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A Note on Determining Confidence Level in Reliability Test for Assuring B_x-Life*

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In this paper, we consider the problem of determining the confidence level in zero-failure reliability sampling plans when the life distribution is Weibull distribution with a shape parameter m and a scale parameter η. We introduce zero-failure reliability sampling plans for Weibull distribution and investigate some characteristics of zero-failure reliability sampling plans. Finally, We propose new guideline for determining the confidence level in zero-failure reliability sampling plans for assuring B_x-life.

Keywords: Reliability Sampling Plans, Zero-Faiure Reliability Sampling Plans, Weibull Distribution, Test Period, Confidence Level, B_x-life

1. Introduction

The purposes of reliability test are either to predict lifetime or to monitor the wafer processes for reliability behaviour (Tielemans, Rongen and De Ceuninck, 2002). Acceptance sampling technique is used to determine whether a lot of products is accepted or not based on only inspecting the quality of a small set of products. In many practical applications, an important quality variable is the lifetime of a product. Acceptance sampling plans used to determine the acceptability of a lot of products with respect to their lifetimes are called reliability sampling plans (RSP) (Tsai, Lu, and Wu, 2008). The various types of designs of reliability sampling plans under different censoring schemes and different techniques have been proposed by many researchers. The reliability sampling plans under different censoring schemes have been studied by Fertig and Mann (1980), Schneider (1989), Balasooriya (1995), Wu and Tsai (2000, 2005) and Tsai and Wu (2006). Bayesian reliability sampling plans considering the prior information from the past experience and engineering knowledge have been researched by Fertig and Mann (1974), Nigm and Ismail (1985), Lam (1988), Lam and Lau (1993), Zhang and Meeker (2005) and Tsai, Lu and Wu (2008). Accelerated life test sampling plans (ALTSP), which is designed to reduce the test time, have been studied by Yum and Kim (1990), Hsiesh (1994), Bai, Kim and Chun (1993, 1995) and Seo, Jung and Kim (2009).

In most literatures, authors have developed the reliability sampling plans for assuring or predicting mean time to failure (MTTF) which is less practical reliability characteristic than B_x-life which is defined as the time at which $x\%$ of the units in a population fail. B_x-life is widely used as the guaranteed life of an item at the design phase in industries.

During the past 10 years, reliability evaluation centers designated by Korean government have conducted the reliability test for assuring the guaranteed life at the request from the industries. In most cases, zero-failure reliability sampling plan (ZFRSP) and B_x-life have been used as test method and the guaranteed life, respectively. ZFRSP for assuring B_x-life mainly consists of the value of x, guaranteed life, confidence level of the test, sample size and test period among which the first three factors are determined by designer of the test. The sample size and the test period are negatively dependent each other. The values of x are typically fixed at 5 or 10 and rarely fixed at 1. It is easy to determine the guaranteed life since the information on the frequency of usage per week, daily operating hours and so on is available. However, it is not easy to determine the confidence level of the ZFRSP since there is no guideline for determining

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the confidence level. The designer of ZFRSP just chooses one of 60%, 70%, 80% and 90% as the confidence level of the test. Hence it is necessary to develop a guideline for determining the confidence level. In this paper, we propose a method for determining the confidence level in ZFRSP for assuring B_x-life when the life distribution is Weibull distribution with a shape parameter m and a scale parameter η. Throughout this paper, m is assumed to be known.

Chapter 2 introduces ZFRSP for assuring B_x-life when the life distribution is Weibull distribution. We investigate the ZFRSP by finding some characteristics numerically and OC curve and comparing two ZFRSPs with different CLs and p in Chapter 3. Chapter 4 is devoted to find the effect of CL on ZFRSP and to propose the method of determining CL.

Notations

- β : Consumer's risk. Probability of accepting a batch from the producer which, in fact, has unacceptable reliability level(URL) so that it should have been rejected.
- T : Test period
- r : Number of failed items
- c : Acceptable number of failures
- CL : Confidence level (=$100(1-\beta)\%$)
- B_x-life : The x-th percentile of a life distribution. That is, $B_x = t_p$ satisfying $F(t_p) = p$, where $p = x/100$.
- $N(t)$: Number of failures in $(0, t)$.

2. ZFRSP for Aassuring B_x-life

2.1 Summary of MIL-HDBK-781A [Alternative Fixed-duration Test Plan]

Suppose that life times of the items being tested are exponentially distributed with a scale parameter λ. Then the probability density function, the reliability function and MTTF(θ) are $f(t) = \lambda e^{-\lambda t}$, $R(t) = \lambda e^{-\lambda t}$ and $\theta = 1/\lambda$, respectively.

The alternative fixed-duration test plan is to test the null hypothesis $H_0 : \theta \le \theta_1$ against the alternative hypothesis $H_0 : \theta > \theta_1$. Then the rejection criteria is to reject H_0 if $N(t) \le c$. Then the sampling plan (n, T, r, c) can be obtained by finding (n, T) satisfying the equation (1) for given β and c.

$$L(\theta_1) = P[N(T) \le c|H_0] = \sum_{r=0}^{c} C_r^n [1-R(T)]^r R(T)^{n-r} \le \beta, \qquad (1)$$

where $R(T) = \exp[-T/\theta_1]$.

2.2 Fixed-Duration Test Plans for Assuring B_x-life When the Life Distribution is Weibull Distribution

Suppose that the lifetimes of the items being tested have a Weibull distribution with a shape parameter m and a scale parameter η with the probability density function and the reliability function, respectively.

$$f(t) = \frac{1}{\eta}\left(\frac{t}{\eta}\right)^{m-1} \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \qquad (2)$$

and

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \qquad (3)$$

Then MTTF and B_x-life are $MTTF = \eta\Gamma(1+1/m)$ and $B_x = \eta[-\ln(1-p)]^{1/m}$, respectively. The shape parameter m is assumed to be known.

Fixed-duration test plan for assuring B_x-life is to test the null hypothesis $H_0 : B_x \le \tau_1$ against $H_1 : B_x > \tau_1$ where τ_1 represents the guaranteed life time. Then the rejection criteria is to reject H_0 if $N(t) \le c$ and the test plan (n, T) can be constructed by finding n and T satisfying the equation (4) for given β and c

$$L(\tau_1) = P[N(T) \le c|H_0] = \sum_{r=0}^{c} C_r^n [1-R(T)]^r R(T)^{n-r} \le \beta \qquad (4)$$

where $R(T) = \exp[-T/\eta_1]$ and $\eta_1 = \tau_1/[-\ln(1-p)]^{1/m}$

Zero-failure reliability sampling plan (hereafter, ZFRSP) for assuring B_x-life can be obtained by substituting 0 for c in the equation (4). And then ZFRSP can be determined by finding (n, T) which satisfies the equation (5) for given β

$$L(\tau_1) = P[N(T) \le 0|H_0] = R(T)^n \le \beta \qquad (5)$$

Note that it follows from the fact that $\eta_1 = \tau_1/[-\ln(1-p)]^{1/m}$ that

$$R(T) = \exp\left[-\left(\frac{T}{\eta_1}\right)^m\right] = \exp\left[-\left(\frac{T[-\ln(1-p)]^{1/m}}{\tau_1}\right)^m\right] = \exp\left[-\left(\frac{T^m[-\ln(1-p)]}{\tau_1^m}\right)\right]$$

For given β, ZFRSP for assuring B_x-life that satisfies (5) is determined by finding (n, T) satisfying the following equation.

$$T \ge \left[\frac{\tau_1^m[-\ln(\beta)]}{n[-\ln(1-p)]}\right]^{\frac{1}{m}} = \tau_1\left[\frac{\ln(1-CL)}{n[\ln(1-p)]}\right]^{\frac{1}{m}}, \qquad (6)$$

where $CL = 1-\beta$.

3. Analysis of ZFRSP

In Chapter 2, we have constructed ZFRSP when the life times of items being tested have a Weibull distribution. In this chapter, we investigate the characteristics of ZFRSP and the operating characteristic (OC) curve in ZFRSP.

3.1 Characteristics of ZFRSP

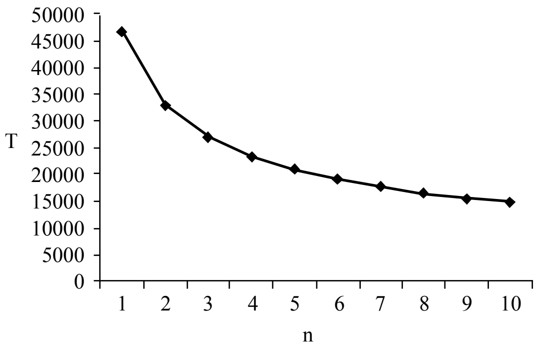
First, we note that there are various combinations of (n, T) for a given confidence level CL) and the guaranteed life time $B_x = \tau_1$. That is, we can find the combinations (n, T) satisfying the following equation for given CL and $B_x = \tau_1$

$$T = \tau_1 \left[\frac{\ln(1-CL)}{n ln(1-p)} \right]^{\frac{1}{m}}, \qquad (7)$$

where $CL = 1-\beta$ and $p = \frac{x}{100}$.

<Figure 1> shows several combinations of (n, T) when the confidence level is 90% and the guaranteed life time is $B_{10} = 10000hr$. It is noted from <Figure 1> that the test period decreases as the sample size increases.

Also we note that there are various combinations of (p, CL) which can determine the same ZFRSP (n, T) for assuring the guaranteed life time $B_x = \tau_1$. <Figure 2> shows various combinations (p, CL) all of which construct the ZFRSP $(n, T) = (5, 20000)$ for assuring the guaranteed life time $B_x = 10000hr$ when the life distribution of items is Weibull distribution with $m = 2.0$



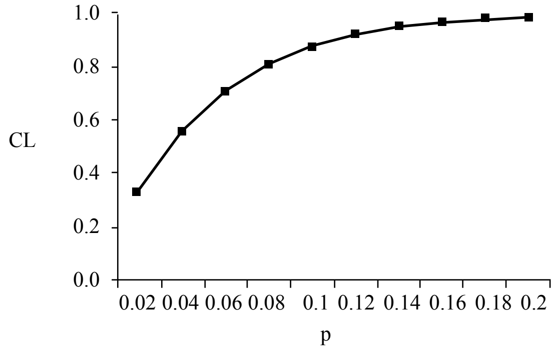
<Figure 1> ZFRSP (n, T) for given $CL = 90\%$ and $B_{10} = 10000hr$ when $m = 2.0$

3.2 Operating Characteristics Curve of ZFRSP

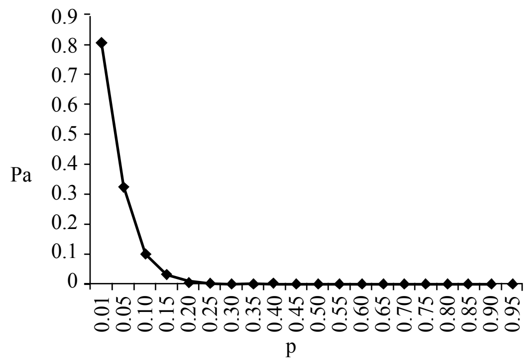
Now we consider OC curve which describes the probability of accepting a lot as a function of the lot's B_x-life. Hence the probability of accepting a lot is

$$P_a = \sup_{H_0} [R(T|H_0)]^n = [\exp(-\left(\frac{T}{\eta_1}\right)^m)]^n, \qquad (8)$$

where $\eta_1 = \frac{\tau_1}{[-\ln(1-p)]^{1/m}}$.



<Figure 2> Combinations of (p, CL) which can construct the ZFRSP $(n, T) = (5, 20000)$ for assuring $B_x = 10000hr$ when $m = 2.0$



<Figure 3> OC curve of ZFRSP $(n, T) = (5, 20000)$ for assuring $B_x = 10000hr$ with $CL = 90\%$ when $m = 2.0$

3.3 Comparison of Two Different Combinations of (p, CL) Constructing the Same ZFRSP

Suppose that the life distribution is Weibull distribution with a shape parameter m and a scale parameter. We consider two ZFRSP among which one is to assure higher B_x-life with lower confidence level (ZFRSP1) while the other is to assure lower B_x-life with higher confidence level (ZFRSP2). Then it is natural that the following question arises.

"When we select one of each from two lots passing ZFRSP 1 and ZFRSP 2, respectively, which one is more likely to fail before the guaranteed B_x-life τ1?"

In order to answer this question, it is necessary to compute the probability that the items fail before the assured B_x-life τ1. It is noted that if a lot of size n passes ZFRSP for assuring the guaranteed life time τ1 with 50% CL, then the estimator of the scale parameter is given by $\hat{\eta} = \left[\frac{2nT^m}{\chi^2_{0.5}(2)}\right]^{1/m}$, where $\chi^2_{0.5}(2)$ is the upper 50th percentile of chi-square distribution with 2 degrees of freedom. Since $T = \tau_1\left[\frac{\ln(1-CL)}{nln(1-p)}\right]^{\frac{1}{m}}$ from the equation (8), the estimator of η can be rewritten as $\hat{\eta} = \tau_1\left[\frac{2\ln(1-CL)}{\chi^2_{0.5}(2)\ln(1-p)}\right]^{1/m}$. Hence the reliability at τ_1 of an item from the lot is as follows.

$$\hat{R}(\tau_1) = \exp\left[\frac{\chi^2_{0.5}(2)\ln(1-p)}{2\ln(1-CL)}\right] \tag{9}$$

Example 1. Suppose that ZFRSP 1 is to assure $B_{10} = 10{,}000hr$ with 70% confidence level while ZFRSP 2 is to assure $B_{20} = 10000hr$ with 90% confidence level. Let items A and B be items selected from two lots which have successfully passed ZFRSP 1 and ZFRSP 2, respectively. Then the probability that the item A fails before the assured $B_{10} = 10000hr$ is

$$\widehat{F(10000)} = 1 - \exp\left[-\frac{\chi^2_{0.5}(2)\ln(1-0.1)}{2\ln(1-0.7)}\right] = 0.0589.$$

And the probability that the item B fails before the assured $B_{20} = 10000hr$ is

$$\widehat{F(10000)} = 1 - \exp\left[-\frac{\chi^2_{0.5}(2)\ln(1-0.2)}{2\ln(1-0.9)}\right] = 0.0650.$$

Hence items from the lot which is accepted under ZFRSP 1 are more reliable than items under ZFRSP 2.

4. Determining the Confidence Level in ZFRSP

It is noted from the equation (9) that the reliability at the guaranteed B_x-life is positively affected by CL. <Table 1> shows numerical results of the reliability at the guaranteed B_x-life for various combinations of CL and p. As we expect, the reliability increases as CL increases and decreases as x-value in B_x-life increases.

Also, we develop a new guideline to determine the value of CL in ZFRSP for assuring B_x-life. The proposed guideline is based on the tolerance limit of the probability that the item from a lot passing ZFRSP for assuring B_x-life fails before the assured B_x-life.

Suppose that the tolerance limit of the probability that the item fails before the assured B_x-life is ξ. Then we can obtain the value of CL for ZFRSP for assuring B_x-life by solving the following inequality for CL.

$$\widehat{F(\tau_1)} = 1 - \exp\left[-\frac{\chi^2_{0.5}(2)\ln(1-p)}{2\ln(1-CL)}\right] \le \xi$$

Then we have

$$CL \ge \exp\left[-\frac{\chi^2_{0.5}(2)\ln(1-p)}{2\ln(1-\xi)}\right] \tag{10}$$

<Table 1> Reliability at the Guaranteed B_x-life

p	CL				
	60%	70%	80%	90%	95%
0.02	0.9848	0.9884	0.9913	0.9939	0.9953
0.04	0.9696	0.9768	0.9826	0.9878	0.9906
0.06	0.9543	0.9650	0.9737	0.9815	0.9858
0.08	0.9389	0.9531	0.9647	0.9752	0.9809
0.10	0.9234	0.9411	0.9556	0.9688	0.9759
0.12	0.9078	0.9290	0.9464	0.9622	0.9709
0.14	0.8922	0.9168	0.9371	0.9556	0.9657
0.16	0.8764	0.9045	0.9277	0.9489	0.9605
0.18	0.8606	0.8920	0.9181	0.9420	0.9551
0.20	0.8447	0.8794	0.9084	0.9350	0.9497

Example 2. Suppose that ZFRSP (n, T) is constructed to assure $B_{10} = 10{,}000hr$. The probability that the item from the lot passing this ZFRSP fails before the assured $B_{10} = 10{,}000hr$ should be less than 0.01. Then CL of this ZFRSP can be obtained by substituting 0.1 and 0.01 for p and ξ in the equation (10), respectively and it should be greater than 99.93%.

5. Conclusion

In this paper, we consider ZFRSP for assuring B_x-life and propose a method for determining the confidence level in ZFRSP for assuring B_x-life when the life distribution is Weibull distribution with a shape parameter m and a scale parameter η. To this end, we study ZFRSP for assuring B_x-life when the life distribution is Weibull distribution and investigate the ZFRSP by finding some characteristics numerically and OC curve and comparing two ZFRSPs with different CLs and p. Also we study the effect of CL on ZFRSP and propose the method of determining CL.

The results we obtain in this research could be used in the design stage of reliability assurance test. For the future study, the lognormal distribution can be considered as the life distribution since the lognormal distribution is gradually considered when the reliability of an item is evaluated.

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