.* (2006), Choi *et al.* (2003), Donohue (2000), Eppen and Iyer (1997a, 1997b), Gurnani and Tang (1999),

Iyer and Bergen (1997) discussed two ordering models where a manufacturer responded to each ordering of a buyer with the make-to-order policy under the different production mode.

In the real business world, there exist many news-vendor-type products like USB flash drivers, fashion apparel, personal computers, etc. Demands of those tend to be affected by the retail price and the advertising cost.

There are some previous studies considering the effect of the unit retail price on the uncertain demand of a single product (Agrawal and Seshadri, 2000; Chen and Xiao, 2011; Lau and Lau, 1988; Petruzzi and Dada, 1999; Polatoglu, 1991).

Also, there are some previous studies considering the effect of advertising cost on uncertainty in demand of a single product (Gerchak and Parkar, 1987; Khouja and Robbins, 2003).

However, the previous papers concerning news-vendor models with two production modes or two ordering opportunities neither considered the case where the random demand was dependent together on selling price and advertising cost, nor did they study the optimal decision of production/ordering, pricing, and advertising simultaneously. Wang *et al.* (2010) discussed a two-echelon production/ordering coordination issues of news-vendor-type products by considering the combined effect of advertising and pricing on demand.

Also, it is necessary to determine the optimal sales strategies to establish a supply chain to obtain its profitability. In a decentralized supply chain (DSC), all members in the supply chain determine the optimal sales strategies so as to maximize their own profits. As one of the optimal decision-making approaches under a DSC, the Stackelberg game has been adopted in several previous papers. In the Stackelberg game, there is a single leader of the decision-making and a single (multiple) follower(s) of the decision-making of the leader. The leader of the decision-making determines the optimal sales strategy so as to maximize the leader's (expected) profit. The follower(s) of the decision-making determine(s) the optimal sales strategy so as to maximize the follower(s)'s (expected) profit under the optimal sales strategy determined by the leader of the decision-making (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al.*, 2009; Esmaili and Zeepongsekul, 2010; Hu *et al.*, 2011; Lee *et al.*, 2011; Leng and Parlar, 2009; Liu *et al.*, 2012).

Also, in a supply chain management, the optimal decisions under an integrated supply chain (ISC) maximizing the whole supply chain's expected profit can bring the more expected profit to the whole supply chain than those under a DSC maximizing the expected profit of a leader of decision-making. So, from the aspect of the total optimization in supply chain management, it is preferable for all members in the supply chain to shift the optimal decisions under the ISC. In this case, it is the absolute requirement for all members under the ISC to obtain the more expected profits than those under the DSC. In order to achieve the increases in profits of all

members under the ISC, a variety of supply chain coordination approaches between all members have been discussed by Arcelus *et al.*, (2011), Berr (2011), Cachon (2003), Cachon and Netessine (2004), Cai *et al.* (2009), Chen and Xiao (2011), Du *et al.* (2011), Esmaili and Zeepongsekul (2010), Lau *et al.* (2008), Nagarajan and Sosis (2008), Zhou and Li (2007), Zhou and Wang (2009), Wang *et al.* (2010), etc.

Wang *et al.* (2010) discussed issues of both a two-echelon production/ordering and a supply chain coordination for news-vendor-type products by considering the combined effect of advertising and pricing on demand. The two-echelon supply chain consists of a buyer and a manufacturer. A buyer has two ordering opportunities: the one happens before the beginning of the season, and the other takes place at the end of the season. The ordered items were produced by the manufacturer in two production modes regarding the unit production cost and the unit wholesale price for different requirements. The optimal sales strategy for two-echelon production/ordering system include the three issues mentioned above: 1) production mode for each ordering of a buyer with the make-to-order policy, 2) uncertainty in demand regarding sales price and advertising cost, and 3) supply chain coordination. Concretely, production/ordering, pricing and advertising were simultaneously optimized under a DSC and an ISC. Supply chain coordination were discussed between a buyer and a manufacturer in order to achieve the optimal decision-making under the ISC by not only adjusting the unit wholesale price, but also sharing some ratio of profit of a retailer with a manufacturer. However, no update of demand information was considered at each order time of a retailer to a manufacturer. Also, the expected profits of both members under the ISC with supply chain coordination were not compared with those under the DSC. So, it is not always guaranteed that the expected profits of all members under an ISC with profit sharing are higher than those under a DSC. Therefore, it is necessary to guarantee more profits to all members under an ISC in order to encourage the shift of the optimal sales strategies under an ISC from those under a DSC.

This paper proposes a 2-stage-ordering-production system referred to as 2SOPS. 2SOPS has not only two production modes with two types of the unit production cost, but also twice ordering opportunity with two types of the unit wholesale price. 2SOPS is incorporated into a supply chain.

Different from the previous papers related to a 2SOPS, this paper verifies the following topics for both academic researchers and real-world policymakers who try to make the optimal production planning in a supply chain adopting a 2SOPS:

- Profitability obtained from a 2SOPS by comparing the optimal sales strategies and the expected profits under DSC and ISC at the first order time t_1 with those at the second order time t_2 ,
- Effect of change of variance σ^2 in random variable

ε from the expected demand on the sales strategies and the expected profits under DSC and ISC,

- Effect of supply chain coordination on not only the adjustment of the unit wholesale prices at the first and second order times t_1 and t_2 , but also the expected profits of all members under ISC in order to guarantee the more profit under the optimal sales strategy of ISC.

This paper incorporated the following topics into a supply chain:

- A 2SOPS with two production modes with two types of the unit production cost and twice ordering opportunity with two types of the unit wholesale price is incorporated into a supply chain,
- The optimal sales strategy for the order quantity of a single product, the advertising cost of product sales and the unit retail price are adjusted based on the demand distribution of a single product updated at the first and second order times before the sales of product,
- The uncertainty in demand is considered as the additive random variable from the expected demand depending on the advertising cost of product sales and the unit retail price,
- As supply chain coordination, the unit wholesale prices at the first and second order times are adjusted as Nash bargaining solutions.

Concretely, this paper presents an optimal operation for a 2SOPS consisting of a retailer and a manufacturer. Here, suppose Mode 1 with a long lead time, cheap wholesale price and cheap production cost at order time t_1 , and Mode 2 with a short lead time, high wholesale price and high production cost at order time t_2 . First, at the first order time t_1 , the retailer determines the optimal first order quantity Q_1^* , the optimal advertising cost a_1^* and the optimal retail price p_1^* of a single product under the demand information estimated at t_1 . The manufacturer produces the quantity Q_1^* of product. Next, at the second order time t_2 , the retailer updates the demand information. If the retailer finds that Q_1^* is dissatisfies the product demand from the demand information updated at t_2 , the retailer not only determines the optimal second order quantity Q_2^* based on the optimal first order quantity Q_1^* , but also adjusts optimally the advertising cost and the retail price to a_2^* and p_2^* from the demand information updated at t_2 .

Here, optimal decisions approaches under DSC and ISC are adopted for the product order quantity, the advertising cost and the retail price.

The optimal sales strategy under DSC is determined by adopting the Stackelberg game (Cachon and Netessine, 2004). This paper assumes that under DSC, the retailer is a leader of the decision-making, and the manufacturer is the follower. The retailer determines the optimal sales strategy for the order quantity, the advertising cost and the retail price at each order time so as to maximize the retailer's expected profit. The manufacturer produces the

optimal product quantity at each order time and sells the products to the retailer with each wholesale price at each order time. The optimal sales strategy under DSC is determined so as to maximize the whole system's profit.

The numerical analysis investigates 1) how the optimal sales strategy for a 2SOPS can bring more profit to all members and the whole system in a supply chain, 2) how change of variance σ^2 in random variable ε from the expected demand affects the sales strategies and the expected profits under DSC and ISC, and 3) how supply chain coordination influences not only the adjustment of the unit wholesale prices at the first and second order times t_1 and t_2 , but also the expected profits of all members under ISC. Concretely, in the numerical analysis, the results of the optimal decisions under DSC are compared with those under ISC. Also, it is discussed how supply chain coordination enable to bring the more profits to a retailer and a manufacturer under ISC and encourage to shift the optimal sales strategy under ISC from that under DSC. In this paper, each of the unit wholesale prices, w_1 and w_2 , is coordinated at first and second order times, t_1 and t_2 , respectively, as Nash Bargaining solutions.

The results of the numerical analysis can show academic researchers and real-world policymakers, who make the optimal production planning in a supply chain, the understanding and the managerial insights into the profitability obtained from the following combination: 1) a 2SOPS with two production modes and two ordering opportunities, 2) the update of demand distribution at each ordering time, and 3) supply chain coordination to adjust the unit wholesale price at each order time.

The rest of this paper is organized as follows: in Section 2, notation used in mathematical model in this paper is defined. Section 3 presents model descriptions of a 2SOPS including the operational flows and the model assumption. Section 4 formulates the expected profits in 2SOPS. Section 5 presents the optimal sales strategy for 2SOPS under DSC and ISC. Section 6 discusses supply chain coordination in SOPS. Section 7 shows the results of numerical examples to illustrate managerial insights for the optimal sales strategy of 2SOPS. In Section 8, conclusions, managerial insights and future researches for this paper are summarized.

2. NOTATION

The following notations are used to develop the mathematical expressions for a 2SOPS under DSC and ISC in this paper.

T	: selling time of a single product
t_1	: first order time from a retailer to a manufacturer ($0 < t_1 < T$)
t_2	: second order time from a retailer to a manufacturer ($0 < t_1 < t_2 < T$)
i	: index indicating order/production time, $i=1$ denotes that the first order/production time,

- $i=2$ denotes the second order/production time
- j : index indicating the type of supply chain, $j=D$ denotes a DSC and $j=I$ denotes an ISC
- k : index indicating the type of member of supply chain, $k=R$ denotes a retailer, $k=M$ denotes a manufacturer and $k=S$ denotes the whole system
- $Q_i(i=1, 2)$: order/production quantity of a single product, referred to order quantity, from a retailer to a manufacturer at t_i
- $p_i(i=1, 2)$: the unit retail price of a single product determined at t_i
- $a_i(i=1, 2)$: advertising cost determined at t_i ($0 \leq a_i < p_i$)
- x : demand of product in a market referred to demand
- x_t : demand in a market at time t ($0 < t < t_i$)
- $D(a_i, p_i)(i=1, 2)$: expected demand depending on a_i and p_i (non-negative value)
- ε : random variable from the expected demand
- $\hat{\mu}_i$: mean of random variable ε forecasted at t_i
- $\hat{\sigma}_i^2$: variance of random variable ε forecasted at t_i
- $\hat{f}_{\varepsilon i}(\varepsilon)(i=1, 2)$: probability density function (pdf) of random variable ε forecasted at order time t_i
- $\hat{F}_{\varepsilon i}(\varepsilon)(i=1, 2)$: cumulative distribution function (cdf) of random variable ε forecasted at order time t_i
- $w_i(i=1, 2)$: the unit wholesale price of a single product at order time t_i
- $c_i(i=1, 2)$: the unit production cost of a single product at order time t_i
- h : the unit inventory holding cost of unsold products per unit of time
- g : the unit shortage penalty cost of the unsatisfied demand of a single product
- $\pi_k(Q_i, a_i, p_i|x)(i=1, 2, k=R, M, S)$: profit of member k for Q_i, a_i and p_i under demand x
- $E[\pi_k(Q_i, a_i, p_i)](i=1, 2, j=D, C, k=R, M, S)$: the expected profit of member k for Q_i, a_i and p_i
- $Q_i^j(i=1, 2, J=D, I)$: optimal order quantity under type j of supply chain at t_i
- $a_i^j(i=1, 2, J=D, I)$: optimal advertising cost under type j of supply chain at t_i
- $p_i^j(i=1, 2, J=D, I)$: optimal retail price under type j of supply chain at t_i
- $w_i^N(i=1, 2)$: wholesale price coordinated between a retailer and a manufacturer under ISC

3. MODEL DESCRIPTIONS OF 2-STAGE ORDERING PRODUCTION (2SOPS)

3.1 Operational Flows of a 2SOPS

Operational flows of a 2SOPS in this paper are shown here. A 2SOPS consisting of a retailer and a manufacturer is considered. The retailer sells a single prod-

uct at selling time T under the uncertainty in product demand. Products are sold during a single period. The retailer incurs the shortage penalty cost per the unsatisfied demand, meanwhile the retailer incurs the inventory holding cost per excess inventory of products per time at the end of the single selling period. The retailer faces a random product demand depending on the advertising cost of product sales and the unit retail price. The higher the unit retail price is, the lower the product demand is. Meanwhile, the higher the advertising cost of product sales is, the higher the product demand is. So, the retailer needs the optimal decisions for the order quantity of a single product, the advertising cost of product sales and the unit retail price. In 2SOPS, not only twice orders with the manufacturer can be placed, but also the advertising cost and the unit retail price can be adjusted at the second order time so as to maximize the expected profit. Concretely, the first product order quantity Q_1 , the initial advertising cost a_1 , and the initial retail price p_1 are determined at the first order time t_1 under the demand information estimated at time period between time 1 ($t=1$) (the start time where a manufacturer produces a single product) and the first order time t_1 . Also, in 2SOPS, at the second order time t_2 , not only the second ordering of a single product with the manufacturer can be placed, but also the advertising cost and the unit retail price can be adjusted under the demand information updated from time period between time 1 ($t=1$) and the second order time t_2 . If the optimal first order quantity Q_1 is unsatisfied with the product demand, the second order quantity Q_2 can be determined at the second order time t_2 under the demand information updated at the second order time t_2 . Also, the optimal advertising cost a_1 and the unit retail price p_1 determined at the first order time t_1 can be adjusted to the optimal advertising cost a_2 and the optimal unit retail price p_2 at the second order time under the total order quantity Q_1+Q_2 and the demand information updated at the second order time t_2 .

3.2 Model Assumptions of a 2SOPS

- (1) It is assumed that the demand x follows a probability distribution, and the expected demand not only depends on the advertising cost $a_i(i=1, 2)$ and the retail price $p_i(i=1, 2)$ at order time t_i , but also has an additive random variable ε following a probability distribution with the pdf $f_\varepsilon(\varepsilon)$. In this case, the demand x is modeled as $x=D(a_i, p_i)+\varepsilon$. As the demand information, mean and variance of the additive random variable ε in the expected demand $D(a_i, p_i)(i=1, 2)$ are estimated by actual demand data observed during time 1 ($t=1$) (the start time where a manufacturer produces a single product) and order time $t_i(i=1, 2)$ as

$$\hat{\mu}_{\varepsilon i} = \bar{\varepsilon} = \frac{1}{t_i} \sum_{t=1}^{t_i} \varepsilon_t (1 < t_1 < t_2 \leq T) \quad (1)$$

$$\hat{\sigma}_{\varepsilon i}^2 = \frac{1}{t_i - 1} \sum_{t=1}^{t_i} (\varepsilon_t - \bar{\varepsilon})^2 (1 < t_1 < t_2 \leq T) \quad (2)$$

Using the estimated values of $\hat{\mu}_{\varepsilon i}$ and $\hat{\sigma}_{\varepsilon i}^2$, the demand information distribution at order time $t_i (i=1, 2)$ is updated. Therefore, the pdf $\hat{f}_{\varepsilon i}(\varepsilon)(i=1, 2)$ forecasted for additive random variable ε at order time t_i is obtained as

$$\hat{f}_{\varepsilon i}(\varepsilon)(i=1, 2) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_{\varepsilon i}} \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon - \hat{\mu}_{\varepsilon i}}{\hat{\sigma}_{\varepsilon i}}\right)^2\right\} \quad (3)$$

From in Eq. (3), the later order time is, the smaller the demand information error is. Using the pdf $\hat{f}_{\varepsilon i}(\varepsilon)$ in Eq. (3), the optimal decisions for order quantity $Q_i (i=1, 2)$, advertising cost of product sales $a_i (i=1, 2)$ and the unit retail price $p_i (i=1, 2)$ at order time $t_i (i=1, 2)$ are made.

- (2) In 2SOPS, two production modes are adopted. In Mode 1, a single product is produced at the unit production cost p_1 and is sold to a retailer at the unit wholesale price w_1 . At first order time t_1 , p_1 and w_1 are cheap, but the product delivery lead-time L_1 is long. In Mode 2, a single product is produced at the unit production cost p_2 and is sold to a retailer at the unit wholesale price w_2 . At the second order time t_2 , p_2 and w_2 are higher than p_1 and w_1 at first order time t_1 , but the product delivery lead-time L_2 is shorter than L_1 at first order time t_1 . Here, it is assumed that each product quality in each production mode is same.
- (3) The condition $p_i > w_i > c_i (i=1, 2)$ is satisfied.

4. EXPECTED PROFITS IN 2SOPS

From 3. Model Descriptions of 2SOPS, first the profits of a retailer, a manufacturer and the whole system for the first order of a single product at the first order time t_1 are formulated.

The retailer's profit at the first order time t_1 consists of the product sales, the first order cost, the advertising cost, the inventory holding cost of excess product inventory, and the anticipated second order cost to supplement the unsatisfied product demands. Concretely, the retailer's profit $\pi_R(Q_1, a_1, p_1|x)$ for the first order quantity Q_1 of a single product, the advertising cost a_1 of product sales and the unit retail price p_1 at the first order time t_1 under the demand x is formulated as

$$\pi_R(Q_1, a_1, p_1|x) = \begin{cases} p_1x - a_1 - h(Q_1 - x) - w_1Q_1 & (0 \leq x \leq Q_1), \\ p_1Q_1 - a_1 - w_1Q_1 - w_2(x - Q_1) & (x > Q_1). \end{cases} \quad (4)$$

The manufacturer's profit at the first order time t_1 consists of the wholesales of product, the first production cost for the first order quantity Q_1 and the anticipated second production cost to supplement the unsatisfied product demands. Concretely, the manufacturer's profit $\pi_M(Q_1|x)$ for the first order quantity Q_1 of a single

product at the first order time t_1 under the demand x is formulated as

$$\pi_M(Q_1|x) = \begin{cases} w_1Q_1 - c_1Q_1 & (0 \leq x \leq Q_1), \\ w_1Q_1 - c_1Q_1 - c_2(x - Q_1) & (x > Q_1). \end{cases} \quad (5)$$

The profit of the whole system at the first order time t_1 is obtained as the sum of the profits of the retailer and the manufacturer at t_1 . Therefore, $\pi_S(Q_1, a_1, p_1|x)$ for the first order quantity Q_1 of a single product, the advertising cost a_1 of product sales and the unit retail price p_1 at the first order time t_1 under the demand x is calculated as

$$\pi_S(Q_1, a_1, p_1|x) = \pi_R(Q_1, a_1, p_1|x) + \pi_M(Q_1|x) \quad (6)$$

Next, we formulate the profits of a retailer, a manufacturer and the whole system for the second order of a single product at the second order time t_2 .

The retailer's profit at the second order time t_2 consists of the product sales, the second order cost, the advertising cost, the inventory holding cost of excess product inventory and the shortage penalty cost for unsatisfied product demand. Concretely, the retailer's profit $\pi_R(Q_2, a_2, p_2|x, Q_1)$ for the second order quantity Q_2 of a single product, the advertising cost a_2 of product sales and the unit retail price p_2 at the second order time t_2 under the first order quantity Q_1 and the demand x is formulated as

$$\pi_R(Q_2, a_2, p_2|x, Q_1) = \begin{cases} p_2x - h(Q_1 + Q_2 - x) - a_2 - w_1Q_1 - w_2Q_2, & (0 < x < Q_1 + Q_2) \\ p_2(Q_1 + Q_2) - g(x - Q_1 - Q_2) - a_2 - w_1Q_1 - w_2Q_2. & (x > Q_1 + Q_2) \end{cases} \quad (7)$$

The manufacturer's profit at the second order time t_2 consists of the wholesales of product and the second production cost for the second order quantity Q_2 . Concretely, the manufacturer's profit $\pi_M(Q_2, a_2, p_2|x, Q_1)$ for the second order quantity Q_2 of a single product at the second order time t_2 under the first order quantity Q_1 and the demand x is formulated as

$$\pi_M(Q_2|Q_1, x) = (w_1 - c_1)Q_1 + (w_2 - c_2)Q_2 \quad (8)$$

The profit of the whole system at the second order time t_2 is obtained as the sum of the profits of the retailer and the manufacturer at t_2 . Therefore, $\pi_S(Q_2, a_2, p_2|x)$ for the second order quantity Q_2 of a single product, the advertising cost a_2 of product sales and the unit retail price p_2 at the second order time t_2 under the first order quantity Q_1 and the demand x is calculated as

$$\pi_S(Q_2, a_2, p_2 | x, Q_1) = \pi_R(Q_2, a_2, p_2 | x, Q_1) + \pi_M(Q_2 | x, Q_1) \quad (9)$$

Next, the expected profits of the retailer, the manufacturer and the whole system for the first order at order time t_1 are derived as follows:

From Eq. (1), the demand is modeled as $x = D(a_1, p_1) + \varepsilon$. Taking the expectation for the additive random variable ε of the demand x in Eqs. (4)–(6), the expected profits of the retailer, the manufacturer and the whole system for the first order quantity Q_1 , the advertising cost a_1 and the unit retail price p_1 at the first order time t_1 can be formulated as

$$\begin{aligned} E[\pi_R(Q_1, a_1, p_1)] &= p_1 D(a_1, p_1) \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ p_1 \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon + p_1 Q_1 \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \\ &- h \{Q_1 - D(a_1, p_1)\} \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ h_1 \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon - a_1 - w_1 Q_1 \\ &- w_2 D(a_1, p_1) \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon - w_2 \int_{Q_1 - D(a_1, p_1)}^{\infty} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon \\ &+ w_2 Q_1 \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \end{aligned} \quad (10)$$

$$\begin{aligned} E[\pi_M(Q_1, a_1, p_1)] &= (w_1 - c_1) Q_1 \\ &+ w_2 D(a_1, p_1) \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon + w_2 \int_{Q_1 - D(a_1, p_1)}^{\infty} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon \\ &- w_2 Q_1 \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \end{aligned} \quad (11)$$

$$\begin{aligned} E[\pi_S(Q_1, a_1, p_1)] &= E[\pi_R(Q_1, a_1, p_1)] + E[\pi_M(Q_1, a_1, p_1)] \\ &= p_1 D(a_1, p_1) \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ p_1 \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon + p_1 Q_1 \int_{Q_1 - D(a_1, p_1)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \\ &- h \{Q_1 - D(a_1, p_1)\} \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ h_1 \int_{-D(a_1, p_1)}^{Q_1 - D(a_1, p_1)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon - a_1 - c_1 Q_1 \end{aligned} \quad (12)$$

Similarly, taking the expectation for the additive random variable ε of the demand x in Eqs. (7)–(9), the expected profits of the retailer, the manufacturer and the whole system for the second order quantity Q_2 , the advertising cost a_2 and the retail price p_2 at the second order time t_2 as

$$\begin{aligned} E[\pi_R(Q_2, a_2, p_2 | Q_1)] &= p_2 D(a_2, p_2) \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \hat{f}_2(\varepsilon) d\varepsilon \\ &+ p_2 \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \varepsilon \hat{f}_2(\varepsilon) d\varepsilon \\ &+ p_2 (Q_1 + Q_2) \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \hat{f}_2(\varepsilon) d\varepsilon \\ &- h \{Q_1 + Q_2 - D(a_2, p_2)\} \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \hat{f}_2(\varepsilon) d\varepsilon \end{aligned}$$

$$\begin{aligned} &+ h \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon - a_2 - w_1 Q_1 - w_2 Q_2 \\ &- g \{D(a_2, p_2) - Q_1 - Q_2\} \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ g \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon \end{aligned} \quad (13)$$

$$E[\pi_M(Q_2, a_2, p_2 | Q_1)] = (w_1 - c_1) Q_1 + (w_2 - c_2) Q_2 \quad (14)$$

$$\begin{aligned} E[\pi_S(Q_2, a_2, p_2 | Q_1)] &= E[\pi_R(Q_2, a_2, p_2 | Q_1)] + E[\pi_M(Q_2, a_2, p_2 | Q_1)] \\ &= p_2 D(a_2, p_2) \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \hat{f}_2(\varepsilon) d\varepsilon \\ &+ p_2 \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \varepsilon \hat{f}_2(\varepsilon) d\varepsilon \\ &+ p_2 (Q_1 + Q_2) \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \hat{f}_2(\varepsilon) d\varepsilon \\ &- h \{Q_1 + Q_2 - D(a_2, p_2)\} \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ h \int_{-D(a_2, p_2)}^{Q_1 + Q_2 - D(a_2, p_2)} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon - a_2 - c_1 Q_1 - c_2 Q_2 \\ &- g \{D(a_2, p_2) - Q_1 - Q_2\} \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \hat{f}_1(\varepsilon) d\varepsilon \\ &+ g \int_{Q_1 + Q_2 - D(a_2, p_2)}^{\infty} \varepsilon \hat{f}_1(\varepsilon) d\varepsilon \end{aligned} \quad (15)$$

5. OPTIMAL SALES STRATEGY FOR 2SOPS

5.1 Decentralized Supply Chain

For the optimal decisions are made under DSC, the optimal decision approach for the Stackelberg game (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al.*, 2009; Esmaili and Zeepongsekul, 2010; Hu *et al.*, 2011; Leng and Parlar, 2009; Liu *et al.*, 2012) is adopted. The reason why the Stackelberg game is adopted under DSC of this paper is shown as follows: the optimal decision in the Stackelberg game is made under a situation consisting of one leader of the decision-making and one (multiple) follower(s). First, a leader of the decision-making makes the optimal decision so as to the leader's profit. Next, one (multiple) follower(s) operate(s) the own activity the optimal decision made by the leader of the decision-making. Suppose that decision variable(s) of supply chain members affect(s) not only the optimal decision so as to maximize the profit of a supply chain member, but also that (those) of the other supply chain member(s), interacting between supply chain members' profit. Under the situation, the optimal decision approach in the Stackelberg game is adopted effectively among supply chain members (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al.*, 2009; Esmaili and Zeepongsekul, 2010; Hu *et al.*, 2011; Leng and Parlar, 2009; Liu *et al.*, 2012). This paper regards a retailer as the leader of the decision-making under DSC and regards a manufacture as the fol-

lower of the decision-making of the retailer under DSC. The reason is due to the following situation: a retailer faces stochastic demands of products in a market, sells the products in the market and earns the most profit in the whole supply chain.

The retailer determines the optimal sales strategy for the first and second order quantities Q_1 and Q_2 , the advertising costs of product sales a_1 and a_2 and the unit retail prices p_1 and p_2 at the first and second order times t_1 and t_2 so as to maximize the retailer's expected profit. The manufacturer produces the optimal $Q_i (i=1, 2)$ at order time $t_i (i=1, 2)$ and sells the products to the retailer with each wholesale price $w_i (i=1, 2)$ at order time $t_i (i=1, 2)$.

First, the decision procedures for the optimal sales strategy under DSC at the first order time t_1 are shown.

Proposition 1: The retailer's expected profit in Eq. (10) is the concave function for the first order quantity Q_1 under a given the advertising cost a_1 and a given the unit retail price p_1 at the first order time t_1 .

Proof: The first- and second-order differential equations between the first order quantity Q_1 and the expected profit $E[\pi_r(Q_1|a_1, p_1)]$ of the retailer at the first order time t_1 in Eq. (10) under a given the advertising cost a_1 and a given the unit retail price p_1 as follows:

$$\begin{aligned} dE[\pi_r(Q_1|a_1, p_1)]/dQ_1 &= -(p_1 + h + w_2)\hat{F}_1(Q_1 - D(a_1, p_1)) \\ &\quad + (h + w_1)\hat{F}_1(-D(a_1, p_1)) + p_1 - w_1 + w_2 \end{aligned} \quad (16)$$

$$d^2E[\pi_r(Q_1|a_1, p_1)]/dQ_1^2 = -(p_1 + h)\hat{f}_1(Q_1 - D(a_1, p_1)). \quad (17)$$

It is derived that Eq. (17) is negative since it is natural to satisfy the conditions $p_1 > 0, h > 0$ and $\hat{f}_1(Q_1 - D(a_1, p_1)) \geq 0$. The theoretical analysis results in Proposition 1.

Proposition 2: The optimal first order quantity Q_1^D under DSC at the first order time t_1 can be obtained as the following unique solution to maximize Eq. (10):

$$Q_1^D = D(a_1, p_1) + \hat{F}_1^{-1}\left(\frac{p_1 - w_1 + w_2}{p_1 + h + w_2}\right). \quad (18)$$

Proof: The solution of $dE[\pi_r(Q_1|a_1, p_1)]/dQ_1 = 0$ substituting 0 into Eq. (16) results in Proposition 2.

Substituting Q_1^D into the retailer's expected profit in Eq. (10), the optimal combination (a_1^D, p_1^D) of the advertising cost and the unit retail price at the first order time t_1 can be determined by the numerical search, satisfying

$$\text{Max}_{a_1, p_1} E[\pi_r(a_1, p_1 | Q_1^D)], \quad (19)$$

where $0 \leq a_1, 0 \leq p_1, D(a_1, p_1) > 0$.

The expected profits of the retailer, the manufacturer and the whole system under DSC at the first order time t_1 can be obtained by substituting the optimal sales strategy (Q_1^D, a_1^D, p_1^D) under DSC into the relevant expected profits in at the first order time t_1 in Eqs. (10)–(12).

Next, the decision procedures for the optimal sales strategy under DSC at the second order time t_2 are shown.

Proposition 3: The retailer's expected profit in Eq. (13) is the concave function for the second order quantity Q_2 under the optimal first order quantity Q_1^D , a given the advertising cost a_2 and a given the unit retail price p_2 at the second order time t_2 .

Proof: The first- and second-order differential equations between the first order quantity Q_2 and the expected profit $E[\pi_r(Q_2|Q_1^D, a_2, p_2)]$ of the retailer at the first order time t_1 in Eq. (13) under the optimal first order quantity Q_1^D , a given the advertising cost a_2 and a given the unit retail price p_2 as follows:

$$\begin{aligned} dE[\pi_r(Q_2|Q_1^D, a_2, p_2)]/dQ_2 &= -(p_2 + h + g)\hat{F}_2(Q_1^D + Q_2 - D(a_2, p_2)) \\ &\quad + p_2 + g - w_2 \end{aligned} \quad (20)$$

$$\begin{aligned} d^2E[\pi_r(Q_2|Q_1^D, a_2, p_2)]/dQ_2^2 &= -(p_2 + h + g)\hat{f}_2(Q_1^D + Q_2 - D(a_2, p_2)). \end{aligned} \quad (21)$$

It is derived that Eq. (21) is negative since it is natural to satisfy the conditions $p_2 > 0, h > 0, g > 0$ and $\hat{f}_2(Q_1^D + Q_2 - D(a_2, p_2)) \geq 0$. The theoretical analysis results in Proposition 3.

Proposition 4: The optimal first order quantity Q_2^D under DSC at the second order time t_2 can be obtained as the following unique solution to maximize Eq. (13):

$$Q_2^D = D(a_1, p_1) + \hat{F}_2^{-1}\left(\frac{p_2 + g - w_2}{p_2 + g + h}\right) - Q_1^D. \quad (22)$$

Proof: The solution of $dE[\pi_r(Q_2|Q_1^D, a_2, p_2)]/dQ_2 = 0$ substituting 0 into Eq. (20) results in Proposition 4.

Substituting Q_1^D and Q_2^D into the retailer's expected profit in Eq. (13), the optimal combination (a_2^D, p_2^D) of the advertising cost and the unit retail price at the second order time t_2 can be determined by the numerical search, satisfying

$$\text{Max}_{a_2, p_2} E[\pi_r(a_2, p_2 | Q_1^D, Q_2^D)], \quad (23)$$

where $0 \leq a_2, 0 \leq p_2, D(a_2, p_2) > 0$.

The expected profits of the retailer, the manufac-

turer and the whole system under DSC at the second order time t_2 can be obtained by substituting the optimal sales strategy $(Q_1^D, Q_2^D, a_2^D, p_2^D)$ under DSC into the relevant expected profits ion at the first order time t_1 in Eqs. (13)–(15).

5.2 Integrated Supply Chain

The optimal sales strategy under ISC is determined so as to maximize the wholes system’s expected profit which is sum of the expected profits of a retailer and a manufacturer.

First, the decision procedures for the optimal sales strategy under ISC at the first order time t_1 are shown.

Proposition 5: The whole system’s expected profit in Eq. (12) is the concave function for the first order quantity Q_1 under a given the advertising cost a_1 and a given the unit retail price p_1 at the first order time t_1 .

Proof: The first- and second-order differential equations between the first order quantity 0 and the expected profit $E[\pi_S(Q_1|a_1, p_1)]$ of the whole system at the first order time t_1 in Eq. (12) under a given the advertising cost a_1 and a given the unit retail price p_1 as follows:

$$\begin{aligned} dE[\pi_S(Q_1|a_1, p_1)]/dQ_1 \\ = -(p_1 + h + c_2)\hat{F}_1(Q_1 - D(a_1, p_1)) + p_1 - c_1 + c_2, \end{aligned} \quad (24)$$

$$\begin{aligned} d^2E[\pi_S(Q_1|a_1, p_1)]/dQ_1^2 \\ = -(p_1 + h + c_2)\hat{f}_1(Q_1 - D(a_1, p_1)). \end{aligned} \quad (25)$$

It is derived that Eq. (25) is negative since it is natural to satisfy the conditions $p_1 > 0, h > 0, c_2 > 0$ and $\hat{f}_1(Q_1 - D(a_1, p_1)) \geq 0$. The theoretical analysis results in Proposition 5.

Proposition 6: The optimal first order quantity Q_1^D under DSC at the first order time t_1 can be obtained as the following unique solution to maximize Eq. (12):

$$Q_1^D = D(a_1, p_1) + \hat{F}_1^{-1}\left(\frac{p_1 - c_1 + c_2}{p_1 + h + c_2}\right). \quad (26)$$

Proof: The solution of $dE[\pi_S(Q_1|a_1, p_1)]/dQ_1 = 0$ substituting 0 into Eq. (24) results in Proposition 6.

Substituting Q_1^D into the whole system’s expected profit in Eq. (12), the optimal combination (a_1^D, p_1^D) of the advertising cost and the unit retail price at the first order time t_1 can be determined by the numerical search, satisfying

$$\text{Max}_{a_1, p_1} E[\pi_S(a_1, p_1 | Q_1^D)], \quad (27)$$

where $0 \leq a_1, 0 \leq p_1, D(a_1, p_1) > 0$.

The expected profits of the retailer, the manufacturer and the whole system under ISC at the first order time t_1 can be obtained by substituting the optimal sales strategy (Q_1^D, a_1^D, p_1^D) under DSC into the relevant expected profits ion at the first order time t_1 in Eqs. (10)–(12).

Next, the decision procedures for the optimal sales strategy under ISC at the second order time t_2 are shown.

Proposition 7: The whole system’s expected profit in Eq. (15) is the concave function for the second order quantity Q_2 under the optimal first order quantity Q_1^D , a given the advertising cost a_2 and a given the unit retail price p_2 at the second order time t_2 .

Proof: The first- and second-order differential equations between the first order quantity Q_2 and the expected profit $E[\pi_S(Q_2|Q_1^D, a_2, p_2)]$ of the whole system at the second order time t_2 in Eq. (15) under the optimal first order quantity Q_1^D , a given the advertising cost a_2 and a given the unit retail price p_2 as follows:

$$\begin{aligned} dE[\pi_S(Q_2|Q_1^D, a_2, p_2)]/dQ_2 \\ = -(p_2 + h + g)\hat{F}_2(Q_1^D + Q_2 - D(a_2, p_2)) + p_2 + g - c_2 \end{aligned} \quad (28)$$

$$\begin{aligned} d^2E[\pi_S(Q_2|Q_1^D, a_2, p_2)]/dQ_2^2 \\ = -(p_2 + h + g)\hat{f}_2(Q_1^D + Q_2 - D(a_2, p_2)). \end{aligned} \quad (29)$$

It is derived that Eq. (29) is negative since it is natural to satisfy the conditions $p_2 > 0, h > 0, g > 0$ and $\hat{f}_2(Q_1^D + Q_2 - D(a_2, p_2)) \geq 0$. The theoretical analysis results in Proposition 7.

Proposition 8: The optimal first order quantity Q_2^D under DSC at the second order time t_2 can be obtained as the following unique solution to maximize Eq. (15):

$$Q_2^D = D(a_2, p_2) + \hat{F}_2^{-1}\left(\frac{p_2 + g - c_2}{p_2 + g + h}\right) - Q_1^D. \quad (30)$$

Proof: The solution of $dE[\pi_S(Q_2|Q_1^D, a_2, p_2)]/dQ_2 = 0$ substituting 0 into Eq. (20) results in Proposition 8.

Substituting Q_1^D and Q_2^D into the whole system’s expected profit in Eq. (15), the optimal combination (a_2^D, p_2^D) of the advertising cost and the unit retail price at the second order time t_2 can be determined by the numerical search, satisfying

$$\text{Max}_{a_2, p_2} E[\pi_S(a_2, p_2 | Q_1^D, Q_2^D)], \quad (31)$$

where $0 \leq a_2, 0 \leq p_2, D(a_2, p_2) > 0$.

The expected profits of the retailer, the manufacturer and the whole system under DSC at the second order time t_2 can be obtained by substituting the optimal sales strategy $(Q_1^D, Q_2^D, a_2^D, p_2^D)$ under ISC into the rele-

vant expected profits ion at the first order time t_1 in Eqs. (13)–(15).

6. SUPPLY CHAIN COORDINATION IN 2SOPS

A supply chain coordination is discussed in order to guarantee that the expected profits of all members under ISC are higher than those under DSC and enable to encourage all members to shift the optimal sales strategy under ISC from that under DSC. Concretely, the Nash bargaining solution (Du *et al.*, 2011; Nagarajan and Sobic, 2008) is adopted as one of reasonable solution to coordinate the unit wholesale prices w_1 and w_2 at the first and second order times t_1 and t_2 between a retailer and a manufacturer.

The reasonable unit wholesale prices w_1^N and w_2^N at the first and second order times t_1 and t_2 can be coordinated so as to satisfy the following objective function and conditions regarding the expected profits of the retailer and the manufacturer at the second order times t_2 by numerical search:

$$\begin{aligned} \max \Pi(w_1^N, w_2^N) &= \left\{ E \left[\pi_R(w_1^N, w_2^N | Q_1^I, Q_2^I, p_2^I, a_2^I) \right] \right. \\ &\quad \left. - E \left[\pi_R(w_1, w_2 | Q_1^D, Q_2^D, p_2^D, a_2^D) \right] \right\} \\ &\quad \times \left\{ E \left[\pi_M(w_1^N, w_2^N | Q_1^I, Q_2^I, p_2^I, a_2^I) \right] \right. \\ &\quad \left. - E \left[\pi_M(w_1, w_2 | Q_1^D, Q_2^D, p_2^D, a_2^D) \right] \right\}, \end{aligned} \quad (32)$$

subject to

$$\begin{aligned} E \left[\pi_R(w_1^N, w_2^N | Q_1^I, Q_2^I, p_2^I, a_2^I) \right] \\ - E \left[\pi_R(w_1, w_2 | Q_1^D, Q_2^D, p_2^D, a_2^D) \right] > 0, \end{aligned} \quad (33)$$

$$\begin{aligned} E \left[\pi_M(w_1^N, w_2^N | Q_1^I, Q_2^I, p_2^I, a_2^I) \right] \\ - E \left[\pi_M(w_1, w_2 | Q_1^D, Q_2^D, p_2^D, a_2^D) \right] > 0. \end{aligned} \quad (34)$$

where Eqs. (33) and (34) are the constraint conditions to guarantee that the expected profit of each member under ISC with supply chain coordination at the second order times t_2 is always higher than that under DSC at the second order times t_2 .

7. NUMERICAL EXPERIMENTS

This section illustrates results of the optimal sales strategies under DSC and ISC adopting a 2SOPS proposed in Section 5 by providing numerical examples.

Also, the effect of supply chain coordination so as to encourage to shift the optimal sales strategy under ISC from that under DSC, guaranteeing the more profit to all members under ISC. The numerical analysis verifies the following topics for both academic researchers and real-world policymakers who try to make the optimal production planning in a supply chain adopting a 2SOPS:

- Profitability obtained from a 2SOPS by comparing the optimal sales strategies and the expected profits under DSC and ISC at the first order time t_1 with those at the second order time t_2 ,
- Effect of change of variance σ^2 in random variable ε from the expected demand on the sales strategies and the expected profits under DSC and ISC,
- Effect of supply chain coordination on not only the adjustment of the unit wholesale prices at the first and second order times t_1 and t_2 , but also the expected profits of all members under ISC.

First, the results of the optimal sales strategy under DSC in 2SOPS is compared with that under ISC in 2SOPS through numerical examples. Concretely, the optimal sales strategy regarding the order quantity, the advertising cost of product sales and the unit retail price and the expected profits of a retailer, a manufacturer and the whole system under DSC in the 2SOPS at the first and second order times t_1 and t_2 are compared with that under ISC in the 2SOPS.

Next, it is investigated how variance of product's demand impact the optimal sales strategy and the expected profits under DSC and ISC in the 2SOPS.

Moreover, it is investigated how the supply chain coordination can encourage all members to shift the strategy under ISC from that under DSC. In this paper, the unit wholesale prices at the first and second order times t_1 and t_2 are adjusted between a retailer and a manufacturer based on Nash Bargaining solution.

Data sources of numerical examples to operate a 2SOPS under DSC and ISC addressed in this paper are provided as follows: $w_1 = 4$, $c_1 = 1$, $w_2 = 5$, $c_2 = 2$, $g = 0.9$, $p_i (i = 1, 2)$, $h = 7$, $t_1 = 50$, $T_2 = 10000$. The expected demand for the advertising cost of product sales and the unit retail price at each order times $t_i (i = 1, 2)$ is provided as

$$\begin{aligned} D(a^j, p^j) (i = 1, 2, j = D, C) \\ = 800 - 7 \exp(0.2p^j) + 4.5(a^j)^{0.5}. \end{aligned} \quad (35)$$

The optimal combination of the advertising costs and the unit retail price under DSC and ISC $a_1^j (j = D, C)$ and $p_1^j (j = D, C)$ at the first order time t_1 are determined by using the expected profits in Eqs. (10) and (12) under the optimal order quantities $Q_1^j (j = D, C)$ in Eqs. (18) and (26) through the numerical search where the step size is 1 in the following ranges $1 \leq a_1 \leq 1000$ and $1 \leq p_1 \leq 30$. Similarly, the optimal combination of the

advertising costs and the unit retail price under DSC and ISC $a_2^j (j = D, C)$ and $p_2^j (j = D, C)$ at the second order time t_2 are determined by using the expected profits in Eqs. (13) and (15) under the optimal order quantities $Q_i^j (i = 1, 2, j = D, C)$ in Eqs. (18), (22), (26), and (30) through the numerical search where the step size is 1 in the following ranges $1 \leq a_i \leq 1000$ and $1 \leq p_i \leq 30$.

Here, the optimal decisions for the advertising cost and the unit retail price $a_i^j (i = 1, 2, j = D, C)$ and $p_i^j (i = 1, 2, j = D, C)$ at each order time $t_i (i = 1, 2)$ under DSC and ISC are substituting into the relative terms in above data sources of numerical examples.

The additive random variable ε from the expected demand, indicating the uncertain demand, follows the normal distribution with mean $\mu_1 = 0$ and variance $\sigma^2 = 10^2, 20^2, 30^2, 100^2, 200^2$.

First, the results of the optimal sales strategy under DSC are compared with those under ISC in 2SOPS at the first and second order times t_1 and t_2 . Table 1 shows the results of the optimal sales strategy under DSC and ISC and the expected profits for a retailer, a manufacturer and the whole system in 2SOPS at the first and second order times t_1 and t_2 when variance σ^2 in random variable ε from the expected demand changes.

From Table 1, the following results can be seen:

Optimal order quantities at first and second order time

The optimal total order quantity at the first order time t_1 and the second order time t_2 under ISC are larger than that under DSC.

The reasons are considered as follows: Q_1^D and Q_2^D are determined from Eqs. (18) and (22). Meanwhile, Q_1^I and Q_2^I are determined from Eqs. (26) and (30). From the comparison of these equations, it can be seen that Q_1^D and Q_2^D are affected by the unit whole prices w_1 and w_2 at the first and second order times t_1 and t_2 . Meanwhile, Q_1^I and Q_2^I are affected by the unit production costs c_1 and c_2 at the first and second order times t_1 and t_2 . In general, it is natural to satisfy the conditions where $c_1 < w_1, c_2 < w_2, c_1 < c_2, w_2 < w_1$ in a 2SOPS. Therefore, it is verified that the optimal total order quantity under ISC at t_1 and t_2 can be determined as a larger values than that under DSC from the theoretical analysis.

At the second order time t_2 , the optimal additional order quantities under DSC and ISC are determined so as to cover the shortage of demand of product as much as possible at the selling time of product T .

Optimal advertising cost of product sales at first and second order time

The optimal advertising costs a_1^j and a_2^j at the first

Table 1. Results of optimal sales strategy under DSC and ISC in 2SOPS

Order time t_i ($i = 1, 2$)	Type of supply chain ($j = D, I$)	Variance of σ^2	Expected demand $D(a_i^j, p_i^j)$	Optimal total order quantity $Q_1^j + Q_2^j$	Optimal first order quantity Q_1^j	Optimal second order quantity Q_2^j	Optimal advertising cost a_i^j	Optimal unit retail price p_i^j
t_1 ($i = 1$)	DSC ($j = D$)	10^2	551.89	555	555	0	201	19
		20^2	557.10	565	565	0	242	19
		30^2	558.10	570	570	0	249	19
		100^2	559.81	599	599	0	254	19
		200^2	555.74	636	636	0	226	19
	ISC ($j = I$)	10^2	646.03	652	652	0	516	18
		20^2	658.28	670	670	0	647	18
		30^2	660.38	678	678	0	671	18
		100^2	662.76	721	721	0	687	18
		200^2	661.24	780	780	0	681	18
t_2 ($i = 2$)	DSC ($j = D$)	10^2	656.04	660	555	105	622	18
		20^2	655.50	663	565	98	616	18
		30^2	655.41	667	570	97	615	18
		100^2	655.23	694	599	95	613	18
		200^2	652.09	731	636	95	579	18
	ISC ($j = D$)	10^2	742.79	750	652	98	1149	17
		20^2	742.46	757	670	87	1144	17
		30^2	742.32	764	678	86	1142	17
		100^2	742.19	814	721	93	1140	17
		200^2	741.39	885	780	105	1128	17

DSC: decentralized supply chain, ISC: integrated supply chain, 2SOPS: 2-stage-ordering-production system.

order time t_1 and the second order time t_2 under ISC are higher than the optimal advertising costs a_1^D and a_2^D at t_1 and t_2 under DSC.

The reasons are considered as follows: In ISC, there is no transaction cost regarding wholesales of product between a retailer and a manufacturer in the expected profits at the first and second order times t_1 and t_2 in Eqs. (12), (27), (15), and (31). This can result in not only the more optimal total order quantity under ISC than that under DSC, but also the higher expected demand of product. Therefore, it is verified that in ISC, the optimal advertising costs under ISC at t_1 and t_2 can be determined as a higher value than those under DSC from the theoretical analysis and the numerical analysis. At the second order time t_2 , the optimal advertising cost under DSC and ISC are adjusted as higher values so as to satisfy the more demand of product as much as possible at the selling time of product T .

Optimal unit retail price at first and second order time

The optimal unit retail prices p_1^I and p_2^I at the first order time t_1 and the second order time t_2 under ISC are lower than the optimal unit retail prices p_1^D and p_2^D at t_1 and t_2 under DSC.

The reasons are considered as follows: In ISC, there is no transaction cost regarding wholesales of product between a retailer and a manufacturer in the expected profits at the first and second order times t_1 and t_2 in Eqs. (12), (27), (15) and (31). This can result in not only the more optimal total order quantity under ISC than that under DSC, but also the higher expected demand of product. Therefore, it is verified that in ISC, the optimal unit retail prices under ISC at t_1 and t_2 can be determined as a higher value than those under DSC from the theoretical analysis and the numerical analysis. At the second order time t_2 , the optimal unit retail prices under DSC and ISC are adjusted as lower values as to satisfy the more demand of product as much as possible at the selling time of product T .

From the results in Table 1, it is verified that it is profitable for policymakers regarding inventory management to incorporate a 2SOPS into inventory management in supply chains.

Next, as the sensitivity analysis, it is discussed how a change of variance σ^2 in random variable ε from the expected demand $D(a_i^j, p_i^j)(i=1, 2, j=D, I)$ impact the optimal sales strategies under DSC and ISC.

From Table 1, the following results can be seen:

Effect of variance σ^2 in ε on the optimal order quantity

Not only the optimal order quantities Q_1^D and Q_1^I under DSC and ISC at the first order time t_1 , but also the optimal total order quantities $Q_1^D + Q_2^D$ under DSC and $Q_1^I + Q_2^I$ under ISC at the second order time, increase as σ^2 increases. This is because the decision-makers under DSC and ISC try to avoid the shortage of demand of product. Meanwhile, as σ^2 increases, it is verified that the optimal order quantities Q_2^D and Q_2^I at the sec-

ond order time t_2 under DSC and ISC are determined from the aspect of the decision-makers under DSC and ISC by observing the increasing tendency or decreasing tendency for the expected demand depending on the advertising cost and the unit retail price at t_2 .

Effect of variance σ^2 in ε on the optimal advertising cost of product sales

The optimal advertising costs a_1^D and a_1^I under DSC and ISC at the first order time t_1 tend to increase as σ^2 increase in the range of small change from 10^2 to 100^2 . However, a_1^D and a_1^I at t_1 tend to decrease as σ^2 increase in the range of large change with more than 100^2 . This is because the decision-makers under DSC and ISC try to deal with the increase tendency of the optimal order quantities Q_1^D and Q_1^I under DSC and ISC at t_1 as the expected demand increases due to increment of σ^2 . Meanwhile, the optimal advertising costs a_2^D and a_2^I under DSC and ISC at the second order time t_2 tend to decrease as σ^2 increase. This is because the decision-makers under DSC and ISC try to deal with the decrease tendency of the expected demand increases due to increment of σ^2 .

Effect of variance σ^2 in ε on the optimal unit retail price

All the optimal unit retail prices $a_i^j (i=1, 2, j=D, C)$ under DSC and ISC, at the first and second order time $t_i (i=1, 2)$ have no change for the increment of σ^2 in the range of change from 10^2 to 200^2 . This implies that the optimal unit retail prices under DSC and ISC at t_i are insulated from the influence of change of variance σ^2 in ε .

Moreover, as the sensitivity analysis, it is discussed how a change of variance σ^2 in random variable ε from the expected demand $D(a_i^j, p_i^j)(i=1, 2, j=D, I)$ impact the expected profits of a retailer, a manufacturer and the whole system under DSC and ISC. Table 2 shows the results of the expected profits for optimal sales strategy under DSC and ISC in 2SOPS when variance σ^2 in random variable ε from the expected demand changes ε .

From Table 2, the following results can be seen:

Effect of variance σ^2 in ε on the expected profits under DSC and ISC

The expected profits of a retailer and the whole system under DSC and ISC decreases as σ^2 increases. This is because not only the optimal total order quantity and the advertising cost of product sales increase under DSC and ISC as σ^2 increases, but also the order cost of product under DSC and the production cost under ISC increase as the optimal total order quantity increase under DSC and ISC due to the increment of σ^2 .

Meanwhile, the expected profits of the manufacturer increase under DSC and ISC as σ^2 increases. This is because the optimal total order quantity increase under DSC and ISC due to the increment of σ^2 .

Table 2. Results of the expected profits for optimal sales strategy under DSC and ISC in 2SOPS

Order time t_i ($i = 1, 2$)	Type of supply chain ($j = D, I$)	Variance σ^2	Expected profits		
			Retailer	Manufacturer	Whole system
t_1 ($i = 1$)	DSC ($j = D$)	10^2	3423	2230	5653
		20^2	3383	2285	5668
		30^2	3255	2318	5573
		100^2	2243	2523	4766
		200^2	809	2787	3596
	ISC ($j = I$)	10^2	3974	2614	6588
		20^2	4022	2698	6720
		30^2	3905	2740	6645
		100^2	2875	2980	5855
		200^2	1378	3309	4687
t_2 ($i = 2$)	DSC ($j = D$)	10^2	7575	2744	10319
		20^2	7430	2751	10181
		30^2	7283	2765	10048
		100^2	6266	2873	9139
		200^2	4827	3017	7844
	ISC ($j = D$)	10^2	7448	3098	10546
		20^2	7316	3114	10430
		30^2	7171	3141	10312
		100^2	6147	3350	9497
		200^2	4692	3648	8340

DSC: decentralized supply chain, ISC: integrated supply chain, 2SOPS: 2-stage-ordering-production system.

Profitability of a 2SOPS under DSC and ISC

The expected profits of a retailer, a manufacturer and the whole system at the first order time t_1 is compared with those at the second order time t_2 . All the expected profits under DSC and ISC at t_2 are higher than those at t_1 . This is because the optimal additional order quantity under DSC and ISC are determined, and the total order quantities under DSC and ISC $Q_1^j + Q_2^j$ ($j = D, I$) at t_2 are larger than the optimal order quantities under DSC and ISC Q_1^j ($j = D, I$) at t_1 from the results of Table 1.

Comparison of the expected profits under DSC and ISC

The expected profits under DSC with 2SOPS are compared with those under ISC with 2SOPS. The expected profits of a manufacturer and the whole system under ISC at the first and second order times t_1 and t_2 are higher than those under DSC. Meanwhile, the expected profit of a retailer under ISC at the first order time t_1 is higher than that under DSC, but the expected profit of a retailer under ISC at the second order time t_2 is lower than that under DSC as variance σ^2 in random variable ε from the expected demand $D(a_i^j, p_i^j)$ ($i = 1, 2, j = D, I$).

From the total optimization of a supply chain, the optimal sales strategy under ISC is recommended since the expected profit of the whole system can be increased.

However, it is difficult for a retailer who is the leader of the decision-making under DSC to shift the optimal sales strategy under ISC from that under DSC.

As a supply chain coordination under ISC, any reasonable profit sharing is necessary for members under ISC to shift the optimal sales strategy under ISC, guaranteeing more profits to members under ISC than those under DSC. This paper adjusts the unit wholesale prices w_i^N ($i = 1, 2$) between a retailer and a manufacturer at the first and second order time t_i ($i = 1, 2$) as the Nash bargaining solutions obtained from Eqs. (24)–(26). It is investigated how the unit wholesale prices at the first and second order time t_i ($i = 1, 2$) are adjusted and the profit sharing adopting Nash bargaining solutions impact the expected profits of a retailer and a manufacturer.

Table 3 shows the effect of supply chain coordination between a retailer and a manufacturer under ISC.

From Table 3, the following results can be seen:

Effect of supply chain coordination adopting Nash bargaining solutions on the expected profits under ISC

It can be seen that the expected profits of both members under ISC with supply chain coordination at the first and second order time t_i ($i = 1, 2$) are higher than those under DSC by adjusting the unit wholesale prices at the first and second order time t_i ($i = 1, 2$) to w_i^N ($i = 1, 2$) between both members.

Table 3. Effect of supply chain coordination between a retailer and a manufacturer under ISC

variance σ^2	Without supply chain coordination		With supply chain coordination (Nash bargaining solution)		Expected profit under DSC		Expected profit under ISC with supply chain coordination	
	w_1	w_2	w_1^N	w_2^N	Retailer	Manufacturer	Retailer	Manufacturer
10^2	5	7	4.0	11.2	7575	2744	7688	2857
20^2	5	7	3.7	14.3	7430	2751	7554	2876
30^2	5	7	4.4	8.9	7283	2765	7415	2897
100^2	5	7	4.6	6.9	6266	2873	6445	3052
200^2	5	7	3.4	15.2	4827	3017	5076	3265

DSC: decentralized supply chain, ISC: integrated supply chain.

Adjustment of the unit wholesale price at the first and second order times $t_i (i=1,2)$

The unit wholesale price w_2^N adjusted at the second order time t_2 is higher than the unit wholesale price w_1^N adjusted at the first order time t_1 . This is because the optimal total order quantity $Q_1^j + Q_2^j (j = D, I)$ at the second order time t_2 is larger than the optimal order quantity $Q_1^j (j = D, I)$ at the first order time t_1 .

Effect of variance σ^2 in ε on adjustment of the unit wholesale prices at the first and second order times $t_i (i=1, 2)$

The unit wholesale price $w_i^N (i=1,2)$ adjusted at the first and second order times $t_i (i=1,2)$ as supply chain coordination under ISC have neither increasing tendency nor decreasing tendency as σ^2 increases. This is because $w_i^N (i=1, 2)$ is adjusted so as to satisfy Eqs. (32)–(34).

Thus, the adjusting the unit wholesale price $w_i^N (i=1, 2)$ at the first and second order times $t_i (i=1, 2)$ as Nash bargaining solution can guarantee the more profits to all members under ISC.

From the results of Tables 1–3, adopting a 2SOPS enables to not only adjust the optimal sales strategies for the order quantity of a single product, the advertising cost of product sales and the unit retail price under DSC and ISC at the second order time t_2 , but also adjust the unit wholesale price at the first and second order times $t_i (i=1, 2)$. These adjustments in 2SOPS can bring more profits to policy-makers under DSC and ISC.

8. CONCLUSIONS

This paper presented the optimal sales strategy for a 2SOPS consisting of a retailer and a manufacturer. The following 2SOPS was incorporated into a supply chain: 1) twice ordering opportunity of a single product so as to reduce the uncertainty in demand of the product and the shortage penalty cost of the unsatisfied demand of the product, 2) two types of production mode for manufactures to respond to each ordering of decision-makers with make-to-order policy. In addition, two optimal decisions for sales strategy regarding the order quantity of a single product, the advertising cost of product sales and the unit retail price were made in the 2SOPS.

One was made under a DSC whose objective was to maximize the retailer’s profit. The other was made under an ISC whose objective was to maximize the whole system’s profit.

In the numerical analysis, the results of the optimal decisions under DSC with a 2SOPS were compared with those under ISC with a 2SOPS. Furthermore, it was investigated how variance of random variable from the expected demand impacted the optimal sales strategies and the expected profits under DSC and ISC. Moreover, supply chain coordination was discussed in order to encourage all members to shift the optimal sales strategy under ISC from that under DSC. The unit wholesale prices at the first and second order times $t_i (i=1, 2)$ were adjusted as Nash bargaining solutions between a retailer and a manufacturer under ISC.

This paper contributed the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to both academic researchers and real-world policymakers who try to make the optimal production planning in a supply chain adopting a 2SOPS:

- Profitability obtained from a 2SOPS by comparing the optimal sales strategies and the expected profits under DSC and ISC at the first order time t_1 with those at the second order t_2 .
- Effect of change of variance σ^2 in random variable ε from the expected demand on the sales strategies and the expected profits under DSC and ISC,
- Comparison of the expected profits under DSC and ISC,
- Effect of supply chain coordination on not only the adjustment of the unit wholesale prices at the first and second order times t_1 and t_2 , but also the expected profits of all members under ISC.

Therefore, it is highly expected that research outcomes in this paper would provide not only the optimal solution and its practices to construct a supply chain in the following situations: 1) a 2SOPS with two ordering opportunities and two production modes at the first and second order times is adopted, 2) the demand of a single product is uncertain for the advertising cost and the unit retail price, 3) supply chain coordination adjusting the

unit wholesale price is adopted. Thus, the above research outcomes can give informative motivations to researchers and policymakers who try to make the optimal inventory management in a green supply chain.

This paper incorporated the following topics into a supply chain:

- A 2SOPS with two production modes with two types of the unit production cost and twice ordering opportunity with two types of the unit wholesale price is incorporated into a supply chain,
- The optimal sales strategy for the order quantity of a single product, the advertising cost of product sales and the unit retail price were adjusted based on the demand distribution of a single product updated at the first and second order times before the sales of product,
- The uncertainty in demand was considered as the additive random variable from the expected demand depending on the advertising cost of product sales and the unit retail price,
- As supply chain coordination, the unit wholesale prices at the first and second order times were adjusted as Nash bargaining solutions.

As the extendable consideration for the proposed model in this paper, it will be necessary to discuss the following issues to analyze the optimal sales strategy in a supply chain with a 2SOPS:

- Consecutive adjustment of the optimal sales strategy not only at each order time, but also during the sales period of product;
- Impact of delivery lead time of products on the order quantity;
- Different type of modeling of the uncertainty in demand;
- Situation where the multiple types of products are handled in a supply chain;
- Impact of supply disruptions regarding either natural disasters or production failures (such as machine breakdowns or human errors);
- Situation where the order quantity of products is different from the production quantity (the order quantity of products is optimized by retailers and the production quantity of products are optimized by manufacturers) and
- Proposal of an alternative approach of profit sharing as supply chain coordination to promote a retailer-manufacturer partnership by combining each member's cost performance with profit sharing between all members in a supply chain.

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