J. Appl. Math. & Informatics Vol. **32**(2014), No. 5 - 6, pp. 687 - 695
http://dx.doi.org/10.14317/jami.2014.687

ZWEIER I-CONVERGENT DOUBLE SEQUENCE SPACES DEFINED BY ORLICZ FUNCTION†

VAKEEL A. KHAN* AND NAZNEEN KHAN

Abstract. In this article we introduce the zweier double sequence spaces ${}_2\mathcal{Z}^I(M)$, ${}_2\mathcal{Z}_0^I(M)$ and ${}_2\mathcal{Z}_\infty^I(M)$ using the Orlicz function M. We study the algebraic properties and inclusion relations on these spaces.

AMS Mathematics Subject Classification : 65H05, 65F10.
Key words and phrases : Double sequences, Ideal, filter, Orlicz function, I-convergent, I-null, solid, Zweier spaces.

1. Introduction

Let $\mathbb{N}$, $\boldsymbol{R}$ and $\mathbb{C}$ be the sets of all natural, real and complex numbers respectively. We write

$$\omega = \{x = (x_{ij}) : x_{ij} \in \boldsymbol{R} \times \boldsymbol{R} \text{ } or \text{ } \mathbb{C} \times \mathbb{C}\},$$

the space of all double sequences real or complex.

Let ℓ_∞, c and c_0 denote the Banach spaces of bounded, convergent and null sequences respectively normed by

$$||x||_\infty = \sup_k |x_k|.$$

At the initial stage the notion of I-convergence was introduced by Kostyrko,Šalát and Wilczyński [1]. Later on it was studied by Šalát, Tripathy and Ziman[2], Demirci [3] and many others. I-convergence is a generalization of Statistical Convergence.

Now we have a list of some basic definitions used in the paper .

Definition 1.1 ([4,5])**.** Let X be a non empty set. Then a family of sets I⊆ 2^X(2^X denoting the power set of X) is said to be an ideal in X if
(i) $\emptyset \in$ I
(ii) I is additive i.e A,B∈I ⇒ A ∪ B∈I.
(iii) I is hereditary i.e A∈I, B⊆A⇒B∈I.

Received March 29, 2014. Revised May 26, 2014. Accepted June 9, 2014. *Corresponding author. †This work was supported by the research grant of Maulana Azad National Fellowship, University Grants Commission, India.

For more details see [6,7,8,9,10]. An Ideal I$\subseteq 2^X$ is called non-trivial if I$\neq 2^X$. A non-trivial ideal I$\subseteq 2^X$ is called admissible if $\{\{x\} : x \in X\} \subseteq$I.

A non-trivial ideal I is maximal if there cannot exist any non-trivial ideal J$\neq$I containing I as a subset.

For each ideal I, there is a filter $\pounds$(I) corresponding to I. i.e

$$\pounds(I) = \{K \subseteq N : K^c \in I\}, \text{ where } K^c = N - K.$$

Definition 1.2. A double sequence of complex numbers is defined as a function $x : \mathbb{N} \times \mathbb{N} \to \mathbb{C}$. We denote a double sequence as (x_{ij}), where the two subscripts run through the sequence of natural numbers independent of each other. A number $a \in \mathbb{C}$ is called a double limit of a double sequence (x_{ij}) if for every $\epsilon >\ 0$ there exists some $N = N(\epsilon) \in \mathbb{N}$ such that

$$|x_{ij} - a| < \epsilon,\ \forall\ i,j \geq\ N\ (see\ [11,12,13])$$

Definition 1.3 ([12])**.** A double sequence $(x_{ij}) \in \omega$ is said to be I-convergent to a number L if for every $\epsilon > 0$,

$$\{i,j \in \mathbb{N} : |x_{ij} - L| \geq \epsilon\} \in I.$$

In this case we write $I - \lim x_{ij} = L$.

Definition 1.4 ([12])**.** A double sequence $(x_{ij}) \in \omega$ is said to be I-null if L = 0. In this case we write

$$I - \lim x_{ij} = 0.$$

Definition 1.5 ([12])**.** A double sequence $(x_{ij}) \in \omega$ is said to be I-Cauchy if for every $\epsilon > 0$ there exist numbers m = m(ϵ), n= n(ϵ) such that

$$\{i,j \in \mathbb{N} : |x_{ij} - x_{mn}| \geq \epsilon\} \in I.$$

Definition 1.6 ([12])**.** A double sequence $(x_{ij}) \in \omega$ is said to be I-bounded if there exists $M > 0$ such that

$$\{i,j \in \mathbb{N} : |x_{ij}| > M\}.$$

Definition 1.7 ([12])**.** A double sequence space E is said to be solid or normal if $(x_{ij}) \in$ E implies $(\alpha_{ij}x_{ij}) \in$ E for all sequence of scalars (α_{ij}) with $|\alpha_{ij}| < 1$ for all i,j $\in \mathbb{N}$.

Definition 1.8 ([12])**.** A double sequence space E is said to be monotone if it contains the canonical preimages of its stepspaces.

Definition 1.9 ([12])**.** A double sequence space E is said to be convergence free if $(y_{ij}) \in E$ whenever $(x_{ij}) \in E$ and $x_{ij} = 0$ implies $y_{ij} = 0$.

Definition 1.10 ([12])**.** A double sequence space E is said to be a sequence algebra if $(x_{ij}.y_{ij}) \in E$ whenever $(x_{ij}), (y_{ij}) \in E$.

Definition 1.11 ([12])**.** A double sequence space E is said to be symmetric if $(x_{ij}) \in E$ implies $(x_{\pi(ij)}) \in E$, where π is a permutation on $\mathbb{N}$.

Any linear subspace of ω, is called a sequence space.

A sequence space λ with linear topology is called a K-space provided each of maps $p_i \longrightarrow \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for all $i \in \mathbb{N}$.

A K-space λ is called an FK-space provided λ is a complete linear metric space.

An FK-space whose topology is normable is called a BK-space.

Let λ and μ be two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n, k \in \mathbb{N}$. Then we say that A defines a matrix mapping from λ to μ, and we denote it by writing $A : \lambda \longrightarrow \mu$.

If for every sequence $x = (x_k) \in \lambda$ the sequence $Ax = \{(Ax)_n\}$, the A transform of x is in μ, where

$$(Ax)_n = \sum_k a_{nk}x_k, \ (n \in \mathbb{N}). \ (1)$$

By $(\lambda : \mu)$, we denote the class of matrices A such that $A : \lambda \longrightarrow \mu$.

Thus, $A \in (\lambda : \mu)$ if and only if the series on the right side of (1) converges for each $n \in \mathbb{N}$ and every $x \in \lambda$. (see[14]).

The approach of constructing the new sequence spaces by means of the matrix domain of a particular limitation method have been recently studied by Başar and Altay[15], Malkowsky[16], Ng and Lee[17] and Wang[18], Başar, Altay and Mursaleen[19].

Şengönül[20] defined the sequence $y = (y_i)$ which is frequently used as the Z^p transform of the sequence $x = (x_i)$ i.e,

$$y_i = px_i + (1-p)x_{i-1}$$

where $x_{-1} = 0, p \neq 1$, $1 < p < \infty$ and Z^p denotes the matrix $Z^p = (z_{ik})$ defined by

$$z_{ik} = \begin{cases} p, (i = k), \\ 1-p, (i-1 = k); (i, k \in \mathbb{N}), \\ 0, \text{otherwise}. \end{cases}$$

Following Basar and Altay [15], Şengönül[20] introduced the Zweier sequence spaces $\mathcal{Z}$ and $\mathcal{Z}_0$ as follows

$$\mathcal{Z} = \{x = (x_k) \in \omega : Z^p x \in\}$$

$$\mathcal{Z}_0 = \{x = (x_k) \in \omega : Z^p x \in c_0\}.$$

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$, which is continuous, non-decreasing and convex with $M(0) = 0, M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.(see[21,22]).

Lindenstrauss and Tzafriri[22] used the idea of Orlicz functions to construct the sequence space

$$\ell_M = \{x \in \ell^0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) < \infty, \text{for some } \rho > 0\}$$

The space ℓ_M is a Banach space with the norm

$$||x|| = \inf\{\rho > 0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) \leq 1\}$$

The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(x) = x^p$ for $1 \leq p < \infty$ (c.f [23],[24],[25]).

The following Lemmas will be used for establishing some results of this article.

Lemma 1.12 ([24])**.** *A sequence space E is solid implies that E is monotone.*

Lemma 1.13 ([26,27,28])**.** *Let $K \in \pounds(I)$ and $M \subseteq N$. If $M \notin I$, then$M \cap K \notin I$.*

Lemma 1.14 ([26,27,28])**.** *If $I \subset 2^N$ and $M \subseteq N$. If $M \notin I$, then $M \cap K \notin I$.*

Recently Vakeel.A.Khan et. al.[29] introduced and studied the following classes of sequence spaces.

$$\mathcal{Z}^I = \{k \in \mathbb{N} : \{x = (x_k) \in \omega : I - \lim Z^p x = L \text{ for some L}\}\} \in I$$
$$\mathcal{Z}_0^I = \{k \in \mathbb{N} : \{x = (x_k) \in \omega : I - \lim Z^p x = 0\}\} \in I$$
$$\mathcal{Z}_\infty^I = \{k \in \mathbb{N} : \{x = (x_k) \in \omega : \sup_k |Z^p x| < \infty\}\} \in I$$

We also denote by

$$m_{\mathcal{Z}}^I = \mathcal{Z}_\infty^I \cap \mathcal{Z}^I$$

and

$$m_{\mathcal{Z}_0}^I = \mathcal{Z}_\infty^I \cap \mathcal{Z}_0^I.$$

2. Main results

In this article we introduce the following classes of zweier I-Convergent double sequence spaces defined by the Orlicz function.

$${}_2\mathcal{Z}^I(M) = \{x = (x_{ij}) \in \omega : I - \lim M(\frac{|x'_{ij} - L|}{\rho}) = 0 \text{ for some L and } \rho > 0\},$$
$${}_2\mathcal{Z}_0^I(M) = \{x = (x_{ij}) \in \omega : I - \lim M(\frac{|x'_{ij}|}{\rho}) = 0 \text{ for some } \rho > 0\},$$
$${}_2\mathcal{Z}_\infty^I(M) = \{x = (x_{ij}) \in \omega : \sup_{i,j} M(\frac{|x'_{ij}|}{\rho}) < \infty \text{ for some } \rho > 0\}.$$

Also we denote by

$${}_2m_{\mathcal{Z}}^I(M) ={}_2 \mathcal{Z}_\infty^I(M) \cap_2 \mathcal{Z}^I(M)$$

and

$${}_2m_{\mathcal{Z}_0}^I(M) ={}_2 \mathcal{Z}_\infty^I(M) \cap_2 \mathcal{Z}_0^I(M).$$

Throughout the article, for the sake of convenience, we will denote by
$Z^p(x_k) = x'$, $Z^p(y_k) = y'$, $Z^p(z_k) = z'$ for $x, y, z \in \omega$.

Theorem 2.1. *For any Orlicz function M, the classes of sequences ${}_2\mathcal{Z}^I(M)$, ${}_2\mathcal{Z}_0^I(M)$, ${}_2m_{\mathcal{Z}}^I(M)$ and ${}_2m_{\mathcal{Z}_0}^I(M)$ are linear spaces.*

Proof. We shall prove the result for the space ${}_2\mathcal{Z}^I(M)$. The proof for the other spaces will follow similarly. Let $(x_{ij}), (y_{ij}) \in {}_2\mathcal{Z}^I(M)$ and let α, β be scalars. Then there exists positive numbers ρ_1 and ρ_2 such that

$$I - \lim M(\frac{|x'_{ij} - L_1|}{\rho_1}) = 0, \text{ for some } L_1 \in \mathbb{C};$$
$$I - \lim M(\frac{|y'_{ij} - L_2|}{\rho_2}) = 0, \text{ for some } L_2 \in \mathbb{C}.$$

That is for a given $\epsilon > 0$, we have

$$A_1 = \{(i,j) \in \mathbb{N} \times \mathbb{N} : M(\frac{|x'_{ij} - L_1|}{\rho_1}) > \frac{\epsilon}{2}\} \in I, \tag{1}$$
$$A_2 = \{(i,j) \in \mathbb{N} \times \mathbb{N} : M(\frac{|y'_{ij} - L_2|}{\rho_2}) > \frac{\epsilon}{2}\} \in I. \tag{2}$$

Let $\rho_3 = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since M is non-decreasing and convex function, we have

$$M(\frac{|(\alpha x'_{ij} + \beta y'_{ij}) - (\alpha L_1 + \beta L_2)|}{\rho_3}) \leq M(\frac{|\alpha||x'_{ij} - L_1|}{\rho_3}) + M(\frac{|\beta||y'_{ij} - L_2|}{\rho_3}).$$
$$\leq M(\frac{|x'_{ij} - L_1|}{\rho_1}) + M(\frac{|y'_{ij} - L_2|}{\rho_2})$$

Now, by (1) and (2),

$$\{(i,j) \in \mathbb{N} \times \mathbb{N} : M(\frac{|(\alpha x'_{ij} + \beta y'_{ij}) - (\alpha L_1 + \beta L_2)|}{\rho_3}) > \epsilon\} \subset A_1 \cup A_2.$$

Therefore $(\alpha x_{ij} + \beta y_{ij}) \in {}_2\mathcal{Z}^I(M)$. Hence ${}_2\mathcal{Z}^I(M)$ is a linear space. □

Theorem 2.2. *The spaces ${}_2m_{\mathcal{Z}}^I(M)$ and ${}_2m_{\mathcal{Z}_0}^I(M)$ are Banach spaces normed by*

$$||x_{ij}|| = \inf\{\rho > 0 : \{i,j \in \mathbb{N}\}| \sup M(\frac{|x_{ij}|}{\rho}) \leq 1\}.$$

Proof. Proof of this result is easy in view of the existing techniques and therefore is omitted. □

Theorem 2.3. *Let M_1 and M_2 be Orlicz functions that satisfy the $\triangle_2$-condition. Then*
(a) $X(M_2) \subseteq X(M_1.M_2)$;
(b) $X(M_1) \cap X(M_2) \subseteq X(M_1 + M_2)$ For $X = {}_2\mathcal{Z}^I$, ${}_2\mathcal{Z}_0^I$, ${}_2m_{\mathcal{Z}}^I$ and ${}_2m_{\mathcal{Z}_0}^I$.

Proof. (a) Let $(x_{ij}) \in {}_2\mathcal{Z}_0^I(M_2)$. Then there exists $\rho > 0$ such that

$$I - \lim_{i,j} M_2(\frac{|x'_{ij}|}{\rho}) = 0 \tag{3}$$

Let $\epsilon > 0$ and choose δ with $0 < \delta < 1$ such that $M_1(t) < \epsilon$ for $0 \leq t \leq \delta$.
Write $y_{ij} = M_2(\frac{|x'_{ij}|}{\rho})$ and consider for all $(i,j) \in \mathbb{N} \times \mathbb{N}$ we have

$$\lim_{0 \leq y_{ij} \leq \delta} M_1(y_{ij}) = \lim_{y_{ij} \leq \delta} M_1(y_{ij}) + \lim_{y_{ij} > \delta} M_1(y_{ij}).$$

We have

$$\lim_{y_{ij} \leq \delta} M_1(y_{ij}) \leq M_1(2) \lim_{y_{ij} \leq \delta} (y_{ij}). \tag{4}$$

For $(y_{ij}) > \delta$, we have

$$(y_{ij}) < (\frac{y_{ij}}{\delta}) < 1 + (\frac{y_{ij}}{\delta}).$$

Since M_1 is non-decreasing and convex, it follows that

$$M_1(y_{ij}) < M_1(1 + (\frac{y_{ij}}{\delta})) < \frac{1}{2}M_1(2) + \frac{1}{2}M_1(\frac{2y_{ij}}{\delta})$$

Since M_1 satisfies the $\triangle_2$-condition, we have

$$M_1(y_{ij}) < \frac{1}{2}K(\frac{y_{ij}}{\delta})M_1(2) + \frac{1}{2}K(\frac{y_{ij}}{\delta})M_1(2) = K(\frac{y_{ij}}{\delta})M_1(2).$$

Hence

$$\lim_{y_{ij} > \delta} M_1(y_{ij}) \leq max(1, K\delta^{-1}M_1(2)) \lim_{y_{ij} > \delta} (y_{ij}). \tag{5}$$

From (3), (4) and (5), we have $(x_{ij}) \in \mathcal{Z}_0^I(M_1.M_2)$. Thus

$$\mathcal{Z}_0^I(M_2) \subseteq \mathcal{Z}_0^I(M_1.M_2).$$

The other cases can be proved similarly.

(b) Let $(x_k) \in \mathcal{Z}_0^I(M_1) \cap \mathcal{Z}_0^I(M_2)$. Then there exists $\rho > 0$ such that

$$I - \lim_{k} M_1(\frac{|x'_k|}{\rho}) = 0 \text{ and } I - \lim_{k} M_2(\frac{|x'_k|}{\rho}) = 0$$

The rest of the proof follows from the following equality

$$\lim_{k \in \mathbb{N}} (M_1 + M_2)(\frac{|x'_k|}{\rho}) = \lim_{k \in \mathbb{N}} M_1(\frac{|x'_k|}{\rho}) + \lim_{k \in \mathbb{N}} M_2(\frac{|x'_k|}{\rho})$$

□

Theorem 2.4. *The spaces ${}_2\mathcal{Z}_0^I(M)$ and ${}_2m_{\mathcal{Z}_0}^I(M)$ are solid and monotone.*

Proof. We shall prove the result for ${}_2\mathcal{Z}_0^I(M)$. For $m_{\mathcal{Z}_0}^I(M)$ the result can be proved similarly. Let $(x_{ij}) \in {}_2\mathcal{Z}_0^I(M)$. Then there exists $\rho > 0$ such that

$$I - \lim_{i,j} M(\frac{|x'_{ij}|}{\rho}) = 0 \tag{6}$$

Let (α_{ij}) be a sequence of scalars with $|\alpha_{ij}| \leq 1$ for all $(i,j) \in \mathbb{N} \times \mathbb{N}$. Then the result follows from (6) and the following inequality for all

$$M(\frac{|\alpha_{ij}x'_{ij}|}{\rho}) \leq |\alpha_{ij}|M(\frac{|x'_{ij}|}{\rho}) \leq M(\frac{|x'_{ij}|}{\rho}).$$

By Lemma 1.12, a sequence space E is solid implies that E is monotone.
We have the space ${}_2\mathcal{Z}_0^I(M)$ is monotone. □

Theorem 2.5. *The spaces ${}_2\mathcal{Z}^I(M)$ and ${}_2m_{\mathcal{Z}}^I(M)$ are neither solid nor monotone in general.*

Proof. Here we give a counter example. Let $I = I_\delta$ and $M(x) = x^2$ for all $x \in [0,\infty)$. Consider the K-step space $X_K(M)$ of $X(M)$ defined as follows,
let $(x_{ij}) \in X(M)$ and let $(y_{ij}) \in X_K(M)$ be such that

$$y_{ij} = \begin{cases} x_{ij}, \text{if (i+j) is even,} \\ \quad 0, \text{otherwise.} \end{cases}$$

Consider the sequence (x_{ij}) defined by $x_{ij} = 1$ for all $(i,j) \in \mathbb{N} \times \mathbb{N}$.
Then $(x_{ij}) \in {}_2\mathcal{Z}^I(M)$ but its K-stepspace preimage does not belong to ${}_2\mathcal{Z}^I(M)$.
Thus ${}_2\mathcal{Z}^I(M)$ is not monotone. Hence ${}_2\mathcal{Z}^I(M)$ is not solid. □

Theorem 2.6. *The spaces ${}_2\mathcal{Z}_0^I(M)$ and ${}_2\mathcal{Z}^I(M)$ are not convergence free in general.*

Proof. Here we give a counter example. Let $I = I_f$ and $M(x) = x^3$ for all $x \in [0,\infty)$. Consider the sequence (x_{ij}) and (y_{ij}) defined by

$$x_{ij} = \frac{1}{i+j} \quad \textit{and } y_{ij} = i + j$$

Then $(x_{ij}) \in {}_2\mathcal{Z}^I(M)$ and ${}_2\mathcal{Z}_0^I(M)$, but $(y_{ij}) \notin {}_2\mathcal{Z}^I(M)$ and ${}_2\mathcal{Z}_0^I(M)$.
Hence the spaces ${}_2\mathcal{Z}^I(M)$ and ${}_2\mathcal{Z}_0^I(M)$ are not convergence free. □

Theorem 2.7. *The spaces ${}_2\mathcal{Z}_0^I(M)$ and ${}_2\mathcal{Z}^I(M)$ are sequence algebras.*

Proof. We prove that ${}_2\mathcal{Z}_0^I(M)$ is a sequence algebra. For the space ${}_2\mathcal{Z}^I(M)$, the result can be proved similarly. Let $(x_{ij}), (y_{ij}) \in {}_2\mathcal{Z}_0^I(M)$. Then

$$I - \lim M(\frac{|x'_{ij}|}{\rho_1}) = 0 \text{ and } I - \lim M(\frac{|y'_{ij}|}{\rho_2}) = 0$$

Let $\rho = \rho_1.\rho_2 > 0$. Then we can show that

$$I - \lim M(\frac{|(x'_{ij}.y'_{ij})|}{\rho}) = 0.$$

Thus $(x_{ij}.y_{ij}) \in {}_2\mathcal{Z}_0^I(M)$. Hence ${}_2\mathcal{Z}_0^I(M)$ is a sequence algebra. □

Theorem 2.8. *If I is not maximal and $I \neq I_f$, then the spaces ${}_2\mathcal{Z}^I(M)$ and ${}_2\mathcal{Z}_0^I(M)$ are not symmetric.*

Proof. Let $A \in I$ be infinite and $M(x) = x$ for all $x = (x_{ij})$. If

$$x_{ij} = \begin{cases} 1, \text{for } i, j \in A, \\ 0, otherwise. \end{cases}$$

Then $(x_{ij}) \in {}_2\mathcal{Z}_0^I(M) \subset {}_2\mathcal{Z}^I(M)$, by lemma 1.14. Let $K \subset \mathbb{N}$ be such that $K \notin I$ and $\mathbb{N} - K \notin I$. Let $\phi : K \to A$ and $\psi : \mathbb{N} - K \to \mathbb{N} - A$ be bijections, then the map $\pi : \mathbb{N} \to \mathbb{N}$ defined by

$$\pi(k) = \begin{cases} \phi(k), \text{ for } k \in K, \\ \psi(k), \text{ otherwise,} \end{cases} \text{ end } \pi_k = \begin{cases} \phi_k, \text{ for k} \in \text{K}, \\ \psi_k, \text{ for k} \in \text{N-K}. \end{cases}$$

is a permutation on $\mathbb{N}$, but $(x_{\pi(i)\pi(j)}) \notin {}_2\mathcal{Z}^I(M)$ and $(x_{\pi(i)\pi(j)}) \notin {}_2\mathcal{Z}_0^I(M)$. Hence ${}_2\mathcal{Z}_0^I(M)$ and ${}_2\mathcal{Z}^I(M)$ are not symmetric. □

Acknowledgment

The authors would like to record their gratitude to the reviewer for his careful reading and making some useful corrections which improved the presentation of the paper.

References

1. P. Kostyrko, T. Šalát, W. Wilczyński, *I-convergence.* Real Anal. Exch.26(2) 669-686(2000).
2. T. Šalát, B.C. Tripathy, M. Ziman, *On some properties of I-convergence.* Tatra Mt. Math. Publ.,(28)279-286(2004).
3. K. Demirci, *I-limit superior and limit inferior*, Math. Commun.6,165-172(2001).
4. P. Das, P. Kostyrko, P. Malik, W. Wilczyński, *I and I^*-Convergence of Double Sequences.*Math. Slovaca, 58,605-620,(2008).
5. M. Gürdal, M.B. Huban, *On I-Convergence of Double Sequences in the Topology induced by Random 2-Norms.* Matematicki Vesnik, vol. 65, no. 3, pp. 113, 2013.
6. Ayhan Esi and Bipan Hazarika, *Lacunary summable sequence spaces of fuzzy numbers defined by ideal convergence and an Orlicz function* , Afrika Matematika November (2012), 1-7.
7. Ayhan Esi and M. Kemal zdemir, *0-strongly summable sequence spaces in n-normed spaces defined by ideal convergence and an Orlicz function*, Mathematica Slovaca 63(4)(2013), 829-838.
8. Ayhan Esi and Bipan Hazarika, *λ-ideal convergence in intuitionistic fuzzy 2-normed linear space*, Journal of Intelligent Fuzzy Systems: Applications in Engineering and Technology, 24(4)(2013), 725-732.
9. Ayhan Esi and S.K Sharma, *Some I-convergent sequence spaces defined by using sequence of moduli and n-normed space*,Journal of the Egyptian Mathematical Society, 21(2)(2013), 103-107.
10. Ayhan Esi, Hemen Dutta and Alias B Khalaf, *Some Orlicz extended I-convergent A-summable classes of sequences of fuzzy numbers*, Journal of Inequalities and Applications, 2013.

11. M. Gürdal, S. Ahmet, *Extremal I-Limit Points of Double sequences.* Applied Mathematics E-Notes,8,131-137,(2008).
12. V.A. Khan and N. Khan, *On a new I-convergent double sequence spaces.* Int J of Anal,Hindawi Publishing Corporation, Article ID-126163 Vol 2013,1-7(2013).
13. V.A. Khan and T. Sabiha, *On Some New double sequence spaces of Invariant Means defined by Orlicz function.* Commun. Fac. Sci. 60,11-21 (2011).
14. V.A. Khan and K. Ebadullah, *On Zweier I-convergent sequence spaces defined by Orlicz functions.* Submitted.
15. F. Başar and B. Altay, *On the spaces of sequences of p-bounded variation and related matrix mappings.*Ukr. Math.J.55.(2003).
16. E. Malkowsky, *Recent results in the theory of matrix transformation in sequence spaces.*Math.Vesnik.(49)187-196(1997).
17. P.N. Ng and P.Y. Lee, *Cesaro sequence spaces of non-absolute type.*Comment.Math. Prace.Mat.20(2)429-433(1978).
18. C.S. Wang, *On Nörlund sequence spaces.*Tamkang J.Math.(9)269-274(1978).
19. B. Altay, F. Başar and M. Mursaleen, *On the Euler sequence space which include the spaces l_p and l_∞*.I,Inform.Sci.176(10)(2006),1450-1462
20. M. Şengönül, *On The Zweier Sequence Space.* Demonstratio Math. Vol.XL No.(1)181-196(2007).
21. V.K. Bhardwaj and N. Singh, *Some sequence spaces defined by Orlicz functions.* Demons. Math.,33(3): 571-582(2000).
22. J. Lindenstrauss and L.Tzafriri, *On Orlicz sequence spaces.* Israel J. Math.,101: 379-390(1971).
23. P.K. Kamthan and M. Gupta, *Sequence spaces and series.* Marcel Dekker Inc,New York.(1980).
24. B.C. Tripathy and B. Hazarika, *Paranorm I-Convergent sequence spaces.*,Math Slovaca.59(4)485-494.(2009).
25. B.C. Tripathy and B. Hazarika, *Some I-Convergent sequence spaces defined by Orlicz function.*,Acta Math. Appl. Sin.,Engl.Ser.27(1)149-154(2011).
26. V.A. Khan and N. Khan, *On some I- Convergent double sequence spaces defined by a sequence of modulii* Ilirias J. Math 4(2), 1-8, (2013).
27. V.A. Khan and N. Khan, *On some I- Convergent double sequence spaces defined by a modulus function*Eng. Sci. Res., 5, 35-40, (2013).
28. V.A. Khan and N. Khan, *I-Pre-Cauchy Double Sequences and Orlicz Functions* Eng.Sci. Res., 5, 52-56, (2013).
29. V.A. Khan, K. Ebadullah, Ayhan Esi, N. Khan, M. Shafiq, *On Paranorm Zweier I-Convergent Sequence Spaces* Journal of Mathematics,Hindawi Publishing Corporation,Vol 2013, Article ID 613501,1-6(2013)

Vakeel A. Khan received M.Sc., M.Phil and Ph.D at Aligarh Muslim University. He is currently an Associate Professor at Aligarh Muslim University. He has published a number of research articles and some books to his name. His research interests include Functional Analysis, sequence spaces, I-convergence, invariant means, zweier sequences and so on.

Department of Mathematics, Aligarh Muslim University, Aligarh, 200-002, India.
e-mail: vakhanmaths@gmail.com

Nazneen Khan received M.Sc. and M.Phil. from Aligarh Muslim University, and is currently a Ph.D. scholar at Aligarh Muslim University. Her research interests are Functional Analysis, sequence spaces and double sequences.

Department of Mathematics, Aligarh Muslim University, Aligarh, 200-002, India.
e-mail: nazneen4maths@gmail.com