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A Phase-shifter for Regulating Circulating Power Flow in a Parallel-feeding AC Traction Power System

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Abstract – A parallel-feeding AC traction power system increases the power supply capacity and decreases voltage fluctuations, but the circulating power flow caused by the phase difference between the traction substations prevents the system from being widely used. A circuit analysis shows that the circulating power flow increases almost linearly as the phase difference increases, which adds extra load to the system and results in increased power dissipation and load unbalance. In this paper, we suggest a phase shifter for the parallel-feeding AC traction power system. The phase shifter regulates the phase difference and the circulating power flow by injecting quadrature voltage which can be obtained directly from the Scott-connection transformer in the traction substation. A case study involving the phase shifter applied to the traction power system of a Korean high-speed rail system shows that a three-level phase shifter can prevent circulating power flow while the phase difference between substations increases up to 12 degrees, mitigate the load unbalance, and reduce power dissipation.

Keywords: AC traction power system, Parallel-feeding, Circulating power, Phase shifter

1. Introduction

AC traction power systems operating in single-end fed mode have an insulation section between the traction substations and feed the track section from the substation to the insulation section separately. The insulation section acts as a barrier because trains should pass through the insulation section without a power supply. A double-end fed, or parallel-feeding, traction power system has no insulation section between the traction substations, and both end substations feed the entire track section concurrently. This increases the power supply capacity and reduces the voltage fluctuations caused by train load variations [1].

A parallel-feeding AC traction power system is not widely used because of the circulating power flowing from the phase-leading substation to the phase-lagging substation. The circulating power flow generally increases as the phase difference increases between the substations and adds extra load to the traction power system facility. This is why a parallel-feeding AC traction power supply is allowed under specific operation conditions so the phase difference is small enough to limit the circulating power flow. Few studies have been made on the circulating power flow in the parallel-feeding traction power system, and an extensive study is needed for wide implementation of the system.

Recently, flexible AC transmission system (FACTS) technologies have been introduced in the AC traction power system to improve the power quality and balance the load [2-3]. Shunt compensating devices such as the static VAR compensator (SVC) and static synchronous compensator (STATCOM) have been investigated to reduce voltage fluctuation and to improve the power factor [4-5]. A STATCOM has been installed at a Korean rolling stock depot to improve the power factor and to decrease harmonics [6]. A unified power flow controller (UPFC) has been studied to provide unified reactive power compensation and balance the power load [7]. A railway power conditioner, a dedicated UPFC for the railway system, has been installed in a Japanese railroad system [8-9]. The circulating power flow of the parallel-feeding traction power system can be controlled by the UPFC, but it consists of high-cost inverters, capacitors, and controllers that may limit broad application in a railway system.

We suggest a phase shifter to reduce the circulating power flow in the parallel-feeding AC traction power system and keep the substation loads in balance. The phase shifter regulates the feeding voltage phase by injecting quadrature voltage in series with the feeding voltage. Several types of phase shifters have been introduced to utility power systems. A phase-shifting transformer was introduced to control the power flow through the power system interconnection corridors [10]. Thyristor-controlled phase-angle regulators have been introduced to increase the power system stability margins [11-12]. These three-phase phase-shifting transformers are costly because they require a regulating transformer, a series transformer and switching devices, which can be complex [13]. The proposed phase shifter does not have a regulating transformer and obtains

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directly the quadrature voltage for phase compensation from the Scott-connection transformer in a traction substation.

This paper provides a preliminary study of a phase-shifter application for a parallel-feeding AC traction power system. After the system is introduced, the circulating power flow in the parallel-feeding traction power system is analyzed in section 2. The phase shifter for the AC traction power system is suggested in section 3. A study of the phase shifter applied to the traction power system of a Korean high-speed rail system is provided in section 4, and the paper concludes in section 5.

2. Circulating Power Flow

The circulating power flow in a parallel-feeding traction power system is analyzed, which depends on not only the phase difference between the traction substations but also the train load and location.

2.1 AC traction power system

Fig. 1 shows the configuration of a traction power system fed by an automatic transformer (AT) [14]. The system consists of traction substations, ATs, and an overhead catenary system. The traction substation has a Scott-connected transformer that converts three-phase 154 kV to double-phase 55 kV. ATs with a turn ratio of 1:1 are installed every 10 km along the track, and the substation feeding voltage is twice the train voltage. The catenary system consists of a contact wire and a feeder. Train current flows through the contact wire, the rail, and the feeder.

A sectioning post (SP) is installed halfway between the substations to separate the power supply section. The traction substations feed their own sections from the substation up to the SP. This is called a single-end fed traction power system, which is common in Korean railway systems. In the parallel-feeding system, the insulation section in the SP is directly connected, and the substations feed the entire power supply section concurrently. This removes the risk of a train passing through the insulation section without powering, increases the power supply capacity, and reduces voltage fluctuations and peak load demand.

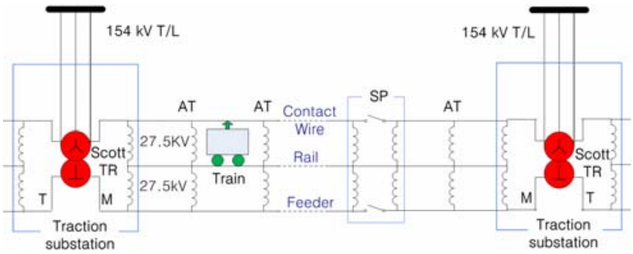


Fig. 1. Configuration of the AC traction power system

Table 1. Recommendation for parallel-feeding operation

Phase difference between substations	Possibility of parallel feeding
0–3°	Applicable
3–6°	Applicable but not recommended
6–10°	Countermeasures required
More than 10°	Not applicable

The AC traction power system should be parallel-fed when the following conditions are met. First, the main transformers in both traction substations have the same voltage, impedance, and turn ratio specifications. Second, the phase angle difference between the traction substation voltages should be small enough to suppress the circulating power flow from one substation to the other. Table 1 shows the recommendations for parallel-feeding in Korean railway systems according to the phase differences.

2.2 Analysis of circulating power flow

Fig. 2 shows an equivalent circuit of the parallel-feeding AC traction power system. $\mathbf{V}_1$ and $\mathbf{V}_2$ are the feeding voltages of traction substations 1 and 2, respectively. Let's assume the $\boldsymbol{V}_1$ phase leads by δ against $\boldsymbol{V}_2$ phase which is set as the reference. $\mathbf{I}_1$ and $\mathbf{I}_2$ are the substation currents. $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ are the impedance of the power supply sections from substation 1 to the train and that of the power supply section from the train to substation 2, respectively. If the power traction system is lossless, $\boldsymbol{Z}_1 = jX_1$ and $\boldsymbol{Z}_2 = jX_2$. $\boldsymbol{Z}_\mathrm{T}$ represents the train load whose power factor is $\cos\alpha_T$:

$$\mathbf{Z}_T = Z_T \varepsilon^{j\alpha_T} = R_T + jX_T \tag{1}$$

When there is no train load, substation power is easily calculated as

$$P_1 = -P_2 = \frac{V_1 V_2}{X_1 + X_2}\sin\delta . \tag{2}$$

If there is a phase difference δ between the substations, then the effective power of phase-lagging substation P_2 becomes negative, which indicates circulating power is flowing into the phase-lagging substation. The magnitude of the circulating power is directly proportional to $\sin\delta$, and inversely proportional to the traction power system

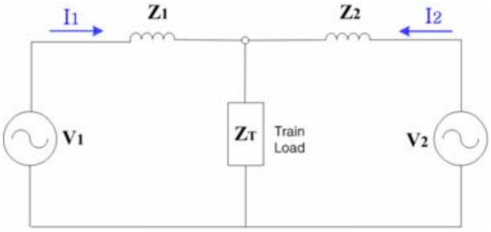


Fig. 2. An equivalent circuit of a parallel-feeding AC traction power system

reactance.

When there is a train in the power supply section, the substation currents change as follows:

$$\boldsymbol{I}_1 = \frac{V_1\boldsymbol{Z}_2 + (V_1 - V_2)\boldsymbol{Z}_T}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + (\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\boldsymbol{Z}_T} \tag{3}$$

$$\boldsymbol{I}_2 = \frac{V_2\boldsymbol{Z}_1 - (V_1 - V_2)\boldsymbol{Z}_T}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + (\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\boldsymbol{Z}_T}. \tag{4}$$

Then, the power supplied from the phase-lagging substation 2 is

$$\begin{aligned} V_2\boldsymbol{I}_2 &= P_2 - jQ_2 \\ &= \frac{V_2^2\boldsymbol{Z}_1 - (V_1V_2 - V_2^2)\boldsymbol{Z}_T}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + (\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\boldsymbol{Z}_T}. \end{aligned} \tag{5}$$

If $\boldsymbol{Z}_1 = jX_1$, $\boldsymbol{Z}_2 = jX_2$, and $\boldsymbol{Z}_T = R_T + jX_T$, (5) can be rewritten as

$$\begin{aligned} P_2 - jQ_2 &= \frac{-jX_1V_2 + (V_1\cos\delta - V_2 + jV_1\sin\delta)(R_T + jX_T)}{-X_1X_2 + j(X_1 + X_2)(R_T + jX_T)} \\ &= V_2\frac{(R_TV_1\cos\delta - X_TV_1\sin\delta - R_TV_2)}{-X_1X_2 - (X_1 + X_2)X_T + j(X_1 + X_2)R_T} \\ &\quad + V_2\frac{j(R_TV_1\sin\delta + X_TV_1\cos\delta - X_1V_2 - X_TV_2)}{-X_1X_2 - (X_1 + X_2)X_T + j(X_1 + X_2)R_T}. \end{aligned} \tag{6}$$

The effective power of substation 2 is

$$\begin{aligned} P_2 &= V_1V_2\frac{R_TX_1^2V_2/V_1 - R_TX_1X_2\cos\delta}{(X_1X_2 + X_T(X_1 + X_2))^2 + R_T^2(X_1 + X_2)^2} \\ &\quad + V_1V_2\frac{(X_1X_2X_T + (X_1 + X_2)Z_T^2)\sin\delta}{(X_1X_2 + X_T(X_1 + X_2))^2 + R_T^2(X_1 + X_2)^2} \\ &= \frac{V_1V_2}{X}\left(R_TX_1^2V_2/V_1 - \sqrt{X}\sin(\delta - \varphi)\right), \end{aligned} \tag{7}$$

Where

$$X = (X_1^2X_2^2 + (X_1 + X_2)X_T)^2 + R_T^2(X_1 + X_2)^2 \tag{8}$$

$$\varphi = \sin^{-1}\frac{R_TX_1X_2}{X}. \tag{9}$$

The effective power of the phase-lagging substation P_2 depends on the train impedance and the train location as well as the phase difference. Let's introduce a parameter k representing the train location. k increases linearly as the train moves to the phase-lagging substation from the phase-leading substation:

$$k = \frac{X_1}{X_1 + X_2}. \tag{10}$$

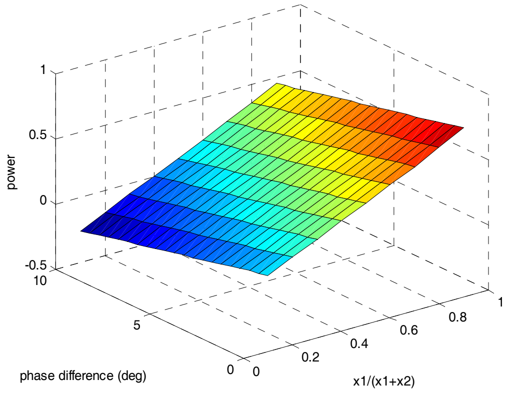


Fig. 3. The normalized effective power of the phase-lagging substation

Train impedance is also normalized by dividing by the total system reactance as follows:

$$\hat{Z}_T = \frac{Z_T}{X_1 + X_2} \tag{11}$$

$$\hat{R}_T = \frac{R_T}{X_1 + X_2} \tag{12}$$

$$\hat{X}_T = \frac{X_T}{X_1 + X_2}. \tag{13}$$

Then, the effective power of the phase-lagging substation P_2 can be normalized as

$$\hat{P}_2 = \frac{\hat{Z}_T^{\,2}}{k(1-k)}\left(\frac{k}{1-k}\frac{V_2}{V_1}\frac{\cos\alpha_T}{Y} - \frac{\sin(\delta - \varphi)}{\sqrt{Y}}\right), \tag{14}$$

Where

$$Y = \left(1 + \frac{\hat{X}_T}{k(1-k)}\right)^2 + \left(\frac{\hat{R}_T}{k(1-k)}\right)^2 \tag{15}$$

$$\varphi = \sin^{-1}\frac{\cos\alpha_T}{\sqrt{(1 + \hat{X}_T/k(1-k))^2 + (\hat{R}_T/k(1-k))^2}}. \tag{16}$$

Fig. 3 shows the normalized effective power $\hat{P}_2$ as a function of the phase difference δ and the train location parameter k. It is assumed that $V_1 = V_2$, $\cos\alpha_T = 1.0$, and $\hat{Z}_T = 1.0$. The circulating power flow increases almost linearly as the phase difference increases and the train moves to the phase-leading substation.

3. A Phase Shifter for Regulating the Circulating Power Flow

Phase shifting principles are briefly discussed, and a dedicated phase shifter for regulating the circulating power

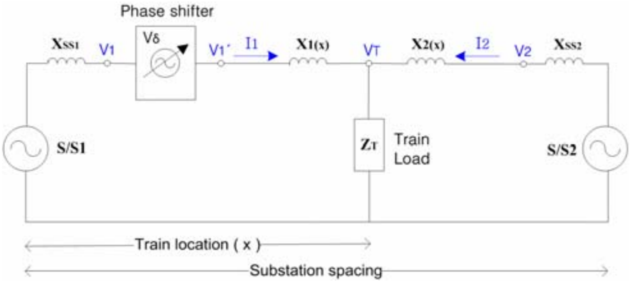
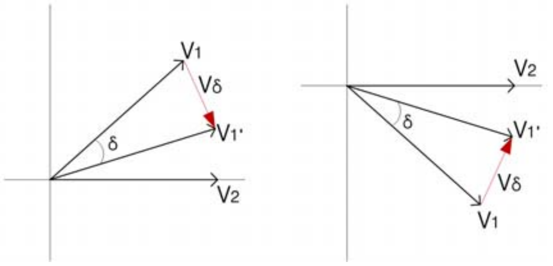


Fig. 4. A phase shifter installed in the parallel-feeding AC traction power system



(a) Phase-lagging (b) Phase-leading

Fig. 5. Phaser diagrams of the phase shifting

flow in a parallel-feeding AC traction power system is suggested.

3.1 Phase-shifting principles

Fig. 4 shows a phase shifter installed at the parallel-feeding AC traction power system. The phase angle of the feeding voltage can be shifted by inserting voltage in series with the feeding voltage. For the largest possible phase angle shift, the injected voltage is usually in quadrature to the feeding voltage. Fig. 5 indicates the relationship between the amount of quadrature voltage added and the resulting phase shift. The magnitude of the series-injected quadrature voltage for phase shifting δ is

$$V_\delta = V_1 \tan\delta \,. \qquad (17)$$

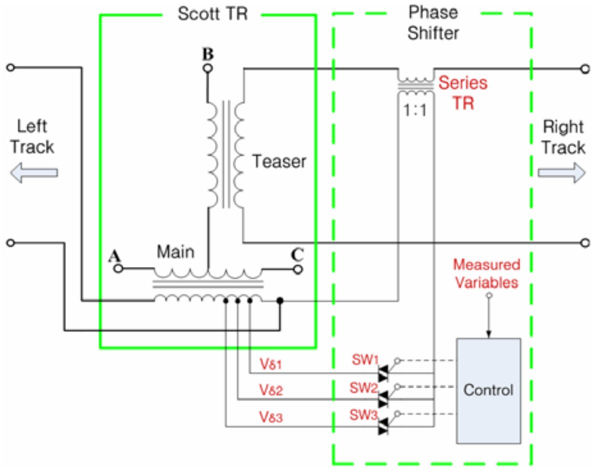
Modification of the phase difference from δ to δ' by applying the quadrature voltage changes the phase-lagging substation power as

$$P_2^{'} = -\frac{V_1 V_2}{\sqrt{X}} \sin(\delta' - \varphi) + \frac{R_T X_1^{\;2}}{X} V_2^{\;2} \,, \qquad (18)$$

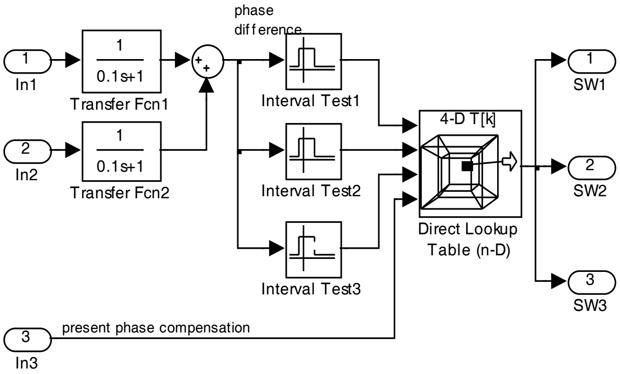
where X and φ are given in (8) and (9), respectively.

3.2 A dedicated phase shifter

A phase shifter is simply a device that injects quadrature voltage in series with the feeding voltage. We suggest a phase shifter for regulating the circulating power flow in the AC traction power system using the quadrature output of the Scott transformer in traction substations. Fig. 6 shows the configuration of the proposed phase shifter for the AC traction power system. The Scott transformer has two voltage outputs in quadrature to each other. This quadrature voltage output is used as a series-injected voltage source for regulating the phase angle. The series transformer is designed to inject the quadrature voltage into the power line in series. The injection of the quadrature voltage is controlled by the switching devices such as a thyristor switch.



(a) Configuration



(b) Control block diagram

Fig. 6. The proposed phase shifter

The magnitude of the series quadrature voltage is determined by the amount of the required phase shifting and the feeding voltage, which is 55 kV in the AT feeding system. For application of the phase shifter to the phase difference between traction substations up to 12 degrees, three levels of phase shifting, including 3, 6, and 9 degrees, are selected. The series-injected quadrature voltages for the three-level phase shifting are calculated with (17), and are shown in Table 2. Fig. 6(b) shows the control block diagram of the controller. After measuring the phase difference between the traction substations, the phase compensation level is determined. The switch to turn-on is

Table 2. Phase-shifting levels of the phase shifter

Phase shifting	Series injecting voltage	Applicable scope of phase difference
3°	2.88kV	1.5°–4.5°
6°	5.78kV	4.5°–7.5°
9°	8.71kV	7.5°–12.0°

selected by comparing the desired compensation level with the present phase compensation level.

4. Application Study

After the circulating power flow in a parallel-feeding AC traction power system is modeled, a decrease in the circulating power due to the proposed phase shifter is evaluated.

4.1 A simulated case

Fig. 7 shows a simulation model of the parallel-feeding AC traction power system for the Korean high-speed rail system. The substations are 50 km apart, and ATs are installed every 10 km along the railway track. The specifications of the Scott transformer, the utility power system, and the transmission line are shown in Tables 3 and Table 4. The impedances per unit kilometer of the contact wire, feeder, and rail are shown in Table 5.

The traction substations and the utility power system are modeled as a voltage source with impedance. The train is modeled as a constant power load as the train has an inverter-controlled traction system that controls the active/reactive power [15]. The maximum load and the power factor of the high-speed train are 15 MVA and 0.98, respectively.

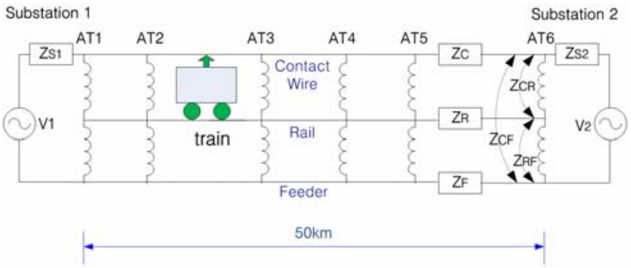


Fig. 7. A simulation model of the parallel-feeding traction power system

Table 3. Transformer specifications

Transformer	Capacity	Transform. ratio	Impedance
Scott	45 MVA	154/55 kV	0.584+j13.4318 Ω
AT	10 MVA	55/27.5 kV	0.0287+j0.44912 Ω

Table 4. Utility power system impedances

Substation	Component	Impedance
1	Utility system	0.289+j1.615 Ω
	154 KV T/L	0.0348+j0.1612 Ω/km
2	Utility system	0.063+j0.945 Ω
	154 KV T/L	0.3351+j2.0059 Ω/km

Table 5. Catenary system impedances

Substation	Component	Impedance
Self	Contact wire	0.1379+j0.3436 Ω/km
	Rail	0.0568+j0.1668 Ω/km
	Feeder	0.1637+j0.4822 Ω/km
Mutual	Contact wire to rail	0.0587+j0.3788 Ω/km
	Contact wire to feeder	0.0582+j0.4699 Ω/km
	Rail to feeder	0.0586+j0.3646 Ω/km

4.2 Simulation of circulating power flow

The circulating power flow in the parallel-feeding traction power system is solved using a circuit analysis, provided in the Appendix. Two types of train load conditions are investigated: a light load condition when a train runs along the entire traction section and a heavy load condition when five trains run in each AT section.

Fig. 8 shows the substation powers in the light load condition. The power flow of the phase-lagging substation decreases almost linearly as the phase difference increases, and the circulating power begins to flow when the phase difference is 4 degrees. The total train load is constant at all phase differences, but the transmission loss increases with the phase differences owing to the circulating power flow.

Fig. 9 shows the substation powers in the heavy load conditions. Circulating power does not flow until the phase difference increases to 12 degrees. This is because the phase-lagging substation supplies part of the train load, which offsets the circulating power flow. However, the load unbalance between the substations increases with the phase difference. The phase-leading substation supplies more train load than the phase-lagging substation does. The load unbalance factor, defined as the ratio of the phase-leading substation power to the phase-lagging substation power, increases to 3.0 when the phase difference is 12 degrees. The transmission loss, or power dissipation, also increases as the phase difference increases because of the increased load unbalance between stations.

4.3 Application of the phase shifter

The power flow between the substations is analyzed again after the phase shifter is installed at the phase-leading substation. The phase shifter has three-level phase shifting, including 3, 6, and 9 degrees, as shown in Table 2.

Fig. 10 shows the substation powers in the light load condition with a phase shifter installed at the phase-leading substation. The phase-lagging substation power comes close to the phase-leading substation power owing to the phase shifting, and the circulating power flow does not appear until the phase difference increases by 12 degrees. The transmission loss also decreases as the circulating

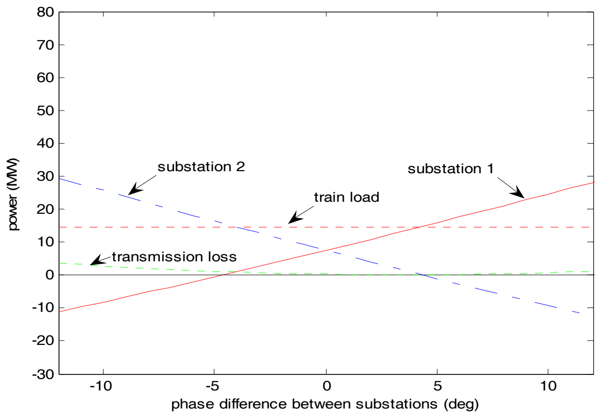


Fig. 8. Substation powers in the light load condition

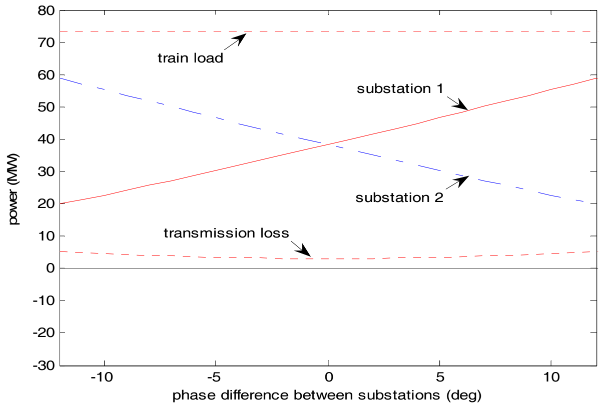


Fig. 9. Substation powers in the heavy load condition

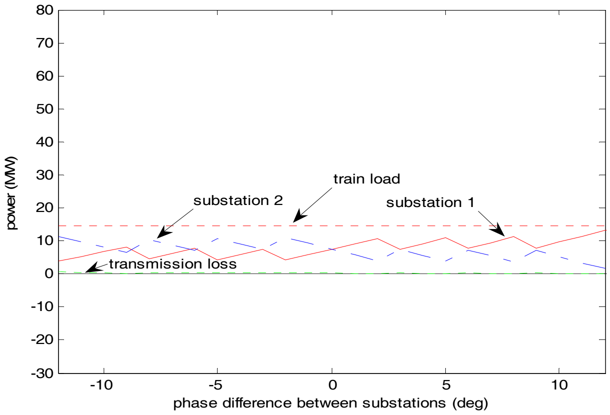


Fig. 10. Substation powers in the light load condition (with a phase shifter installed)

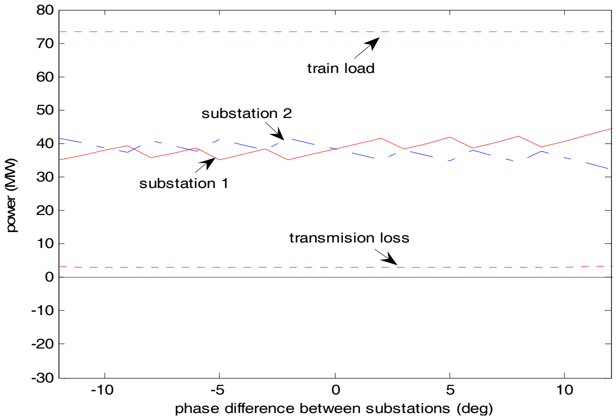


Fig. 11. Substation powers in the heavy load condition (with a phase shifter installed)

power does not flow. The transmission loss at the phase difference of 12 degrees decreases to 0.34 MW, which would be 1.43 MW without the phase shifter.

Fig. 11 shows the substation powers in the heavy load condition with a phase shifter installed at the phase-leading substation. The phase-lagging substation power comes close to the phase-leading substation power, which results in decreased load unbalance between the substations. The load unbalance factor decreases to 1.2 when the phase difference is 12 degrees, which would be 3.0 without the phase shifter. In addition, the transmission loss decreases to 3.2 MW when the phase difference is 12 degrees. Without the phase shifter, the loss would be 5.0 MW.

5. Conclusion

The parallel-feeding traction power system has many advantages, but is not widely used because of the circulating power flow. It is proposed to apply a phase shifter, for the first time, to regulate the circulating power flow and keep both the substation loads in balance. Modeling and simulation studies revealed the following findings about the phase shifter and its application to the parallel-feeding traction power system:

An elaborated circuit analysis deduced the characteristics of the circulating power flow in the parallel-feeding traction power system fed by auto transformers (ATs). The circulating power flow increases almost linearly as the phase difference between substations increases. The circulating power adds extra load to the phase-leading substation and increases power dissipation. The train load may offset the circulating power flow, but the load unbalance between the substations and the power dissipation still increases as the phase difference increases.

A dedicated phase shifter for the parallel-feeding traction power system is devised. The phase shifter has a simple configuration compared to the conventional phase shifters which have a high-power regulating transformer to generate quadrature voltage which is injected in series with the feeding voltage for phase shifting. The phase shifter obtains quadrature voltage directly from the Scott transformer connection in the traction power system, and does not have a high-cost regulating transformer.

The simulation study indicates the phase shifter is very effective for wide implementation of the parallel-feeding traction power system. The phase shifter having three levels of phase shifting including 3, 6, and 9 degrees ensures successful implementation of the parallel-feeding traction power system for the Korean high-speed train where the measured phase difference between substations are lower than 12 degrees. Simulation results show that applying the three levels of phase shifting can prevent circulating power flow, reduce power dissipation, and mitigate the load unbalance between substations while the phase difference between substations increases by 12 degrees.

Acknowledgements

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Appendix

Circulating Power Flow Calculation

The circulating power flow in a parallel-feeding traction power system is calculated from a network analysis based on the mesh current method. Fig. A1 shows a network analysis model of the parallel-feeding AC traction power system. V_{S1}, V_{S2} are substation voltages, and Z_{S1}, Z_{S2} are power system impedances. AT is modeled as a dependent voltage source with impedance. Z_{AT}. Z_C, Z_R, and Z_F are impedances of the contact wire, rail, and feeder, respectively. Z_{CR}, Z_{RF}, and Z_{CF} are mutual impedances among the contact wire, rail, and feeder, respectively. Z_{T1} and Z_{T2} are train impedances.

The power flow can be calculated using the mesh current method. The loop equations for mesh currents i_1 - i_6 are

$$V_{S1} = (Z_{S!} + 2Z_{AT})i_1 - Z_{AT}i_2 - Z_{AT}i_5 + 2V_1 \qquad \text{(A1)}$$

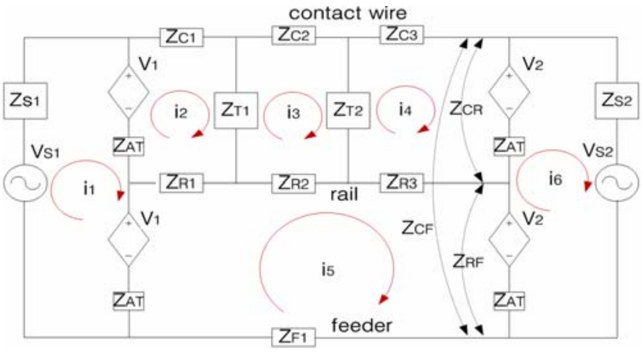

Fig. A1. A network analysis model for the parallel-feeding AC traction power system

$$V_1 = -Z_{AT}i_1 + (Z_{C!} + Z_{R1} - 2Z_{CR1} + Z_{T1} + Z_{AT})i_2 - Z_{AT}i_3 - (Z_{R1} - Z_{RF1} + Z_{CF1})i_5 \quad \text{(A2)}$$

$$0 = -Z_{AT}i_2 + (Z_{C2} + Z_{R2} - 2Z_{CR2} + Z_{T1} + Z_{T2})i_3 - Z_{T2}i_4 - (Z_{R2} - Z_{RF2} + Z_{CF2})i_5 \quad \text{(A3)}$$

$$0 = -Z_{T2}i_3 + (Z_{C3} + Z_{R3} - 2Z_{CR3} + Z_{T2} + Z_{AT})i_4 - Z_{AT}i_6 - (Z_{R3} - Z_{RF3} + Z_{CF3})i_5 \quad \text{(A4)}$$

$$V_1 = -Z_{AT}i_1 - (Z_{R!} - Z_{RF1} + Z_{CF1})i_2 - (Z_{R2} - Z_{RF2} + Z_{CF2})i_3 - (Z_{R3} - Z_{RF3} + Z_{CF3})i_4 + (Z_{R!} + Z_{R2} + Z_{R3} + Z_F - 2Z_{RF1} + 2Z_{AT})i_5 - Z_{AT}i_6 \quad \text{(A5)}$$

$$2V_2 = (Z_{S2} + 2Z_{AT})i_6 - Z_{AT}i_4 - Z_{AT}i_5 + 2V_{S2} \quad \text{(A6)}$$

AT voltages V_1 and V_2 are dependent variables, and two equations are needed. The two equations can be deduced from the AT circuit characteristics that the same current flows through the up and down sides of AT in the opposite direction as follows:

$$i_1 - i_5 = i_2 - i_1 \quad \text{(A7)}$$

$$i_5 - i_6 = i_6 - i_4 \quad \text{(A8)}$$

Solving the eight equations (A1)-(A8), the mesh currents and the substation powers can be calculated. When the trains have an inverter-controlled traction system working as a constant power load, the train impedance should be adjusted following the train voltage fluctuation to maintain the specified train power. This is an optimization problem to minimize the errors of the train power calculation. The multi-objective goal attain approach provided by MATLAB showed good convergence results within four iterations.

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