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POLARIZATION AND UNCONDITIONAL CONSTANTS OF $\mathcal{P}(^2d_*(1,w)^2)$

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Abstract. We explicitly calculate the polarization and unconditional constants of $\mathcal{P}(^2d_*(1,w)^2)$.

1. Introduction

According to the Krein-Milman Theorem, every nonempty convex set in a Banach space is fully described by the set of its extreme points. We recall that if C is a convex set in a Banach space, a point $e \in C$ is said to be extreme if $x, y \in C$ and $e = \lambda x + (1-\lambda)y$ for some $0 < \lambda < 1$ implies that $x = y = e$. Let $n \in \mathbb{N}$. We write B_E for the closed unit ball of a real Banach space E. We denote by $extB_E$ the sets of all the extreme points of B_E. We denote by $\mathcal{L}(^nE)$ the Banach space of all continuous n-linear forms on E endowed with the norm $\|T\| = \sup_{\|x_k\|=1} |T(x_1,\ldots,x_n)|$. A n-linear form T is symmetric if $T(x_1,\ldots,x_n) = T(x_{\sigma(1)},\ldots,x_{\sigma(n)})$ for every permutation σ on $\{1,2,\ldots,n\}$. We denote by $\mathcal{L}_s(^nE)$ the Banach space of all continuous symmetric n-linear forms on E. A mapping $P : E \to \mathbb{R}$ is a continuous n-homogeneous polynomial if there exists a unique $T \in \mathcal{L}_s(^nE)$ such that $P(x) = T(x,\ldots,x)$ for every $x \in E$. In this case it is convenient to write $T = \check{P}$. We denote by $\mathcal{P}(^nE)$ the Banach space of all continuous n-homogeneous polynomials from E into $\mathbb{R}$ endowed with the norm $\|P\| = \sup_{\|x\|=1} |P(x)|$. It is well-known that

$$\|P\| \le \|\check{P}\| \le \frac{n^n}{n!}\|P\| \ (\forall P \in \mathcal{P}(^nE)).$$

For more details about the theory of multilinear mappings and polynomials on a Banach space, we refer to [7].

We will denote by $T((x_1,y_1),\ (x_2,y_2)) = ax_1x_2 + by_1y_2 + c(x_1y_2 + x_2y_1)$ and $P(x,y) = ax^2 + by^2 + cxy$ a symmetric bilinear form and a 2-homogeneous

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polynomial on a real Banach space of dimension 2, respectively. We denote the predual of two dimensional real Lorentz sequence space with a positive weight $0 < w < 1$ by

$$d_*(1,w)^2 := \{(x,y) \in \mathbb{R}^2 : \|(x,y)\|_{d_*} := \max\{|x|,\ |y|,\ \frac{|x|+|y|}{1+w}\}\}.$$

In [22] *the nth polarization constant* of E is defined by

$$c_{\text{pol}}(n : E) = \inf\{M > 0 : \|\check{P}\| \le M\|P\| \text{ for every } P \in \mathcal{P}(^nE)\}.$$

Let X^α denote the monomial $x_1^{\alpha_1} \cdots x_m^{\alpha_m}$, where $X = (x_1, \ldots, x_m) \in \mathbb{R}^m$ and $\alpha = (\alpha_1, \ldots, \alpha_m)$ with $\alpha_k \in \mathbb{N} \cup \{0\}$, $1 \le k \le m$. If $P(X) = \sum_{|\alpha| \le n} a_\alpha X^\alpha$ is a polynomial of degree n on $\mathbb{R}^m$, we define its modulus $|P|$ by $|P|(X) = \sum_{|\alpha| \le n} |a_\alpha| X^\alpha$. We define the nth unconditional constant of $d_*(1,w)^2$ by

$$c_{\text{unc}}(n : d_*(1,w)^2) = \inf\{M > 0 : \||P|\| \le M\|P\| \text{ for every } P \in \mathcal{P}(^nd_*(1,w)^2)\}.$$

Gamez-Merino et al. [9] classify the extreme points of the unit ball of $\mathcal{P}(^2\square)$ and, using its extreme points, compute the polarization and unconditional constants of $\mathcal{P}(^2\square)$, where $\square$ is the unit square of vertices $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$. The author [14] characterized the extreme points of the unit ball of $\mathcal{P}(^2d_*(1,w)^2)$. Recently, the author [16] calculated the norm of symmetric bilinear form of $\mathcal{L}_s(^2d_*(1,w)^2)$ and classified the extreme points of the unit ball of $\mathcal{L}_s(^2d_*(1,w)^2)$. We refer to ([1–6], [8–22]) and references therein for some recent work about extremal properties of multilinear mappings and homogeneous polynomials on some classical Banach spaces. By the Krein-Milman Theorem, a convex function (like a polynomial norm, for instance) defined on a convex set (like the unit ball of a finite dimensional polynomial space) attains its maximum at one extreme point of the convex set. In this paper, using the results of [14] and [16] with the Krein-Milman Theorem, we explicitly calculate $c_{\text{pol}}(2 : d_*(1,w)^2)$ and $c_{\text{unc}}(2 : d_*(1,w)^2)$ as follows:

(a) If $w \le \sqrt{2} - 1$, then $c_{\text{pol}}(2 : d_*(1,w)^2) = \frac{2(1+w^2)}{(1+w)^2}$;
(b) If $w > \sqrt{2} - 1$, then $c_{\text{pol}}(2 : d_*(1,w)^2) = 1 + w^2$;
(c) If $w \le \sqrt{2} - 1$, then $c_{\text{unc}}(2 : d_*(1,w)^2) = \frac{1+w^2+\sqrt{2(1+w^4)}}{(1+w)^2}$;
(d) If $w > \sqrt{2} - 1$, then $c_{\text{unc}}(2 : d_*(1,w)^2) = \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}$.

2. The results

Theorem 2.1 ([16])**.** *Let* $T((x_1,y_1),\ (x_2,y_2)) := (a,b,c,c) \in \mathcal{L}_s(^2d_*(1,w)^2)$ *with* $|b| \le a,\ c \ge 0$. *Then*:
Case $1 : b \ge 0$
Subcase $1 : c > a$
If $w \le \frac{c-a}{c-b}$, *then* $\|T\| = (a+b)w + c(1+w^2)$.
If $w > \frac{c-a}{c-b}$, *then* $\|T\| = bw^2 + 2cw + a$.
Subcase $2 :$ *If* $c \le a$, $\|T\| = bw^2 + 2cw + a$.

Case $2: b<0$
Subcase $1: c<|b|$
If $w \le \frac{c}{|b|}$, *then* $\|T\| = \max\{bw^2+2cw+a,\ (a-b)w+c(1-w^2)\}$.
If $w > \frac{c}{|b|}$, *then* $\|T\| = \max\{a-bw^2,\ (a-b)w+c(1-w^2)\}$.
Subcase $2: c \ge |b|$
If $w \le \frac{|b|}{c}$, *then* $\|T\| = \max\{bw^2+2cw+a,\ (a-b)w+c(1-w^2)\}$.
If $w > \frac{|b|}{c}$, *then* $\|T\| = \max\{bw^2+2cw+a,\ (a+b)w+c(1+w^2)\}$.

Theorem 2.2 ([14]). *Let* $P \in \mathcal{P}(^2d_*(1,w)^2)$ *with* $P(x,y) = ax^2+by^2+cxy$ *for* $(x,y) \in d_*(1,w)^2$ *with* $a \ge |b| \ge 0,\ c \ge 0$. *Then*
Case $1: 0 \le c < 2|b|$
Subcase $1: b<0$
(a) *If* $\frac{c}{2|b|} \le w$, *then*

$$\|P\| = a + \frac{c^2}{4|b|}.$$

(b) *If* $\frac{c}{2|b|} > w$, *then*

$$\|P\| = bw^2+cw+a.$$

Subcase 2 : *If* $b>0$, *then*

$$\|P\| = bw^2+cw+a.$$

Case 2 : *If* $2|b| \le c \le 2a$, *then*

$$\|P\| = bw^2+cw+a.$$

Case $3: 2a<c$
(a) *If* $\frac{c-2a}{c-2b} < w$, *then*

$$\|P\| = bw^2+cw+a.$$

(b) *If* $\frac{c-2a}{c-2b} \ge w$, *then*

$$\|P\| = \frac{(c^2-4ab)(1+w)^2}{4(c-a-b)}.$$

Theorem 2.3 ([14]).

$$\begin{aligned} & extB_{\mathcal{P}(^2d_*(1,w)^2)} \\ &= \{\pm x^2,\ \pm y^2,\ \pm\frac{1}{1+w^2}(x^2+y^2),\ \pm\frac{1}{(1+w)^2}(x\pm y)^2 \\ &\quad \pm[t(x^2-y^2)\pm 2\sqrt{t(1-t)}xy]\ (\frac{1}{1+w^2} \le t \le 1), \\ &\quad \pm[t(x^2-y^2)\pm \frac{2+2\sqrt{1-t^2(1+w)^4}}{(1+w)^2}xy]\ (0 \le t \le \frac{1-w}{(1+w)(1+w^2)})\}. \end{aligned}$$

Form now on we will use the following notations for the extreme points of $B_{\mathcal{P}(^2 d_*(1,w)^2)}$:

$$P_t(x,y) = \pm[t(x^2-y^2) \pm 2\sqrt{t(1-t)}xy] \ (\frac{1}{1+w^2} \leq t \leq 1),$$
$$Q_s(x,y) = \pm[s(x^2-y^2) \pm \frac{2+2\sqrt{1-s^2(1+w)^4}}{(1+w)^2}xy] \ (0 \leq s \leq \frac{1-w}{(1+w)(1+w^2)}),$$
$$R_1(x,y) = \pm\frac{1}{(1+w)^2}(x \pm y)^2,$$
$$R_2(x,y) = \pm\frac{1}{1+w^2}(x^2+y^2),$$
$$R_3(x,y) = \pm x^2,$$
$$R_4(x,y) = \pm y^2.$$

Notice that if $0 < w < 1$ and $w^* = \frac{1-w}{1+w}$, then $0 < w^* < 1$ and $(w^*)^* = w$.

Lemma 2.4. *Let $w^* = \frac{1-w}{1+w}$. Then, there is an isometry $\phi : d_*(1,w) \to d_*(1,w^*)$ such that*
$$\phi(x,y) := \left(\frac{x+y}{1+w}, \frac{x-y}{1+w}\right).$$

Proof. By definition, the norms of $(x,y) \in d_*(1,w)$ and $(X,Y) \in d_*(1,w^*)$ are given by
$$\|(x,y)\|_{d_*(1,w)} = \max\left\{|x|, |y|, \frac{|x|+|y|}{1+w}\right\},$$
$$\|(X,Y)\|_{d_*(1,w^*)} = \max\left\{|X|, |Y|, \frac{|X|+|Y|}{1+w^*}\right\}.$$

Now, let $(X,Y) = \phi(x,y) = \left(\frac{x+y}{1+w}, \frac{x-y}{1+w}\right)$. Then
$$\begin{aligned}\|(X,Y)\|_{d_*(1,w^*)} &= \max\left\{\left|\frac{x+y}{1+w}\right|, \left|\frac{x-y}{1+w}\right|, \left(\frac{\left|\frac{x+y}{1+w}\right| + \left|\frac{x-y}{1+w}\right|}{1+w^*}\right)\right\} \\ &= \max\left\{\frac{|x|+|y|}{1+w}, \frac{|x+y|+|x-y|}{2}\right\} \\ &= \max\left\{\frac{|x|+|y|}{1+w}, \max\{|x|,|y|\}\right\} \\ &= \|(x,y)\|_{d_*(1,w)}.\end{aligned}$$
□

Lemma 2.5. *Let $0 < w < 1$, $w^* = \frac{1-w}{1+w}$. Define $\Phi : \mathcal{P}(^2 d_*(1,w)^2) \to \mathcal{P}(^2 d_*(1,w^*)^2)$ by $\Phi(P) = P \circ \phi^{-1}$, where ϕ is the isometry in Lemma 2.4. Then Φ is an isometrically isomorphism. Moreover, $P \in extB_{\mathcal{P}(^2 d_*(1,w)^2)}$ if and only if $\Phi(P) \in extB_{\mathcal{P}(^2 d_*(1,w^*)^2)}$.*

Remark. Note that, for $\frac{1}{1+w^2} \le t \le 1$,

$$\begin{aligned}\Phi(P_t)(X,Y) &= \frac{2\sqrt{t(1-t)}}{(1+w^*)^2}(X^2-Y^2) \pm (\frac{2}{1+w^*})^2 XY \\ &= Q_s(X,Y) \text{ for a unique } 0 \le s \le \frac{1-w^*}{(1+w^*)(1+(w^*)^2)},\end{aligned}$$

where $w^* = \frac{1-w}{1+w}$ and $X = \frac{x+y}{1+w}$, $Y = \frac{x-y}{1+w}$.

Theorem 2.6. (a) *If* $w \le \sqrt{2}-1$, *then* $c_{pol}(2 : d_*(1,w)^2) = \frac{2(1+w^2)}{(1+w)^2}$;
(b) *If* $w > \sqrt{2}-1$, *then* $c_{\rm pol}(2 : d_*(1,w)^2) = 1 + w^2$.

Proof. By the Krein-Milman Theorem,

$$\begin{aligned}c_{\rm pol}(2 : d_*(1,w)^2) = \max\{&\|\check{P}_t\|, \|\check{Q}_s\|, \|\check{R}_k\| : \frac{1}{1+w^2} \le t \le 1, \\ &0 \le s \le \frac{1-w}{(1+w)(1+w^2)},\ k = 1,2,3,4\}.\end{aligned}$$

Note that $\|\check{R}_k\| = 1$ for $k = 1,2,3,4$. We claim that $\max\{\|\check{P}_t\| : \frac{1}{1+w^2} \le t \le 1\} = 1+w^2$. Note that $\check{P}_t((x_1,y_1),(x_2,y_2)) = tx_1x_2 - ty_1y_2 \pm \sqrt{t(1-t)}(x_1y_1 + x_2y_1)$ for $\frac{1}{1+w^2} \le t \le 1$. Simple calculation shows that, for $\frac{1}{1+w^2} \le t \le 1$,

$$t \le \frac{(1+w)^2}{2(1+w^2)} \Leftrightarrow 2wt + \sqrt{t(1-t)}(1-w^2) \ge (1+w^2)t.$$

Let $g(t) = 2wt + \sqrt{t(1-t)}(1-w^2)$ for $\frac{1}{1+w^2} \le t \le 1$. Then $g'(t) = 0$ for $\frac{1}{1+w^2} \le t \le 1$ implies that $t = \frac{(1+w)^2}{2(1+w^2)}$.

Case 1: $w \ge \sqrt{2}-1$

Obviously, $\frac{1}{1+w^2} \le \frac{(1+w)^2}{2(1+w^2)} < 1$ and $\frac{(1+w)^2}{2} < \frac{3w-w^3}{1+w^2} < 1+w^2$. It follows that, by Theorem 2.1 (Case 2, Subcase 1),

$$\begin{aligned}&\max\{\|\check{P}_t\| : \frac{1}{1+w^2} \le t \le 1\} \\ =\ &\max\{\max\{\|\check{P}_t\| : \frac{1}{1+w^2} \le t \le \frac{(1+w)^2}{2(1+w^2)}\}, \max\{\|\check{P}_t\| : \frac{(1+w)^2}{2(1+w^2)} \le t \le 1\}\} \\ =\ &\max\{\max\{(1+w^2)t, 2wt + \sqrt{t(1-t)}(1-w^2) : \frac{1}{1+w^2} \le t \le \frac{(1+w)^2}{2(1+w^2)}\}, \\ &\max\{(1+w^2)t, 2wt + \sqrt{t(1-t)}(1-w^2) : \frac{(1+w)^2}{2(1+w^2)} \le t \le 1\}\} \\ =\ &\max\{\max\{2wt + \sqrt{t(1-t)}(1-w^2) : \frac{1}{1+w^2} \le t \le \frac{(1+w)^2}{2(1+w^2)}\}, \\ &\max\{(1+w^2)t : \frac{(1+w)^2}{2(1+w^2)} \le t \le 1\}\}\end{aligned}$$

$$= \max\{\max\{g(\frac{1}{1+w^2}), g(\frac{(1+w)^2}{2(1+w^2)})\}, 1+w^2\}$$
$$= \max\{\frac{3w-w^3}{1+w^2}, \frac{(1+w)^2}{2}, 1+w^2\}$$
$$= 1+w^2.$$

Case 2: $w < \sqrt{2}-1$
Obviously, $\frac{(1+w)^2}{2(1+w^2)} < \frac{1}{1+w^2}$. Since $\frac{1}{1+w^2} \le t \le 1$,

$$\max\{(1+w^2)t, 2wt + \sqrt{t(1-t)}(1-w^2) : \frac{1}{1+w^2} \le t \le 1\}$$
$$= \max\{(1+w^2)t : \frac{1}{1+w^2} \le t \le 1\} = 1+w^2.$$

It follows that by Theorem 2.1 (Case 2, Subcase 1),

$$\max\{\|\check{P}_t\| : \frac{1}{1+w^2} \le t \le 1\}$$
$$= \max\{(1+w^2)t, 2wt + \sqrt{t(1-t)}(1-w^2) : \frac{1}{1+w^2} \le t \le 1\}$$
$$= \max\{(1+w^2)t : \frac{1}{1+w^2} \le t \le 1\}$$
$$= 1+w^2.$$

We claim that $\max\{\|\check{Q}_s\| : 0 \le s \le \frac{1-w}{(1+w)(1+w^2)}\} = \frac{2(1+w^2)}{(1+w)^2}$.

By Lemmas 2.4–5, $Q_s(x,y) = P_t(X,Y)$ for some $\frac{1}{1+(w*)^2} \le t \le 1$, where $w^* = \frac{1-w}{1+w}$ and $X = \frac{x+y}{1+w}, Y = \frac{x-y}{1+w}$. By the above claim, it follows that

$$\max\{\|\check{Q}_s\| : 0 \le s \le \frac{1-w}{(1+w)(1+w^2)}\}$$
$$= \max\{\|\check{P}_t((X_1,Y_1),(X_2,Y_2))\| : \frac{1}{1+(w*)^2} \le t \le 1\} = 1+(w^*)^2$$
$$= \frac{2(1+w^2)}{(1+w)^2}.$$

Therefore,

$$c_{\text{pol}}(2 : d_*(1,w)^2) = \max\{1+w^2, \frac{2(1+w^2)}{(1+w)^2}\}.$$

Since $\frac{2(1+w^2)}{(1+w)^2} \ge 1+w^2 \Leftrightarrow w \le \sqrt{2}-1$, we complete the proof. □

Theorem 2.7. (a) *If $w \le \sqrt{2}-1$, then $c_{unc}(2 : d_*(1,w)^2) = \frac{1+w^2+\sqrt{2(1+w^4)}}{(1+w)^2}$;*

(b) *If $w > \sqrt{2}-1$, then $c_{unc}(2 : d_*(1,w)^2) = \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}$.*

Proof. By the Krein-Milman Theorem,

$$c_{\text{unc}}(2 : d_*(1,w)^2) = \max\{|||P_t|||, |||Q_s|||, |||R_k||| : \frac{1}{1+w^2} \le t \le 1, \\ 0 \le s \le \frac{1-w}{(1+w)(1+w^2)},\ k = 1,2,3,4\}.$$

Note that $|||R_k||| = 1$ for $k = 1,2,3,4$. We claim that $\max\{|||P_t||| : \frac{1}{1+w^2} \le t \le 1\} = \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}$. Note that $|P_t|((x_1,y_1),(x_2,y_2)) = tx^2 + ty^2 \pm \sqrt{t(1-t)}xy$ for $\frac{1}{1+w^2} \le t \le 1$. Let $f(t) = (1+w^2)t + 2w\sqrt{t(1-t)}$ for $\frac{1}{1+w^2} \le t \le 1$. Then $0 = f^{'}(t)$ for $\frac{1}{1+w^2} \le t \le 1$ implies that $t = \frac{1}{2} + \frac{1+w^2}{2\sqrt{(1+w^2)^2+4w^2}}$. Clearly, $\frac{1}{1+w^2} < \frac{1}{2} + \frac{1+w^2}{2\sqrt{(1+w^2)^2+4w^2}} < 1$. It follows that, by Theorem 2.2 (Case 1, Subcase 2),

$$\begin{aligned} |||P_t||| &= \max\{(1+w^2)t + 2w\sqrt{t(1-t)} : \frac{1}{1+w^2} \le t \le 1\} \\ &= \max\{f(\frac{1}{1+w^2}), f(1), f(\frac{1}{2} + \frac{1+w^2}{2\sqrt{(1+w^2)^2+4w^2}})\} \\ &= \max\{\frac{1+3w^2}{1+w^2}, 1+w^2, \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}\} \\ &= \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}. \end{aligned}$$

By Lemmas 2.4–5, $Q_s(x,y) = P_t(X,Y)$ for some $\frac{1}{1+(w*)^2} \le t \le 1$, where $w^* = \frac{1-w}{1+w}$ and $X = \frac{x+y}{1+w}, Y = \frac{x-y}{1+w}$. By the above claim, it follows that

$$\begin{aligned} &\max\{|||Q_s||| : 0 \le s \le \frac{1-w}{(1+w)(1+w^2)}\} \\ = &\max\{|||P_t(X,Y)||| : \frac{1}{1+(w*)^2} \le t \le 1\} = 1 + (w^*)^2 \\ = &\frac{1+(w^*)^2 + \sqrt{(1+(w^*)^2)^2 + 4(w^*)^2}}{2} \\ = &\frac{1+w^2+\sqrt{2(1+w^4)}}{(1+w)^2}. \end{aligned}$$

Therefore,

$$c_{\text{unc}}(2 : d_*(1,w)^2) = \max\{\frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2}, \frac{1+w^2+\sqrt{2(1+w^4)}}{(1+w)^2}\}.$$

Since $\frac{1+w^2+\sqrt{2(1+w^4)}}{(1+w)^2} \ge \frac{1+w^2+\sqrt{(1+w^2)^2+4w^2}}{2} \Leftrightarrow w \le \sqrt{2}-1$, we complete the proof. □

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