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THERMAL EFFECTS OF EYELID IN HUMAN EYE TEMPERATURE MODEL[†]

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Abstract. Presence of eyelid on anterior ocular surface and its thermal effects play significant role in maintaining eye temperature. In most of the literatures of thermal modeling in human eye, the eyelid is not considered as an eye component. In this paper, finite element model is developed to investigate the thermal effects of eyelid closure and opening in human eye. Based on different properties and parameter values reported in literatures, the bio-heat transfer process is simulated and compared with experimental results in steady and transient state cases. The sensitivity analysis using various ambient temperatures, evaporation rates, blood temperatures and lens thermal conductivities is carried out. The temperature values so obtained in open eye show a good agreement with past results. The closure of eyelid is found to increase/decrease the eye temperature significantly than its opening, when the parameter values are considered to be at extreme.

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Key words and phrases : Bio-heat transfer, Eyelid, Finite Element Method.

1. Introduction

Temperature is of vital importance for all biological functions. The rate of metabolic process increases with temperature. Temperature changes can affect tissues in several ways; it can kill cells, denature proteins, slow down or speed up metabolism, involve in pathological changes etc. Due to the lack of sufficient blood flow in the inner part and lack of skin as a protecting layer in anterior part, human eye is assumed to be the most vulnerable organ in human body, even for small thermal interactions. Exposure to elevated ambient temperature has been proposed to be a risk factor for presbyopia and cataract and low temperature for dry eye, eyelid spams, excess tearing etc[10]. Various environmental conditions like airflow, temperature, humidity and thermal radiation can influence on ocular temperature.

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The human eye is relatively a small and complex organ, consists of several sub domains with different material properties and having complex geometry. Direct in vivo temperature monitoring in the interior of a live human eye is not yet completely possible. Invasive or direct contact techniques in measuring the eye temperature is now confined to animal experiments due to its damaging nature of test procedures[15].

Bolometer was used to measure corneal surface temperature in early 1960s. Mapstone[14] used bolometer to measure corneal temperature and obtained a mean temperature 34.8°C. Later on, the infrared radiation thermometry has been used in measuring corneal surface temperature ever since. Fujishima et al.[5] reported mean corneal surface temperature 34.2°C using infrared radiation technique. All these non invasive techniques measure corneal surface temperature only. The information of corneal surface temperature is not sufficient to predict intraocular temperature variations in case of medical treatment processes like hyperthermia, laser etc. Therefore, mathematical modeling is the alternative and appropriate approach to obtain the temperature distribution within the components of human eye.

Lagendijk[11] used a finite difference method to calculate the temperature distribution in human and rabbit eyes during hyperthermia treatment. The heat transport from the sclera to the surrounding anatomy is described by a single heat transfer coefficient which includes the impact of blood flow in choroid and sclera. Scott[18] utilized finite element method to obtain the temperature profile based on heat conduction using various heat transfer coefficients given by Lagendijk. Flyckt et al.[4] studied the impact of choroidal blood flow and scleral convection on heat transfer coefficient in human eye. Ng and Ooi[15] presented a 2D finite element model and simulated ocular surface temperature using bioheat equation. Ooi et al.[16] studied the effect of aqueous humor hydrodynamics on heat transfer within human eye. Li et al.[13] studied the bio-heat transfer in the human eye using 3D alpha finite element method. Shafai and Vafai[19] studied the eye response to thermal disturbances and analyzed the role of primary thermal transport mechanisms on the eye subject to different conditions.

All these past models have neglected the effects of eyelid in temperature distribution in human eye. Some authors[16, 18] acknowledged their deficiency of the model is excluding the eyelid as an eye component. Eyelid is closed while sleeping. During sleep, with the eyelid closed, the temperature of the anterior segment is increased about 2°C by the blood flow in the eyelid, although surface body temperature also declines with sleep[21]. Even in wake up, eyelid covers cornea surface for 7 seconds in a minute (on an average) while blinking, but it varies by individual and depends on different environmental and health conditions. When the eyes are focused on an object for long time, eyelid closure time decreases and when the eye is exposed on high speed wind flow, extreme hot/cold temperatures etc the eyelid closure time increases. Although, eyelid closure time is much shorter than opening, however, this change may contribute to the temperature of ocular surface[5].

There are four types of temperature effects on anterior corneal surface: heat loss/gain between blinks, heat transfer between cornea and environment, heating/cooling due to spread of tear across the surface of cornea, and the heating/cooling effect caused by the movement of the eyelid. Closure and opening of the eyelid show different variations in corneal temperature. During the opening of eyelid, the cornea surface temperature is controlled by convection, radiation and tear evaporation. When eyelid is closed these heat losses are prevented and the cornea is influenced by another thermal environment, heat conduction from vascular palpebral conjunctivamapstone1968.

Therefore, modeling of heat transport is needed in order to completely explain the actual temperature variation in human eye in presence of eyelid. In the present paper, finite element method is used as a tool to predict the steady and transient temperature distribution in human eye in case of eyelid closure and opening. The eye temperature along pupillary axis using different values of evaporation rates, blood temperatures, ambient temperatures and lens thermal conductivities are studied. The temperature values obtained are also compared with past results.

2. Model Formulation

2.1. Eye geometry and properties. A schematic diagram of the human eye when eyelid is opened is presented in figure 1. For modeling purpose, the opened eye is divided into six regions: cornea, aqueous humor, lens, vitreous humor, retina, and sclera. In several literatures[20, 19, 18, 15, 16, 9] choroid/retina/sclera were modeled as a single layer as sclera. We separated choroid/retina from sclera and assumed as a different layer for three reasons:

(1) Sclera is avascular but choroid/retina is vascular[3]
(2) The blood perfusion rate in the choroid is the highest among any other tissue[9]
(3) The metabolic rate is the highest in the retina among any other tissue[2]

The diameter of the eye along pupillary axis is about $25.10mm$[7]. The thickness of cornea, aqueous humor, lens, vitreous humor, retina and sclera have been considered as $l'_1, l'_2 - l'_1, l'_3 - l'_2, l'_4 - l'_3, l'_5 - l'_4$ and $l'_6 - l'_5$ respectively. Similarly, $T'_0, T'_1, T'_2, T'_3, T'_4$ and T'_5 are the nodal temperatures at a distances $x = 0, x = l'_1, x = l'_2, x = l'_3, x = l'_4$ and $x = l'_5$ respectively and $T'_6 = T_c$ is the body core temperature at $x = l'_6$.

Similarly, a schematic diagram of the eye when eyelid is closed is displayed in fig. 2. The eyelid consists of four layers: skin, orbicularis muscle, tarsus plate and palpebral conjunctiva. Palpebral conjunctiva is very thin compare to other layers, so we model palpebral conjunctiva and tarsal plate as a single layer. Hence, the eyelid is further divided into three layers: dermis (skin surface), orbicularis oculi muscle and tarsal plate. Orbicularis oculi is a kind of skeletal muscle that is responsible for closing of eyelid. Tarsal plate, considered to be skeleton of eyelid, consists not of cartilage, but of dense fibrous tissue that

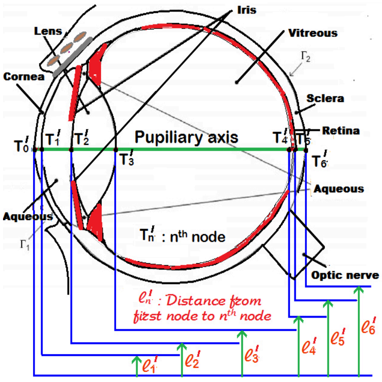

FIGURE 1. Schematic diagram of human eye when eyelid is opened

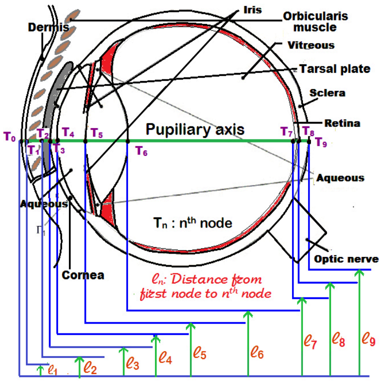

FIGURE 2. Schematic diagram of human eye when eyelid is closed

gives the eyelid its contour, and functions as stiffener. The eyelid thickness along pupillary axis is $4.25mm$[12].

The eye in this case is divided into nine regions: dermis, orbicularis oculi, tarsal plate, cornea, aqueous humor, lens, vitreous humor, retina, and sclera. Therefore, the diameter of the closed eye along pupillary axis is about $29.35mm$. The thickness of dermis, orbicularis oculi, tarsal plate, cornea, aqueous humor, lens, vitreous humor, retina and sclera have been considered as $l_1, l_2 - l_1, l_3 - l_2, l_4 - l_3, l_5 - l_4, l_6 - l_5, l_7 - l_6, l_8 - l_7$ and $l_9 - l_8$ respectively. Similarly, $T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8$ and $T_9 = T_c$ (body core temperature) are the nodal temperatures at a distances $x = 0, x = l_1, x = l_2, x = l_3, x = l_4, x = l_5, x = l_6, x = l_7, x = l_8$ and $x = l_9$. Due to the lack of appropriate data, the parameter values of cartilage are used for tarsal plate. The parameter values for different parts of eye are presented in table 1.

2.2. Governing equation and boundary conditions. The governing differential equation representing the bio-heat transfer in the human eye can be written by the well known Pennes equation addressing the effect of blood perfusion and metabolism[17] is given by:

$$\rho c \frac{\partial T}{\partial t} = \nabla.(K \nabla T) + \omega \rho_b c_b (T_b - T) + Q_m + Q \tag{1}$$

where, ρ_b= blood density (Kgm^{-3}), c_b = blood specific heat ($JKg^{-1 \circ}C^{-1}$), k = tissue thermal conductivity ($Wm^{-1 \circ}C^{-1}$), ω = volumetric blood perfusion rate per unit volume (s^{-1}), T_b = blood temperature (°C), T = tissue temperature (°C), Q_m = heat generation due to metabolism (Wm^{-3}) and Q = heat generation due to external heat source(Wm^{-3}).

Boundary conditions for the system can be defined as follows:

TABLE 1. Thermal properties of human eye tissues

Tissue Type	Thermal Conductivity K $(Wm^{-1o}C^{-1})$	Blood Perfusion ω (s^{-1})	Metabolic Rate Q_m (Wm^{-3})	Density ρ (Kgm^{-3})	Specific heat C $(JKg^{-1o}C^{-1})$
Dermis	0.34[4]	0.0087[4]	1620[9]	1070[4]	3662[4]
Orbicularis	0.56[4]	0.0034[4]	480[9]	1050[4]	3639[4]
Tarsal	0.47[9]	0.0082[9]	1600[9]	1250[9]	3600[9]
Cornea	0.58[15]	0[4]	0[9]	1050[15]	4178[15]
Aqueous	0.58[15]	0[4]	0[9]	996[15]	3997[15]
Lens	0.40[15]	0[4]	0[9]	1050[15]	3000[15]
Vitreous	0.603[15]	0[4]	0[9]	1000[15]	4178[15]
Retina	0.565[3]	0.0222[4]	22000[9]	1050[3]	3680[3]
Sclera	1.0042[15]	0[3]	0[3]	1100[15]	3180[15]

(1) On the outer surface of the sclera, the heat flows run into the eye with the complicated network of ophthalmic vessels which are located inside the choroidal layer acting as a heating source to the sclera. This heat exchange between the eye and the surrounding is modeled using the following convection boundary condition:

$$\Gamma_2 : -k_s \frac{\partial T}{\partial \eta} = h_b(T - T_b) \tag{2}$$

where η is the normal direction to the surface boundary, k_s is the thermal conductivity of sclera, h_b is the heat transfer coefficient between blood and eye $(Wm^{-2\circ}C^{-1})$, and T_b is blood temperature (°C).

(2) Since outer surface of the eye (cornea or skin) is exposed to the environment, the heat loss caused via convection, radiation, and evaporation. This loss is modeled using the following boundary condition :

$$\Gamma_1 : -k_c \frac{\partial T}{\partial \eta} = h_a(T - T_a) + \sigma\epsilon(T^4 - T_a^4) + E \tag{3}$$

where $h_a = \begin{cases} h_c, & \text{When eyelid is opened} \\ h_s, & \text{When eyelid is closed} \end{cases}$, h_c represents heat transfer coefficient between environment and cornea and h_s represents heat transfer coefficient between skin and environment $(Wm^{-2\circ}C^{-1})$, T_a is the ambient temperature (°C), σ is the Stefan Boltzmann constant $(5.67 \times 10^{-8} Wm^{-2\circ}C^{-4})$, ϵ is the emissivity, E is evaporative heat loss (Wm^{-2}).

The nonlinear radiation term in the boundary condition (3) is treated by using simple iterative procedure as follows:

$$-k_c \frac{\partial T_1}{\partial \eta} = [h_a + \sigma\epsilon(T_1 + T_a)(T_1^2 + T_a^2)](T_1 - T_a) + E \tag{4}$$

$$-k_c\frac{\partial T_1^m}{\partial \eta} = h_{cr}(T_1^m - T_a) + E \tag{5}$$

$$h_{cr} = h_a + \sigma\epsilon(T_1^{m-1} + T_a)((T_1^{m-1})^2 + T_a^2) \tag{6}$$

$$h_{cr} = h_{convection} + h_{radiation} \tag{7}$$

where $T_1^m$are temperature sequences for $m = 1, 2, 3,$and T_1^0 represents an initial guess of temperature. The iteration is completed when the convergent condition is satisfied:

$$\|T_1^m - T_1^{m-1}| < \delta \tag{8}$$

where δ is iteration tolerance.

The inner body core temperature T_c is assumed to be 37°C. Therefore, the initial boundary condition is

$$T_c = 37^\circ C \tag{9}$$

The partial differential equation (1) together with boundary conditions (2) and (5) in one dimensional variational form is:

$$\begin{aligned} I &= \frac{1}{2}\int_0^L [K(\frac{dT}{dx})^2 + \omega\rho_b c_b(T_b - T)^2 - 2Q_m T + \rho c\frac{\partial T^2}{\partial t}]dx \\ &+ \frac{1}{2}h_b(T - T_b)^2 + \frac{1}{2}h_{cr}(T - T_a)^2 + ET \end{aligned} \tag{10}$$

To optimize I, we differentiate I partially with respect to T_i and equating to zero as follows,

$$\frac{\partial I}{\partial T_i} = 0 \tag{11}$$

Equation (11) is the system of linear equations which can be written in matrix form as

$$[C]\{\dot{T}\} + [K]\{T\} = \{R\} \tag{12}$$

where $\{\dot{T}\} = \{\frac{\partial T_i}{\partial t}\}$, $\{T\} = \{T_i\}$ and $\{R\} = \{R_i\}$ are 6×1 and 9×1 vectors in case of eyelid opening and closure respectively. Similarly, [C] and [K] are 6×6 and 9×9 matrices in case of eyelid opening and closure respectively.

Now, to get the steady state temperature distribution, we remove the time derivative from (12) and solve the following system of equation:

$$[K]\{T\} = \{R\} \tag{13}$$

To get the transient temperature distribution, we apply Crank-Nicolson method to solve the system (12) with respect to time

$$(\frac{1}{\Delta t}[C] + \frac{1}{2}[K])\{T\}_{n+1} = (\frac{1}{\Delta t}[C] - \frac{1}{2}[K])\{T\}_n + \{R\} \tag{14}$$

where Δt is time interval.

The temperature increases from outer surface of the eye towards core, when ambient temperature is less than 37°C and vice versa. Hence, we consider the temperature increases/decreases in linear order towards eye core with regard to thickness. For initial nodal temperatures $\{T\}_0$ at time $t = 0$, we assume the following initial condition

$$T(x = l_i, t = 0) = T(0, 0) + rl_i \tag{15}$$

Where i ranges from $1, 2, 3, \cdots, 6$ and $1, 2, 3, \cdots, 9$ in case of eyelid opening and closure respectively, $T(0, 0) = 20°\text{C}$ and $r =$ constant to be determined. The equation(13) and (14) are solved to get the required nodal temperatures in steady and transient state cases.

3. Results

3.1. Sensitivity analysis. In this section, the influences of various parameters on eye temperature are studied in case of eyelid closure and opening. These parameters are ambient temperatures, blood temperatures, evaporation rates and lens thermal conductivities. The aim is to determine the dominant affecting parameters on temperature distribution in human eye. For analysis, $h_s = 6.28Wm^{-2°}C^{-1}$[18], $h_c = 10Wm^{-2°}C^{-1}$[17], $h_b = 65Wm^{-2°}C^{-1}$[17] and the parameter values in table 1 are used.

3.1.1. Effects of evaporation rates. When eyelids are opened the tear evaporation occurs from corneal surface. On the surface of corneal epithelium, there is usually a thin lipid layer produced by the Meibomian glands of the tarsal plate[18]. The function of this layer is to prevent evaporation of tear from the corneal surface[22]. When this layer is destroyed, the evaporation rate increases dramatically and can reach as high as $320Wm^{-2}$. In normal condition evaporation rate ranges from $20Wm^{-2}$ to $100Wm^{-2}$[18]. Therefore, in this study E equals $0Wm^{-2}$ and $120Wm^{-2}$ are chosen for extreme cold and hot environment and $40Wm^{-2}$ for room temperature respectively.

When the eyelids are closed, the sweat evaporation occurs from eyelid skin surface. Although the skin layer in the eyelid is thinnest within whole body, same parameter values as in normal skin are used for analysis. A resting man doesn't sweet when atmospheric temperature is below 20°C. A man at 21°C can lose 25% of his/her heat by sweat evaporation[8]. Thus, in this study E equals $0Wm^{-2}$ and $192Wm^{-2}$ are chosen for extreme cold and hot environment and $96Wm^{-2}$ for room temperature respectively[1]. The temperature values so obtained are shown in figure 3.

From figure 3 it is observed that the corneal surface temperature is dropped by 3.69°C and 4.75°C respectively when eyelids are opened and closed. The corneal surface temperature difference between eyelid closure and opening are $1.73, 0.67°\text{C}$ and $0.58°\text{C}$ respectively at low, average and high evaporation rates. The corneal temperature is dropped from 35.03°C to 30.28°C and from 33.30°C to 29.61°C, when the evaporation rate varies from low to high. The results show that when evaporation rate increases corneal temperature decreases but decreasing rate is slower in eyelid opening than closure.

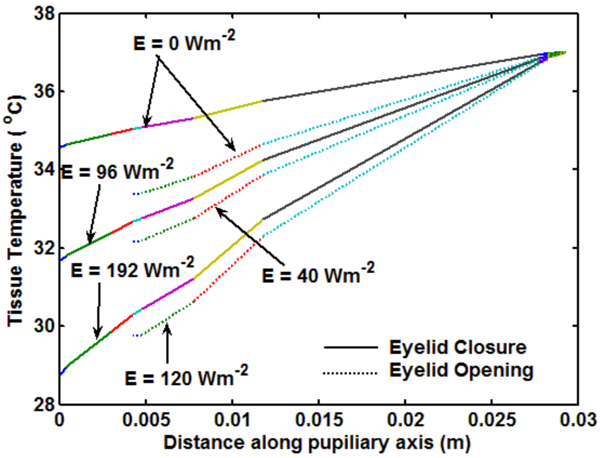

Figure 3. Temperature variation for different evaporation rates

3.1.2. Effects of ambient temperatures. Heat loss/gain from corneal surface depends on the temperature difference between the eye tissue (cornea or skin) and the surrounding air. This loss/gain occurs due to conduction, convection, radiation and evaporation. About $3\%, 15\%, 60\%$ and 22% of total heat is lost by conduction, convection, radiation and evaporation respectively from human body[8]. In this study, the ambient temperatures 0°C and 50°C are used to study the thermal response to extreme cold and extreme hot environments and 25°C for normal. The numerical results are shown in figure 4.

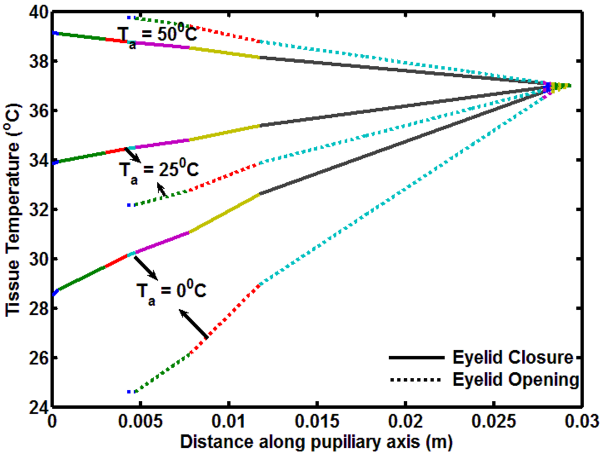

Figure 4. Temperature variation for different ambient temperatures

It is observed from figure 4 that corneal temperature is increased from 24.37°C to 39.78°C and from 30.11°C to 38.76°C respectively in case of eyelid opening and closure. That is, an increase of 8.65°C and 15.41°C is observed at corneal surface when ambient temperature varies from 0°C to 50°C. The temperature difference at cornea between eyelid closure and opening are 5.74, 2.36°C and 1.02°C at ambient temperatures 0, 25°C and 50°C respectively. The eye is heated, when environmental temperature drops below normal level and vice versa by blood flow in eyelid.

3.1.3. Effects of blood temperatures. The blood flow in the choroid/retina/iris acts as the major source of heat for opened eye. In addition, blood flow in skin/orbicularis oculi/tarsal plate of eyelid acts as a heating source for anterior eye. Although, blood flow occurs only in a few parts, its temperature significantly affects the temperature throughout the human eye. The blood temperatures 35°C and 39°C are used to study the thermal response due to sickness and 37°Cfor normal case. The numerical result is presented in figure 5.

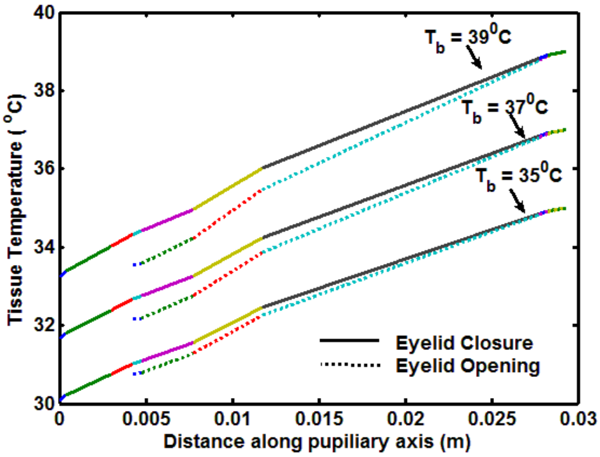

FIGURE 5. Temperature variation for different blood temperatures

It is observed from figure 5 that corneal temperature is increased from 31°C to 34.31°C and from 30.69°C to 33.45°C respectively in case of eyelid closure and opening. The corneal temperature is increased to 3.31°C and 2.76°C when blood temperature is increased from 35°C to 39°C. The corneal temperature differences are 0.31°C, 0.58°C and 0.86°C respectively between eyelid closure and opening at different blood temperatures 35°C, 37°C and 39°C respectively. Increase in blood temperature increases corneal temperature but the rate is higher in closed eye than opened eye.

3.1.4. Effects of lens thermal conductivities. The lens contains 65 percent of water. The water content of the lens decreases as age of individual increases[15]. Different water contents will produce different thermal conductivities. In this study, the lens thermal conductivities $0.30Wm^{-1\circ}C^{-1}$ and $0.54Wm^{-1\circ}C^{-1}$ are used as extreme values and $0.4Wm^{-1\circ}C^{-1}$ as a normal value. The numerical results are presented in figure 6.

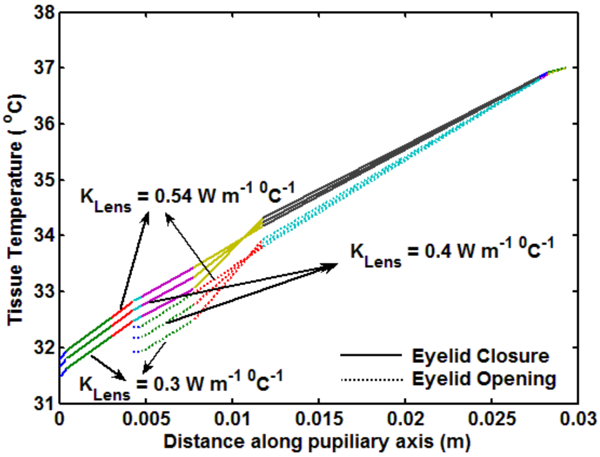

Figure 6. Temperature variation for different lens thermal conductivities

It is observed that the corneal temperature increases from 32.46°C to 32.81°C and from 31.82°C to 32.27°C respectively in case of eyelid closure and opening, when the lens thermal conductivity is increased from $0.3Wm^{-1\circ}C^{-1}$ to $0.54Wm^{-1\circ}C^{-1}$. The corneal surface temperature is increased by 0.35°C and 0.45°C respectively, when lens thermal conductivity varies from $0.3Wm^{-1\circ}C^{-1}$ to $0.54Wm^{-1\circ}C^{-1}$. The corneal temperature difference between eyelid closure and opening are 0.64°C, 0.58°C and 0.54°C at lens thermal conductivities 0.3, 0.4, $0.54Wm^{-1\circ}C^{-1}$ respectively.

3.2. Transient results. The transient temperature distribution cornea for $t = 3600$ seconds is presented in figure 7. The parameter values $E = 0, 40$ and $120Wm^{-2}$ for cornea surface, $E = 0, 96$ and $192Wm^{-2}$ for eyelid skin surface are used to find the temperature profile at $T_a = 0, 25$ and 50°C respectively.

In eyelid opening, figure 7, the temperature rises rapidly over the first 1500 seconds, this increasing rate then slows down and steady state is achieved in around 1969, 2267 and 2767 seconds at $T_a = 0, 25$ and 50°C respectively. Similarly, the temperature rises rapidly over the first 8-9 min, this increasing rate then slows down and steady state is achieved in around 1326, 1374 and 1529 seconds at $T_a = 0, 25$ and 50°C respectively when eyelid is closed. The cornea surface temperature differences between eyelid opening and closure are 9.33,

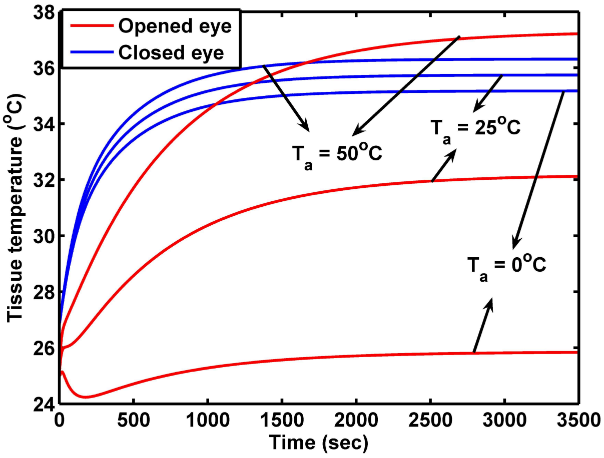

FIGURE 7. Transient temperature variations of cornea

3.62 and 0.9°C at ambient temperatures $T_a = 0, 25$ and 50°C respectively. The corneal surface temperature varies from 25.84°C to 37.21°C in case of eyelid opening and from 35.17°C to 36.31°C in case of eyelid closure respectively, when ambient temperature varies from 0°C to 50°C.

4. Discussion

The steady state results show that when evaporation increases the eye temperature decreases. More heat is lost during closure of eyelid via evaporation. The skin is vascular where as cornea is avascular. Vascular skin has higher temperature than avascular cornea. Evaporative water molecules pick up high amount of heat energy from vascular skin than avascular cornea. Also the lipid layer of cornea prevents evaporation of tear film causing less heat transfer from cornea to environment. But eyelid skin doesn't contain such layer to prevent evaporation. In addition, anterior part of eyelid contains glands of zeis and glands of moll, which are sweat glands that open into the hair of follicles and in a row near the base of the eyelashes. Hence more evaporation occurs from eyelid skin surface and causes rapid temperature drop in eyelid closure than opening.

Increase in ambient temperature increases corneal temperature. Rapid increase in corneal temperature is observed in eyelid opening. The ambient temperature has direct effect on cornea in eyelid opening. At low ambient temperature, high amount of heat loss occurs from cornea to environment via convection and radiation, but the heating mechanism is only conduction from body core. Similarly at high ambient temperature, heat loss occurs from eye surface via radiation; tear evaporation and conduction process from cornea to body core. Hence at extreme hot/cold temperatures corneal temperature is increased /decreased rapidly during eyelid opening. However, in eyelid closure blood flow in vascular eyelid prevents cornea from high degree of heating/cooling in extreme environments. Hence, corneal surface temperature is less affected by ambient temperatures when eyelids are closed than opened.

When blood temperature increases eye temperature also increases. The blood temperature on eyelid, when closed, has direct effect on anterior corneal surface temperature. The blood flow in anterior eyelid and posterior retina/choroid shielded the eyeball by blood temperature, when closed. So, anterior corneal surface temperature varies according to blood temperature in eyelid closure. But in eyelid opening, corneal surface temperature depends on posterior choroidal blood flow and ambient temperature. Conduction of heat from choroid/ retina is the only source of heating cornea. Hence increase in blood temperature increases corneal temperature but impact is less in case of eyelid opening.

Increase in lens thermal conductivity increases corneal temperature and decreases posterior lens temperature in both cases. Higher thermal conductivities of the lens permits more heat transfer from the posterior region (high temperature region) to anterior region (low temperature region) of the eye, and thus causes the corneal surface temperature to increase. But there is no significant effect of lens thermal conductivities in case of eyelid closure and opening for corneal temperature.

Hence the dominant parameters affecting eye surface temperature are blood temperatures, ambient temperatures and evaporation rates. In all these analysis the corneal center is noticed as the most sensitive point for thermal response. From figures 3, 4, 5 and 6, it is seen that the temperature changes are dominant in cornea and anterior part, very small in vitreous and almost not in sclera.

The transient results show that the steady state temperature is achieved earlier in case of eyelid closure. In eyelid closure, the eye temperature increases rapidly for 600 seconds and then plateaus in around 1400 seconds. The variation in ambient temperatures has less effect on eye temperature when closed because the temperature starts to plateau from same time in all three cases and variation is only 1.14°C. In eyelid opening, temperature is increased mostly in the first 1500 seconds then plateaus in around 2300 seconds. The corneal temperature reaches in steady state earlier at low ambient temperature than high, when opened, and the variation 11.37°C is also high. High volume of blood flow in eyelid prevents rise/drop of corneal temperature in extreme ambient conditions and balance eye temperature faster in case of eyelid closure than opening.

In this model the steady state corneal temperature is found to be 32.17°C and 34.43°C respectively in eyelid opening and closure. Ng and Ooi[15] summarized the corneal temperatures obtained from different experimental setup, which vary from 32°C to 36.6°C. Thus, our steady state results are in a good agreement with past experimental results. Mapstone[14] studied the effect of eyelid opening and closure experimentally using bolometer. He found a fall in temperature with a range of 0.6°C to 1.6°C when the eyelids are opened from closed position. Similarly, a rise in temperature with a range of 1.1°C to 2.2°C when the eyelids are closed from opened position. He obtained the results at $19 - 24$°C variation of ambient temperatures. Our result at ambient temperature 25°C shows a rise in temperature of 2.36°C when the eyelids are closed from open position.

Kessel et al.[10] found experimentally that steady state corneal temperature is achieved between 33°C to 35°C, when ambient temperature increased from 22°C to 28°C. Also they found that 20°C increase in ambient temperature, from 2°C to 22°C, increases corneal temperature by 3°C. In our model, the steady state corneal temperature is achieved 32.17°C at ambient temperature 25°C. Also, 20°C increase in ambient temperature increases corneal temperature by 3.38°C. The slight variation in temperature may be due to the consideration of different environmental temperatures and due to the consideration of various parameter values at the layers of human eye.

5. Conclusion

We presented a one dimensional finite element model of human eye and simulate its steady and transient temperature distribution along pupillary axis in case of eyelid opening and closure. The blood flow in eyelid increases anterior temperature of human eye when eyelid is closed. The eyelid not only acts as heater/cooler of the human eye but also helps to maintain eye temperature constant. In sensitivity analysis, the effect of various blood temperatures, evaporation rates, ambient temperatures and lens thermal conductivities were analyzed. It is found that blood temperature, ambient temperature and evaporation rate are the dominant factors affecting corneal surface temperature strongly. The variation in the thermal conductivity of lens leads to minor changes in anterior corneal temperature. The transient analysis shows that, steady state temperature is achieved earlier and the variation in temperature is very less in eyelid closure than opening. The results obtained in this study are compared with the available experimental data and are in good agreement with them.

The present model is one dimensional which underestimates the real temperature distribution in eye geometry. The iris is not included because it doesn't lie on pupillary axis, but iris is assumed to be the heating/cooling source for anterior eye and it also prevents lens from extreme temperature conditions. The model did not include the effects of aqueous humor flow and tear flow in temperature distribution in human eye.

Beside these disadvantages, we separated the retina/choroid region from sclera and the blood perfusion and metabolism effects were analyzed. We also modeled eyelid as a three layer structure and effects of those layers were analyzed. At last, we can conclude that eyelid is one of the significant eye component that affects eye temperature.

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