

# A Secure MQAM Scheme Based on Signal Constellation Hopping

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## Abstract

In this paper, a secure multilevel quadrature amplitude modulation (MQAM) scheme is proposed for the physical layer security (PLS) of the wireless communications. In the proposed scheme, each transmitted symbol's signal constellation (SC) is hopping with the control of two unique factors: *amplitude distortion (AD) factor* and *phase hopping (PH) factor*. With unknown the two factors, the eavesdropper cannot extract effective information from the received signal. We first introduce a security metric, referred to as *secrecy gain*, and drive a lower bound on the gain that the secrecy capacity can be improved. Then, we investigate the relationship among the secrecy gain, the signal to noise power ratios (SNRs) of the main and wiretap channels, and the secrecy capacity. Next, we analyze the security of the proposed scheme, and the results indicate that the secrecy capacity is improved by our scheme. Specifically, a positive secrecy capacity is always obtained, whether the quality of the main channel is better than that of the wiretap channel or not. Finally, the numerical results are provided to prove the analytical work, which further suggests the security of the proposed scheme..

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**Keywords:** Secure MQAM scheme, signal constellation hopping (SCH), amplitude distortion (AD) factor, phase hopping (PH) factor, secrecy gain

## 1. Introduction

In 1975, Wyner introduced a general model for the physical layer security (PLS) of communications [1], where a transmitter (Alice) sends information to a legitimate user (Bob) through the main channel. Meanwhile, the eavesdropper (Eve) can extract the information from the output signal of the wiretap channel. He proposed a security metric for the system, i.e., *secrecy capacity*, which is defined as the difference capacity of main and wiretap channel, and its general expression was further proposed in [2]. Since then, PLS has received considerable attentions [3]-[27].

In [3]-[7], authors proved that the system with multiple-input multiple-output (MIMO) channel can obtain the secrecy capacity by optimizing beam-forming [3]-[5], proper signal power allocation [6]-[7] and space-time coding [7], when the channel state information (CSI) is known. While in [8]-[12], secure cooperation communications were investigated. With known the perfect CSI, authors proposed optimal relay selection and weight schemes for different numbers of eavesdroppers and different relay strategies. In [13]-[14], authors further proved that the orthogonal frequency division multiplexing (OFDM) modulation system can also achieve the secrecy capacity by optimizing the carrier power. And in [15]-[16], the radio fingerprinting (RF) techniques were applied to the communications authorization, where the RF feature of the received signal is utilized to detect the malicious intruders. In [17]-[19], artificial noise (AN) were explored to the PLS of the wireless communications. Authors in [17] proved that, when Alice and her helping relays have more antennas than Eve, PLS can be guaranteed by injecting the AN at the transmitter. And authors in [18]-[19] proposed the schemes that the AN is sent by Bob or relays. The methods are both robust for their security do not depend on the feedback of CSI and the condition that Eve's antennas are less than Bob's. In addition, some researchers propose the usage of coding to achieve the maximum secrecy rate. As shown in [20]-[24], authors proved that we can utilize the existing coding schemes, e.g., low density parity check (LDPC) coding [20], network coding [21], polar coding [22]-[23] and lattice coding [24], to design constructive codes which satisfy both reliability and security conditions [1], while the secrecy rate approaches the secrecy capacity. Some researchers also investigate cryptography security enhancement schemes based on the physical layer techniques [25]-[26]. As shown in [25]-[26], the secret key can be extracted from the fading channel coefficients. Based on the difference of the main and wiretap channels, Eve cannot intercept the key. And in [27], a joint encryption, error correction and modulation scheme was proposed to improve both the security and reliability of the communications.

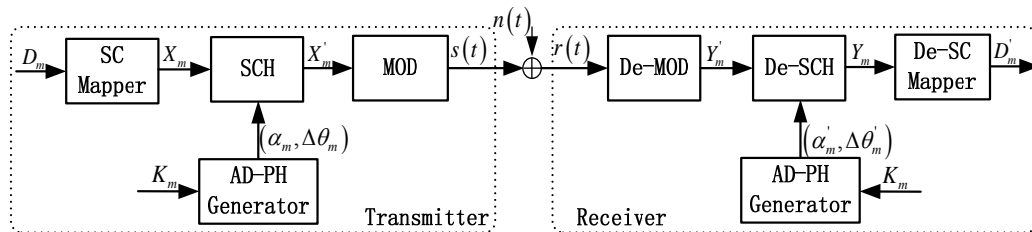
In deed, all the techniques in [3]-[27] have provided effective solutions for different secure communication scenarios. However, there exist some challenges for these techniques. As shown in [3]-[14], CSI is needed for transmission strategy optimization, while it is usually difficult in practice. Due to the dynamic environment, the signal RF feature is always changing, which makes it hard to extract the signal RF [15]-[16]. When AN is utilized for the security, it needs additional power to transmit AN. However, it is not desirable for some power constraint communication scenario, e.g., satellite communication. While coding schemes will lead to the decrease of transmission efficiency, as shown in [20]-[24]. When it exploits the cross-layer techniques proposed in [25]-[26], the system complexity will increase significantly.

Motivated by above observations, we propose a secure multilevel quadrature amplitude modulation (MQAM) scheme based on signal constellation hopping (SCH) in this paper. Different from traditional modulation scheme, each transmitted symbols' signal constellation

(SC) is hopping with the control of two unique factors: *amplitude distortion (AD) factor* and *phase hopping (PH) factor*, which are both generated by an AD-PH generator. At the receiver, with known the two factors, the demodulator can rebuild the correct SC. Otherwise, the receiver will get uncorrect SC and decision regions, which will lead to the demodulation failure, i.e., the communication security is guaranteed. The main contributions of this paper are summarized as follows.

- 1) We introduce a security metric, referred to as *secrecy gain*, and drive a lower bound on the gain that the secrecy capacity can be improved. And the relationship among the secrecy gain, the signal to noise power ratios (SNRs) of the main and wiretap channels, and the secrecy capacity is also investigated.
- 2) We analyze the security of our scheme and prove that, when the secrecy gain is larger enough, a positive secrecy capacity can be always obtained, whether the quality of the main channel is better than that of the wiretap channel or not. Additionally, we also discuss the anti-attack ability of the proposed scheme with a conclusion that, it is difficult to estimate AD and PH factors when it exists noise in the channel.
- 3) Three sets of simulations with 16QAM and 64QAM are provided to prove the analytical results, where both the main channel and the wiretap channel are assumed as additive white Gaussian noise (AWGN) channels.

The rest of the paper is organized as follows. In Section 2, we introduce the proposed scheme, and analyze the theory error performance. And in Section 3, we first propose the secrecy gain and drive a lower bound on the gain that the secrecy capacity can be improved. Then, relationship among the secrecy gain and secrecy capacity is investigated. And the anti-attack ability of the proposed scheme is also discussed. Next, in Section 4, three sets of simulations are set up to evaluate the security performance of our scheme. Finally, we make some concluding in Section 5.



**Fig.1.** Block diagram of the SCH MQAM system.

## 2. Proposed SCH MQAM Scheme

### 2.1 System Model

The general ideal of our proposed scheme is to change each transmitted symbol's SC with the AD and PH factors and to map all the symbols onto hopping SCs. With unknown the AD and PH factors, the receiver cannot make correct decision on each received symbols, i.e., it fails to extract the information from the received signal. The block diagram SCH MQAM system is illustrated in **Fig.1**.

We can observe from the figure that, at the transmitter, the information bits ( $D_m$ ) are first mapped onto MQAM symbols, i.e.,  $X_m$ . Then, through the processing of SCH with the AD and PH factors, i.e.,  $(\alpha_m, \Delta\theta_m)$ , we can get the transmitted SCH MQAM symbol as  $X'_m$ . Next, through modulating, the final transmitted signal  $s(t)$  is given by

$$\begin{aligned}
s(t) &= \sum_{m=0}^{\infty} \text{Re} \left[ X_m \bullet \alpha_m e^{j\Delta\theta_m} \bullet g(t - mT_s) e^{j(2\pi f_c(t - mT_s))} \right] \\
&= \sum_{m=0}^{\infty} \text{Re} \left[ X'_m g(t - mT_s) e^{j(2\pi f_c(t - mT_s))} \right] \\
&= \sum_{m=0}^{\infty} \text{Re} \left[ \alpha_m A_m e^{j(\theta_m + \Delta\theta_m)} g(t - mT_s) e^{j(2\pi f_c(t - mT_s))} \right] \\
&= \sum_{m=0}^{\infty} \alpha_m A_m g(t - mT_s) \cos(2\pi f_c(t - mT_s) + \theta_m + \Delta\theta_m),
\end{aligned} \tag{1}$$

where  $A_m$ ,  $\theta_m$  are amplitude and phase of  $X_m$ ,  $g(t)$  is the shape filter,  $T_s$  is the symbol period, and  $f_c$  is the carrier frequency. For each MQAM symbol, there has

$$X_m = A_m e^{j\theta_m} = A_{mi} + jA_{mq}, A_m = \sqrt{A_{mi}^2 + A_{mq}^2}, \text{ and } \theta_m = \tan^{-1}(A_{mq}/A_{mi}), \tag{2}$$

where  $A_{mi}$ ,  $A_{mq}$  are the information-bearing signal amplitudes of quadrature carriers.

At the receiver, the received signal  $r(t)$  will be

$$r(t) = h(t) * s(t) + n(t), \tag{3}$$

where ‘ $*$ ’ is the convolution,  $h(t)$  is the channel impulse response,  $n(t)$  is the additive Gaussian noise with power spectral density of  $N_0/2$ . For the AWGN channel, there has

$$h(t) \equiv 1. \tag{4}$$

From (1), (3) and (4),  $r(t)$  can be written as

$$r(t) = \sum_{m=0}^{\infty} \alpha_m A_m g(t - mT_s) \cos(2\pi f_c(t - mT_s) + \theta_m + \Delta\theta_m) + n(t). \tag{5}$$

Suppose that there is no frequency and phase offsets, the received SCH MQAM symbol, denoted as  $Y'_m$ , will be

$$Y'_m = r_m(\alpha_m, A_m, \theta_m, \Delta\theta_m) + n'_m = \alpha_m A_m e^{j(\theta_m + \Delta\theta_m)} + n'_m, \tag{6}$$

where  $n'_m$  is a complex i.i.d Gaussian variable with mean 0 and variance  $N_0$ .

Finally, through the processing of the de-SCH with the synchronization AD and PH factors, i.e.,  $(\alpha'_m, \Delta\theta'_m)$ , the received MQAM symbol, denoted as  $Y_m$ , is given by

$$Y_m = Y'_m \alpha'_m e^{j\Delta\theta'_m} = \alpha'_m \alpha_m A_m e^{j(\theta_m + \Delta\theta_m + \Delta\theta'_m)} + n'_m \alpha'_m e^{j\Delta\theta'_m}. \tag{7}$$

Then, we can get the estimated information bits ( $D'_m$ ) based on  $Y_m$  and the decision rules of the MQAM.

It is noticed from the above presentation that, the security of our scheme is closely related to  $(\alpha_m, \Delta\theta_m)$  and  $(\alpha'_m, \Delta\theta'_m)$ , which are all obtained from the AD-PH generator. In this paper, we assume that the AD-PH generator is driven by a random sequence, and its  $m$ th element is denoted as  $K_m$ . And it is also assumed that  $K_m$  is known to both Alice and Bob but unknown to Eve.

**Remark 1:** The role of  $K_m$  in our scheme is the same as that of the secret key in the cryptography scheme. While in this paper, we do not introduce how to keep  $K_m$  secret in detail.

## 2.2 Signal Constellation Hopping

In this subsection, we introduce how to make the MQAM symbols' SC hopping with the control of the AD and PH factors. As shown in **Fig.2**, with the modification of  $(\alpha_m, \Delta\theta_m)$ , i.e.,

$X'_m = \alpha_m A_m e^{j(\theta_m + \Delta\theta_m)}$ , the constellation of a 16QAM symbol is changed. Since the AD and PH factors are both unique to each 16QAM symbol, its constellation will be different. Hence, when the symbol number increases, the transmitted SC will be fuzzy, correspondingly, just like the SC is hopping, as shown in Fig.3.

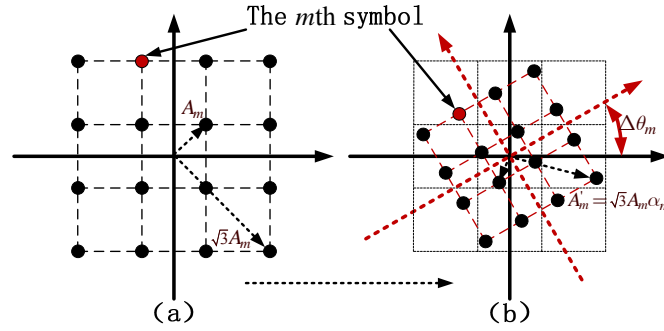


Fig.2. (a) Original 16QAM symbol constellation. (b) SCH 16QAM symbol constellation.

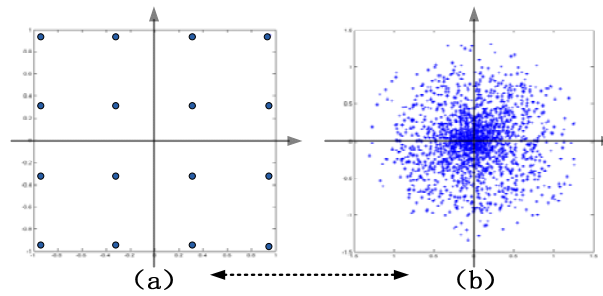


Fig.3. (a) Original 16QAM SC. (b) Fuzzy 16QAM SC.

### 2.3 Error Performance

In this subsection, we analyze the error performance of the legitimate receiver (Bob) and the eavesdropper (Eve), where both the main channel and the wiretap channel are assumed as AWGN channels, and their noise samples are denoted as  $n_m^B$  and  $n_m^E$ , respectively. In addition, we refer the synchronization AD and PH factors of Bob and Eve to be different as  $(\alpha_m^{B'}, \Delta\theta_m^{B'})$  and  $(\alpha_m^{E'}, \Delta\theta_m^{E'})$ .

Since the AD and PH factors are known to Bob, we have

$$\alpha_m^{B'} \alpha_m = 1 \text{ and } \Delta\theta_m + \Delta\theta_m^{B'} = 0. \quad (8)$$

According to (7) and (8), Bob's received symbol, denoted as  $Y_m^B$ , is given by

$$Y_m^B = A_m e^{j(\theta_m)} + n_m^B \alpha_m^{B'} e^{j\Delta\theta_m^{B'}} = A_m e^{j(\theta_m)} + n_m^{B'}, \quad (9)$$

where  $n_m^{B'}$  is the equivalent noise sample of the Bob's channel.

Suppose that Eve know Alice's signal processing method, and she will try to recover the SC with  $(\alpha_m^{E'}, \Delta\theta_m^{E'})$ . Therefore, from (7), Eve's received symbol, denoted as  $Y_m^E$ , will be given by

$$Y_m^E = \alpha_m^{E'} \alpha_m A_m e^{j(\theta_m + \Delta\theta_m + \Delta\theta_m^{E'})} + n_m^E \alpha_m^{E'} e^{j\Delta\theta_m^{E'}} = \beta A_m e^{j(\theta_m + \phi)} + n_m^{E'}, \quad (10)$$

where  $n_m^{E'}$  is the equivalent noise sample of the Eve's channel, and  $\beta, \phi$  are the estimation offsets of the AD and PH factors, i.e.,

$$\beta = \alpha_m^{E'} \alpha_m \text{ and } \phi = \Delta\theta_m + \Delta\theta_m^{E'}, \quad (11)$$

In order to analyze the error performance of Bob and Eve, we first introduce the statistic characteristics of  $n_m^{B'}$  and  $n_m^{E'}$  with the following theorem.

**Theorem 1:** Suppose that the random variables  $X'$ ,  $Y$  and  $\theta$  are mutual independent, and  $X' \sim N(0, N_0)$ ,  $Y \sim U(0, A)$ ,  $\theta \sim U(0, 2\pi)$ . The complex variable  $V' = X'Ye^{j\theta}$  will be a Gaussian variable with mean 0 and variance  $A^2N_0$ .

**Proof:** The proof of this theorem is implemented in Appendix-A.

**Theorem 1** shows that both  $n_m^{B'}$  and  $n_m^{E'}$  follow the Gaussian distribution. Hence, we can exploit the analysis method presented in 0 to evaluate the error performance of Bob and Eve.

It is shown in 0 that, the symbol error rate (SER) of a digital modulation scheme is equal to the average pairwise error probability, i.e., the probability of event that the transmit symbol is  $X_m$  but detected symbol is  $X_n$ .

According to (9), the pairwise error probability of Bob, denoted as  $p_{m \rightarrow n}^B$ , is given by

$$p_{m \rightarrow n}^B = P\left[\left\|Y_m^B - A_m e^{j(\theta_m)}\right\| > \left\|Y_m^B - A_n e^{j(\theta_n)}\right\|\right] = P\left[\left\|n_m^{B'}\right\| > \left\|A_m e^{j(\theta_m)} + n_m^{B'} - A_n e^{j(\theta_n)}\right\|\right] \quad (12)$$

To simplify (12), we define three vectors as  $\mathbf{n} = (n_1, n_2)$ ,  $\mathbf{s}_m = (s_{m1}, s_{m2})$  and  $\mathbf{s}_n = (s_{n1}, s_{n2})$ , where  $n_1 = \text{Re}[n_m^{B'}]$ ,  $n_2 = \text{Im}[n_m^{B'}]$ ,  $s_{m1} = \text{Re}[A_m e^{j(\theta_m)}]$ ,  $s_{m2} = \text{Im}[A_m e^{j(\theta_m)}]$ ,  $s_{n1} = \text{Re}[A_n e^{j(\theta_n)}]$ ,  $s_{n2} = \text{Im}[A_n e^{j(\theta_n)}]$ . Then, (12) can be written as

$$p_{m \rightarrow n}^B = P[\mathbf{n}(\mathbf{s}_n - \mathbf{s}_m) > d_{mn}^2/2], \quad (13)$$

where  $d_{mn}$  is the Euclidean distance of  $X_m$  to  $X_n$ , and  $\mathbf{n}(\mathbf{s}_n - \mathbf{s}_m)$  follows Gaussian distribution with mean 0 and variance  $d_{mn}^2 A^2 N_0 / 2$ . Therefore, the SER of Bob will be

$$p_e^B = \frac{1}{M} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} \mathcal{Q}\left(\sqrt{d_{mn}^2 / 2A^2 N_0}\right). \quad (14)$$

And from (10), the pairwise error probability of Eve, denoted as  $p_{m \rightarrow n}^E$ , is given by

$$p_{m \rightarrow n}^E = P\left[\left\|\beta A_m e^{j(\theta_m + \phi)} + n_m^{E'} - A_m e^{j(\theta_m)}\right\| > \left\|\beta A_m e^{j(\theta_m + \phi)} + n_m^{E'} - A_n e^{j(\theta_n)}\right\|\right]. \quad (15)$$

Based on the similar simplification as (12), we can get the SER of Eve, i.e.,

$$p_e^E(\beta, \phi) = \frac{1}{M} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} \mathcal{Q}\left(\left|(d_{en}^2 - d_{em}^2)/2\right| / \sqrt{(d_{em}^2 + d_{en}^2)A^2 N_0 / 2}\right), \quad (16)$$

where  $d_{em}$ ,  $d_{en}$  are the Euclidean distances of  $\beta A_m e^{j(\theta_m + \phi)}$  to  $X_m$  and  $X_n$ , respectively, and  $d_{en}^2 - d_{em}^2$ ,  $d_{en}^2 + d_{em}^2$  are obtained by

$$d_{en}^2 - d_{em}^2 = A_m^2 - A_n^2 - 2\beta A_m^2 \cos\phi + 2\beta A_m A_n \cos(\theta_m + \phi - \theta_n), \quad (17)$$

$$d_{en}^2 + d_{em}^2 = 2\beta^2 A_m^2 + (A_m^2 + A_n^2) - 2\beta A_m [A_m \cos\phi + A_n \cos(\theta_m + \phi - \theta_n)]. \quad (18)$$

It is noticed from (16)-(18) that the SER of Eve depends on  $\beta$ ,  $\phi$ . We assume that  $\beta$  and  $\phi$  both follow the uniform distribution, i.e.,  $\beta \sim U(0, T_0)$  and  $\phi \sim U(0, 2\pi)$ . Then, the average SER of Eve will be

$$p_e^{EAV} = \iint p_e^E(\beta, \phi) / (2\pi T_0) d\beta d\phi, \quad (19)$$

where  $T_0$  is the maximum estimation offset of the AD factor.

### 3. Security Analysis

In this section, we analyze the security of the proposed scheme, in which the secrecy gain is introduced and the secrecy capacity is investigated. Additionally, we also discuss the anti-attack ability of our scheme.

#### 3.1 Secrecy Gain

It is proven in [2] that the secrecy capacity of the wiretap system depends the SNR difference between the main channel and wiretap channel, which are both considered as AWGN channels. We can observe from (14) and (16) that the minimum Euclidean distance of transmitted symbol is changed by the proposed scheme, just as the SNR of channel has been modified. To quantify this “modified” SNR, here, we introduce a metric, referred to as *secrecy gain*, and its definition is given as follows.

**Definition 1:** Let  $SNR_B, SNR_E$  be the actual SNRs of the main and wiretap channel, and  $SNR'_B, SNR'_E$  be their *equivalent* SNRs through the processing of the PLS technique. Suppose that  $SNR'_B/SNR_B = \gamma_B, SNR'_E/SNR_E = \gamma_E$ , the secrecy gain is defined as  $\gamma_s = \gamma_B/\gamma_E$ .

According to above definition and (14), (16), we can obtain the secrecy gain of our scheme as

$$\gamma_s = \frac{1}{M(M-1)} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} d_{mn}^2 (d_{em}^2 + d_{en}^2) / (d_{en}^2 - d_{em}^2)^2. \quad (20)$$

Substituting (17) and (18) into (20), we have

$$\gamma_s = \frac{1}{M(M-1)} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} \frac{(A_m^2 + A_n^2 - 2A_m A_n \cos(\theta_m - \theta_n))(2\beta^2 A_m^2 + (A_m^2 + A_n^2) - 2\beta A_m [A_n \cos \phi + A_n \cos(\theta_m + \phi - \theta_n)])}{(A_m^2 - A_n^2 - 2\beta A_m^2 \cos \phi + 2\beta A_m A_n \cos(\theta_m + \phi - \theta_n))^2}. \quad (21)$$

For the 4QAM, the secrecy gain will be

$$\gamma_{s,M=4} = \frac{\beta^2 + 1 - \beta(\cos \phi - \sin \phi)}{\beta^2(1 - \sin 2\phi)}. \quad (22)$$

Furthermore, when  $M \rightarrow \infty$ , there has

$$\lim_{M \rightarrow \infty} \gamma_s = \lim_{\substack{\theta_m - \theta_n \rightarrow 0 \\ A_m - A_n \rightarrow 0}} \gamma_s = \infty. \quad (23)$$

Additionally, from (19), (20), we can get the average secrecy gain as

$$\gamma_s^{AV} = \frac{1}{M(M-1)} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} \iint d_{mn}^2 (d_{em}^2 + d_{en}^2) / (2\pi T_0 (d_{en}^2 - d_{em}^2)^2) d\beta d\phi \quad (24)$$

We notice from **Definition 1** that the security of the PLS technique can be measured via its secrecy gain. To elaborate this point, we introduce the following theorem to give a lower bound on the secrecy gain that the secrecy capacity can be improved.

**Theorem 2:** If the secrecy gain satisfy  $\gamma_s > 1$ , the secrecy capacity of the wiretap system can be improved.

**Proof:** The proof of this theorem is implemented in Appendix-B.

Based on (22), (23) and **Theorem 2**, we can conclude that the secrecy capacity of the wiretap system can be improved by the proposed secure MQAM scheme.

### 3.2 Secrecy Capacity

It is shown in [2] that, the secrecy capacity of the AWGN wiretap system is given by

$$C_s = (\log(1 + \text{SNR}_B) - \log(1 + \text{SNR}_E), 0)^+ \quad (25)$$

Therefore, from (14), (16) and (25), we can obtain the secrecy capacity of our scheme as

$$C_s(\beta, \phi) = \frac{1}{M(M-1)} \sum_{m=1}^M \sum_{\substack{1 \leq n \leq M \\ n \neq m}} \left( \log \left( 1 + \frac{1}{A^2} \text{SNR}_B \right) - \log \left( 1 + \frac{(d_{en}^2 - d_{em}^2)^2}{A^2 d_{mn}^2 (d_{em}^2 + d_{en}^2)} \text{SNR}_E \right), 0 \right)^+ \quad (26)$$

Then, the average secrecy capacity will be

$$C_s^{AV} = \iint C_s(\beta, \phi) / (2\pi T_0) d\beta d\phi \quad (27)$$

We notice from (26), (27) that the average secrecy capacity of the proposed scheme depends on its secrecy gain and  $\text{SNR}_B, \text{SNR}_E$ . To elaborate the relationship among them, here, we introduce another theorem as follows.

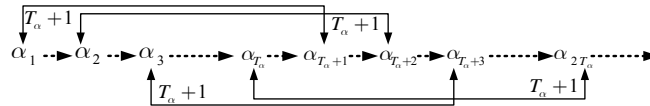
**Theorem 3:** When the secrecy gain and  $\text{SNR}_B, \text{SNR}_E$  satisfy the condition of  $\gamma_s \text{SNR}_B > \text{SNR}_E$ , a positive secrecy capacity can be achieved.

**Proof:** The proof of this theorem is implemented in Appendix-C.

**Theorem 3** shows that, when the secrecy gain is large enough, a positive secrecy capacity can be always obtained, even  $\text{SNR}_E$  is larger than  $\text{SNR}_B$ . Hence, we can conclude from (23) that our SCH MQAM scheme can achieve a positive secrecy capacity.

### 3.3 Anti-Attack Ability

It is noticed from the previous subsections that the security of our scheme depends on the assumption that Eve is unknown about the AD and PH factors. To achieve this goal, we exploit an AD-PH generator driven by a random sequence to generate these factors. In general, the driven sequence can be obtained from the coefficients of the main channel [25]-[26], or from a homogenous, stationary and acyclic sequence. For the former case, Eve needs to know the CSI of the main channel, which is difficult in practice. While for the latter case, Eve needs to estimate the whole acyclic sequence, and it is more difficult. In fact, even the driven sequence is obtained from a periodical sequence, our scheme is still security. The reason is that, due to the existence of the channel noise, it cannot accurately estimate the AD and PH factors. Here, we take the estimation of AD sequence for example, as shown in Fig.4.



**Fig.4.** Estimation of AD factor sequence, and  $T_\alpha$  is the sequence period.

We can observe from Fig.4 that, with accurate AD factors, we can estimate the AD sequence correctly by

$$\alpha_1 = \alpha_{T_\alpha+1}, \alpha_2 = \alpha_{T_\alpha+2}, \alpha_3 = \alpha_{T_\alpha+3}, \dots, \alpha_{T_\alpha} = \alpha_{2T_\alpha} \quad (28)$$

Though it needs long time to observe the AD factor sample. However, with the noised AD factors, there may exists

$$\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \dots \neq \alpha_{T_\alpha} \neq \alpha_{T_\alpha+1} \neq \alpha_{T_\alpha+2} \neq \alpha_{T_\alpha+3} \neq \dots \neq \alpha_{2T_\alpha} \neq \dots, \quad (29)$$

which means that it is impossible to estimate the AD sequence accurately. Hence, we can conclude that our proposed scheme is robust for Eve's passive attacking.



## 4. Numerical Results

In this section, three sets of simulations are set up to evaluate the performance of our scheme. In these simulations, 16QAM and 64QAM are exploited, both the main and wiretap channels are assumed as AWGN channels, the maximum value of  $\beta$  is set to 1, i.e.,  $T_0 = 1$ .

### 4.1 Simulations for Secrecy Gain

In the first set, we investigate relationship between the secrecy gain and  $\beta, \phi$ , where the results are shown in Fig.5. It is noticed from the figure that, the secrecy gain of SCH 16QAM scheme is larger than 10dB. Specifically, when  $\beta < 0.1$ , the secrecy gain is more than 30dB. Even  $\beta$  is close to 1, the positive secrecy gain still exists, i.e., 12.5dB. In addition, Table 1 provides the average secrecy gains of the SCH 16QAM and SCH 64QAM schemes. We can observe that the average secrecy gains of the two schemes are both larger than 450dB.

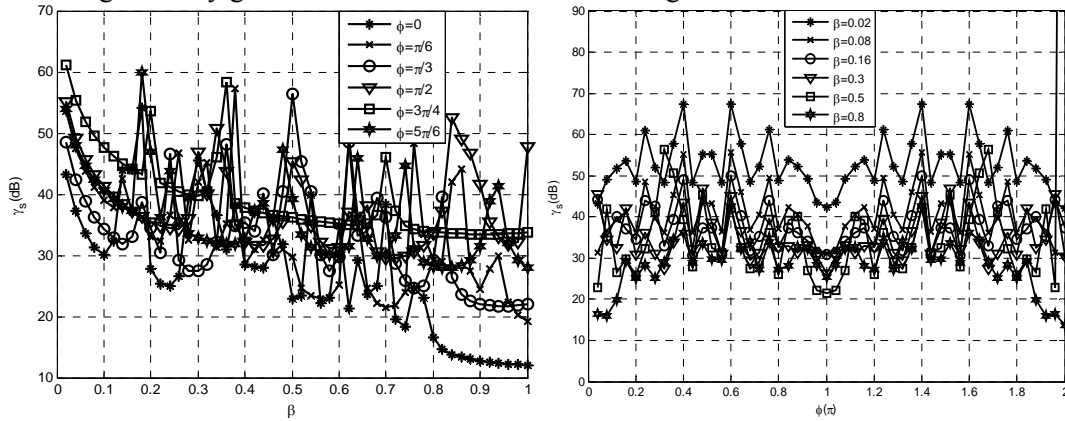


Fig.5. Secrecy gain of the SCH 16QAM scheme with different  $\beta$  and  $\phi$ .

Table 1. Secrecy Gains of SCH 16QAM and SCH 64QAM Schemes

Modulation		Average Secrecy Gain(dB)
MQAM	16QAM	458.7506
	64QAM	466.7669

### 4.2 Simulations for Secrecy Capacity

In the second set, we first investigate the relationship between the secrecy capacity and  $\beta, \phi$ , where the results are shown in Fig.6. It is assumed that  $SNR_B = SNR_E = 6dB$ , we know from (25) that, with the original MQAM scheme, the secrecy capacity will be zero in this case. However, it is noticed from Fig.6 that, there exists a positive secrecy capacity with the SCH 16QAM scheme. Specifically, even  $\beta$  is close to 1 and  $\phi$  is close to 0, the positive secrecy capacity still exists, e.g., when  $\phi = 0.04\pi, \beta = 1$ , the secrecy capacity is 0.048, and when  $\beta = 0.02, \phi = 2\pi$ , it will be 0.435.

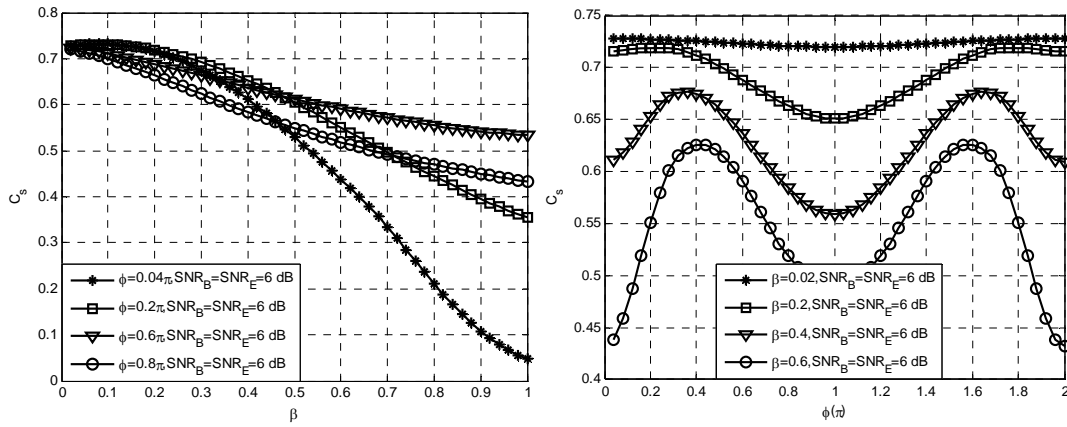


Fig.6. Secrecy capacity of the SCH 16QAM scheme with different  $\beta$  and  $\phi$  .

Then, we explore the average secrecy capacities of the SCH 16QAM and SCH 64QAM schemes with  $SNR_E = SNR_B$  and  $SNR_E > SNR_B$ , where the results are illustrated in Fig.7. We observe from the figure that, the average secrecy capacities of two schemes are both larger than 0.27. When  $SNR_E = SNR_B = SNR > 0$  dB, the average secrecy capacities are increasing with  $SNR$ . It is also noticed that, even  $SNR_E > SNR_B$ , there still exists a positive secrecy capacities for the two schemes, e.g., when  $SNR_E - SNR_B$  is equal to 50dB, the corresponding secrecy capacities are 0.0022 and 0.0018, respectively. And when  $SNR_E - SNR_B$  increases, the secrecy capacity will decrease.

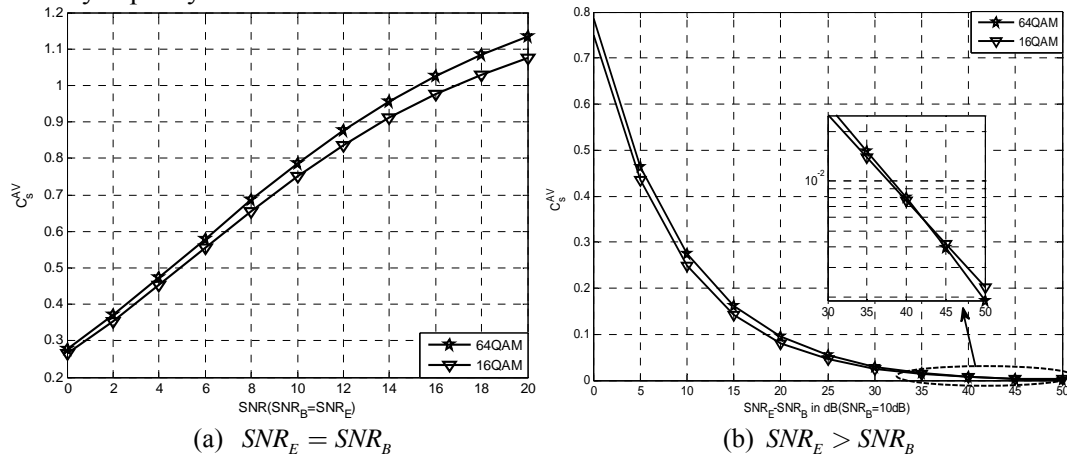
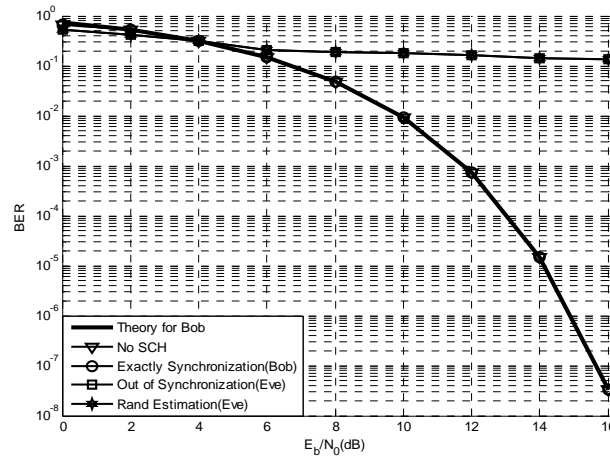


Fig.7. Average secrecy capacities of SCH 16QAM and SCH 64QAM schemes.

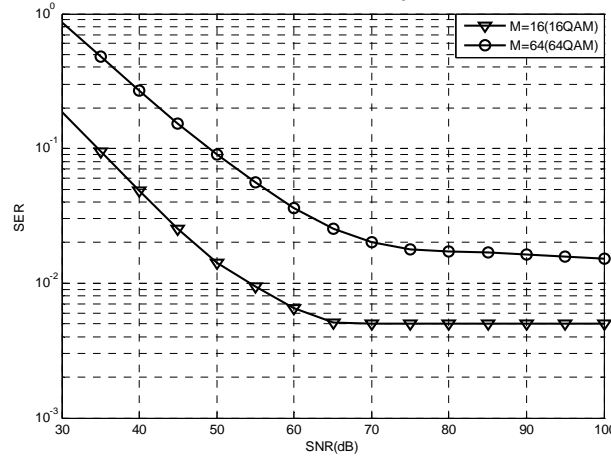
### 4.3 Simulations for Error Performance

In the third set, we first investigate the error performance of our scheme via bit error rate (BER), where the results are shown in Fig.8. It is noticed from the figure that, when SC is not hopping or the AD and PH factors are exactly synchronized, the BER is almost equal to the theory value. However, when the factors are not synchronized, the BER is around 0.13, which means that the receiver cannot extract the information. In addition, to evaluate the robust of our scheme, we also consider the case that Eve can get partial AD and PH factors. We notice that, even the ratio of the factors obtained by Eve is high to 60%, its BER is still close to the case that the factors are not synchronized.



**Fig.8.** BER comparison among the SCH 16QAM scheme with different cases.

Then, we explore the theory bounds on the SERs of the SCH 16QAM and SCH 64QAM schemes without the AD and PH factors, where the results are shown in Fig.9. It is noticed from the figure that, when SNR is less than 65 dB, the SER is increasing with SNR. Otherwise, it will keep at around 0.01, which indicates the security of the SCH MQAM scheme.



**Fig.9.** Bounds on the SERs of the SCH MQAM scheme without the AD and PH factors.

## 5. Conclusion

In this paper, a secure MQAM scheme was proposed for the PLS of the wireless communications. In our scheme, each transmitted symbol's SC was hopping with the control of two unique factors: amplitude distortion factor and phase hopping factor. With unknown those two factors, the eavesdropper could not extract the information from the received signal. We first introduced a security metric, i.e., secrecy gain, and drove a lower bound on the gain that the secrecy capacity can be improved. Then, we investigated the relationship among the secrecy gain, the SNRs of the main and wiretap channels, and the secrecy capacity. The analytical and simulation results showed that the secrecy capacity can be improved by our scheme. Specifically, a positive secrecy capacity can be always obtained, whether the quality of the main channel is better than that of the wiretap channel or not, which indicates the

security of our scheme.

Finally, One important point should be noted that, the security of our scheme is closely related to the security of the driven sequence of the AD-PH generator. While in this paper, we did not introduce the detailed methods to generate the driven sequence and to keep it secret. Hence, the research on the driven sequence should be further conducted.

## Appendix

### A. Proof of Theorem 1

Let  $X$  and  $V$  be the real parts of  $X'$  and  $V'$ . Suppose that the probability density functions (PDFs) of  $X$  and  $Y$  are

$$f_X(x) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{x^2}{N_0}}, \text{ and } f_Y(y) = \begin{cases} \frac{1}{A}, & y \in (0, A) \\ 0, & \text{others} \end{cases} \quad (30)$$

Since  $\theta \sim U(0, 2\pi)$ , the PDF of  $z = \sin \theta$  will be

$$f_z(z) = \begin{cases} \frac{1}{\pi\sqrt{1-z^2}}, & z \in (-1, 1) \\ 0, & \text{others} \end{cases} \quad (31)$$

To prove **Theorem 1**, we need the PDF of  $V = XYZ$ , which can be obtained by its cumulative distribution function (CDF), i.e.,

$$F_V(v) = P(V \leq v) = P(XYZ \leq v) = \iiint_{\substack{XYZ \leq v \cap z \in (-1, 1) \\ \cap y \in (0, A) \cap x \in (-\infty, +\infty)}} \frac{e^{-\frac{x^2}{N_0}}}{A\pi\sqrt{\pi N_0}\sqrt{1-z^2}} dx dy dz \quad (32)$$

Through a complexity derivation, we obtain

$$F_V(v) = \begin{cases} \int_{-\infty}^{v/A} \frac{e^{-\frac{x^2}{N_0}}}{\sqrt{\pi N_0}} dx + 2 \cdot \int_{-1}^0 \frac{e^{-\frac{v^2}{N_0 A^2 u^2}}}{\pi\sqrt{\pi N_0}} \left( \arcsin u - u \left( \ln|u| - \ln(1 + \sqrt{1-u^2}) \right) \right) \left( \frac{-v du}{Au^2} \right), & v \leq 0 \\ \int_0^{v/A} \frac{e^{-\frac{x^2}{N_0}}}{\sqrt{\pi N_0}} dx + 2 \cdot \int_1^0 \frac{e^{-\frac{v^2}{N_0 A^2 u^2}}}{\pi\sqrt{\pi N_0}} \left( \arcsin u - u \left( \ln|u| - \ln(1 + \sqrt{1-u^2}) \right) \right) \left( \frac{-v du}{Au^2} \right), & v > 0 \end{cases} \quad (33)$$

It is easy to prove that the second part of (33) approaches to 0 for any  $v, A, N_0, u$ . Hence, we can get the PDF of  $V$ , i.e.,  $f_V(v) = e^{-v^2/N_0 A^2} / A\sqrt{\pi N_0}$ , which indicates that  $V$  is a Gaussian variable with mean 0 and variance  $A^2 N_0 / 2$ . Hence,  $V'$  will be a complex Gaussian variable with mean 0 and variance  $A^2 N_0$ .

### B. Proof of Theorem 2

To prove **Theorem 2**, we first define a function as

$$f(x, y) = (1 + Ax)(1 + y) - (1 + x)(1 + By), A > 0, B > 0, x > 0, y > 0. \quad (34)$$

And if  $A = B$ , there has

$$f(x, y) = (1 + Ax)(1 + y) - (1 + x)(1 + Ay) = -A(x - y)^2 \leq 0. \quad (35)$$

While if  $A \neq B$ , we can obtain the stationary point of  $f(x, y)$  as

$$\begin{cases} \frac{\partial f}{\partial x} = A(1+y) - (1+By) = A - 1 + (A - B)y = 0 \\ \frac{\partial f}{\partial y} = (1+Ax) - B(1+x) = 1 - B + (A - B)x = 0 \end{cases} \Rightarrow \begin{cases} x_0 = \frac{B-1}{A-B} \\ y_0 = \frac{1-A}{A-B} \end{cases}. \quad (36)$$

Since  $f(x, y) > f(x_0, y_0)$ , in order to ensure  $f(x, y) > 0$  always satisfied for any  $x, y$ , there must has

$$f(x_0, y_0) = x_0 y_0 (A - B) > 0 \Rightarrow A > B. \quad (37)$$

Therefore, when  $\gamma_B > \gamma_E$ , we have

$$\log(1 + \gamma_B \text{SNR}_B) - \log(1 + \gamma_E \text{SNR}_E) > \log(1 + \text{SNR}_B) - \log(1 + \text{SNR}_E). \quad (38)$$

That is to say, when  $\gamma_s > 1$ , the secrecy capacity is improved.

### C. Proof of Theorem 3

When a positive secrecy capacity exists, we must have  $C_s > 0$ . According to [Definition 1](#) and (25), there has

$$\begin{aligned} \log(1 + \gamma_B \text{SNR}_B) - \log(1 + \gamma_E \text{SNR}_E) &= \log\left(\frac{1 + \gamma_B \text{SNR}_B}{1 + \gamma_E \text{SNR}_E}\right) \\ &= \log\left(\frac{1/\gamma_E + \gamma_B/\gamma_E \text{SNR}_B}{1/\gamma_E + \text{SNR}_E}\right) = \log\left(\frac{1/\gamma_E + \gamma_s \text{SNR}_B}{1/\gamma_E + \text{SNR}_E}\right) > 0. \end{aligned} \quad (39)$$

Therefore, it is easy to obtain the conclusions of [Theorem 3](#) from (39), i.e.,

$$\frac{1/\gamma_E + \gamma_s \text{SNR}_B}{1/\gamma_E + \text{SNR}_E} > 1 \Rightarrow \gamma_s \text{SNR}_B > \text{SNR}_E. \quad (40)$$

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