On the Lightning Induced Voltage Along Overhead Power Distribution Line

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Abstract – Lightning induced voltage is a major factor that causes interruptions on distribution lines. In this paper, analytical expressions are proposed to evaluate a lightning induced voltage on power lines directly in the time domain without the need to apply any extra conversions. The proposed expressions can consider the widely used current functions and models in contrast to the earlier analytical expressions which had a number of limitations related to the simplification of the channel base current and the current along the lightning channel. The results show that the simulated values based on the proposed method are in good agreement with the previous studies and the proposed expressions can be used for optimizing the insulation and protection level of existing and new lines being designed.

Keywords: Lightning induced voltage, Lightning, Electromagnetic field, Return stroke

1. Introduction

Lightning is an important natural phenomenon that can cause interruptions on power lines[1-3]. Therefore, studying the effects of lightning is an important issue for the coordination of insulation of power systems. The lightning effect can be categorized in two aspects i.e. direct and indirect effects[4-5]. For the direct effect, the lightning directly strikes the power network while for the indirect effect the lightning strikes the surface of the ground or any object around a power line, and the lightning induces a voltage due to coupling between the lightning electromagnetic fields and the power line. In this paper, the indirect effect of lightning is studied when the lightning strikes the ground around a power line [6-9]. Several studies have been undertaken to evaluate the effect of the lightning induced voltage on power networks which is dependent on the wave shape of the current at different heights along the lightning channel, the lightning field components and the coupling model applied [8, 10-17]. In order to study the current wave shapes at the channel base and at different heights along the lightning channel, different current functions and models are proposed respectively. In previous studies, non-realistic current functions and models have been applied in the analytical voltage expressions [11, 18-19].

In this study, a number of algorithms are proposed for

realistic current functions whereas previous studies usually evaluated the field components in the frequency domain and in order to enter the field values into coupling models they have to be converted again to the time domain [9, 20-24]. The coupling models are usually expressed by a system of partial differential equations such that an algorithm is needed to solve this system. In this paper, a set of analytical voltage expressions are proposed to evaluate the lightning induced voltage on the power lines which can cover the widely used current functions and different current models directly in the time domain without the need to apply any extra conversions in the field or coupling calculations, unlike some of the previous methods. The lightning induced voltage on the power line will be evaluated based on the proposed method and the results compared to the corresponding values from previous studies. The basic assumptions in this study are as follows:

 i. The lightning channel is perpendicular to the surface of the ground without any branches.
 ii. The surface of the ground is flat.
 iii. The ground conductivity is assumed to be perfect.

2. Return stroke current

The return stroke current can be considered from two aspects i.e. the channel base and different heights along the lightning channel that can be simulated using current functions and current models. In order to generalize the field and voltage expressions, the widely used current functions are tabulated in Table 1 where A, A_1, A_2, B, B_1, B_2, Γ_1, Γ_2, Γ_{11}, Γ_{12}, Γ_{21}, Γ_{22}, η, n_1, n_2 are constant coefficients and I_p, I_{p1}, I_{p2}, I_{01}, I_{02} are current peaks. Note that similar symbols in different current functions do not have a similar concept [25]. In this paper, two main

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Table 1. The widely used channel base current functions

Current Model	Channel base current function
Bruce and Golde [26]	$I_p[\exp(-At) - \exp(-Bt)]$
Improvement of Uman and McLain on Bruce and Golde function [27]	$\dfrac{I_p}{\eta}[\exp(-At) - \exp(-Bt)]$
Improvement of Jones on Bruce and Golde function [28]	$\dfrac{I_p}{\eta}\left[\exp\left(-\dfrac{t^*}{\Gamma_1}\right) - \exp\left(-\dfrac{t^*}{\Gamma_2}\right)\right]$ $t^* = \dfrac{\Gamma_2^2}{\Gamma_1} + t$
Pierce and Ciones [29]	$I_{p1}[\exp(-A_1t) - \exp(-B_1t)]$ $+I_{p2}[\exp(-A_2t) - \exp(-B_2t)]$
Heidler function [30]	$\dfrac{I_p}{\eta_1}\dfrac{\left(\dfrac{t}{\Gamma_1}\right)^{n_1}}{1+\left(\dfrac{t}{\Gamma_1}\right)^{n_1}}\exp\left(\dfrac{-t}{\Gamma_2}\right)$
Sum of two Heidler functions [31]	$\dfrac{i_{01}}{\eta_1}\dfrac{\left(\dfrac{t}{\Gamma_{11}}\right)^{n_1}}{1+\left(\dfrac{t}{\Gamma_{11}}\right)^{n_1}}\exp\left(\dfrac{-t}{\Gamma_{12}}\right)$ $+\dfrac{i_{02}}{\eta_2}\dfrac{\left(\dfrac{t}{\Gamma_{21}}\right)^{n_2}}{1+\left(\dfrac{t}{\Gamma_{21}}\right)^{n_2}}\exp\left(\dfrac{-t}{\Gamma_{22}}\right)$
Improvement of Nucci on Heidler function [32]	$\dfrac{i_{01}}{\eta_1}\dfrac{\left(\dfrac{t}{\Gamma_{11}}\right)^{n_1}}{1+\left(\dfrac{t}{\Gamma_{11}}\right)^{n_1}}\exp\left(\dfrac{-t}{\Gamma_{12}}\right)$ $+i_{02}[\exp(-\Gamma_{21}t) - \exp(-\Gamma_{22}t)]$

current functions are proposed in Eqs. (1) and (2) and the general expressions of the field components and lightning induced voltage are proposed based on these two expressions. All the functions mentioned in Table 1 can be made by a combination of these two functions.

$$I(0,t) = \frac{I_0}{\eta_0}(c_1 e^{-\alpha t} - c_2 e^{-\gamma t}) \tag{1}$$

$$I(0,t) = \frac{I_{01}}{\eta_{01}}\frac{\left(\frac{t}{t_1}\right)^n}{1+\left(\frac{t}{t_1}\right)^n}\exp\left(\frac{-t}{t_2}\right) \tag{2}$$

where:

$$\eta_{01} = \exp\left[-\left(t_1/t_2\right)\left(n\frac{t_2}{t_1}\right)^{\frac{1}{n}}\right]$$

In order to study the current wave shape at different heights along the lightning channel, the general form of the engineering current model is selected in this work as presented in Eq. (3). It should be mentioned that the widely used current models such as the Transmission Line model (TL), Modified Transmission line with Exponential Decay model (MTLE), Modified Transmission Line with Linear Decay model (MTLL) can all be expressed by substituting different functions instead of the P(z') term in Eq. (3) [33-34].

$$I(z',t) = I\left(0, t - \frac{z'}{v}\right) \times P(z') \times u\left(t - \frac{z'}{v_f}\right) \tag{3}$$

where:

z' is the temporary charge height along lightning channel,
I(z', t) is current distribution along lightning channel at any height z' and any time t,
I(0,t) is channel base current,
P(z') is the attenuation height dependent factor,
v is the current-wave propagation velocity,
v_f is the upward propagating front velocity,
u is the Heaviside function as defined by

$$u\left(t - \frac{z'}{v_f}\right) = \begin{cases} 1 \text{ for } t \geq \dfrac{z'}{v_f} \\ 0 \text{ for } t < \dfrac{z'}{v_f} \end{cases}$$

3. Lightning Induced Voltage

The lightning induced voltage on a power line can be evaluated using coupling models. The most common coupling models can be classified into five groups as follows [8-9, 15, 24]:

   i.   Rusck model
  ii.   Chowduhari model
 iii.   Taylor model
 iv.   Rachidi model
  v.   Agrawal model

The Taylor coupling model is applied in this study. Therefore, by assuming a line with infinite length and by ignoring line impedance, the lightning induced voltage at different distances along the power line can be obtained from Eq. (4) and the geometry of problem is shown in Fig. 1 [11-12, 18-19].

$$V(x,t)=$$
$$\left[-\int_0^h E_z(x,z,d,t)dz - \frac{1}{2}\int_x^{\frac{(ct+x)^2-h^2-d^2}{2(ct+x)}} E_x\left(\eta, h, d, t - \frac{\eta-x}{c}\right)d\eta \right.$$
$$\left. + \frac{1}{2}\int_{-\frac{(ct-x)^2-h^2-d^2}{2(ct-x)}}^x E_x\left(\eta, h, d, t + \frac{\eta-x}{c}\right)d\eta \right]$$
$$U\left(t - \frac{\sqrt{x^2+d^2+h^2}}{c}\right) \tag{4}$$

where:

$\vec{E}_x$ is the electric field at x-axis,
$\vec{E}_z$ is the electric field at z-axis,
h is height of line,
c is light speed in free space,
d is radial distance between striking point and line,
x is observation point along power line,
t is time.

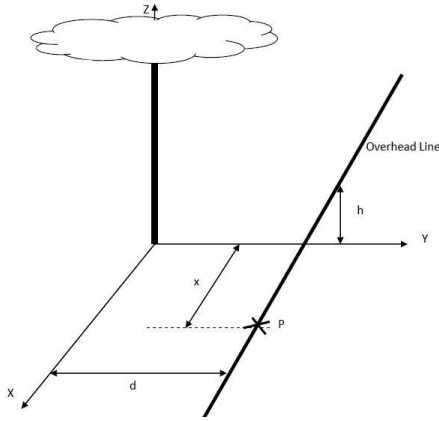As presented in Eq. (4), the lightning induced voltage is

Fig. 1. Geometry of problem

highly dependent on the value of the lightning electric field in the x and z directions. Therefore, the corresponding electric field components in Eq. (1) and Eq. (2) can be expressed by Eq. (5) to Eq. (8) in which Maxwell's equations, the Dipole method, the Trapezoid algorithm and the FDTD method are applied. The internal terms of Eq. (5) to Eq. (8) are listed in Table 2. These terms can support a wide range of current models by using the $P(z')$ factor [25]. Moreover, Eqs. (5) to (8) can support the different current functions in Table 1 by superposition of the electric fields due to the two fundamental equations, namely Eqs. (1) and (2)[10, 35-36].

$$\vec{E}_{x\_RS1}(x, y, z, t_n) = \vec{E}_{x\_RS1}(x, y, z, t_{n-1}) +$$
$$\Delta t \times \sum_{i=1}^{n} \sum_{m=1}^{k+1} \{a_m F_1(x, y, z, t = t_n, z' = h_{m,i}) -$$
$$a'_m F_1(x, y, z, t = t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c}) \quad (5)$$

$$\vec{E}_{z\_RS1}(x, y, z, t_n) = \vec{E}_{z\_RS1}(x, y, z, t_{n-1}) +$$
$$\Delta t \times \sum_{i=1}^{n} \sum_{m=1}^{k+1} \{a_m F_2(x, y, z, t = t_n, z' = h_{m,i}) -$$
$$a'_m F_2(x, y, z, t = t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c}) \quad (6)$$

$$\vec{E}_{x\_RS2}(x, y, z, t_n) = \vec{E}_{x\_RS2}(x, y, z, t_{n-1}) +$$
$$\Delta t \times \sum_{i=1}^{n} \sum_{m=1}^{k+1} \{a_m F_3(x, y, z, t = t_n, z' = h_{m,i}) -$$
$$a'_m F_3(x, y, z, t = t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c}) \quad (7)$$

$$\vec{E}_{z\_RS2}(x, y, z, t_n) = \vec{E}_{z\_RS2}(x, y, z, t_{n-1}) +$$
$$\Delta t \times \sum_{i=1}^{n} \sum_{m=1}^{k+1} \{a_m F_4(x, y, z, t = t_n, z' = h_{m,i}) -$$
$$a'_m F_4(x, y, z, t = t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c}) \quad (8)$$

where:

$\vec{E}_{x\_RS1}$ is the electric field at x-axis due to return stroke current function from Eq. (1),

$\vec{E}_{z\_RS1}$ is the electric field at z-axis due to return stroke current function from Eq. (1),

$\vec{E}_{x\_RS2}$ is the electric field at x-axis due to return stroke current function from Eq. (2),

$\vec{E}_{z\_RS2}$ is the electric field at z-axis due to return stroke current function from Eq. (2),

Δt is the time step,

n is the number of time steps,

$$t_n = (n - 1)\Delta t \quad n = 1, 2, \dots, n_{max}$$

$$a_m = \begin{cases} \frac{\Delta h_i}{2 \times k} \text{ for } m = 1 \text{ and } m = k + 1 \\ \\ \frac{\Delta h_i}{k} \text{ for others} \end{cases}$$

$$a'_m = \begin{cases} \frac{\Delta h'_i}{2 \times k} \text{ for } m = 1 \text{ and } m = k + 1 \\ \\ \frac{\Delta h'_i}{k} \text{ for others} \end{cases}$$

k is division factor(>=2),

Δh_i
$$= \begin{cases} \beta \chi^2 \{(ct_i - ct_{i-1}) - \sqrt{(\beta ct_i - z)^2 + (\frac{r}{\chi})^2} + \sqrt{(\beta ct_{i-1} - z)^2 + (\frac{r}{\chi})^2}\} \\ \\ \beta \chi^2 \{-(\beta z - ct_i) - \sqrt{(\beta ct_i - z)^2 + (\frac{r}{\chi})^2}\} \text{ for } i = 1 \end{cases}$$

$\Delta h'_i$
$$= \begin{cases} \beta \chi^2 \{(ct_{i-1} - ct_i) + \sqrt{(\beta ct_i + z)^2 + (\frac{r}{\chi})^2} - \sqrt{(\beta ct_{i-1} + z)^2 + (\frac{r}{\chi})^2}\} \\ \\ \beta \chi^2 \{-(\beta z + ct_i) + \sqrt{(\beta ct_i + z)^2 + (\frac{r}{\chi})^2}\} \text{ for } i = 1 \end{cases}$$

$$h_{m,i} = \begin{cases} \frac{(m - 1) \times \Delta h_i}{k} + h_{m=k+1, i-1} \\ \\ \frac{(m - 1) \times \Delta h_i}{k} \text{ for } i = 1 \end{cases}$$

$$h'_{m,i} = \begin{cases} \frac{(m - 1) \times \Delta h'_i}{k} + h'_{m=k+1, i-1} \\ \\ \frac{(m - 1) \times \Delta h'_i}{k} \text{ for } i = 1 \end{cases}$$

Therefore, the first term of Eq. (4) due to the current functions from Eqs. (1) and (2) can be obtained from Eqs. (9) and (10), respectively.

$$A_{RS1}^1 = - \int_0^h E_z(x, d, z, t = t_n) dz$$
$$= \frac{-\Delta h}{2} \sum_{m'=1}^{k'+1} b_m \times [\Delta t$$
$$\times \sum_{i=1}^{n} \sum_{m=1}^{k+1} \{a_m F_2(x, y, z$$
$$= (m' - 1) \times \Delta h, t = t_n, z' = h_{m,i})$$
$$- a'_m F_2(x, y, z = (m' - 1) \times \Delta h, t$$
$$= t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c})]$$
$$\quad (9)$$

$$A_{RS2}^1 = -\int_0^h E_z(x, d, z, t = t_n)dz$$

$$= \frac{-\Delta h}{2} \sum_{m'=1}^{k'+1} b_m \times [\Delta t$$

$$\times \sum_{i=1}^n \sum_{m=1}^{k+1} \{a_m F_4(x, y, z$$

$$= (m'-1) \times \Delta h, t = t_n, z' = h_{m,i})$$

$$- a'_m F_4(x, y, z = (m'-1) \times \Delta h, t$$

$$= t_n, z' = h'_{m,i})\} U(t - \frac{\sqrt{r^2 + z^2}}{c})]$$

(10)

where:

$$\Delta h = \frac{h}{k'},$$

k′ is division factor(>=2) ,

$$b_m = \begin{cases} 2 \text{ for } m' = 1 \text{ and } m' = k' + 1 \\ 1 \text{ for others} \end{cases}$$

Similarly, the second term of Eq. (4) due to the current functions from Eqs. (1) and (2) can be obtained from Eqs. (11) and (12), respectively.

$$A_{RS1}^2 = -\frac{1}{2}\int_x^{\frac{(ct_n+x)^2-h^2-d^2}{2(ct_n+x)}} E_x\left(\eta, d, h, t_n - \frac{\eta-x}{c}\right) d\eta$$

$$= -\frac{\Delta x}{4} \times \sum_{m'=1}^{q+1} c_m \times$$

$$[\Delta t \times \sum_{i=1}^n \sum_{m=1}^{k+1} \{a_m F_1\left(x = x_{m'}, y = d, z\right.$$

$$= h, t = t_n - \frac{x_{m'}-x}{c}, z' = h_{m,i}\Big)$$

$$- a'_m F_1\left(x = x_{m'}, y = d, z = h, t\right.$$

$$= t_n - \frac{x_{m'}-x}{c}, z' = h'_{m,i}\Big)\}$$

$$U(t_n - \frac{x_{m'}-x}{c} - \frac{\sqrt{x_{m'}^2+d^2+h^2}}{c})]$$

(11)

$$A_{RS2}^2 = -\frac{1}{2}\int_x^{\frac{(ct_n+x)^2-h^2-d^2}{2(ct_n+x)}} E_x\left(\eta, d, h, t_n - \frac{\eta-x}{c}\right) d\eta$$

$$= -\frac{\Delta x}{4} \times \sum_{m'=1}^{q+1} c_m \times [\Delta t$$

$$\times \sum_{i=1}^n \sum_{m=1}^{k+1} \{a_m F_3\left(x = x_{m'}, y = d, z = h, t\right.$$

$$= t_n - \frac{x_{m'}-x}{c}, z' = h_{m,i}\Big) - a'_m F_3$$

$$\left(x = x_{m'}, y = d, z = h, t = t_n - \frac{x_{m'}-x}{c}, z' = h'_{m,i}\right)\}$$

$$U(t_n - \frac{x_{m'}-x}{c} - \frac{\sqrt{x_{m'}^2+d^2+h^2}}{c})]$$

(12)

where:

$$\Delta x = \frac{\frac{(ct_n+x)^2-h^2-d^2}{2(ct_n+x)}-x}{q} ,$$

$$x_{m'} = x + (m'-1) \times \Delta x ,$$

q is division factor(>=2) ,

$$c_m = \begin{cases} 2 \text{ for } m' = 1 \text{ and } m' = q + 1 \\ 1 \text{ for others} \end{cases}$$

Moreover, the third term of Eq. (4) due to the current functions from Eqs. (1) and (2) can be obtained from Eqs. (13) and (14), respectively.

$$A_{RS1}^3 = \frac{1}{2}\int_{-\frac{(ct_n-x)^2-h^2-d^2}{2(ct_n-x)}}^x E_x\left(\eta, d, h, t_n + \frac{\eta-x}{c}\right) d\eta$$

$$= \frac{\Delta x'}{4} \times \sum_{m'=1}^{q+1} c_m \times [\Delta t$$

$$\times \sum_{i=1}^n \sum_{m=1}^{k+1} \{a_m F_1\left(x = x'_{m'}, y\right.$$

$$= d, z = h, t = t_n + \frac{x'_{m'}-x}{c}, z' = h_{m,i}\Big)$$

$$- a'_m F_1\left(x = x'_{m'}, y = d, z = h, t\right.$$

$$= t_n + \frac{x'_{m'}-x}{c}, z' = h'_{m,i}\Big)\} U(t_n$$

$$+ \frac{x'_{m'}-x}{c} - \frac{\sqrt{[x'_{m'}^2+d^2+h^2}}{c})]$$

(13)

$$A_{RS2}^3 = \frac{1}{2}\int_{-\frac{(ct_n-x)^2-h^2-d^2}{2(ct_n-x)}}^x E_x\left(\eta, d, h, t_n + \frac{\eta-x}{c}\right) d\eta$$

$$= \frac{\Delta x'}{4} \times \sum_{m'=1}^{q+1} c_m \times [\Delta t$$

$$\times \sum_{i=1}^n \sum_{m=1}^{k+1} \{a_m F_3\left(x = x'_{m'}, y\right.$$

$$= d, z = h, t = t_n + \frac{x'_{m'}-x}{c}, z' = h_{m,i}\Big)$$

$$- a'_m F_3\left(x = x'_{m'}, y = d, z = h, t\right.$$

$$= t_n + \frac{x'_{m'}-x}{c}, z' = h'_{m,i}\Big)\} U(t_n$$

$$+ \frac{x'_{m'}-x}{c} - \frac{\sqrt{x'_{m'}^2+d^2+h^2}}{c})]$$

(14)

Where:

$$\Delta x' = \frac{x+\frac{(ct-x)^2-h^2-d^2}{2(ct-x)}}{q} ,$$

Table 2. The internal parameters of proposed electric field expressions

Parameter	Expression
F_1	$\dfrac{I_0 P(z')}{4\pi\varepsilon_0\eta_0}(z-z')x\left\{\dfrac{-3(c_1\alpha\exp(-\alpha A_1)-c_2\gamma\exp(-\gamma A_1))}{cR^4}+\dfrac{c_1\alpha^2\exp(-\alpha A_1)-c_2\gamma^2\exp(-\gamma A_1)}{c^2R^3}\right.$ $\left.+\dfrac{3(c_1\exp(-\alpha A_1)-c_2\exp(-\gamma A_1))}{R^5}\right\}$
F_2	$\dfrac{I_0 P(z')}{4\pi\varepsilon_0\eta_0}\left\{\dfrac{3(x^2+y^2)(c_1\alpha\exp(-\alpha A_1)-c_2\gamma\exp(-\gamma A_1))}{cR^4}+\dfrac{2(-c_1\alpha\exp(-\alpha A_1)+c_2\gamma\exp(-\gamma A_1))}{cR^2}\right.$ $+\dfrac{(x^2+y^2)(-c_1\alpha^2\exp(-\alpha A_1)+c_2\gamma^2\exp(-\gamma A_1))}{c^2R^3}-\dfrac{3(x^2+y^2)(c_1\exp(-\alpha A_1)-c_2\exp(-\gamma A_1))}{R^5}$ $\left.+\dfrac{2(c_1\exp(-\alpha A_1)-c_2\exp(-\gamma A_1))}{R^3}\right\}$
F_3	$\dfrac{I_{01}P(z')x(z-z')A_2}{4\pi\varepsilon_0\eta_{01}\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}\left\{\dfrac{\left(\frac{A_1}{t_1}\right)^n(3c^2t_2^2-3ct_2R+R^2)}{c^2t_2^2R^5}\right.$ $+\dfrac{2n^2\left(\frac{A_1}{t_1}\right)^{3n-2}+n(n-1)\left(\frac{A_1}{t_1}\right)^{n-2}\left[\left(\frac{A_1}{t_1}\right)^n+1\right]^2+(-3n^2+n)\left(\frac{A_1}{t_1}\right)^{2n-2}\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}{c^2t_1^2R^3\left[\left(\frac{A_1}{t_1}\right)^n+1\right]^2}$ $\left.+\dfrac{3n\left(\left[\left(\frac{A_1}{t_1}\right)^n+1\right]\left(\frac{A_1}{t_1}\right)^{n-1}-\left(\frac{A_1}{t_1}\right)^{2n-1}\right)}{ct_1R^4\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}+\dfrac{2n\left(\left(\frac{A_1}{t_1}\right)^{2n-1}-\left[\left(\frac{A_1}{t_1}\right)^n+1\right]\left(\frac{A_1}{t_1}\right)^{n-1}\right)}{c^2t_1t_2R^3\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}\right\}$
F_4	$\dfrac{I_{01}P(z')A_2}{4\pi\varepsilon_0\eta_{01}\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}\left\{\dfrac{(x^2+y^2)\left[\left(\frac{A_1}{t_1}\right)^n(-3c^2t_1t_2^2+3ct_1t_2R-t_1R^2)+n\left(\frac{A_1}{t_1}\right)^{n-1}(-3ct_2^2R+2t_2R^2)\right]}{c^2t_1t_2^2R^5}\right.$ $+\dfrac{2\left(\frac{A_1}{t_1}\right)^n(ct_1t_2-t_1R)+2n\left(\frac{A_1}{t_1}\right)^{n-1}t_2R}{ct_1t_2R^3}-\dfrac{n(n-1)(x^2+y^2)\left(\frac{A_1}{t_1}\right)^{n-2}}{c^2t_1^2R^3}$ $+\dfrac{(x^2+y^2)\left[2n^2t_2R\left(\frac{A_1}{t_1}\right)^{2n-2}+3nct_1t_2\left(\frac{A_1}{t_1}\right)^{2n-1}+n(n-1)t_2R\left(\frac{A_1}{t_1}\right)^{2n-2}-2nt_1R\left(\frac{A_1}{t_1}\right)^{2n-1}\right]-2nct_1t_2R^2\left(\frac{A_1}{t_1}\right)^{2n-1}}{c^2t_1^2t_2R^4\left[\left(\frac{A_1}{t_1}\right)^n+1\right]}$ $\left.-\dfrac{2n^2(x^2+y^2)\left(\frac{A_1}{t_1}\right)^{3n-2}}{c^2t_1^2R^3\left[\left(\frac{A_1}{t_1}\right)^n+1\right]^2}\right\}$
	$R=\sqrt{x^2+y^2+(z-z')^2};\ A_1=t-\dfrac{R}{c}-\dfrac{|z'|}{v};\ A_2=\exp\left(\dfrac{\frac{R}{c}-t+\frac{|z'|}{v}}{t_2}\right)$

$$x'_{m'}=-\frac{(ct_n-x)^2-h^2-d^2}{2(ct_n-x)}+(m'-1)\times\Delta x',$$

q is division factor($>=2$) ,

$$c_m=\begin{cases}2\text{ for }m'=1\text{ and }m'=q+1\\1\text{ for others}\end{cases}$$

4. Results and Discussion

In order to study the behaviour of the lightning induced voltage on a typical power line, a sample of the channel base current based on the sum of two Heidler functions is selected for which the current parameters are listed in Table 3 as follows(v=c/3):

Table 3. The channel base current parameters based on the sum of two Heidler functions

I_{01}(kA)	τ_{11}(μs)	τ_{21}(μs)	n_1
19.5	1	2	2
I_{02}(kA)	**τ_{21}(μs)**	**τ_{22}(μs)**	**n_2**
12.3	8	30	3

The lightning induced over voltage (LIOV) in the middle of the power line is shown in Fig. 2 for which the height of the conductor and the radial distance between the lightning channel and the power line are assumed to be 10m and 50m, respectively, and the proposed induced voltage expression of Eq. (16) was applied. The current model in this case is the MTLE model with a value of λ of 1500 m.
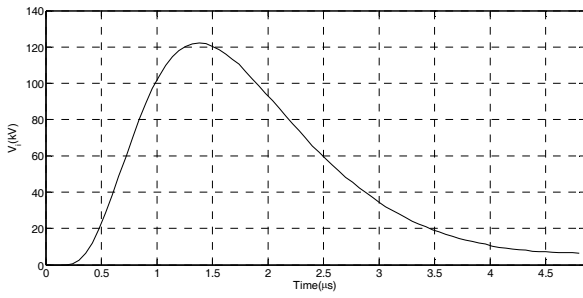
Fig. 2. The lightning induced voltage wave shape (d=50m, x=0, h=10m, λ=1500m, $v = 1 \times 10^8$ m/s)
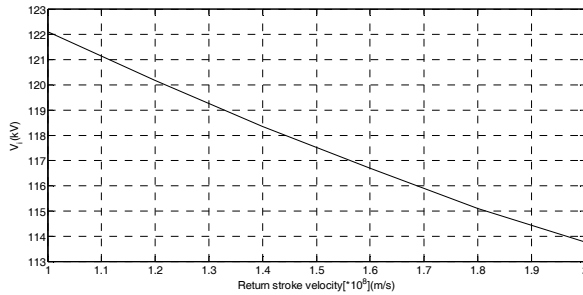
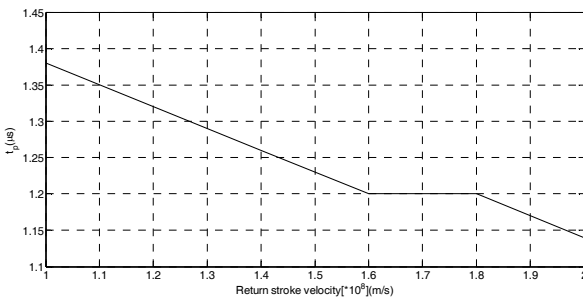**Fig. 3.** The behavior of LIOV peak versus return stroke velocity changes (d=50m, x=0, h=10m, λ=1500m)

Fig. 4. The behavior time to peak of LIOV versus return stroke velocity changes (d=50m, x=0, h=10m, λ= 1500m)

Fig. 2 shows that the peak value and time to peak of the LIOV are about 122 kV and 1.38 μs, respectively and the corresponding channel base current wave shape is shown in Fig. 11. In order to study the effect of the return stroke velocity on the peak value of the LIOV, Fig. 3 illustrates the variation of LIOV peaks versus return stroke velocity changes whereby all the other parameters are under the same conditions as in Fig. 2. Fig. 3 shows that when the return stroke velocity increases, the peak values of the LIOV decrease with a decreasing trend.

The effect of the velocity on the value of the time to peak of the lightning induced voltage is shown in Fig. 4 which indicates that by increasing the return stroke velocity, the value of the time to peak shows a decreasing trend.

Moreover, the effect of conductor height on the peak value of the LIOV for different distances with respect to the lightning channel is demonstrated in Fig. 5.
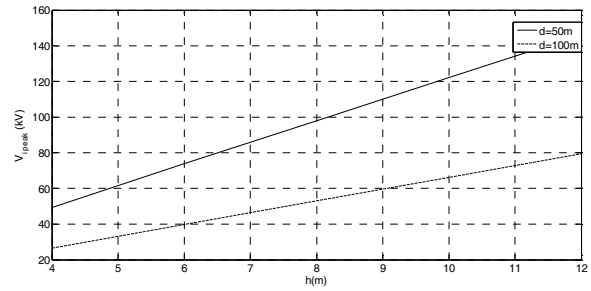

Fig. 5. The behavior of LIOV peak versus conductor height changes (x=0, λ=1500m, $v = 1 \times 10^8$ m/s)

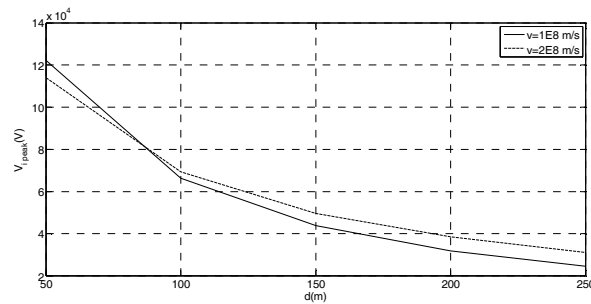**Fig. 6.** The behavior of LIOV peaks versus striking point (d) changes based on two different values of return stroke velocities(x=0, h=10m, λ=1500m)

Fig. 5 shows that by increasing the height of the conductor with respect to the surface of the ground, the LIOV peaks are increased with a linear trend under two different values of the 'd' parameter. Moreover, it illustrates that the voltage peak has an inverse relationship with the radial distance between the striking point and the line. Therefore, in order to design a power line, especially as part of a distribution network and also to set an appropriate protection level on the line, the conductor height can play an important role in the design. However other critical issues related to the safety of lines should also be considered.

The behaviour of the LIOV peaks versus the changes in the striking distance with respect to the middle of line is considered as illustrated in Fig. 6. Two different return stroke velocities are applied in this figure which shows that by increasing the radial distance between the lightning and the power line, the peak value of the LIOV demonstrates a decreasing and nonlinear trend.

Likewise, the behaviour of the lightning induced voltage at different points along the power line is considered as demonstrated in Fig. 7 whereby the simulated voltages are based on the proposed voltage expressions and the initial data obtained from Table 2. The values of k, k' and q are assumed to be 3, 10 and 100, respectively.

Fig. 7 shows that by increasing the distance along the power line with respect to the middle of the line, the peak values of the lightning induced voltage decrease and also the initial delay times are increased. Further, the initial delay time is directly dependent on the propagation speed of the lightning electromagnetic fields and also the wave
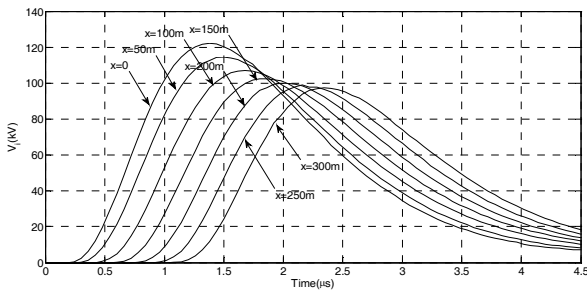
Fig. 7. The behavior of LIOV at different distances with respect to middle of line (d=50m, h=10m, λ=1500 m, $v = 1 \times 10^8$ m/s)
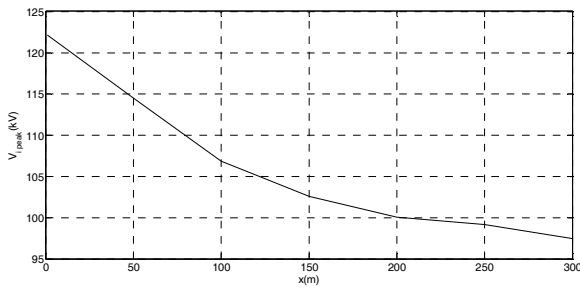
Fig. 8. The behavior of LIOV peaks versus 'x' changes (d=50m, h=10m, λ=1500m, $v = 1 \times 10^8$ m/s)
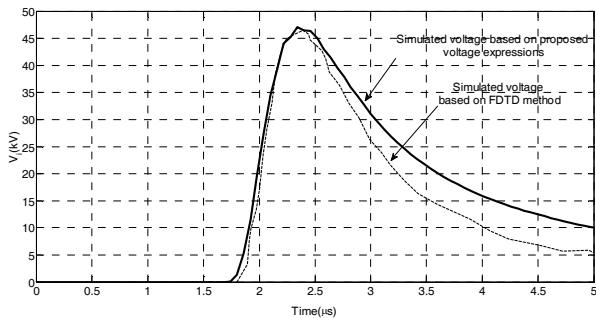

Fig. 9. Comparison between simulated induced voltages based on proposed method and FDTD method from reference [37]

Table 4. The typical channel base current parameters (based on sum of two Heidler functions)

I_{01}(kA)	τ_{11}(µs)	τ_{21}(µs)	n_1
10.7	0.25	2.5	2
I_{02}(**kA**)	τ_{21}(**µs**)	τ_{22}(**µs**)	n_2
6.5	2.1	230	3

propagation speed along the power line. Moreover, Fig. 8 shows the behaviour of the voltage peak versus changes in 'x' whereby increasing the distance with respect to the reference point, the value of the lightning induced voltage decreases with a nonlinear trend.

In order to validate the proposed method, a sample of the return stroke current is used for evaluation of the lightning induced voltage at a distance of 500 m with respect to the middle of a single line with h=7.5 m whereas the d
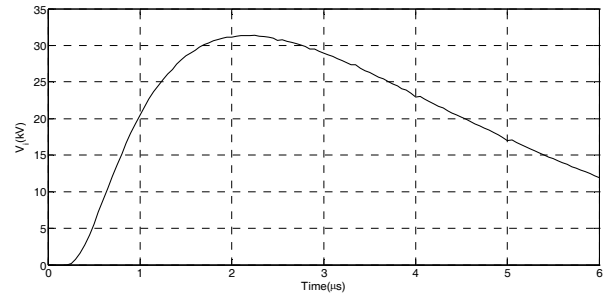

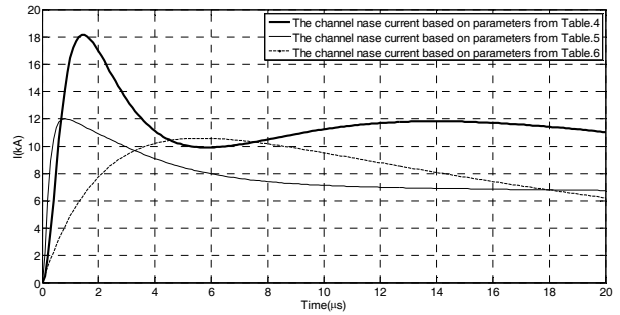**Fig. 10.** The lightning induced voltage wave shape due to BG current function at the middle of line

Fig. 11. The channel base current wave shapes based on current parameters from Tables.4-6

Table 5. The typical channel base current parameters (based on Bruce &Golde function)

I_p(kA)	**A**	**B**	λ(m)	**v[$\times 10^8$]m/s**
15	4.4×10^4	4.6×10^5	1500	1

parameter is assumed to be 50 m and the current parameters are listed in Table 4 as follows[37]:

Fig. 9 shows a comparison between the simulated induced voltage based on the proposed voltage expression and the corresponding voltage based on the FDTD model from sources [37]. The figure illustrates that the evaluated induced voltage is in good agreement with respect to the previous method. It should be mentioned that the length of the line is assumed to be infinite in the proposed method and the corresponding channel base current wave shape is as shown in Fig. 11.

Fig. 10 shows the proposed voltage expressions applied to a sample of channel base current based on the Bruce and Golde function and the LIOV evaluated at the middle of line. The current parameters are tabulated in Table 5 and also the MTLE current model is applied to study the wave shapes of the current at different heights along the channel. Moreover, the h and d parameters are assumed to be 10 m and 50 m, respectively. Fig. 10 also shows that the voltage peak and the time to peak of the LIOV are about 31.5 kV and 2.2 µs, respectively. The corresponding channel base current wave shape is shown in Fig. 11.

In order to consider on the behaviour of proposed functions, the evaluated induced voltage using proposed method were compared to another simulated induced
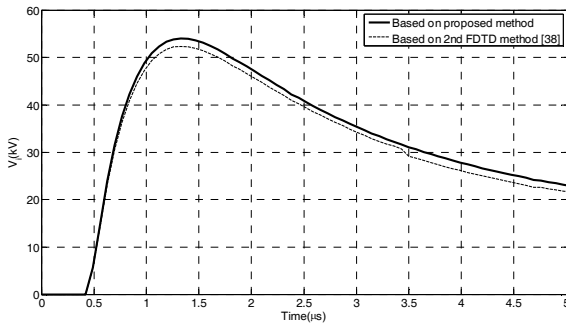
Fig. 12. Comparison between simulated induced voltages based on proposed method and 2nd FDTD method from reference [38]

Table 6. The channel base current parameters [38]

I_{01}(kA)	τ_{11}(µs)	τ_{21}(µs)	n_1
13.1	0.22	88	2
I_{02}(kA)	τ_{21}(µs)	τ_{22}(µs)	n_2
8.7	0.21	61	2

voltage based on 2nd FDTD method from reference [38] as shown in Fig. 12 while the current parameters were tabulated in Table 6 as follows;

The result illustrates, the simulated voltage in a good agreement with corresponding one based on 2nd FDTD method.

The proposed method can support a wide range of channel base current functions and current models directly in the time domain without the need to apply any extra conversions. This is unlike some previous methods which require extra conversions to the frequency domain and vice versa and yet other previous methods which use simplified analytical expressions for the step and linear rising currents and the TL current model.

5. Conclusion

In this paper, an analytical set of induced voltage expressions are proposed which can evaluate the lightning induced voltage on power lines directly in the time domain without the need to apply any extra conversions. Moreover, the proposed voltage expressions can support a wide range of current functions and current models. The simulated induced voltage obtained from the proposed method is compared to the corresponding voltage based on the FDTD method from the literature which shows that the two values are in a good agreement. Similarly, the effect of different parameters on the peak value of the lightning induced voltage and the time to peak voltage are considered and discussed accordingly. The proposed analytical voltage expressions can be useful for study of the effects of lightning on a power network and can also be helpful for line designers to improve and optimize the protection level of power lines versus the indirect effect of lightning.

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