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PRICING OF POWER OPTIONS UNDER THE REGIME-SWITCHING MODEL†

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Abstract. Power options have payoffs that are determined by the price of the underlying asset raised to some power. In this paper, power options are considered under a regime-switching model which can capture complex asset dynamics by permitting switching between different regimes. The pricing formulas for the Laplace transforms of power options are obtained. The prices of power options are calculated using the formulas and compared with the results of the Monte Carlo simulation.

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1. Introduction

Power options are one of exotic options and have payoffs that are determined by the stock price raised to some power, whereas plain vanilla options have linear payoffs. Because of the non-linear characteristics of these power options, the power option is appropriate for hedging non-linear price risks [11]. When the exponent of power option is greater than 1, the power option provides a greater leverage than a plain vanilla option. Therefore, for an investor with a strong view of the market, a power call option brings more benefits than a plain vanilla option [7]. On the other hand, it can lead an option seller to high losses. Thus the power call options are usually capped at some predefined level [4].

Power options, also known as polynomial options and leveraged options, are widely traded in financial markets. For example, an option whose payoff is a polynomial function of the Nikkei level at the maturity was issued in Tokyo [7]. Bankers Trust in Germany has issued capped foreign-exchange power options with power exponent two [12]. For more examples, see Tompkins [11], and Macovschi and Quittard-Pinon [9].

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Under the Black-Scholes model, Heynen and Kat [7] obtained the pricing formula for power options. Esser [3] and Macovschi and Quittard-Pinon [9] derived a quasi-closed form pricing equation for power options in a general framework. Under the Heston model, pricing formulas for power options were derived analytically in Kim et al. [8].

In this paper, power option prices are considered in a regime-switching model. The regime-switching model, first introduced by Hamilton [6], is one of the most popular nonlinear time series models in the literature. This model can capture complex asset dynamics by permitting switching between different regimes [5]. Su et al. [10] studied this model and obtained a conditional power option value, which can be computed only for a two-state regime-switching model. In this paper, explicit formulas for the Laplace transforms of power option prices are derived under the general regime-switching model.

The rest of the paper is organized as follows. In Section 2, the regime-switching model and power options are described. Then explicit formulas for the Laplace transforms of power option prices are derived in Section 3. Section 4 calculates prices of power options using the formulas in Section 3, and compares them with the results of the Monte Carlo simulation.

2. The model and power options

Under the risk-neutral measure $\mathbb{P}$, the underlying asset price S_t is given by

$$\frac{dS_t}{S_t} = r_{Z(t)}dt + \sigma_{Z(t)}dW_t$$

where $\{Z(t) : t \geq 0\}$ represents the market regime as an irreducible continuous-time Markov process with a finite state space $E = \{1, \ldots, m\}$ and an infinitesimal generator Q. Here, $\{W(t) : t \geq 0\}$ is standard Brownian motion, and $\{Z(t) : t \geq 0\}$ and $\{W(t) : t \geq 0\}$ are independent under $\mathbb{P}$. We consider a filtration $\{\mathcal{F}_t : t \geq 0\}$, where $\mathcal{F}_t$ is the smallest σ-algebra generated by $\{W(u), Z(u) : u \leq t\}$. For each $i \in E$, r_i and σ_i represent the expected rate of return and the volatility of the stock price at regime i, respectively.

There are three kinds of power options: standard power option, capped power option and powered option. The payoffs of all power options depend on the price of the underlying asset raised to the power $\alpha > 0$. For a standard power call, the payoff is $\max\{S_T^\alpha - K^\alpha, 0\}$, and for a standard power put, it is $\max\{K^\alpha - S_T^\alpha, 0\}$ with strike K and maturity T. Standard power call options can be lead to very high risk for a option seller and thus the call options are usually capped at some predefined level L. The payoff at maturity for a capped power call is $\min\{\max\{S_T^\alpha - K^\alpha, 0\}, L\}$. A powered call has the payoff $(\max\{S_T - K, 0\})^\alpha$, and a powered put has the payoff $(\max\{K - S_T, 0\})^\alpha$.

It is not necessary to obtain all of these price. The payoff of the capped power call option can be represented as the difference between the payoffs of two standard power call options. In addition, a price of a standard power call option can be obtained from a price of a standard power put option by the put-call parity with the conditional α-th moment of the underlying asset. Hence, in the remainder of this paper, standard power put, powered call and powered put options will be considered. The values of the power options are given as follows:

- The value of the α-th standard power put option with strike price K and maturity T is given by

$$\mathrm{Pow}_p(\alpha, S_0, K, T, i) \equiv \mathbb{E}\left[e^{-\int_0^T r_{Z(t)}dt} \max\{K^\alpha - S_T^\alpha, 0\}\Big| Z(0) = i\right].$$

- The value of the α-th powered call and put options with strike price K and maturity T, respectively, are given by

$$\mathrm{Powd}_c(\alpha, S_0, K, T, i) \equiv \mathbb{E}\left[e^{-\int_0^T r_{Z(t)}dt} (\max\{S_T - K, 0\})^\alpha \Big| Z(0) = i\right]$$

and

$$\mathrm{Powd}_p(\alpha, S_0, K, T, i) \equiv \mathbb{E}\left[e^{-\int_0^T r_{Z(t)}dt} (\max\{K - S_T, 0\})^\alpha \Big| Z(0) = i\right].$$

3. Laplace transforms of power option prices

For $w \in \mathbb{C}$ and $t > 0$, define $\psi_i(w, t)$ as

$$\psi_i(w, t) = \mathbb{E}_i[e^{-\int_0^t r_{Z(u)}du + w \log S_t}],$$

with $\mathbb{E}_i[\cdot] = \mathbb{E}[\cdot|Z(0) = i]$ the conditional expectation given $Z(0) = i$.

Lemma 3.1. *For $w \in \mathbb{C}$, $\psi_i(w, t)$ is given by*

$$\psi_i(w, t) = S_0^w \boldsymbol{e}_i^\top e^{t\Lambda(w)} \mathbf{1}, \tag{1}$$

where $\boldsymbol{e}_i$ is the m-dimensional column vector with all 0s except for a 1 in the ith component, $\mathbf{1}$ is the m-dimensional column vector with all its components equal to one, and

$$\Lambda(w) = Q + \mathrm{diag}\Big(- r_i + (r_i - \frac{1}{2}\sigma_i^2)w + \frac{\sigma_i^2}{2} w^2\Big)_{i\in E}$$

where $\mathrm{diag}(a_i)_{i\in E}$ *is an $m \times m$ diagonal matrix whose diagonal entries are a_i, $i \in E$.*

Proof. Define $Y(t) = -\int_0^t r_{Z(u)}du + w \log \frac{S_t}{S_0}$. Then a bivariate Markov process $\{(Y(t), Z(t)) : t \geq 0\}$ is a continuous time Markov addictive process. By Proposition XI.2.2 in Asmussen [2], the $m \times m$ matrix $\Phi(t)$ with (i, j)th entry $\phi_{ij}(t) = \mathbb{E}_i[e^{Y(t)} \mathbb{1}_{\{Z(t)=j\}}]$ is given by

$$\Phi(t) = e^{t\Lambda(w)}.$$

Since $\psi_i(w, t) = S_0^w \sum_{j\in E} \phi_{ij}(t) = S_0^w \boldsymbol{e}_i^\top \Phi(t)\mathbf{1}$, (1) is obtained. □

Now analytic formulas for the Laplace transform of the standard power options and the powered options are presented.

Standard power option. Recall that the $\mathrm{Pow}_p(\alpha, S_0, K, T, i)$ is the price of the standard power put option with strike K and maturity T given that the price of the underlying asset is S_0 and the current regime is i. Let

$$f(k) \quad = \quad \mathrm{Pow}_p(\alpha, S_0, e^k, T, i).$$

The Laplace transform $\tilde{f}$ of f with respect to k is defined as

$$\tilde{f}(w) \quad = \quad \int_{-\infty}^{\infty} e^{-wk} f(k) dk$$

for complex number w such that the right hand side is well-defined. The following theorem provides an explicit formula for the transform $\tilde{f}(w)$.

Theorem 3.2. *Suppose that w is a complex number such that $\mathrm{Re}(w) > \alpha$. Then the Laplace transform $\tilde{f}(w)$ is given by*

$$\tilde{f}(w) \quad = \quad \frac{\alpha}{w(w-\alpha)} S_0^{\alpha - w} \boldsymbol{e}_i^\top e^{T\Lambda(\alpha - w)} \mathbf{1}$$

where $\boldsymbol{e}_i$, $\mathbf{1}$ and $\Lambda(w)$ are given in Lemma 3.1.

Proof. Let $X(t) = \log S(t)$. For $\mathrm{Re}(w) > \alpha$, $\tilde{f}(w)$ can be written as

$$\begin{aligned} \tilde{f}(w) \quad &= \quad \mathbb{E}_i[\int_{-\infty}^{\infty} e^{-wk - \int_0^t r_{Z(u)} du} \max\{e^{\alpha k} - e^{\alpha X_T}, 0\} dk] \\ &= \quad \mathbb{E}_i[e^{-\int_0^t r_{Z(u)} du} \int_{X_T}^{\infty} e^{-wk} (e^{\alpha k} - e^{\alpha X_T}) dk]. \end{aligned} \tag{2}$$

The integration in (2) is

$$\int_{X_T}^{\infty} e^{-wk}(e^{\alpha k} - e^{\alpha X_T}) dk = \frac{\alpha}{w(w-\alpha)} e^{(\alpha - w) X_T}, \quad \text{for } \mathrm{Re}(w) > \alpha. \tag{3}$$

Substituting (3) into (2) yields

$$\begin{aligned} \tilde{f}(w) \quad &= \quad \frac{\alpha}{w(w-\alpha)} \mathbb{E}_i[e^{-\int_0^t r_{Z(u)} du + (\alpha - w) X_T}] \\ &= \quad \frac{\alpha}{w(w-\alpha)} \psi_i(\alpha - w, T). \end{aligned}$$

By (1) in Lemma 3.1, the proof is completed. □

Powered option. Recall that $\mathrm{Powd}_c(\alpha, S_0, K, T, i)$ and $\mathrm{Powd}_p(\alpha, S_0, K, T, i)$ are the prices of the powered call and put options, respectively, with strike K and maturity T given that the price of the underlying asset is S_0 and the current regime is i. Let

$$\begin{aligned} g_c(k) \quad &= \quad \mathrm{Powd}_c(\alpha, S_0, e^k, T, i), \\ g_p(k) \quad &= \quad \mathrm{Powd}_p(\alpha, S_0, e^k, T, i). \end{aligned}$$

Laplace transforms $\tilde{g}_c$ and $\tilde{g}_p$ with respect to k are defined, respectively, as

$$\tilde{g}_c(w) = \int_{-\infty}^{\infty} e^{-wk} g_c(k) dk \quad \text{and} \quad \tilde{g}_p(w) = \int_{-\infty}^{\infty} e^{-wk} g_p(k) dk$$

for complex number w such that the right-hand side of each equation is well-defined.

If α is a positive integer, the payoff of a powered option is the sum of payoffs for standard power options by the binomial theorem. For a general $\alpha > 0$, the Newton's generalized binomial theorem is used. The theorem is stated as follows: For any complex numbers w, x and y such that $|x| > |y|$,

$$(x+y)^w = \sum_{n=0}^{\infty} \binom{w}{n} x^{w-n} y^n, \tag{4}$$

where $\binom{w}{n} = \frac{(w)_n}{n!}$ with falling factorial $(w)_n = w(w-1)\cdots(w-n+1)$.

The complete beta function is used in Theorem 3.3. It is defined as

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$$

for $\mathrm{Re}(x) > 0$ and $\mathrm{Re}(y) > 0$. Note that $B(x,y)$ is symmetric by its definition, and has another form as follows:

$$B(x,y) = \sum_{n=0}^{\infty} \binom{n-y}{n} \frac{1}{x+n}. \tag{5}$$

Theorem 3.3. (a) *Suppose that w is a complex number such that $\mathrm{Re}(w) < 0$. Then the Laplace transform $\tilde{g}_c$ is given by*

$$\tilde{g}_c(w) = B(-w, \alpha+1) S_0^{\alpha-w} \boldsymbol{e}_i^\top e^{T\Lambda(\alpha-w)} \mathbf{1}, \tag{6}$$

where $\boldsymbol{e}_i$, $\mathbf{1}$ and $\Lambda(w)$ are given in Lemma 3.1.

(b) *Suppose that w is a complex number such that $\mathrm{Re}(w) > \alpha$. Then the Laplace transform $\tilde{g}_p$ is given by*

$$\tilde{g}_p(w) = B(w-\alpha, \alpha+1) S_0^{\alpha-w} \boldsymbol{e}_i^\top e^{T\Lambda(\alpha-w)} \mathbf{1}.$$

Proof. Let $X(t) = \log S(t)$. For $\mathrm{Re}(w) < 0$, $\tilde{g}_c(w)$ can be written as

$$\begin{aligned} \tilde{g}_c(w) &= \mathbb{E}_i\Big[\int_{-\infty}^{\infty} e^{-wk-\int_0^t r_{Z(u)}du} (\max\{e^{X_T} - e^k, 0\})^\alpha dk\Big] \\ &= \mathbb{E}_i\Big[e^{-\int_0^t r_{Z(u)}du} \int_{-\infty}^{X_T} e^{-wk}(e^{X_T} - e^k)^\alpha dk\Big]. \end{aligned} \tag{7}$$

Using (4), for $k < X_T$

$$(e^{X_T} - e^k)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} (-1)^n e^{(\alpha-n)X_T + nk}. \tag{8}$$

Combine (7) and (8),

$$\tilde{g}_c(w) = \mathbb{E}_i\Big[\int_{-\infty}^{X_T} \sum_{n=0}^{\infty} \binom{\alpha}{n} (-1)^n e^{-\int_0^t r_{Z(u)}du + (\alpha-n)X_T + (n-w)k} dk\Big]. \tag{9}$$

Since $|\binom{\alpha}{n}| < |\binom{\alpha}{\lfloor(\alpha+1)/2\rfloor}|$ with $\lfloor x \rfloor$ the largest integer less than or equal to x,

$$\left|\binom{\alpha}{n}(-1)^n e^{-\int_0^t r_{Z(u)}du+(\alpha-n)X_T+(n-w)k}\right| < \left|\binom{\alpha}{\lfloor(\alpha+1)/2\rfloor}\right| e^{-\mathrm{Re}(w)k}$$

for $k \leq X_T$. By the dominated convergence theorem, (9) follows

$$\tilde{g}_c(w) = \mathbb{E}_i[\sum_{n=0}^{\infty}\binom{\alpha}{n}(-1)^n e^{-\int_0^t r_{Z(u)}du} e^{(\alpha-n)X_T}\int_{-\infty}^{X_T} e^{(n-w)k}dk].$$

For $\mathrm{Re}(w) < 0$ and $n > 0$, the integration in the above equation is $\int_{-\infty}^{X_T} e^{(n-w)k}dk = \frac{1}{n-w}e^{(n-w)X_T}$. Then

$$\begin{aligned}\tilde{g}_c(w) &= \mathbb{E}_i[\sum_{n=0}^{\infty}\binom{\alpha}{n}(-1)^n e^{-\int_0^t r_{Z(u)}du} e^{(\alpha-n)X_T}\frac{1}{n-w}e^{(n-w)X_T}] \\ &= \mathbb{E}_i[e^{-\int_0^t r_{Z(u)}du+(\alpha-w)X_T}]\sum_{n=0}^{\infty}\binom{\alpha}{n}\frac{(-1)^n}{n-w} \\ &= \psi_i(\alpha-w,T)\sum_{n=0}^{\infty}\binom{\alpha}{n}\frac{(-1)^n}{n-w}. \qquad (10)\end{aligned}$$

Since $\binom{\alpha}{n}(-1)^n = \binom{n-1-\alpha}{n}$ and (5), the summation in (10) can be written as

$$\begin{aligned}\sum_{n=0}^{\infty}\binom{\alpha}{n}(-1)^n\frac{1}{n-w} &= \sum_{n=0}^{\infty}\binom{n-1-\alpha}{n}\frac{1}{n-w} \\ &= B(-w, 1+\alpha).\end{aligned}$$

Substituting the above equation into (10) yields (6). The proof of (b) is omitted since it is similar to that of (a). □

4. Numerical examples

In this section, the numerical examples of power options are provided. The Laplace transforms for the prices of the standard power put and powered put options are given by Theorem 3.2 and Theorem 3.3, respectively. To invert the Laplace transforms, the Euler inversion method in Abate and Whitt [1] is used. The model in the case of two regimes are considered. Suppose that Z is a Markov process with state space $\{1, 2\}$ and infinitesimal generator

$$Q = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}.$$

For the calculation of the data in Tables 1 and 2, the parameters used for underlying asset prices are: $S_0 = 1$, $r_1 = 0.05$, $r_2 = 0.03$, $\sigma_1 = 0.2$ and $\sigma_2 = 0.1$.

In Tables 1 and 2, the transform method and the Monte Carlo simulation results are presented for the prices of standard power put options $\mathrm{Pow}_p(\alpha, S_0, K, T, i)$ and powered put options $\mathrm{Powd}_p(\alpha, S_0, K, T, i)$ with power α=2, 3, 5, maturity T=1, 2, and strike price K =0.9, 1, 1.1. For each simulation result in Tables 1 and 2, 10^7 replications are generated. Tables 1 and 2 show that the values of transform method are included in 95% confidence intervals. It is observed that the price of each standard power put and powered put option for $Z(0) = 2$ is lower than that for $Z(0) = 1$, and the difference between these values decreases as T increases. As α increases, the standard power put option value increases in Table 1, and the powered put option value decreases in Table 2.

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Table 1. Comparison of Theorem 3.2 and the Monte Carlo results for the values of standard power put options with $S_0 = 1$.

				Theorem 3.2	Monte Carlo Simulation	
T	K	α	i	Option Value	Option Value	95% Confidence Interval
1	0.9	2	1	2.4629×10^{-2}	2.4618×10^{-2}	$(2.4577\times10^{-2}, 2.4658\times10^{-2})$
			2	1.4365×10^{-2}	1.4348×10^{-2}	$(1.4319\times10^{-2}, 1.4377\times10^{-2})$
		3	1	3.0974×10^{-2}	3.0960×10^{-2}	$(3.0911\times10^{-2}, 3.1010\times10^{-2})$
			2	1.8289×10^{-2}	1.8269×10^{-2}	$(1.8233\times10^{-2}, 1.8306\times10^{-2})$
		5	1	3.6724×10^{-2}	3.6708×10^{-2}	$(3.6651\times10^{-2}, 3.6765\times10^{-2})$
			2	2.2151×10^{-2}	2.2132×10^{-2}	$(2.2090\times10^{-2}, 2.2175\times10^{-2})$
	1	2	1	8.1102×10^{-2}	8.1092×10^{-2}	$(8.1013\times10^{-2}, 8.1171\times10^{-2})$
			2	6.4053×10^{-2}	6.4082×10^{-2}	$(6.4017\times10^{-2}, 6.4148\times10^{-2})$
		3	1	1.1206×10^{-1}	1.1204×10^{-1}	$(1.1194\times10^{-1}, 1.1215\times10^{-1})$
			2	8.9789×10^{-2}	8.9836×10^{-2}	$(8.9746\times10^{-2}, 8.9925\times10^{-2})$
		5	1	1.6085×10^{-1}	1.6084×10^{-1}	$(1.6070\times10^{-1}, 1.6099\times10^{-1})$
			2	1.3209×10^{-1}	1.3218×10^{-1}	$(1.3205\times10^{-1}, 1.3230\times10^{-1})$
	1.1	2	1	1.9305×10^{-1}	1.9303×10^{-1}	$(1.9291\times10^{-1}, 1.9315\times10^{-1})$
			2	1.8058×10^{-1}	1.8063×10^{-1}	$(1.8052\times10^{-1}, 1.8074\times10^{-1})$
		3	1	2.8912×10^{-1}	2.8909×10^{-1}	$(2.8891\times10^{-1}, 2.8927\times10^{-1})$
			2	2.7423×10^{-1}	2.7431×10^{-1}	$(2.7415\times10^{-1}, 2.7447\times10^{-1})$
		5	1	4.8968×10^{-1}	4.8963×10^{-1}	$(4.8935\times10^{-1}, 4.8992\times10^{-1})$
			2	4.7510×10^{-1}	4.7524×10^{-1}	$(4.7499\times10^{-1}, 4.7550\times10^{-1})$
2	0.9	2	1	3.4573×10^{-2}	3.4573×10^{-2}	$(3.4521\times10^{-2}, 3.4625\times10^{-2})$
			2	2.7085×10^{-2}	2.7098×10^{-2}	$(2.7053\times10^{-2}, 2.7142\times10^{-2})$
		3	1	4.2647×10^{-2}	4.2647×10^{-2}	$(4.2585\times10^{-2}, 4.2710\times10^{-2})$
			2	3.3712×10^{-2}	3.3725×10^{-2}	$(3.3670\times10^{-2}, 3.3779\times10^{-2})$
		5	1	4.8985×10^{-2}	4.8984×10^{-2}	$(4.8915\times10^{-2}, 4.9053\times10^{-2})$
			2	3.9298×10^{-2}	3.9309×10^{-2}	$(3.9248\times10^{-2}, 3.9369\times10^{-2})$
	1	2	1	8.8857×10^{-2}	8.8862×10^{-2}	$(8.8772\times10^{-2}, 8.8952\times10^{-2})$
			2	7.8156×10^{-2}	7.8168×10^{-2}	$(7.8087\times10^{-2}, 7.8250\times10^{-2})$
		3	1	1.2046×10^{-1}	1.2047×10^{-1}	$(1.2035\times10^{-1}, 1.2058\times10^{-1})$
			2	1.0698×10^{-1}	1.0699×10^{-1}	$(1.0688\times10^{-1}, 1.0710\times10^{-1})$
		5	1	1.6773×10^{-1}	1.6774×10^{-1}	$(1.6758\times10^{-1}, 1.6789\times10^{-1})$
			2	1.5130×10^{-1}	1.5131×10^{-1}	$(1.5116\times10^{-1}, 1.5145\times10^{-1})$
	1.1	2	1	1.8421×10^{-1}	1.8424×10^{-1}	$(1.8411\times10^{-1}, 1.8437\times10^{-1})$
			2	1.7460×10^{-1}	1.7463×10^{-1}	$(1.7450\times10^{-1}, 1.7475\times10^{-1})$
		3	1	2.7120×10^{-1}	2.7124×10^{-1}	$(2.7105\times10^{-1}, 2.7143\times10^{-1})$
			2	2.5950×10^{-1}	2.5954×10^{-1}	$(2.5936\times10^{-1}, 2.5972\times10^{-1})$
		5	1	4.4738×10^{-1}	4.4746×10^{-1}	$(4.4717\times10^{-1}, 4.4776\times10^{-1})$
			2	4.3448×10^{-1}	4.3453×10^{-1}	$(4.3424\times10^{-1}, 4.3481\times10^{-1})$

TABLE 2. Comparison of Theorem 3.3 and the Monte Carlo results for the values of powered put options with $S_0 = 1$.

				Theorem 3.3	Monte Carlo Simulation	
T	K	α	i	Option Value	Option Value	95% Confidence Interval
1	0.9	2	1	1.9303×10^{-3}	1.9343×10^{-3}	$(1.9294\times10^{-3}, 1.9391\times10^{-3})$
			2	2.2687×10^{-4}	2.2630×10^{-4}	$(2.2521\times10^{-4}, 2.2739\times10^{-4})$
		3	1	3.3083×10^{-4}	3.3171×10^{-4}	$(3.3051\times10^{-4}, 3.3291\times10^{-4})$
			2	2.4127×10^{-5}	2.4016×10^{-5}	$(2.3833\times10^{-5}, 2.4198\times10^{-5})$
		5	1	1.5737×10^{-5}	1.5773×10^{-5}	$(1.5662\times10^{-5}, 1.5885\times10^{-5})$
			2	5.0923×10^{-7}	5.0359×10^{-7}	$(4.9489\times10^{-7}, 5.1228\times10^{-7})$
	1	2	1	7.4593×10^{-3}	7.4669×10^{-3}	$(7.4557\times10^{-3}, 7.4781\times10^{-3})$
			2	2.4832×10^{-3}	2.4813×10^{-3}	$(2.4772\times10^{-3}, 2.4854\times10^{-3})$
		3	1	1.5922×10^{-3}	1.5948×10^{-3}	$(1.5914\times10^{-3}, 1.5982\times10^{-3})$
			2	3.2112×10^{-4}	3.2065×10^{-4}	$(3.1981\times10^{-4}, 3.2149\times10^{-4})$
		5	1	1.1155×10^{-4}	1.1182×10^{-4}	$(1.1137\times10^{-4}, 1.1227\times10^{-4})$
			2	9.4565×10^{-6}	9.4190×10^{-6}	$(9.3609\times10^{-6}, 9.4771\times10^{-6})$
	1.1	2	1	2.1240×10^{-2}	2.1251×10^{-2}	$(2.1230\times10^{-2}, 2.1273\times10^{-2})$
			2	1.3414×10^{-2}	1.3413×10^{-2}	$(1.3403\times10^{-2}, 1.3424\times10^{-2})$
		3	1	5.6311×10^{-3}	5.6365×10^{-3}	$(5.6285\times10^{-3}, 5.6445\times10^{-3})$
			2	2.3780×10^{-3}	2.3771×10^{-3}	$(2.3742\times10^{-3}, 2.3799\times10^{-3})$
		5	1	5.7089×10^{-4}	5.7194×10^{-4}	$(5.7046\times10^{-4}, 5.7341\times10^{-4})$
			2	1.0970×10^{-4}	1.0953×10^{-4}	$(1.0925\times10^{-4}, 1.0982\times10^{-4})$
2	0.9	2	1	3.5493×10^{-3}	3.5461×10^{-3}	$(3.5381\times10^{-3}, 3.5540\times10^{-3})$
			2	2.4753×10^{-3}	2.4791×10^{-3}	$(2.4729\times10^{-3}, 2.4853\times10^{-3})$
		3	1	7.6581×10^{-4}	7.6480×10^{-4}	$(7.6236\times10^{-4}, 7.6724\times10^{-4})$
			2	4.8843×10^{-4}	4.8962×10^{-4}	$(4.8785\times10^{-4}, 4.9140\times10^{-4})$
		5	1	5.5903×10^{-5}	5.5742×10^{-5}	$(5.5409\times10^{-5}, 5.6075\times10^{-5})$
			2	3.0853×10^{-5}	3.0937×10^{-5}	$(3.0717\times10^{-5}, 3.1156\times10^{-5})$
	1	2	1	1.0357×10^{-2}	1.0351×10^{-2}	$(1.0335\times10^{-2}, 1.0366\times10^{-2})$
			2	8.1331×10^{-3}	8.1388×10^{-3}	$(8.1258\times10^{-3}, 8.1518\times10^{-3})$
		3	1	2.7099×10^{-3}	2.7074×10^{-3}	$(2.7016\times10^{-3}, 2.7132\times10^{-3})$
			2	1.9464×10^{-3}	1.9491×10^{-3}	$(1.9446\times10^{-3}, 1.9535\times10^{-3})$
		5	1	2.7831×10^{-4}	2.7782×10^{-4}	$(2.7675\times10^{-4}, 2.7889\times10^{-4})$
			2	1.7228×10^{-4}	1.7270×10^{-4}	$(1.7195\times10^{-4}, 1.7344\times10^{-4})$
	1.1	2	1	2.4524×10^{-2}	2.4515×10^{-2}	$(2.4488\times10^{-2}, 2.4542\times10^{-2})$
			2	2.1017×10^{-2}	2.1025×10^{-2}	$(2.1002\times10^{-2}, 2.1049\times10^{-2})$
		3	1	7.7157×10^{-3}	7.7107×10^{-3}	$(7.6988\times10^{-3}, 7.7226\times10^{-3})$
			2	6.0899×10^{-3}	6.0946×10^{-3}	$(6.0850\times10^{-3}, 6.1042\times10^{-3})$
		5	1	1.0929×10^{-3}	1.0915×10^{-3}	$(1.0886\times10^{-3}, 1.0945\times10^{-3})$
			2	7.4679×10^{-4}	7.4808×10^{-4}	$(7.4592\times10^{-4}, 7.5023\times10^{-4})$