An Applicability of Teager Energy Operator and Energy Separation Algorithm for Waveform Distortion Analysis: Harmonics, Inter-harmonics and Frequency Variation

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Abstract – This paper deals with an application of Teager Energy Operator (TEO) and Energy Separation Algorithm(ESA) to detect and determine various voltage waveform distortions like harmonics, inter-harmonics and frequency variation. Because the TEO and DESA algorithm was initially proposed for speech or communication analysis, its applications are limited to some types of waveform in the power quality analysis area. For example, an undistorted voltage signal is similar with a pure sinusoid. A voltage fluctuation is very similar with an amplitude-modulated signal, from the viewpoint of signal theory. And a continuous frequency variation is similar with a frequency-modulated signal, which is also known as a chirp signal. This paper is written to show that the TEO and DESA algorithm can be used for detecting occurrences of the representative waveform distortions and determining their instantaneous information of amplitude and frequency.

Keywords: Event detection, Teager Energy Operator (TEO), Waveform distortions, Harmonics & Inter-harmonics, Frequency variation

1. Introduction

Many advanced techniques, which are widely used in the signal processing or automatic control theories, such as Fast Fourier transform (FFT), Short-time Fourier transform (STFT), Wavelet Transform (WT), Hilbert Transform (HT), Gabor-Wigner Transform (GWT) and Kalman Filter (KF) and so on, can be applied to detection and classification areas of power quality disturbances. Reference [1] provides a useful and general review on various methodologies used for the power quality analysis and the event classification. However, the calculation burdens will cause a relatively slow response, since most of them are implemented in the computer based processors.

A non-linear operator, Teager Energy Operator(TEO), can provide the fastest time response enabling to be implemented in the real-time and on-line measurement of PQ event detection [2]. The TEO was initially developed for a speech production modeling by H.M. Teager [3]. Some of previous papers have dealt with the TEO in order to apply to power quality detection and tracking [4-8]. References [4-6] focused on only tracking of voltage fluctuation, because a voltage fluctuation signal can be considered as one of the amplitude modulated signals. Reference [7] considered three different kinds of instantaneous frequency estimation methods such as TEO,

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Prony's method and modified covariance method and compared their performances of detecting instantaneous frequency of voltage sags, swells and harmonics. It used a discrete energy separation algorithm I (DESA I) for AM-FM signal to analyze frequency components. And, Reference [8] dealt with a combined TEO and threshold algorithm for PQ events detection such as impulsive and oscillatory transients, notches and voltage sags. In the study, a simple TEO calculation has been used for detecting instants when the events occur.

This paper is organized as follows. In section II, the mathematical expressions of TEO and DESA are introduced in details. In section III, PQ events of waveform distortions such as harmonics, inter-harmonics and frequency variations that have not dealt in the previous researches will be analyzed through TEO and DESA. And MATLAB simulations for each case will be provided to show an applicability of the TEO and DESA algorithm to determine the waveform distortions. Finally, we conclude the paper.

2. Definition of TEO and DESA

2.1 Teager Energy Operator(TEO)

For an oscillating continuous-time signal, $x(t) = A \cos(\omega_c t + \theta)$, TEO, $\psi[x(t)]$, is defined as

$$\Psi[x(t)] = [\dot{x}(t)]^2 - x(t)\ddot{x}(t) = A^2 \omega_c^2$$
 (1)

which is shown as:

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$$x(t) = A\cos(\omega_c t + \theta)$$

$$\dot{x}(t) = -A\omega_c \sin(\omega_c t + \theta)$$

$$\ddot{x}(t) = -A\omega_c^2 \cos(\omega_c t + \theta)$$

$$\Psi[x(t)] = [\dot{x}(t)]^2 - x(t)\ddot{x}(t) = A^2\omega_c^2$$
(2)

When Eq. (1) is transferred to an equivalent discretetime form by using an approximation of $\dot{x}[n] \approx \frac{x[n] - x[n-1]}{T}$ for small T, it becomes

$$\Psi[x[n]] = ([x[n]]^2 - x[n-1]x[n+1])/T^2$$
 (3)

where T is the sampling period between two adjacent samples, x[i] and x[i+1]. In many cases, an assumption of T=1 is very acceptable, because the digital frequency in radian/sample has the information of T. As a result, we obtain a final discrete form of TEO

$$\Psi[x[n]] = (x[n])^2 - x[n-1]x[n+1] \tag{4}$$

This result gives us a very meaningful significance that the magnitude and frequency components of a timevarying sinusoidal signal can be calculated by only three consecutive sampled data.

2.2 Discrete energy separation algorithm

For tracking of individual amplitude and frequency of a signal, the energy operator $\psi[x[n]]$ needs to be separated into two parts, amplitude (A) and frequency (Ω_c) , by using energy separation algorithm (ESA) [9].

Reference [10] has shown that TEO can approximately estimate the amplitude envelope of amplitude modulated (AM) signals and the instantaneous frequency component of frequency modulated (FM) signals. It can be also applied to AM-FM signals which mean the signals with variable amplitude and variable frequency.

2.2.1 Constant amplitude and constant frequency signals

Consider a signal with constant amplitude and constant frequency which is expressed as

$$x[n] = A\cos(\Omega_c n + \theta) \tag{5}$$

Then, using (4), the discrete time energy operator becomes

$$\Psi[x[n]] = A^2 \cos^2(\Omega_c n + \theta)$$

$$-A^2 \cos(\Omega_c (n+1) + \theta) \cos(\Omega_c (n-1) + \theta)$$
(6)

By using the following relation,

$$\cos(\Omega_c(n+1) + \theta)\cos(\Omega_c(n-1) + \theta)$$

$$= \frac{1}{2} \left[\cos(2\Omega_c n + 2\theta) + \cos(2\Omega_c)\right]$$

$$= \cos^2(\Omega_c n + \theta) - \sin^2(\Omega_c)$$
(7)

We can write (6) as

$$\Psi[x[n]] = A^2 \sin^2(\Omega_c) \tag{8}$$

From (8), the constant amplitude of signal is determined as

$$A = \sqrt{\frac{\Psi[x[n]]}{\sin^2(\Omega_c)}} \tag{9}$$

To obtain the frequency component, we use the backward difference approximation for the first derivative of x(t)

$$y[n] = x[n] - x[n-1]$$

$$= A\cos(\Omega_c n + \theta) - A\cos(\Omega_c (n-1) + \theta)$$

$$= -2A\sin(\frac{\Omega_c}{2})\sin(\Omega_c n + \theta - \frac{\Omega_c}{2})$$
(10)

By the definition of TEO,

$$\Psi[y[n]] = 4A^2 \sin^2\left(\frac{\Omega_c}{2}\right) \sin^2(\Omega_c)$$
 (11)

From (8) and (11),

$$\frac{\Psi[y[n]]}{2\Psi[x[n]]} = 2\sin^2\left(\frac{\Omega_c}{2}\right) = 1 - \cos(\Omega_c)$$
 (12)

Finally, a signal frequency can be found as

$$\Omega_c = \cos^{-1} \left(1 - \frac{\Psi[y[n]]}{2\Psi[x[n]]} \right)$$
 (13)

where Ω_c means the discrete-domain frequency. In order to convert it into the corresponding continuous-domain frequency, it needs to be divided by $2\pi T$ (T: sampling time interval(sec) or a reciprocal of sampling speed(Hz)).

Amplitude and frequency of the analyzed signal can be determined by using (9) and (13), respectively.

2.2.2 Variable amplitude and variable frequency signals

Now, let us consider a signal with variable amplitude and variable frequency which is expressed as An Applicability of Teager Energy Operator and Energy Separation Algorithm for Waveform Distortion Analysis

$$x[n] = A[n]\cos(\Omega_i[n]n + \theta)$$
 (14)

In [8], the formula to estimate the amplitude, A[n], and frequency, $\Omega_i[n]$, of AM-FM signals are introduced, also known as DESA-1 algorithm.

$$A[n] \approx \sqrt{\frac{\Psi[x[n]]}{1 - \left(1 - \frac{\Psi[y[n]] + \Psi[y[n+1]]}{4\Psi[x[n]]}\right)^2}}$$
(15)

$$\Omega_i[n] \approx \cos^{-1} \left(1 - \frac{\Psi[y[n]] + \Psi[y[n+1]]}{4\Psi[x[n]]} \right)$$
(16)

Generally, the RMS magnitudes of voltage and current are calculated every 12 cycles in 60Hz system and their frequency are also estimated every 12 cycles with Fast Fourier Transform or every 8.33 msec by checking the instants of zero-crossings.

As a result, the TEO and DESA method, operated with only three consecutive points, is an information extractor much faster than other traditional methods.

3. TEO and DESA for PQ Analysis

According to previous researches, it is known that DESA of a signal with constant amplitude and constant frequency can be used for the analysis of voltage sag, voltage swell and interruption. Also, DESA of an AM signal with variable amplitude and constant frequency can be used for the voltage fluctuation analysis, and DESA of an AM-FM signal with variable amplitude and variable frequency is used for the analysis of frequency variation and complicated voltage fluctuation in which the system frequency varies [5].

In this paper, the last DESA algorithm of AM-FM signals with variable amplitude and variable frequency will be took into account, because it is very probable that both amplitude and frequency of the supplied voltage signal tend to change slightly within the acceptable range of nominal values.

Only waveform distortions such as harmonics, interharmonics, frequency variation and multiple event with time varying amplitude and frequency will be studied, because other PQ events, e.g. sag, swell, transient, notch and flicker, have already been studied in [4-8].

3.1 TEO and DESA for Harmonics

For a signal containing harmonic distortions, it should be transferred to a form of AM-FM signal.

Owing to the natural characteristic of symmetric sinusoidal wave, it mainly contains odd harmonics. The general form of harmonic distorted signal is expressed as

$$x(t) = \sum_{n=1}^{N} A_n \cos(n\omega_c t)$$
 (17)

where *n* means the order of harmonics $(n = 1, 3, 5, \cdots)$. Using the general mathematical relation shown below

$$\cos^{2}\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\cos\theta_{1}\cos\theta_{2} = \frac{1}{2}\left\{\cos(\theta_{1} - \theta_{2}) + \cos(\theta_{1} + \theta_{2})\right\}$$
(18)

the odd powers of $cos\theta$ are given by

$$\cos^{3}\theta = \cos\theta\cos^{2}\theta = \frac{1}{2}\cos\theta(1+\cos2\theta)$$

$$= \frac{1}{2}(\cos\theta + \cos\theta\cos2\theta)$$

$$= \frac{1}{2}\left\{\cos\theta + \frac{1}{2}(\cos\theta + \cos3\theta)\right\}$$

$$\cos^{5}\theta = \cos^{3}\theta\cos^{2}\theta$$

$$= \frac{1}{4}(3\cos\theta + \cos3\theta)\frac{1}{2}(1+\cos2\theta)$$

$$= \frac{1}{16}(10\cos\theta + 5\cos3\theta + \cos5\theta)$$
(19)

From (18) and (19), it can be easily generalized that any odd power of $cos\theta$ is written as a sum of odd harmonics.

And we can rewrite them as follows

$$\cos 3\theta = 4\cos^{3}\theta - 3\cos\theta = \{2^{2}\cos^{2}\theta - 3\}\cos\theta$$

$$\cos 5\theta = \{2^{4}\cos^{4}\theta - 5\cdot 2^{2}\cos^{2}\theta + 5\}\cos\theta$$

$$\cos 7\theta = \{2^{6}\cos^{6}\theta - 7\cdot 2^{4}\cos^{4}\theta + 7\cdot 2^{3}\cos^{2}\theta - 7\}\cos\theta$$
(20)

Finally, the original signal of (17) becomes

$$x(t) = \left\{ \sum_{n=1}^{N} A_n(t) \right\} \cos(\omega_c t)$$
 (21)

This means that the voltage signal contaminated by harmonics is also interpreted as a special case of AM signals.

Fig. 1 (a) shows a waveform of 220V and 60Hz voltage signal containing 5-th and 7-th harmonics of 7% and 3%, respectively. The harmonic currents had been inserted for only 6 cycle, from 0.05 sec to 0.15 sec. The signal has been acquired or discretized at a rate of 64 samples/cycle, corresponding to 3,840Hz. The values of TEO calculated by Eq. (4) are shown in Fig. 1 (b) and the outputs of amplitude and instantaneous frequency extracted by using DESA-1 algorithm expressed by Eq. (15) and (16) are shown in Figs. 1 (c) and (d), respectively.

As you can see in Fig. 1 (b), TEO values for pure sinusoidal parts are almost constant. They have very minute oscillations which are caused by a truncation error of sampled numbers. If the similar shapes in TEO are

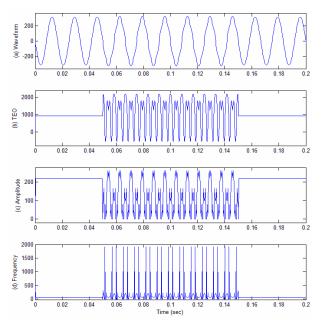


Fig. 1. For sampling rate of 64 Samples/cycle: (a) Signal waveform contaminated by harmonics lasting only for 6 cycles; (b) Calculating results of TEO; (c) Instantaneous amplitude values and (d) Instantaneous frequency values extracted by DESA-1

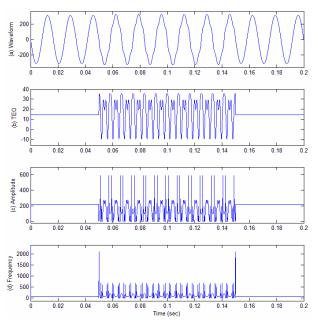


Fig. 2. For sampling rate of 512 Samples/cycle: (a) Signal waveform contaminated by harmonics lasting only for 6 cycles; (b) Calculating results of TEO; (c) Instantaneous amplitude values and (d) Instantaneous frequency values extracted by DESA-1

repeated, then it means that the original waveform has a regularity.

The instant RMS values of voltage shown in Fig. 1 (c) can be obtained by dividing A[n] in Eq. (15) with $\sqrt{2}$.

Since the values of TEO are calculated with three consecutive sampled data with Eq. (14), the TEO can be different according to the sampling rate. The higher the sampling rate is, the smaller the TEO is.

Fig. 2 shows a case of 512 Samples/cycle, that is, 30,720 Hz. From Figs. 1 (b) and Fig. 2 (b), we can come to the conclusion that even though the sampling rates are different, the patterns of TEO can be same.

3.2 TEO and DESA for Inter-harmonics

A signal containing inter-harmonic distortions is also analyzed by the TEO and DESA algorithm.

Unlike the previous harmonic distortions, this case containing inter-harmonic distortions is difficult to generalize its patterns, since the inter-harmonics tend to irregularly distort the fundamental signal.

Fig. 3 (a) shows a waveform of signal containing interharmonic component of 10% at 290Hz.

The values of TEO calculated by Eq. (4) are shown in Fig. 2 (b) and the outputs of amplitude and instantaneous frequency extracted by using DESA-1 algorithm expressed by Eqs. (15) and (16) are shown in Figs. 2 (c) and (d), respectively.

Even though the outputs of TEO and DESA fail to reveal regular patterns in case of inter-harmonics, it is

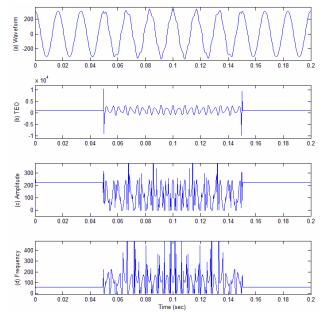


Fig. 3. For sampling rate of 64 Samples/cycle: (a) Signal waveform contaminated by inter-harmonics lasting only for 6 cycles; (b) Calculating results of TEO; (c) Instantaneous amplitude values and (d) Instantaneous frequency values extracted by DESA-1

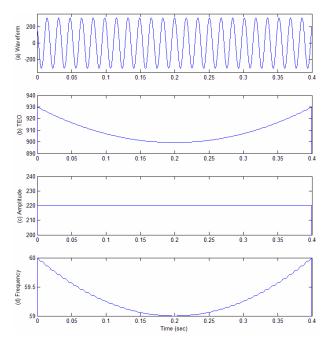


Fig. 4. For sampling rate of 64 Samples/cycle: (a) Signal waveform containing the frequency change between 60Hz and 59Hz; (b) Calculating results of TEO; (c) Instantaneous amplitude values and (d) Instantaneous frequency values extracted by DESA-1

obvious that they can give us a good indication of the event occurrence.

3.3 TEO and DESA for frequency variation

Thirdly, a signal containing frequency variation, also recognized as a slight chirp signal, is analyzed by the TEO and DESA algorithm.

An initial frequency of the test signal is 60Hz and it decreases to 59Hz for 0.2 sec and then recovers to 60Hz for next 0.2 sec, without the change of amplitude.

Fig. 4 (a) shows a waveform of signal containing frequency variation between 60Hz and 59Hz. The values of TEO calculated by Eq. (4) are shown in Fig. 4 (b) and the outputs of amplitude and instantaneous frequency extracted by using DESA-1 algorithm expressed by Eqs. (15) and (16) are shown in Figs. 4 (c) and (d), respectively. In case of the increasing frequency variation from 60Hz to 61Hz, the shape of TEO will be reversed to a convex type.

Fig. 5 is intended to explain the reason. A part of (a) in Fig. 5 exaggerates higher frequency variation and (b) is for lower frequency case. When the sampling interval is sufficiently small, the instantaneous slope of x[n] can be considered linear. A difference between x[n-I] and x[n+I] for higher frequency is greater than that of lower frequency, that is, $\Delta x_a > \Delta x_b$.

$$\begin{cases} x_a[n+1] = x_a[n] + \Delta x_a \\ x_a[n-1] = x_a[n] - \Delta x_a \end{cases}$$
 (22)

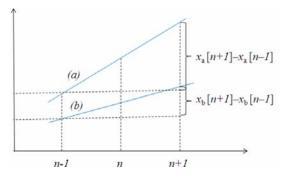


Fig. 5. Graphical approach of the TEO estimation for (a) increasing and (b) decreasing frequency variations

$$\begin{cases} x_b[n+1] = x_b[n] + \Delta x_b \\ x_b[n-1] = x_b[n] - \Delta x_b \end{cases}$$
 (23)

From (22) and (23), we calculate their TEO's.

$$\Psi[x_a[n]] = (x_a[n])^2 - x_a[n-1]x_a[n+1]
= (x_a[n])^2 - (x_a[n] - \Delta x_a)(x_a[n] + \Delta x_a)$$

$$= (\Delta x_a)^2
\Psi[x_b[n]] = (\Delta x_b)^2$$
(25)

As a result, the TEO of lower frequency, $\psi[x_b[n]]$, is less than that of higher frequency, $\psi[x_a[n]]$. This makes a shape of the TEO output concave, as shown in Fig. 4 (b).

3.4 TEO and DESA for Amplitude and Frequency variations

Lastly, a signal containing multiple events of both amplitude and frequency variations is analyzed by the TEO and DESA algorithm.

This is a complicated signal, containing voltage fluctuation and frequency variation, as shown in Fig. 6 (a).

In this study, we have considered a sinusoidal voltage fluctuation, of which magnitude is 10% and frequency is 5Hz, with a slight change in the system frequency between 59Hz and 60Hz for 0.4 sec. The voltage fluctuation can be easily recognized from the signal envelope.

Fig. 6 (a) shows a waveform of the complicated signal. The values of TEO calculated by Eq. (4) are shown in Fig. 6 (b) and the outputs of amplitude and instantaneous frequency extracted by using DESA-1 algorithm expressed by Eqs. (15) and (16) are shown in Figs. 6 (c) and (d), respectively. As known from the simulations, the TEO and DESA algorithm proposed in this paper shows a good tracking performance in real-time amplitude and frequency estimations.

4. Conclusion

This paper is written to show an applicability of the

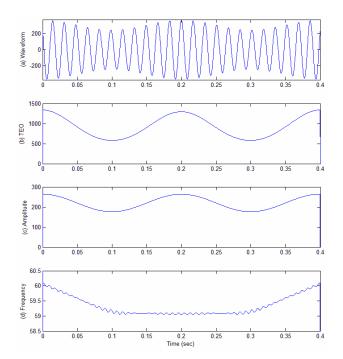


Fig. 6. For sampling rate of 64 Samples/cycle: (a) Signal waveform containing both voltage fluctuation and frequency variation; (b) Calculating results of TEO; (c) Instantaneous amplitude values and (d) Instantaneous frequency values extracted by DESA-1

Teager Energy Operator(TEO), to detecting and determining waveform distortions such as harmonics, inter-harmonics and frequency variations. The TEO, which contains information of both amplitude and frequency, can give us a good intuition for the occurrence of events. So, the DESA should be used to obtain each detailed information.

Under the smart grid circumstances, we will have more chances to experience various PQ disturbances than under the centralized power distribution system. Since TEO and DESA are very efficient and fast algorithms to detect PQ events and can be easily implemented, it will help to improve the quality of power supplied by switching on the voltage compensators as quickly as possible. Combined with other detection techniques such as Kalman filter, Hilbert transform and Wavelet transform, the TEO and DESA algorithm can be widely and efficiently used for many applications related to the power system control and the practical areas.

Based on this research result, the authors will prepare a comparative study on PQ event detection performances with other methods such as general method, half-cycle RMS method and Kalman filter, in the near future.

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