Efficient Resource Allocation with Multiple Practical Constraints in OFDM-based Cooperative Cognitive Radio Networks

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Abstract

This paper addresses the problem of resource allocation in amplify-and-forward (AF) relayed OFDM based cognitive radio networks (CRNs). The purpose of resource allocation is to maximize the overall throughput, while satisfying the constraints on the individual power and the interference induced to the primary users (PUs). Additionally, different from the conventional resource allocation problem, the rate-guarantee constraints of the subcarriers are considered. We formulate the problem as a mixed integer programming task and adopt the dual decomposition technique to obtain an asymptotically optimal power allocation, subcarrier pairing and relay selection. Moreover, we further design a suboptimal algorithm that sacrifices little on performance but could significantly reduce computational complexity. Numerical simulation results confirm the optimality of the proposed algorithms and demonstrate the impact of the different constraints.

Keywords: CRN, OFDM, AF, cooperative communication, joint resource allocation

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1. Introduction

The cognitive radio (CR) technology has been proposed to improve the spectrum utilization and provide adaptability for wireless transmission on licensed spectrums [1]. The CR performance and the spectrum utilization can be further improved by adopting cooperative communications and orthogonal frequency division multiplexing (OFDM) technology [2,3].

With the internal flexibility of OFDM in power loading across subcarriers, a lot of works have been already done on the resource allocation in non-cognitive relay systems [4-6]. However, resource allocation in OFDM-based cooperative CRNs is more complex than that in a conventional OFDM system because more constraints must be considered to protect the performance of primary users (PUs). Thus, many existing resource allocation algorithms are not suitable for OFDM-based cooperative CRNs. Resource allocation for OFDM-based cooperative CRNs have attracted much attention recently. The authors in [7] proposed a resource allocation algorithm in cognitive wireless networks based on game theory. The problem of relay selection and optimal resource allocation for two-way relaying CRN was investigated in [8]. The authors in [9] proposed a joint subcarrier matching and power loading algorithm in relay-aided CRN. A problem of throughput maximization in a multi-carrier underlay CRN with constrained transmission power and interference threshold had been studied in [10]. An extension of [10] that employed an amplify-and-forward (AF) relay to aid transmission could be found in [11]. The authors in [12] proposed a resource allocation algorithm for multiuser OFDM-based CRNs, in which the proportional rate constraints were considered. However, the works in [7-12] considered limited practical constraints.

The motivation for this paper is twofold. Firstly, the rate provided by allocated subcarriers may be too low for practical usage. Therefore, it is important to guarantee that the rate of each subcarrier is not below a certain threshold. Secondly, the resource allocation problems are generally NP-Hard, so the designation of low-complexity and high-performance algorithm is a great challenge. The main contributions of this paper are the following aspects. 1) The joint resources (powers, subcarriers, relays) allocation problem is investigated in AF relayed OFDM-based CRNs. 2) Besides the power and interference constraints considered in traditional schemes, the rate-guarantee constraints are also considered to adapt to practical usage. 3) An asymptotically optimal resource allocation algorithm that adopts dual decomposition technique is proposed. 4) A low-complexity heuristic algorithm with little performance degradation is designed to solve the problem.

The remaining of this paper is organized as follows. Section 2 introduces the system model while the problem is formulated and the optimal scheme is presented in section 3. Section 4 gives a suboptimal algorithm, of which the computer simulation and numerical analysis are provided in section 5. Finally, we draw conclusions in section 6.

2. System Model

The system model of the OFDM-based cooperative CRN is shown in **Fig.1**, where the cognitive users share radio spectrum with a primary system. The available spectrum bandwidth of CRN is divided into N subcarriers, and the bandwidth of each subcarrier is Δf ($\Delta f = B / N$). We assume that the source transmits signals to destination through M relays and there is no direct link between them. Moreover, the relays operate in half-duplex mode with AF-protocol, where AF-protocol is divided into two time slots. In the first time slot, one relay is selected to receive the signals from the source on the j^{th} subcarrier. In the second time slot, the selected relay amplifies the signals and forwards them to the destination via subcarrier k.

The j^{th} subcarrier in the source should be paired with only one subcarrier k in the destination, which may not be the same as j. It is assumed that perfect instantaneous fading gains are available at the transceivers of both CRN and PU.

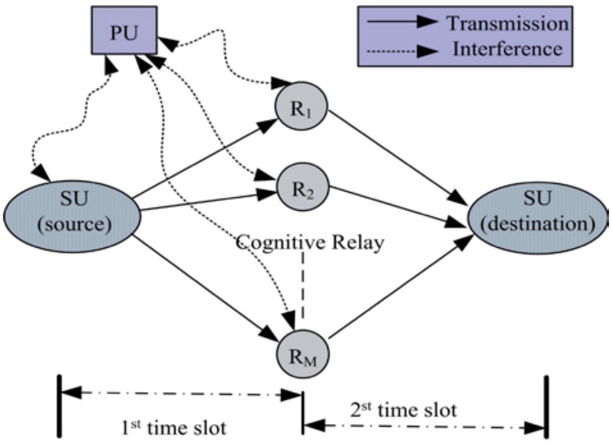

Fig. 1. System model of OFDM-based cooperative cognitive radio network

In OFDM-based CRN, the interference introduced to the PU on the i^{th} subcarrier is described as [13]

$$I_i = \int_{d_i - B/2}^{d_i + B/2} G_i P_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df \; = P_i \rho_i \tag{1}$$

where d_i represents the spectral distance between the i^{th} subcarrier and the PU band. G_i denotes the square of the channel gain between the i^{th} subcarrier and the PU band. P_i is the total transmission power emitted by the i^{th} subcarrier, and T_s is the symbol duration. Let ρ_i denote the interference factor of the i^{th} subcarrier to the PU band. Similarly, the interference power introduced by PU signal with power spectrum density $\vartheta(e^{jw})$ into the band of the i^{th} subcarrier is [13]

$$J_i = \int_{d_i - \Delta f/2}^{d_i + \Delta f/2} G_i \vartheta(e^{jw}) dw \tag{2}$$

J_i can be modeled as additive white Gaussian noise (AWGN) by applying the law of large number or by assuming that the primary and cognitive system are using an independent and random Gaussian codewords [14,15].

From the source to the m^{th} relay, denote the channel coefficient over the j^{th} subcarrier by h^j_{S,R_m} and the power allocation by P^j_{S,R_m} and the transmitted symbol by x^j_{S,R_m}. The signal received at the m^{th} relay is

$$Y^j_{S,R_m} = \sqrt{P^j_{S,R_m}} h^j_{S,R_m} x^j_{S,R_m} + \upsilon^j_{S,R_m} \tag{3}$$

υ^j_{S,R_m} is the summation of AWGN and the interference introduced by the PU signal into the j^{th} subcarrier, where AWGN~$CN(0, \sigma^2_{AWGN_{j,m}})$. If the m^{th} relay is chosen to amplify and forward the signal on the k^{th} subcarrier, the received signal at the receiver will be

$$Y_{R_m,D}^k = \frac{\sqrt{P_{R_m,D}^k} h_{R_m,D}^k \sqrt{P_{S,R_m}^j} h_{S,R_m}^j x_{S,R_m}^j}{\sqrt{P_{S,R_m}^j H_{S,R_m}^j + \sigma_{j,m}^2}} + \frac{\sqrt{P_{R_m,D}^k} h_{R_m,D}^k \upsilon_{S,R_m}^j}{\sqrt{P_{S,R_m}^j H_{S,R_m}^j + \sigma_{j,m}^2}} + \omega_{R_m,D}^k \tag{4}$$

where $P_{R_m,D}^k$ is the power that the relay allocates to transmit the received signal on the k^{th} subcarrier. $h_{R_m,D}^k$ is the channel gain between the m^{th} relay and the destination on the k^{th} subcarrier. $\sigma_{j,m(m,k)}^2 = \sigma_{AWGN_{j,m(m,k)}}^2 + J_{j(k)}$, and H_{S,R_m}^j ($H_{R_m,D}^k$) is the square of the j^{th} (k^{th}) subcarrier fading gain over source to R_m (R_m to destination). $\omega_{R_m,D}^k$ is the summation of AWGN and the interference introduced by the PU signal into the k^{th} subcarrier, where AWGN~$CN(0, \sigma_{AWGN_{m,k}}^2)$.

The transmission rate over subcarrier pair (j,m,k) at high signal-to-noise radio can be approximated as (5), such an approximated is reasonable as discussed in [16].

$$R_{(j,m,k)} \approx \frac{1}{2}\log_2(1+\frac{P_{S,R_m}^j H_{S,R_m}^j P_{R_m,D}^k H_{R_m,D}^k}{\sigma_{j,m}^2 P_{S,R_m}^j H_{S,R_m}^j + \sigma_{m,k}^2 P_{R_m,D}^k H_{R_m,D}^k}) \tag{5}$$

3. Problem Formulation and Optimal Solution

The optimization objective is to maximize the overall throughput of CRN by optimizing the power allocation, subcarrier pairing and relays assignment, while satisfying multiple practical constraints. Accordingly, the optimal problem is formulated as

$$\begin{aligned}
&\max_{P_{S,R_m}^j \geq 0, P_{R_m,D}^k \geq 0, \phi_{(j,k)}, \varphi_{(j,m,k)}} \sum_{m=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N} \phi_{(j,k)} \varphi_{(j,m,k)} R_{(j,m,k)} \\
&\text{s.t.} \\
&C1: \sum_{m=1}^{M}\sum_{j=1}^{N} P_{S,R_m}^j \leq P_S \\
&C2: \sum_{k=1}^{N} P_{R_m,D}^k \leq P_{R_m}, \quad \forall m \\
&C3: \sum_{m=1}^{M}\sum_{j=1}^{N} P_{S,R_m}^j \rho_j \leq I_{th} \\
&C4: \sum_{m=1}^{M}\sum_{k=1}^{N} P_{R_m,D}^k \rho_{m,k} \leq I_{th} \\
&C5: R_{(j,m,k)} \geq R_{th}, \forall j,m,k \\
&C6: \sum_{k=1}^{N} \phi_{(j,k)} \leq 1, \quad \forall j; \quad \sum_{j=1}^{N} \phi_{(j,k)} \leq 1, \quad \forall k; \\
&C7: \sum_{m=1}^{M} \varphi_{(j,m,k)} \leq 1, \quad \forall j,k; \\
&C8: \phi_{(j,k)} \in \{0,1\}, \quad \varphi_{(j,m,k)} = \{0,1\}
\end{aligned} \tag{6}$$

where N denotes the total number of subcarriers while I_{th} is the interference threshold prescribed by PU. P_S and P_{R_m} are the available power budget in the source and the m^{th} relay respectively. ρ_j and $\rho_{m,k}$ are the j^{th} (k^{th}) subcarrier interference factor to the PU band from the source and the m^{th} relay respectively. R_{th} is the rate-guarantee threshold. $\phi_{(j,k)}=1$ if the j^{th} subcarrier from the source is paired with the k^{th} to the destination, and zero otherwise. Additionally, $\varphi_{(j,m,k)}$ is the relay assignment indicator which equals to one if the pair (j,k) is assigned to the m^{th} relay and zero otherwise.

Finding the optimal variables in (6) is a mixed binary integer programming problem. It is difficult to find the global optimal solution. However, under the time sharing condition the duality gap is asymptotically zero for sufficiently large N [17]. Since the time sharing condition is readily satisfied in our case, we can solve the dual problem of the original problem to obtain an asymptotically optimal solution. To make the analysis more clearly and without loss of generality, the noise variance is assumed to be constant for all the subcarriers and users, i.e. $\sigma^2_{j,m}=\sigma^2_{m,k}=\sigma^2$. The dual problem associated with the primal problem (6) can be written as

$$\min_{\tau\geq 0,\upsilon\geq 0,\overline{\tau}_m\geq 0,\overline{\upsilon}\geq 0,\eta_{(j,m,k)}\geq 0} f(\tau,\upsilon,\overline{\tau}_m,\overline{\upsilon},\eta_{(j,m,k)}) \tag{7}$$

where τ and $\overline{\tau}_m$ are the dual variables associated with the power constraint at the source and the different relays respectively. The dual variables υ and $\overline{\upsilon}$ are related to the interference constraints during the first and second time slots respectively. Moreover, $\eta_{(j,m,k)}$ is the dual variable associated with the rate-guarantee constraint. The dual function $f(\tau,\upsilon,\overline{\tau}_m,\overline{\upsilon},\eta_{(j,m,k)})$ is defined as follows

$$\begin{aligned} f(\tau,\upsilon,\overline{\tau}_m,\overline{\upsilon},\eta_{(j,m,k)}) &= \max_{\substack{P^j_{S,R_m}\geq 0,P^k_{R_m,D}\geq 0,\\ \phi_{(j,k)},\varphi_{(j,m,k)}}} L \\ s.t. \quad & (C6,C7,C8) \end{aligned} \tag{8}$$

where the Lagrangian function L is defined as

$$\begin{aligned} L = &\sum_{m=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N}(\phi_{(j,k)}\varphi_{(j,m,k)}R_{(j,m,k)}+\eta_{(j,m,k)}(R_{th}-R_{(j,m,k)}))+\tau(P_S-\sum_{m=1}^{M}\sum_{j=1}^{N}P^j_{S,R_m}) \\ &+\upsilon(I_{th}-\sum_{m=1}^{M}\sum_{j=1}^{N}P^j_{S,R_m}\rho_j)+\sum_{m=1}^{M}\overline{\tau}_m(P_{R_m}-\sum_{k=1}^{N}P^k_{R_m,D})+\overline{\upsilon}(I_{th}-\sum_{m=1}^{M}\sum_{k=1}^{N}P^k_{R_m,D}\rho_{m,k}) \end{aligned} \tag{9}$$

The dual function in (8) can be rewritten as (10)

$$\begin{aligned} f(\tau,\upsilon,\overline{\tau}_m,\overline{\upsilon},\eta_{(j,m,k)}) = &\sum_{m=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N}\phi_{(j,k)}\varphi_{(j,m,k)}g(P^j_{S,R_m},P^k_{R_m,D}) \\ &+\sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{N}\eta_{(j,m,k)}R_{th}+\tau P_S+\sum_{m=1}^{M}\overline{\tau}_m P_{R_m}+I_{th}(\upsilon+\overline{\upsilon}) \\ s.t. \quad & (C6,\ C7,\ C8) \end{aligned} \tag{10}$$

where

$$g(P^j_{S,R_m},P^k_{R_m,D}) = R_{(j,m,k)}(1-\eta_{(j,m,k)})-P^j_{S,R_m}(\tau+\upsilon\rho_j)-P^k_{R_m,D}(\overline{\tau}_m+\overline{\upsilon}\rho_{k,m}) \tag{11}$$

Thus the problem (10) is decomposed into three sub-problems.

1) Sub-problem 1: Power allocation scheme. We assume that (j,m,k) is a valid subcarrier pair, and set the different dual variables with the initial values. Therefore, the optimal power allocation can be determined by solving the following problem for every (j,m,k) pair,

$$\begin{aligned} &\max_{P^j_{S,R_m},P^k_{R_m,D}} \quad g(P^j_{S,R_m},P^k_{R_m,D}) \\ &s.t.\ P^j_{S,R_m} \geq 0,\ P^k_{R_m,D} \geq 0 \end{aligned} \tag{12}$$

Solving (12) for the optimal power, we can obtain the optimal power allocation P^{*j}_{S,R_m} and $P^{*k}_{R_m,D}$.

2) Sub-problem 2: Relay assighnment scheme. The power variables can be eliminated by substituting the optimal power allocation found by (12) to (10). Correspondingly, we can solve the following problem for every (j,k) to get the best relay,

$$\begin{aligned} &\max_{\varphi_{(j,m,k)}} \sum_{m=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N}\phi_{(j,k)}\varphi_{(j,m,k)}g(P^{*j}_{S,R_m},P^{*k}_{R_m,D}) + \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{N}\eta_{(j,m,k)}R_{th} + \tau P_S + \sum_{m=1}^{M}\overline{\tau}_m P_{R_m} + I_{th}(\upsilon+\overline{\upsilon}) \\ &s.t. \quad (C7,\ C8) \end{aligned} \tag{13}$$

Therefore, the optimal relay assignment strategy is achieved by allocating the (j,k) pair to the relay which maximizes the function $g(P^{*j}_{S,R_m},P^{*k}_{R_m,D})$. If $m = \arg\max_m g(P^{*j}_{S,R_m},P^{*k}_{R_m,D})$, then set $\varphi_{(j,m,k)} = 1$ and zero otherwise. By performing this allocation, the best relay is determined for every possible subcarrier pair.

3) Sub-problem 3: Subcarriers pairing scheme. The optimal subcarriers pair can be obtained by the following problem after the powers and relay allocation are determined for every subcarrier pair,

$$\begin{aligned} &\max_{\phi_{(j,k)}} \sum_{m=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N}\phi_{(j,k)}g(P^{*j}_{S,R_m},P^{*k}_{R_m,D}) + \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{N}\eta_{(j,m,k)}R_{th} + \tau P_S + \sum_{m=1}^{M}\overline{\tau}_m P_{R_m} + I_{th}(\upsilon+\overline{\upsilon}) \\ &s.t. \quad (C6,\ C8) \end{aligned} \tag{14}$$

In order to get the solution of (14), the Hungarian method is used.

Once the optimal solutions, i.e. P^{*j}_{S,R_m}, $P^{*k}_{R_m,D}$, $\phi^*_{(j,k)}$ and $\varphi^*_{(j,m,k)}$, are obtained, substituting them into (9) and then the result back into (8), we can get the optimal dual function for the given values of the dual variables. The subgradient method can be used to solve the dual problem with guaranteed convergence. For any initial values τ^0, υ^0, $\overline{\tau}^0_m$, $\overline{\upsilon}^0$ and $\eta^0_{(j,m,k)}$, the dual variables at the $(i+1)^{th}$ iteration can be updated as

$$\begin{aligned} &\tau^{(i+1)} = \tau^{(i)} - \delta^{(i)}(P_S - \sum_{m=1}^{M}\sum_{j=1}^{N}P^{*j}_{SR_m}) \\ &\upsilon^{(i+1)} = \upsilon^{(i)} - \delta^{(i)}(I_{th} - \sum_{m=1}^{M}\sum_{j=1}^{N}P^{*j}_{SR_m}\rho_j) \\ &\overline{\tau}^{(i+1)}_m = \overline{\tau}^{(i)}_m - \delta^{(i)}(P_{R_m} - \sum_{k=1}^{N}P^{*k}_{R_mD}),\ \forall m \\ &\overline{\upsilon}^{(i+1)} = \overline{\upsilon}^{(i)} - \delta^{(i)}(I_{th} - \sum_{m=1}^{M}\sum_{k=1}^{N}P^{*k}_{R_mD}\rho_{m,k}) \\ &\eta^{(i+1)}_{(j,m,k)} = \eta^{(i)}_{(j,m,k)} - \delta^{(i)}(R_{th} - R_{(j,m,k)}),\ \forall j,m,k \end{aligned} \tag{15}$$

where δ^i is the step size that can be updated according to the nonsummable diminishing step size policy [18]. With the updated values of the dual variables, the optimal power allocation and subcarrier matching are evaluated again. The iterations are repeated until convergence is reached.

4. Suboptimal Algorithm

In order to decrease the computational complexity without sacrificing performance, we propose a suboptimal heuristic algorithm for the aforementioned optimization problem by which the different resources are allocated jointly with lower computational complexity than that of the optimal solution. The proposed algorithm takes into consideration the different channel qualities, the rate-guarantee constraint, the available power budgets, the interference introduced to the PU and the limitation introduced by using AF-protocol.

The suboptimal algorithm addresses the optimization problem in two steps. Firstly, subcarrier pairing and relay selection scheme is proposed with initial power values. Then, the optimal power allocation scheme is used to improve the system performance. We commence the description of the proposed schemes by defining the sets S1 and $S2$ to include all the non-assigned subcarriers in the source and the destination sides respectively. Moreover, define the set C to contain all the relays in the network.

4.1 Proposed Subcarrier Pairing and Relay Selection Scheme

In the proposed subcarrier pairing and relay selection scheme, we are going to use the harmonic mean criterion to select the best relay for each pair of subcarriers. In the source side, assume that the available source power is distributed uniformly over the subcarriers, i.e. $P_j^{uni} = P_S / N$, and also assume that the interference introduced to the PUs by every subcarrier is equal; from (1), the maximum allowable power that can be allocated to the j^{th} subcarrier is $P_j^{\max} = I_{th} / (N\rho_j)$. Therefore, the allocated power to the j^{th} subcarrier in the source side is $P_{S,R_m}^j = \min(P_S / N, I_{th} / (N\rho_j))$. To complete the algorithm, we define the harmonic mean criterion as

$$H = \frac{1}{(P_{S,R_m}^j H_{S,R_m}^j)^{-1} + (P_{R_m,D}^k H_{R_m,D}^k)^{-1}} \tag{16}$$

For every subcarrier in the source side, we search over all the relays and all the non-assigned subcarriers in the destination sides to find the subcarrier-relay pair with the largest *H*. Moreover, the rate and the power budget of the selected relay are updated while the rate-guarantee constraint is considered. This trend continues until the set S1 becomes empty. The assigning procedures of a particular subcarrier $j \in S1$ are as follows

1) For every relay $m \in C$ and subcarrier $k \in S2$, evaluate $P_{m,k}^{uni} = P_{R_m} / |S2|$ and $P_{m,k}^{\max} = I_{th} / (N\rho_{m,k})$ where $|S2|$ means the cardinality of the set $S2$. Therefore, the allocated power to the k subcarrier in the m^{th} relay is $P_{R_m,D}^k = \min(P_{R_m} / S2, I_{th} / (N\rho_{m,k}))$.

2) Find the best relay m^* and the subcarrier pair k^* satisfying $(m^*, k^*) = \arg\max_{m,\,k}\{H\}$.

Compute the communication rate $R_{(j,m^*,k^*)}$ for the selected link (j, m^*, k^*).

Case1: If $R_{(j,m^*,k^*)} < R_{th}$, the rate and the power budget of the m^{*th} relay need not update. Remove the subcarrier j and k^* from the sets $S1$ and $S2$ respectively and do not make any allocation.

Case2: If $R_{(j,m^*,k^*)} \geq R_{th}$, then set $\phi_{(j,k^*)} = 1$, $\varphi_{(j,m^*,k^*)} = 1$ and $P^k_{R_m,D} = P^{k^*}_{R_m^*,D}$. Additionally, update the power budget of the m^{th} relay as $P_{R_{m^*}} = P_{R_{m^*}} - P^{k^*}_{R_m^*,D}$. Remove the subcarrier j and k^* from the sets $S1$ and $S2$ respectively.

3) This trend continues until all the subacarriers in the source are processed, i.e $S1 = \varnothing$.

4.2 Proposed Power Allocation Scheme

The proposed subcarrier pairing and relay selection scheme determines the best paring link (j, m_j^*, k_j^*) for every $j \in \overline{S1}$, where $\overline{S1} = \{ j \mid R_{(j,m_j^*,k_j^*)} \geq R_{th} \}$. m_j^* and k_j^* are the best relay and paring subcarrier at destination for subcarrier j at source. Consequently, the initial optimization problem (6) can be redescribed as

$$\max_{P^j_{SR_{m_j^*}}, P^{k_j^*}_{R_{m_j^*}D}} \sum_{\{j \mid j \in \overline{S1}\}} R_{(j,m_j^*,k_j^*)}$$
$$s.t.$$
$$\overline{C1}: \sum_{\{j \mid j \in \overline{S1}\}} P^j_{S,R_{m_j^*}} \leq P_S$$
$$\overline{C2}: \sum_{\{j \mid m_j^* = m\}} P^{k_j^*}_{R_{m_j^*},D} \leq P_{R_{m_j^*}} \quad \forall m \in M \tag{17}$$
$$\overline{C3}: \sum_{\{j \mid j \in \overline{S1}\}} P^j_{S,R_{m_j^*}} \rho_j \leq I_{th}$$
$$\overline{C4}: \sum_{\{j \mid j \in \overline{S1}\}} P^{k_j^*}_{R_{m_j^*},D} \rho_{m_j^*,k_j^*} \leq I_{th}$$

The Lagrangian of (17) is defined as

$$\begin{aligned} L = {} & R_{(j,m_j^*,k_j^*)} - \lambda\Big(P_S - \sum_{\{j \mid j \in \overline{S1}\}} P^j_{S,R_{m_j^*}}\Big) - \mu\Big(I_{th} - \sum_{\{j \mid j \in \overline{S1}\}} P^j_{S,R_{m_j^*}} \rho_j\Big) \\ & - \sum_{\{m_j^* \mid j \in \overline{S1}\}} \overline{\lambda}_{m_j^*}\Big(P_{R_{m_j^*}} - \sum_{\{j \mid j \in \overline{S1}\}} P^{k_j^*}_{R_{m_j^*},D}\Big) - \overline{\mu}\Big(I_{th} - \sum_{\{j \mid j \in \overline{S1}\}} P^{k_j^*}_{R_{m_j^*},D} \rho_{m_j^*,k_j^*}\Big) \end{aligned} \tag{18}$$

where $\lambda, \mu, \overline{\lambda}_{m_j^*}$ and $\overline{\mu}$ are Lagrangian variables. The optimal solution can be obtained from the KKT conditions as follows,

$$\hat{P}^j_{S,R_{m_j^*}} = \frac{\alpha_j P^{k_j^*}_{R_{m_j^*},D}}{2\beta_j} \left[\frac{\sqrt{\alpha_j^2 (P^{k_j^*}_{R_{m_j^*},D})^2 + D1 - 1}}{1 + \alpha_j P^{k_j^*}_{R_{m_j^*},D}} - 1 \right]^+ \tag{19}$$

$$\hat{P}^{k_j^*}_{R_{m_j^*},D} = \frac{\beta_j P^j_{S,R_{m_j^*}}}{2\alpha_j} \left[\frac{\sqrt{\beta_j^2 (P^j_{S,R_{m_j^*}})^2 + D2 - 1}}{1 + \beta_j P^j_{S,R_{m_j^*}}} - 1 \right]^+ \tag{20}$$

where $\alpha_j = H^j_{S,R_m} / \sigma^2$, $\beta_j = H^k_{R_m,D} / \sigma^2$, $D1 = 2\beta_j (1 + \alpha_j P^{k_j^*}_{R_{m_j^*},D}) / (\lambda + \mu \rho_j)$ and

$D2 = 2\alpha_j(1+\beta_j P^j_{S,R_{m_j^*}})/(\bar{\lambda}_{m_j^*} + \bar{\mu}\rho_{m_j^*,k_j^*})$.

In order to determine the optimal power values, we divide the objective function of the (17) into two sub-problems equivalently and apply an alternating optimization method to work out the $\hat{P}^j_{S,R_{m_j^*}}$ and $\hat{P}^{k_j^*}_{R_{m_j^*},D}$ efficiently.

1)sub-problem 1: Optimal power allocation for $\hat{P}^j_{S,R_{m_j^*}}$. As shown in (19), $\hat{P}^j_{S,R_{m_j^*}}$ is regarded as a function of $P^{k_j^*}_{R_{m_j^*},D}$, where $[x]^+ = \max(x,0)$. We can adjust λ and μ to satisfy $\overline{C1}$ and $\overline{C3}$, so we can obtain the $\hat{P}^j_{S,R_{m_j^*}}$ with (19) by setting $P^{k_j^*}_{R_{m_j^*},D}$ to an initial value.

The power allocation scheme at the source is described in **Table 1**.

Table 1. The power allocation at the source

Algorithm 1
1: Initialize λ=0 , $\mu = 0$ and $P^{k_j^*}_{R_{m_j^*},D}$=$\min(P_{R_{m_j^*}}/\lvert\overline{S1}\rvert, I_{th}/(\lvert\overline{S1}\rvert\rho_{m_j^*,k_j^*}))$
2: *while* (true)
3: According to (19), find the minimal λ^* to satisfy $\overline{C1}$ and $\overline{C3}$
4: Update $\lambda = \lambda^*$
5: According to (19), find the minimal μ^* to satisfy $\overline{C1}$ and $\overline{C3}$
6: Update $\mu = \mu^*$
7: *if* convergent
8: break
9: *end if*
10: *end while*
11: Obtain the optimal $\hat{P}^j_{S,R_{m_j^*}}$

2)sub-problem 2: Optimal power allocation for $P^{k_j^*}_{R_{m_j^*},D}$. The power allocation scheme at the relay is the same with algorithm 1 except that λ, μ, $P^j_{S,R_{m_j^*}}$, $\mathrm{P_S}$, ρ_j, (19), $\overline{C1}$ and $\overline{C3}$ are replaces by $\bar{\lambda}_{m_j^*}$, $\bar{\mu}$, $P^{k_j^*}_{R_{m_j^*},D}$, $P_{R_{m_j^*}}$, $\rho_{m_j^*,k_j^*}$, (20), $\overline{C2}$ and $\overline{C4}$.

The proposed power allocation scheme is an iteratively algorithm. The convergence of the proposed scheme is guaranteed. Rewrite the objective function in (17) as follows,

$$\max_{P_{S,R_m},P_{R_m,D}} f(P_{S,R_m}, P_{R_m,D}) \tag{21}$$

where $f(P_{S,R_m}, P_{R_m,D}) = \sum_{\{j \mid j\in\overline{S1}\}} R_{(j,m_j^*,k_j^*)}$, $P_{S,R_m} = \{\hat{P}^j_{S,R_{m_j^*}} \mid j\in\overline{S1}\}$ and $P_{R_m,D} = \{\hat{P}^{k_j^*}_{R_{m_j^*},D} \mid j\in\overline{S1}\}$.

Denote the global optimal point as f^*, we can get the following theorem,

Theorem 1: $\lim_{n\to\infty} f(P^n_{S,R_{m_j^*}}, P^n_{R_{m_j^*},D}) = f^*$

Proof: Start with an arbitrary initial point P_{S,R_m} , for $n \ge 1$, we can get

$$\begin{cases} P^n_{S,R_{m_j^*}} = \arg\max_{P_{S,R_{m_j^*}}} f(P_{S,R_{m_j^*}}, P^{n-1}_{R_{m_j^*},D}) \\ P^n_{R_{m_j^*},D} = \arg\max_{P_{R_{m_j^*},D}} f(P^n_{S,R_{m_j^*}}, P_{R_{m_j^*},D}) \end{cases} \quad (22)$$

Denote $f^n = f(P^n_{S,R_{m_j^*}}, P^n_{R_{m_j^*},D})$, from (22), we can get

$$f^n \geq f(P^n_{S,R_{m_j^*}}, P^{n-1}_{R_{m_j^*},D}) \geq f(P^{n-1}_{S,R_{m_j^*}}, P^{n-1}_{R_{m_j^*},D}) = f^{n-1} \quad (23)$$

Equation (23) demonstates the sequence f^n is non-decreasing. So, it must converge to f^* because f is bounded from above. The above proof shows that the alternating optimization scheme for power allocation converges, and the converged value is the optimal solution.

For the optimal solution derived in the section 3, MN^2 function evaluations are performed to find the power allocation in every iteration. Afterwards, M function evaluations are performed for every possible subcarrier pair where there are N^2 different subcarrier pairs. By including the computational complexity of the Hungarian methord, the optimal algorithm has a complexity of $O(T(MN^2)+N^3)$ where T is the number of iterations required to converge. For the proposed suboptimal algorithm, every subcarrier in the source side requires no more than $(M+MN)$ function evaluations to be paired and assigned to the relay in the subcarrier pairing and relay selection scheme, and the complexity of the power allocation scheme is $T|\overline{S1}|$. Therfore, the complexity of the proposed suboptimal algorithm, is $O(MN^2+T|\overline{S1}|)$, where $|\overline{S1}|$ means the cardinality of the set $|\overline{S1}|$ ($|\overline{S1}| \leq N$). The proposed suboptimal algorithm achieves much lower computational complexity.

5. Simulation results

The simulations are performed under the scenario given in Fig.1. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1. All the results have been averaged over 10000 iterations. In the simulations, *Optimal* and *Suboptimal* schemes apply the dual decomposition technique presented in Sec.3 and the proposed method presented in Sec.4 respectively. Furthermore, *RRA* and *RSA* refer to the method by which the relays and the subcarriers are assigned and matched randomly respectively, while *IFPA* allocates power inversely proportional to the interference level [19]. All the parameters in the simulations are described in **Table 2**.

Table 2. Simulation parameters

Name	N	M	T_s (μs)	B (MHz)	σ^2	$P_S = P_{R_m}$ (dBm)	I_{th} (dBm)	R_{th} (Bit/Hz/Sec)
Fig.2	64	5	5	20	0.0001	-20—10	3	0
Fig.3	64	5	5	20	0.0001	3	-20—20	0
Fig.4	64	5	5	20	0.0001	-3	3	0—0.1
Fig.5	64	1,5, 10,15	5	20	0.0001	-20—10	3	0
Fig.6	32,64, 128	5	5	20	0.0001	-20—10	3	0

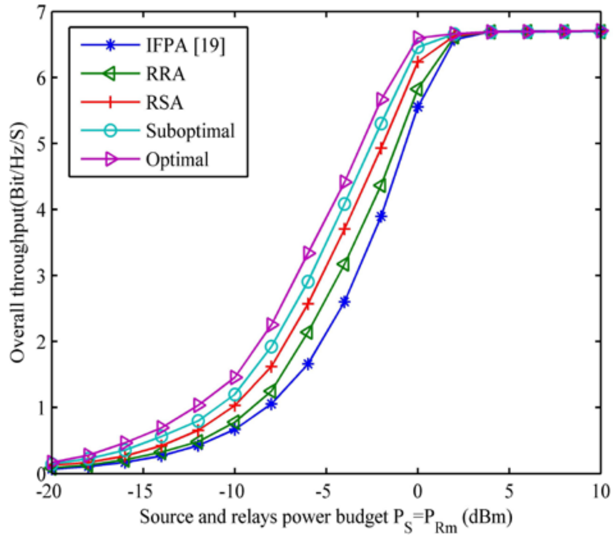

Fig. 2. Overall throughput vs. available power budgets with $P_S = P_{R_m}$.

The comparisons for our proposed algorithms and the other algorithms are shown in **Fig.2**, **Fig.3** and **Fig.4**. It can be found that our proposed algorithms, the *Optimal* and the *Suboptimal* perform better than the others. This is because that the power allocation, subcarrier pairing and relay assignment are performed jointly in our proposed algorithms, while the others take only some of them into consideration. It is worth noticing that the gap between the *Optimal* and the *Suboptimal* is small, suggesting that the suboptimal algorithm provides a good approximation to the optimal. From **Fig.2** we can see that the overall throughput grows with the increase of power budgets, and all the algorithms obtain close solutions when power budgets are large. From **Fig.3** we can observe that the overall throughput grows with the increase of interference threshold, and all the algorithms have a near performance in the low interference threshold region. The same interpretation can be applied on **Fig.4** in which the overall throughput decreases as rate-guarantee threshold grows for all the algorithms.

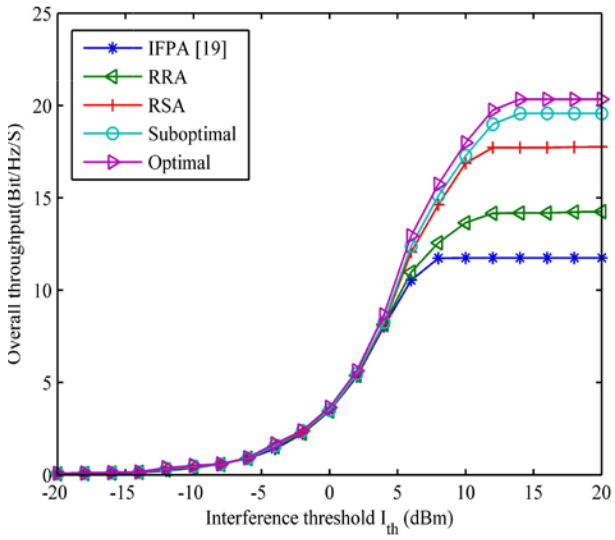

Fig. 3. Overall throughput vs. allowed interference threshold.

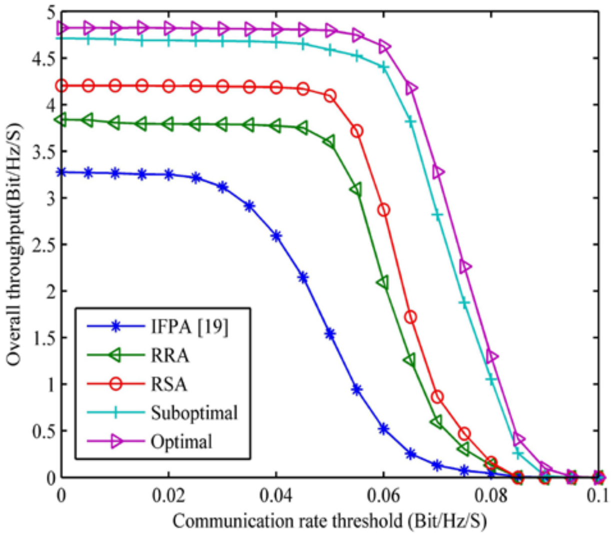

Fig. 4. Overall throughput vs. rate-guarantee threshold.

Fig. 5 illustrates the overall throughput of the suboptimal schemes vs. the available power budgets in source and relay under different number of relays. In the same power budget, the overall throughput increases with the number of relays increases. Moreover, the gain of overall throughput decreases with the number of relays increases.

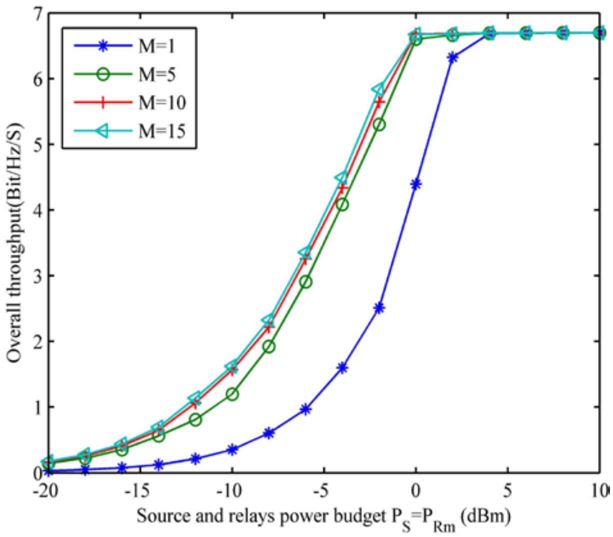

Fig. 5. Overall throughput vs. the available power budgets under different number of relays.

In **Fig.6**, depicts the overall throughput of the suboptimal schemes vs. the available power budgets in source and relay under different number of subcarriers. It can be noted that the overall throughput grows with the number of subcarriers, and the gain of overall throughput increases with the increase of power budgets.

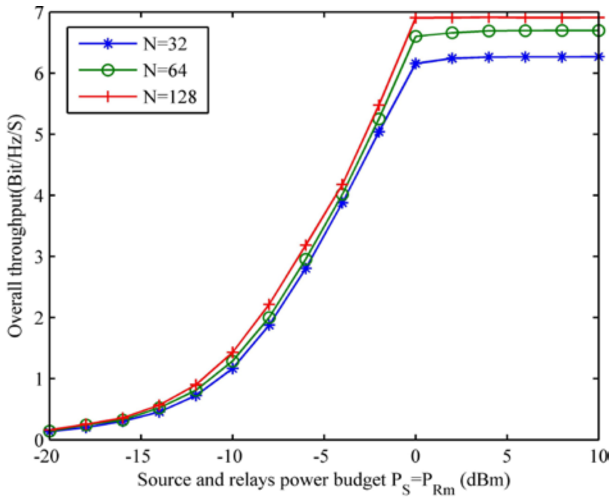

Fig. 6. Overall throughput vs. the available power budgets under different number of subcarriers.

6. Conclusion

In this paper, we have investigated the resource allocation problem in OFDM-based cooperative CRN. To maximize the overall throughput under the consideration of multiple practical limitations, the joint subcarrier pairing, best relay selection and power allocation scheme has been proposed by using the dual decomposition technique. Due to the high computational complexity of the optimal scheme, a heuristic suboptimal algorithm is presented. In the first step, the subcarrier pairing and relay selection scheme is proposed to satisfy the rate-guarantee constraint, and remove the integer constraints from the problem. In the second step, the power allocation scheme is considered to improve the system performance. The suboptimal algorithm shows to perform almost equally well as the optimal scheme with a much lower complexity. Moreover, the performance of the proposed algorithms outperform the others algorithms, *RSA*, *RRA* and *IFPA*.

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