THERMAL DIFFUSION AND RADIATION EFFECTS ON UNSTEADY MHD FREE CONVECTION HEAT AND MASS TRANSFER FLOW PAST A LINEARLY ACCELERATED VERTICAL POROUS PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

M. VENKATESWARLU^{1,†}, G. V. RAMANA REDDY², AND D. V. LAKSHMI³

 $^1\mathrm{Department}$ Of Mathematics, V. R. Siddhartha Engineering College, Kanur, Krishna (Dist), Andhra Prades, India 520 007

E-mail address: mvsr2010@gmail.com, mvenkateswarlu@vrsiddhartha.ac.in

 2 Department Of Mathematics, K. L University, Vaddeswaram, Guntur (Dist), Andhra Pradesh, India 522 502

 3 Department Of Mathematics, Bapatla Women's Engineering College, Bapatla, Guntur (Dist). Andhra Pradesh, India 522 102

ABSTRACT. The objective of the present study is to investigate thermal diffusion and radiation effects on unsteady MHD flow past a linearly accelerated vertical porous plate with variable temperature and also with variable mass diffusion in presence of heat source or sink under the influence of applied transverse magnetic field. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. At time t>0, the plate is linearly accelerated with a velocity $u=u_0t$ in its own plane. And at the same time, plate temperature and concentration levels near the plate raised linearly with time t. The dimensionless governing equations involved in the present analysis are solved using the closed analytical method. The velocity, temperature, concentration, skin-friction, the rate or heat transfer and the rate of mass transfer are studied through graphs in terms of different physical parameters like magnetic field parameter (M), radiation parameter (R), Schmidt parameter (Sc), Soret number (So), Heat source parameter (S), Prandtl number (Pr), thermal Grashof number (Gr), mass Grashof number (Gm) and time (t).

1. Introduction

The study of magnetohydrodynamics with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. In engineering we find its applications like in MHD

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[†] Corresponding author.

pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow the study of effects of magnetic field play a major rule in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radioactive flows are encountered in many industrial and environment processes, e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Steady free convection flow of incompressible viscous fluid past an infinite or semi-infinite vertical plate is studied since long because of its technological importance. Pohlhausen [1] was the first to study it for a flow past or semi- infinite vertical plate by integral method. Similarity solution to this problem was given by Ostrach [2]. Siegel [3] studied the transient free convective flow past a semi- infinite vertical plate by integral method. The same problem was studied by Gebhart [4] by an approximate method. The study of magneto-hydrodynamic flow for electrically conducting fluid past heated surface has attracted the interest of many researches in view of its important applications in many engineering problems such as plasma studies, petroleum industries MHD power generations, cooling of nuclear reactors, the boundary layer control in aerodynamics and crystal growth. Until recently this study has been largely concerned with flow and heat transfer characteristics in various physical situations. Watanabe and Pop [5] investigated the heat transfer in the thermal boundary layer of magneto-hydrodynamic flow over a flat plate. Michiyochi *et al.* [6] considered natural convection heat transfer from a horizontal cylinder to the mercury under a magnetic field. Vajravelu and Nayfeh [7] studied hydro magnetic convection at a cone and a wedge.

The study of magnetic-hydrodynamic free convection through a viscous fluid past a semiinfinite plate is considered very essential to understand the behavior of the performance of the fluid motion in several applications. It serves as the basis for understanding some of the important phenomena occurring in heat exchange devices. MHD free convection flows past a semi- infinite vertical plate have been studied in different physical condition by Sparrow and Cess [8], Riley [9] and others. The Problems mentioned above are concerned with thermal convection only. But in nature along with free convection currents caused by the temperature differences, the flow is also affected by the differences in material constitution, for example, in atmospheric flows there exist differences in H₂O concentration and hence the flow is affected by such concentration difference. In many engineering applications, the foreign gases are injected. This causes a reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Usually, H₂O, CO₂, etc are the foreign gases, which are injected in the air flowing past bodies. The effects of foreign mass, also known as diffusing species concentration were studied under different conditions by Somers [10], Mathers et al. [11], and others either by integral method or by asymptotic analysis. But the first systematic study of mass transfer effects on free convection flow past a semi-infinite vertical plate was presented by Gebhart and Pera [12] who presented a similarity solution to this problem and introduced a parameter N which is a measure of relative importance of chemical and thermal diffusion causing a density difference that drives the flow the parameter N is positive when both effects combined to drive the flow and it is negative when these effects are opposed.

Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [13]. Callahan and Manner [14] first considered the transient free convection flow past a semiinfinite plate by explicit finite difference method. They also considered the presence of species concentration. However this analysis is not applicable for other fluids whose Prandtl number is different from unity. Soundalgekar and Ganesan [15] analyzed transient free convective flow past a semi-infinite vertical flat plate, taking into account mass transfer by an implicit finite difference method of Crank-Nicolson type. In their analysis they observed that an increase in the N leads to an increase in the velocity but a decrease in the temperature and concentration. Elbashbeshy [16] studied heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of magnetic field. Aboeldahab and Elbarbary [17] took into account the Hall current effect on the MHD free convection heat and mass transfer over a semi-infinite vertical plate upon which the flow subjected to a strong external magnetic field. Chen [18] studied heat and mass transfer in HD flow by natural convection from a permeable inclined surface with variable temperature and concentration using Keller box finite difference method and found that an increase in the value of temperature exponent m leads to a decrease in the local skin friction, Nusselt and Sherwood numbers. Takhar et al. [19] considered the unsteady free convection flow over a semi-infinite vertical plate. Ganesan and Rani [20] studied the unsteady free convection on vertical cylinder with variable heat and mass flux.

Heat transfer by simultaneous radiation and convection has applications in numerous technological problems, including combustion, furnace design, the design of high temperature gas cooled nuclear reactors, nuclear reactor safety, fluidized bed heat exchanger, fire spreads, advance energy conversion devices such as open cycle coal and natural gas fired MHD, solar fans, solar collectors natural convection in cavities, turbid water bodies, photo chemical reactors and many others when heat transfer by radiation is of the same order of magnitude as by convection, a separate calculation of radiation and convection and their superposition without considering the interaction between them can lead to significant errors in the results, because of the presence of the radiation in the medium, which alters the temperature distribution within the fluid. Therefore, in such situation heat transfer by convection and radiation should be solved for simultaneously. In this context, Abd El-Naby et al. [21] studied the effects of radiation on unsteady free convective flow past a semi-infinite vertical plate with variable surface temperature using Crank-Nicolson finite difference method. They observed that, both the velocity and temperature are found to decrease with an increase in the temperature exponent. Chamkha et al. [22] analyzed the effects of radiation on free convection flow past a semi-infinite vertical plate with mass transfer by taking into account the buoyancy ratio parameter N. In their analysis they found that, as the distance from the leading edge increase, both the velocity and temperature decrease, whereas the concentration increases.

Ganesan and Loganadhan [23] studied the radiation and mass transfer effects on flow of incompressible viscous fluid past a moving vertical cylinder using Resseland approximation

by The Crank-Nicolson finite difference method. Takhar et al. [24] considered the effects of radiation on MHD free convection flow of a radiating gas past a semi-infinite vertical plate. In most of the studies mentioned above, viscous dissipation is neglected. Gebhart and Mollendorf [26] considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [27] analyzed viscous dissipative heat on the two dimensional unsteady flow past an infinite vertical porous plate when the temperature oscillates in time there is constant suction on the plate. Israel-Cookey et al. [28] investigated the influence of viscous dissipation and radiation on MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Gokhaleand Samman [29] studied the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Ramana Reddy et al. [30] considered effects of radiation and mass transfer on MHD free convective dissipative fluid in the presence of heat source/sink. Recently, Ramana Reddy et al. [31] investigated the chemical and radiation absorption effects on MHD convective heat and mass transfer flow past a semi-infinite vertical moving porous plate with time dependent suction. Turkyilmazoglu et al [32] were analysed the Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate. Gangadhar and Bhaskar Reddy [33] proposed the chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction.

In spite of all the previous studies, the effects of thermal radiation on unsteady MHD flow past an impulsively started linearly accelerated infinite vertical porous plate with variable temperature and also with variable mass diffusion in the presence of heat source or sink and transverse applied magnetic field. The dimensionless governing equations involved in the present analysis are solved using closed analytical method. The behavior of the Velocity, Temperature, Concentration, Skin-friction, Nusselt number and Sherwood number has been discussed qualitatively for variations in the governing parameters.

2. FORMATION OF THE PROBLEM

We consider thermal-diffusion and radiation effects on unsteady MHD flow past of a viscous incompressible, electrically conducting, radiating fluid past an impulsively started linearly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of Heat source/sink under the influence of applied transverse magnetic field. The plate is taken along x-axis in vertically upward direction and y-axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the same temperature T'_{∞} and concentration level C'_{∞} in stationary condition for all the points. At time t'>o, the plate is linearly accelerated with a velocity $u=u_0t'$ in the vertical upward direction against to the gravitational field. And at the same time the plate temperature is raised linearly with time t and also the mass is diffused from the plate to the fluid is linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The viscous dissipation and induced magnetic field are assumed to be negligible. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Then under by usual Boussinesq's approximation, the

unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta \left(T' - T'_{\infty} \right) + g\beta^* \left(C' - C'_{\infty} \right) + \frac{\sigma B_0}{\rho} u' - \frac{\nu}{K'} u' \tag{2.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho C_p} \left[k \frac{\partial^2 T'}{\partial y'^2} - Q_0 \left(T' - T_{\infty}' \right) \right] - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'}$$
 (2.2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} + D_1 \frac{\partial^2 T'}{\partial y'^2} \tag{2.3}$$

where x',y' are the dimensional distance along and perpendicular to the plate, respectively. u' and v' are the velocity components in the x',y' directions respectively, g is the gravitational acceleration, ρ is the fluid density, β and β^* are the thermal and concentration expansion coefficients respectively, K' is the Darcy permeability, ν is the kinematic viscosity, k is the thermal diffusivity of the fluid, B_0 is the magnetic induction, T' is the thermal temperature inside the thermal boundary layer and C' is the corresponding concentration, σ is the electric conductivity, C_p is the specific heat at constant pressure, D is the molecular diffusion coefficient, q_r is the heat flux, Q_0 is the dimensional heat absorption coefficient, and D_1 is the coefficient of thermal diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{array}{ll} t' \leq 0: & u' = 0, & T' = T_{\infty}' \quad C' = C_{\infty}' \text{ for all } y' \\ t' > 0: & \begin{cases} u' = u_0 t', \, T' = T_{\infty}' + \varepsilon \left(T_w' - T_{\infty}' \right) A t', \, C' = C_{\infty}' + \varepsilon \left(C_w' - C_{\infty}' \right) A t' \text{ at } y' = 0 \\ u' = 0, \quad T' \to T_{\infty}', \qquad C' \to C_{\infty}', \qquad \text{as } y' \to \infty \end{cases} \end{aligned} \tag{2.4}$$

where $\varepsilon \ll 1$ is a positive constant, T_w' —the fluid temperature at the plate, T_∞' —the fluid temperature in the free stream, C_∞' —Species concentration in the free stream, C_w' —Species concentration at the surface and $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$q_r = -4a^* \sigma^* T_{\infty}^{'3} \left(T_{\infty}^{'4} - T_{\infty}^{'4} \right)$$
 (2.5)

where σ^* and a^* are the Stefan-Boltzmann constant and the Mean absorption coefficient respectively.

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus we get

$$T'^{4} \cong 4T'^{3}_{\infty}T' - 3T'^{4}_{\infty}. \tag{2.6}$$

From equations (2.5) and (2.6), equation (2.2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma^* T'^3_{\infty} \left(T'_{\infty} - T' \right). \tag{2.7}$$

On introducing the following non-dimensional quantities:

$$y = \frac{u_0 y'}{\nu}, u = \frac{u'}{u_0}, t = \frac{t' u_0^2}{4\nu}, \theta = \frac{T' - T'_{\infty}}{T'_{w} - T'_{\infty}}, C = \frac{C' - C'_{\infty}}{C'_{w} - C'_{\infty}}, K = \frac{K' u_0^2}{\nu^2},$$

$$Gr = \frac{g\beta\nu(T'_{w} - T'_{\infty})}{v_0^3}, \Pr = \frac{\mu\rho C_p}{k}, Gm = \frac{g\beta^*\nu(C'_{w} - C'_{\infty})}{u_0^3}, Sc = \frac{\nu}{D},$$

$$R = \frac{16a^*\nu^2\sigma^*T'_{\infty}^3}{ku_0^2}, M = \frac{\sigma B_0^2\nu}{\rho u_0^2}, S_0 = \frac{D_1(T'_{w} - T'_{\infty})}{\nu(C'_{w} - C'_{\infty})}, H = \frac{Q'\nu^2}{\kappa u_0^2}.$$
(2.8)

The governing equations for the momentum, the energy, and the concentration in a dimensionless form are

$$\frac{\partial u}{\partial t} = Gr\theta + GmC + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u\tag{2.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - \left(\frac{R+H}{\Pr}\right) \theta \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial u^2} + S_0 \frac{\partial^2 \theta}{\partial u^2} \tag{2.11}$$

where Gr, Gm, M, K, Pr, R, H, Sc, and S_0 are the Grashof number, modified Grashof number, magnetic parameter, permeability parameter, Prandtl number, radiation parameter, heat source parameter, Schmidt number and Soret number, respectively.

The relevant corresponding boundary conditions for t > 0 are transformed to:

$$u=t, \quad \theta=t, \quad C=t \quad \text{at} \quad y=0 \\ u\to 0, \quad \theta\to 0, \quad C\to 0 \quad \text{as} \quad y\to \infty.$$
 (2.12)

3. SOLUTION OF THE PROBLEM

In order to solve equations (2.9) - (2.11) with respect to the boundary conditions (2.12) for the flow, let us take

$$u(y,t) = u_0(y) e^{i\omega t}$$
(3.1)

$$\theta(y,t) = \theta_0(y) e^{i\omega t}$$
(3.2)

$$C(y,t) = C_0(y)e^{i\omega t}$$
(3.3)

where ω - is the frequency of oscillation.

Substituting the Equations (3.1) - (3.3) in Equations (2.9) - (2.11), we obtain:

$$u_0'' - \left(M + i\omega + \frac{1}{K}\right)u_0 = -\left[Gr\theta_0 + GmC_0\right]$$
 (3.4)

$$\theta_0'' - A_1^2 \theta_0 = 0 (3.5)$$

$$C_0'' - A_2^2 C_0 = -ScS_0 \theta_0'' \tag{3.6}$$

where prime denotes the ordinary differentiation with respect to y.

The corresponding boundary conditions can be written as

$$u_0 = te^{-i\omega t}, \ \theta_0 = te^{-i\omega t}, C_0 = te^{-i\omega t} \text{ at } y = 0$$

$$u_0 \to 0, \quad \theta_0 \to 0, \quad C_0 \to 0 \quad \text{as } y \to \infty.$$

$$(3.7)$$

Solving equations (3.4) - (3.6) under the boundary conditions (3.7), we obtain the velocity, temperature and concentration distribution in the boundary layer as:

$$u(y,t) = A_{10}e^{-A_1y} + A_7e^{-A_2y} + A_9e^{-A_5y}$$
(3.8)

$$\theta\left(y,t\right) = te^{-A_{1}y}\tag{3.9}$$

$$C(y,t) = A_4 e^{-A_2 y} + A_3 e^{-A_1 y}$$
(3.10)

where

$$\begin{split} A_1 &= \sqrt{R + H + i\omega \Pr}, \quad A_2 = \sqrt{i\omega Sc}, \\ A_3 &= -\frac{ScS_0A_1^2t}{A_1^2 - A_2^2}, \quad A_4 = t - A_3, \\ A_5 &= \sqrt{M + i\omega + \frac{1}{K}}, \quad A_6 = -\frac{tGr}{A_1^2 - A_2^2}, \\ A_7 &= -\frac{A_4Gm}{A_2^2 - A_5^2}, \quad A_8 = -\frac{A_3Gm}{A_1^2 - A_5^2}, \\ A_9 &= -\left[A_6 + A_7 + A_8 + t\right], \quad A_{10} = A_6 + A_8. \end{split}$$

SKIN FRICTION:

Knowing the velocity field, the skin – friction at the plate can be obtained, which in non –dimensional form is given by

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = A_1 A_{10} + A_2 A_7 + A_5 A_9.$$
 (3.11)

NUSSELT NUMBER:

From temperature field, now we study Nusselt number (rate of change of heat transfer) which is given in non-dimensional form as

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = tA_1. \tag{3.12}$$

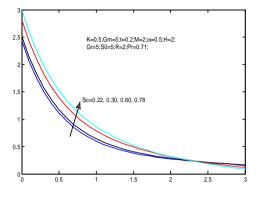
SHERWOOD NUMBER:

From concentration field, now we study Sherwood number (rate of change of mass transfer) which is given in non-dimensional form as

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = A_2 A_4 + A_1 A_3.$$
 (3.13)

4. RESULTS AND DISCUSSION

In order to get the physical insight into the problem, we have plotted velocity, temperature, concentration, the rate of heat transfer and the rate of mass transfer for different values of the physical parameters like Radiation parameter (R), Magnetic parameter (M), Heat source parameter (H), Soret number (S_0) , Schmidt number (Sc), Thermal Grashof number (Gr), Mass Grashof number (Gm) and Prandtl number (Pr) in Fig. 1 to 12. In the present study following default parameter values are adopted for computations: Gm = 5.0, Gr = 5.0, $\omega = 0.5$, R = 5.0, t = 0.4, t = 0.4, t = 0.5, t = 0.5, t = 0.7, t = 0.

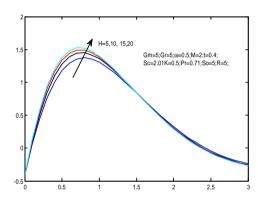


R=1, 2, 3, 4

Gm=5;Gr=5;ω=0.5;M=2;t=0.4;
Sc=2.01;K=0.5;Pr=0.71;So=5;H=5;

FIGURE 1. Effects of Magnetic parameter on velocity profiles

FIGURE 2. Effects of Radiation parameter on velocity profiles



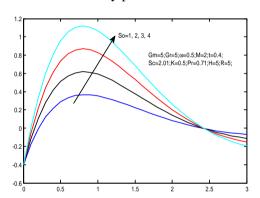


FIGURE 3. Effects of Heat source parameter on velocity profiles

FIGURE 4. Effects of Soret number on velocity profiles

The contribution of the Magnetic field on the velocity profiles is noticed in Fig. 1. It is observed that as the magnetic intensity increases, the velocity field decreases throughout the analysis as long as the radiation parameter is held constant. Further, it is also noticed that the velocity of the fluid medium raises within the boundary layer region and thereafter, it decreases which clearly indicates that the radiation parameter has not that much of significant effect as was in the initial stage.

From Fig. 2 and Fig. 3 are observed that with the increase of radiation parameter (R) or Heat source parameter(H), the velocity increases up to certain y value (distance from the plate) and decreases later for the case of cooling of the plate. But a reverse effect is observed in the case of heating of the plate.

It is seen that from Fig. 4, the velocity increases with an increasing Soret number (S_0) and a reverse effect is also identified.

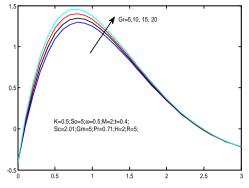


FIGURE 5. Effects of Grashof number on

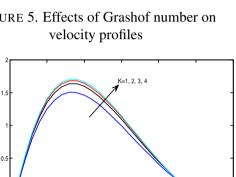


FIGURE 7. Effects of Permeability parameter on velocity profiles

Gr=5:So=5:\u00f3=0.5:M=2:t=0.4:

Sc=2.01;Gm=5;Pr=0.71;H=2;R=5;

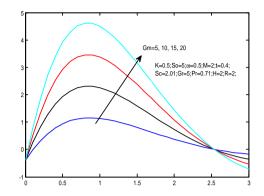


FIGURE 6. Effects of solutal Grashof number on velocity profiles

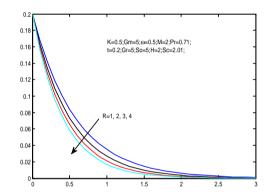


FIGURE 8. Effects of Radiation parameter on temperature profiles

The influence of thermal Grashof number (Gr) and mass Grashof number (Gm) on the velocity field are illustrated graphically in Fig. 5 and Fig. 6. When all the parameter are held constant, and as the thermal Grashof number or mass Grashof number is increased, in general the fluid velocity increases and also reverse effect is observed when mass Grashof number is increases.

Fig. 7 illustrates that the velocity profiles for the different values of permeable parameter K. it is observed that the increasing values of permeability parameter (K) the velocity profiles is also increases.

The temperature of the flow field are mainly affected by the flow parameters, namely, radiation and heat source parameter (H) are observed in Fig. 8 and Fig. 9 It is observed that as radiation parameter R or heat source parameter H are increases the temperature of the flow field decreases at all the points in flow region.

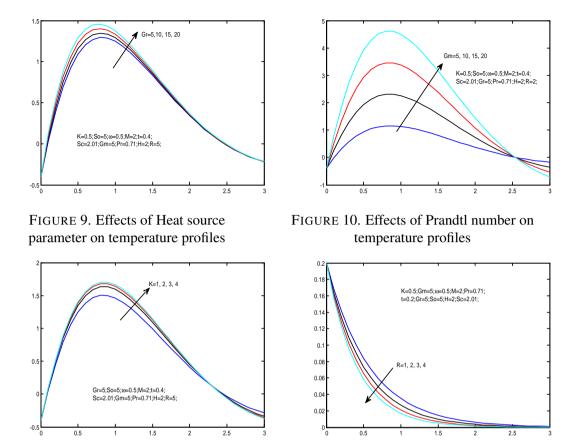


FIGURE 11. Effects of Schmidt number on concentration profiles

FIGURE 12. Effects of Soret number on concentration profiles

Fig. 10 shows the plot of temperature of the flow field against for different values of Prandtl number (Pr). It is observed that the temperature of the flow field decreases in magnitude as Pr increases. It is also observed that the temperature for air (Pr=0.71) is greater than that of water (Pr=7.0). This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting decreases in thermal boundary layer.

The concentration distributions of the flow field are displayed through figures 11 & 12. It is affected by two flow parameters, namely Schmidt number (Sc), Soret number (So) respectively. In Fig. 11, it is observed that the Schmidt number increases the concentration field increases. From the Fig. 12, it is clear that the concentration increases with an increasing of Soret number.

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