Paper
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Adaptive nonsingular sliding mode based guidance law with terminal angular constraint

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Abstract

In this paper, a new adaptive nonsingular terminal sliding mode control theory based impact angle guidance law for intercepting maneuvering targets was documented. In the design procedure, a new adaptive law for target acceleration bound estimation was presented, which allowed the proposed guidance law to be used without the requirement of the information on the target maneuvering profiles. With the aid of Lyapunov stability criteria, the finite-time convergent characteristics of the line-of-sight angle and its derivative were proven in theory. Numerical simulations were also performed under various conditions to demonstrate the effectiveness of the proposed guidance law.

Key words: Missile guidance, Sliding mode control, Impact angle, Maneuvering target

1. Introduction

Proportional navigation guidance (PNG) law and its variants have been widely used for tactical missiles in the past few decades due to their efficient and easy implementation [1-3]. For non-maneuverable or weakly maneuverable targets, the classical PNG law is proven to be optimal for minimization of both the miss distance and energy consumption [4]. However, as the maneuverability of a target increases, the performance of PNG becomes worse, and lacks robustness [2]. For the interception of agile targets, many effective guidance laws have been reported in the literature. One study proposed a novel H_∞ nonlinear guidance law, which does not require the information about the target acceleration [5]. However, the implementation of this guidance law requires the associated Hamilton-Jacobi partial differential inequality to be solved. By regarding target acceleration as a completely unknown but bounded disturbance, a disturbance attenuation L_2 index was formulated in [6] to derive a robust guidance law based on nonlinear missile-target relative dynamics. In [7], by solving the linear matrix inequalities from a pole placement problem, the authors examined a Lyapubov-based nonlinear guidance law, where the state of the guidance system was proven to converge into a compact set. However, although the convergent rate could be tuned by appropriate pole selections, only asymptotic stability was demonstrated for both the non-maneuvering and constantly maneuvering targets. The authors in [8] utilized the receding horizon control method to obtain a general guidance law, demonstrating the possibility for online optimization of the guidance law by using a differential flatness concept to decrease dimensional space and b-spline curves to approximate the flat output. In [9], the authors researched a robust proportional navigation guidance (RPNG) based on the sliding mode control (SMC) theory for intercepting maneuvering targets, in which a first order autopilot lag was also considered. In [10], the authors adopted integral sliding mode control (ISMC) theory, which does not require any reaching phase, to derive an augmented 3D true PNG law for highly maneuverable targets.

For modern application, guidance law with terminal impact angle constraint has been extensively studied by many researchers to increase the effectiveness of the warhead carried by the missiles. The authors in [11] proposed a suboptimal guidance law for reentry

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vehicles with impact angle constraint to intercept non-maneuvering targets, which seems to be the first attempt to address the problem in this area. For stationary targets, a polynomial impact angle guidance law was proposed in [12], which was derived from the solution of a linear quadratic optimal control problem with the energy cost weighted by the power of time-to-go. The authors in [13] extended the results in [12] to enhance target observability and seeker's field-of-view limit. In [14], the authors derived a generalized impact angle guidance law, which consists of the position and velocity error feedback terms. In [15], a novel sliding mode control based impact angle guidance law was presented for maneuvering targets. With the aid of the state-dependent-riccati-equation (SDRE) technique, the authors in [16] derived a new guidance law to satisfy the terminal angle constraint. In [17], the authors used the newly-developed Model Predictive Static Programming (MPSP) technique to derive a suboptimal 3D impact angle guidance law. A novel circular navigation guidance (CNG) impact angle guidance law was developed in [18], which directs the missile to follow a pre-designated circular arc to the target. For large impact angles, the authors in [19] designed a new bias-PNG law along with a novel time-to-go estimation algorithm.

This paper considers the application of nonsingular terminal sliding mode (NTSM) control [20] to guidance law design with a terminal angle constraint for intercepting maneuvering targets. Compared with the traditional terminal sliding mode (TSM)-based guidance law, the proposed method does not exhibit any singularity problem in the control signal. By virtue of the new adaptive law, no information on the target maneuvering profile is required. Using Lyapunov stability criteria, we proved that both the line-of-sight (LOS) angle and its derivative can converge to their corresponding desired value in finite time. Since the flight time is usually very short during the terminal guidance phase, the finite-time convergent characteristic is crucial for precise interception. Because of the principle of NTSM, the proposed guidance law has satisfactory performance of compensation for target maneuvering. Compared with some other existing impact angle guidance laws, the proposed method does not require the estimation of time-to-go, which plays an important role in most of the optimal impact angle guidance laws mentioned above. These aspects set this work apart from the existing literature.

This paper was organized as follows. Sec. 2 provides the nonlinear homing engagement geometry. The main results are then presented in Sec. 3, while numerical simulations are given in Sec. 4. Finally, some conclusions are offered.

2. Problem Formulation

The planar homing engagement geometry between the missile and the target is depicted in Fig. 1, where the subscripts M and T denote the missile and the target; γ_M and γ_T denote the flight path angle of the missile and the target; λ and r denote the LOS angle and the missile-target relative range; a_M and a_T denote acceleration of the missile and the target; and V_M and V_T denote the velocity of the missile and target, respectively. Acceleration of both the missile and target were assumed to be perpendicular to their own velocities. The corresponding engagement dynamic equations were formulated as follows:

$$\dot{r} = V_T \cos\left(\gamma_T - \lambda\right) - V_M \cos\left(\gamma_M - \lambda\right) \tag{1}$$

$$\dot{\lambda} = \left[V_T \sin\left(\gamma_T - \lambda\right) - V_M \sin\left(\gamma_M - \lambda\right)\right] / r \tag{2}$$

$$\dot{\gamma}_M = \frac{a_M}{V_M} \tag{3}$$

$$\dot{\gamma}_T = \frac{a_T}{V_T} \tag{4}$$

For simplifying the design procedure, the following assumptions are needed.

Assumption 1. The velocities of both the missile and the target are constant.

Assumption 2. Both the missile and the target are ideal systems, i.e. their corresponding autopilots are sufficiently fast.

Assumption 3. The target velocity and acceleration satisfy the following inequality:

$$V_T < V_M, \quad \left|a_T\right| \le \Delta \tag{5}$$

where $\Delta > 0$ denotes the target acceleration bound.

The impact angle, denoted by θ_{imp}, is defined as:

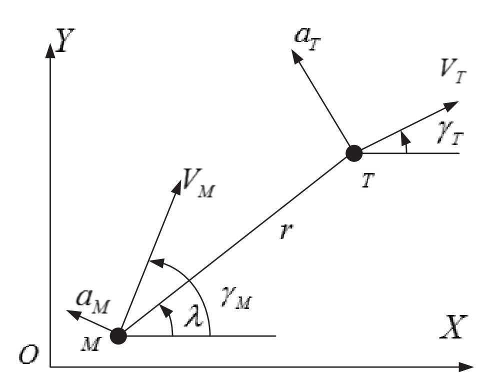

Fig. 1. Homing engagement geometry

$$\theta_{imp} = \gamma_{Tf} - \gamma_{Mf} \tag{6}$$

where γ_{Tf} and γ_{Mf} denote the final flight path angle of the target and the missile, respectively. By accepting the intuition that a zero LOS angular rate will lead to a perfect interception with zero miss distance, Eq. (2), on the collision course [2], can be rewritten as:

$$V_T \sin\left(\gamma_{Tf} - \lambda_f\right) - V_M \sin\left(\gamma_{Mf} - \lambda_f\right) = 0 \tag{7}$$

where λ_f denotes the final LOS angle.

Substituting Eq. (6) into (7) yields:

$$V_T \sin\left(\gamma_{Tf} - \lambda_f\right) - V_M \sin\left(\gamma_{Tf} - \theta_{imp} - \lambda_f\right) = 0 \tag{8}$$

Since $V_T < V_M$, it can easily be deduced that:

$$\lambda_f = \gamma_{Tf} - \tan^{-1}\left(\frac{\sin\theta_{imp}}{\cos\theta_{imp} - \dfrac{V_T}{V_M}}\right) \tag{9}$$

For a pre-designated target, both γ_{Tf} and V_T are unique, hence, the terminal LOS angle and impact angle have a one-to-one correspondence. Without loss of generality, we considered the terminal LOS angle constraint throughout this paper.

3. Guidance Law Design

Differentiating Eq. (2) with respect to time yields the following:

$$\ddot{\lambda} = -\frac{2\dot{r}\dot{\lambda}}{r} + \frac{\cos\left(\gamma_T - \lambda\right)}{r} a_T - \frac{\cos\left(\gamma_M - \lambda\right)}{r} a_M \tag{10}$$

Let e represent the LOS angle error, i.e. $e = \lambda - \lambda_f$. Denote $x_1 = e,\ x_2 = \dot{e}$, then, the LOS angle error dynamic equation can be written as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{r}\left(-2\dot{r}\dot{\lambda} + \cos\left(\gamma_T - \lambda\right) a_T - \cos\left(\gamma_M - \lambda\right) a_M\right) \end{aligned} \tag{11}$$

The NTSM surface is then selected as:

$$s = x_1 + \alpha \left|x_2\right|^{\eta} \operatorname{sgn}\left(x_2\right) \tag{12}$$

where $\alpha > 0$, $1 < \eta = p/q < 2$, p, q are two positive integers, and $\operatorname{sgn}(\cdot)$ denotes the sign function.

The acceleration command is designed as:

$$a_M = \frac{r}{\cos\left(\gamma_M - \lambda\right)}\left[-\frac{2\dot{r}\dot{\lambda}}{r} + \frac{q}{\alpha p}\left|x_2\right|^{2-\frac{p}{q}} \operatorname{sgn}\left(x_2\right) + \frac{\hat{\Delta}\rho \operatorname{sgn}(s)}{r} + \frac{K \operatorname{sgn}(s)}{r}\right] \tag{13}$$

where K>0, $\rho \geq 1$, and $\hat{\Delta}$ denotes the estimation of Δ, which is governed by the following adaptive law:

$$\dot{\hat{\Delta}} = \alpha\rho \frac{p}{qr}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right|, \qquad \hat{\Delta}(0) > 0 \tag{14}$$

Theorem 1. Let $\tilde{\Delta} = \Delta - \hat{\Delta}$ be the estimation error. If the NTSM surface is as (12) and the control law is designed as (13), subject to the adaptive law (14), then the states of system (11) converge to zero in finite time.

Proof. Differentiating Eq. (12) with respect to time yields:

$$\dot{s} = \dot{x}_1 + \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1} \dot{x}_2 \tag{15}$$

Substituting Eqs. (11) and (13) into Eq. (15) gives the following:

$$\begin{aligned} \dot{s} &= x_2 + \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left(\frac{\cos\left(\gamma_T - \lambda\right) a_T}{r} - \frac{q}{\alpha p}\left|x_2\right|^{2-\frac{p}{q}} \operatorname{sgn}\left(x_2\right) - \frac{\hat{\Delta}\rho \operatorname{sgn}(s)}{r} - \frac{K \operatorname{sgn}(s)}{r}\right) \\ &= \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left(\frac{q}{\alpha p}\left|x_2\right|^{2-\frac{p}{q}} \operatorname{sgn}\left(x_2\right) + \frac{\cos\left(\gamma_T - \lambda\right) a_T}{r} - \frac{q}{\alpha p}\left|x_2\right|^{2-\frac{p}{q}} \operatorname{sgn}\left(x_2\right) - \frac{\hat{\Delta}\rho \operatorname{sgn}(s)}{r} - \frac{K \operatorname{sgn}(s)}{r}\right) \\ &= \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left(\frac{\cos\left(\gamma_T - \lambda\right) a_T}{r} - \frac{\hat{\Delta}\rho \operatorname{sgn}(s)}{r} - \frac{K \operatorname{sgn}(s)}{r}\right) \end{aligned} \tag{16}$$

Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\Delta}^2 \tag{17}$$

Differentiating Eq. (17) with respect to time and substituting Eqs. (14) and (16) into the equation gives:

$$\begin{aligned} \dot{V}_1 &= s\dot{s} + \tilde{\Delta}\dot{\tilde{\Delta}} \\ &= s\alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left(\frac{\cos\left(\gamma_T - \lambda\right) a_T}{r} - \frac{\hat{\Delta}\rho \operatorname{sgn}(s)}{r} - \frac{K \operatorname{sgn}(s)}{r}\right) - \tilde{\Delta}\alpha\rho \frac{p}{qr}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right| \\ &\leq \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right|\left(\frac{\left|\cos\left(\gamma_T - \lambda\right) a_T\right|}{r} - \frac{\hat{\Delta}\rho}{r} - \frac{K}{r} - \frac{\tilde{\Delta}\rho}{r}\right) \\ &= \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right|\left(\frac{\left|\cos\left(\gamma_T - \lambda\right) a_T\right|}{r} - \frac{\Delta\rho}{r} - \frac{K}{r}\right) \\ &\leq \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right|\left(\frac{\Delta(1-\rho)}{r} - \frac{K}{r}\right) \end{aligned} \tag{18}$$

Since $\dot{r} < 0$ in the terminal guidance phase, one can imply that:

$$r < r_0 \tag{19}$$

where r_0 denotes the initial relative range between the missile and the target.

Note that 1-ρ<0, we can obtain:

$$\begin{aligned} \dot{V}_1 &\leq \alpha \frac{p}{q}\left|x_2\right|^{\frac{p}{q}-1}\left|s\right|\left(\frac{\Delta(1-\rho)}{r_0} - \frac{K}{r_0}\right) \\ &\leq 0 \end{aligned} \tag{20}$$

which implies that both s and $\tilde{\Delta}$ are ultimately uniformly bounded.

Next, choosing the following Lyapunov function candidate:

$$V_2 = \frac{1}{2}s^2 \tag{21}$$

Differentiating Eq. (21) with respect to time and substituting Eq. (16) into it yields:

$$\begin{aligned}
\dot{V}_2 &= s\dot{s} \\
&= s\alpha\frac{p}{q}|x_2|^{\frac{p}{q}-1}\left(\frac{\cos(\gamma_T-\lambda)a_T}{r} - \frac{\hat{\Delta}\rho\,\mathrm{sgn}(s)}{r} - \frac{K\,\mathrm{sgn}(s)}{r}\right) \\
&\le \alpha\frac{p}{q}|x_2|^{\frac{p}{q}-1}|s|\left(\frac{|\cos(\gamma_T-\lambda)a_T|}{r} - \frac{\hat{\Delta}\rho}{r} - \frac{K}{r}\right) \\
&\le \alpha\frac{p}{q}|x_2|^{\frac{p}{q}-1}|s|\left(\frac{\Delta}{r} - \frac{\hat{\Delta}\rho}{r} - \frac{K}{r}\right)
\end{aligned} \tag{22}$$

From Eq. (14) one can note that $\hat{\Delta}(t) > \hat{\Delta}(0)$, so if a large enough $\hat{\Delta}(0)$ is chosen, i.e. $\hat{\Delta}(0) > |\tilde{\Delta}(0)|$, and

$$\rho \ge 1 + \frac{\sqrt{\hat{\Delta}^2(0)+\varepsilon^2}}{\hat{\Delta}(0)} \tag{23}$$

where ε is an arbitrarily small constant. Then, one can imply that:

$$\begin{aligned}
\Delta - \hat{\Delta}\rho &< \Delta - \hat{\Delta}(0) - \sqrt{\hat{\Delta}^2(0)+\varepsilon^2} \\
&= \tilde{\Delta}(0) - \sqrt{\hat{\Delta}^2(0)+\varepsilon^2} \\
&\le |\tilde{\Delta}(0)| - \sqrt{\hat{\Delta}^2(0)+\varepsilon^2} \\
&\le \hat{\Delta}(0) - \sqrt{\hat{\Delta}^2(0)+\varepsilon^2} \\
&\le 0
\end{aligned} \tag{24}$$

Then, we have:

$$\dot{V}_2 \le -\alpha\frac{p}{q}\frac{K}{r}|x_2|^{\frac{p}{q}-1}|s| \tag{25}$$

Therefore, for the case $x_2 \ne 0$, $V_2 \le 0$, which in turn proves that the trajectories of system (11) can reach the sliding surface (12) in finite time. If x_2=0, substituting Eq. (13) into (11) to yields:

$$\dot{x}_2 = \frac{\cos(\gamma_T-\lambda)a_T}{r} - \frac{\hat{\Delta}\rho\,\mathrm{sgn}(s)}{r} - \frac{K\,\mathrm{sgn}(s)}{r} \tag{26}$$

If s>0, it follows from Eq. (24) that:

$$\begin{aligned}
\dot{x}_2 &= \frac{\cos(\gamma_T-\lambda)a_T}{r} - \frac{\hat{\Delta}\rho}{r} - \frac{K}{r} \\
&\le \frac{\Delta}{r} - \frac{\hat{\Delta}\rho}{r} - \frac{K}{r} \\
&< 0
\end{aligned} \tag{27}$$

If s<0, it also follows from Eq. (24) that:

$$\begin{aligned}
\dot{x}_2 &= \frac{\cos(\gamma_T-\lambda)a_T}{r} + \frac{\hat{\Delta}\rho}{r} + \frac{K}{r} \\
&\ge -\frac{\Delta}{r} + \frac{\hat{\Delta}\rho}{r} + \frac{K}{r} \\
&> 0
\end{aligned} \tag{28}$$

Therefore, $\dot{x}_2 \ne 0$ for all values of $s \ne 0$, which shows that $\dot{x}_2 = 0$ is not an attractor.

In the sliding phase (s=0),

$$x_1 + \alpha|x_2|^{\eta}\,\mathrm{sgn}(x_2) = 0 \tag{29}$$

If $x_2 \ne 0$, it is easily to verify that

$$x_2 = -\gamma|x_1|^{\beta}\,\mathrm{sgn}(x_1) \tag{30}$$

where $\gamma = \alpha^{-\frac{q}{p}} > 0$, $\beta = \frac{q}{p} > 0$.

Finally, considering $V_3 = x_1^2/2$ as a Lyapunov function candidate, evaluating $\dot{V}_3$ along the trajectory (30) gives:

$$\dot{V}_3 = -\gamma|x_1|^{\beta+1} \le 0 \tag{31}$$

Therefore, the states of system (11) can converge to zero along the sliding surface (12) in finite time. This completes the proof.

Remark 1. Due to the existence of a discontinuous sign function in the control law, high frequency chattering will be excited in real application. To address this problem, we chose a continuous saturation function to replace the sign function, as follows:

$$\mathrm{sat}(\delta, s) = \begin{cases} \mathrm{sgn}(s), & |s| > \delta \\ \dfrac{s}{\delta}, & |s| \le \delta \end{cases} \tag{32}$$

Remark 2. Since the boundary layer technique is used in Remark 1 to suppress chattering, the sliding variable s would converge into a boundary layer instead of zero. To avoid the adaptation parameter $\hat{\Delta}$ increase boundlessly, a modified adaptive law is presented as:

$$\dot{\hat{\Delta}} = \begin{cases} \alpha\rho\dfrac{p}{qr}|x_2|^{\frac{p}{q}-1}|s|, & \text{if } |s| > \upsilon \\ 0, & \text{if } |s| < \upsilon \end{cases} \tag{33}$$

where υ is a small positive constant.

Remark 3. Note that the first two terms in equation (13) can be rewritten as follows:

$$\frac{r}{\cos(\gamma_M-\lambda)}\left[-\frac{2\dot{r}\dot{\lambda}}{r} + \frac{q}{\alpha p}|x_2|^{2-\frac{p}{q}}\,\mathrm{sgn}(x_2)\right] = \frac{-\dot{r}\dot{\lambda}}{\cos(\gamma_M-\lambda)}\left(2 - \frac{q}{\alpha p}t_{go}|x_2|^{1-\frac{p}{q}}\right) \tag{34}$$

where $t_{go} = -r/\dot{r}$ denotes the remaining flight time. Thus, the acceleration command (13) can be regarded as a pseudo proportional navigation guidance (PNG) law containing a time-varying navigation ratio $2-\frac{q}{\alpha p}t_{go}|x_2|^{1-\frac{p}{q}}$ and some additional terms.

4. Simulation Results

In this section, the performance of the proposed guidance law was verified via numerical simulations under various conditions. The design parameters in Eq. (13) were selected as: α=1, p=7, q=5, K=1800, υ=0.05, δ=0.01, ρ=2, $\hat{\Delta}(0)$=100. These parameters were observed to provide satisfactory results in our simulations. The initial conditions were selected as: 1) missile velocity: 800m/s; 2) target velocity: 450m/s; 3) missile initial flight path angle: 45°; 4) target initial flight path angle: 180°; 5) missile initial position: (0, 0); 6) target initial position: (20000m, 20000m). In all simulations, the acceleration command was saturated at 20g, where g denotes the gravitational acceleration constant. The following 4 cases were considered in the simulations.

Case 1): the target maneuvering profile was selected as a_T=100sin(t)m/s^2, and the desired terminal LOS angle was 0°;

Case 2): the target maneuvering profile was selected as a_T=100sin(t)m/s^2, and the desired terminal LOS angle was 90°;

Case 3): the target performs sudden maneuvers as in Fig. 2, and the desired terminal LOS angle is 0°;

Case 4): the target performs sudden maneuvers as in Fig. 2, and the desired terminal LOS angle is 90°.

The simulation results for cases 1)-4) are plotted in Figs. 3-5, Figs. 6-8, Figs. 9-11 and Figs. 12-14, respectively. From these figures, it can be seen that interception can be achieved, whatever the desired terminal LOS angles and the target maneuvering profiles are. Clearly, the LOS angle and the LOS angular rate could converge to their

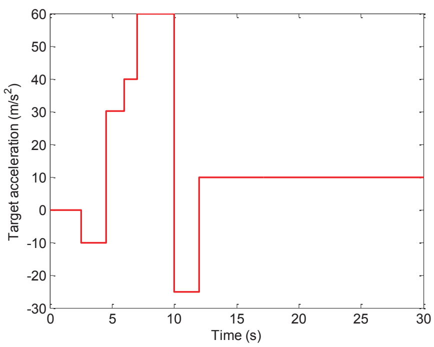

Fig. 2. Target maneuvering profile

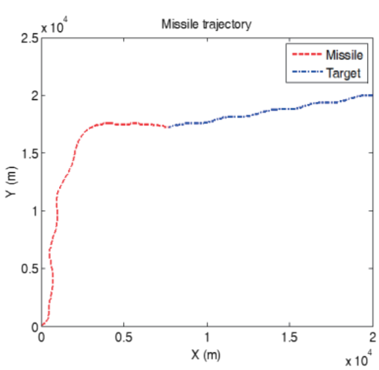

Fig. 3. Missile flight trajectory

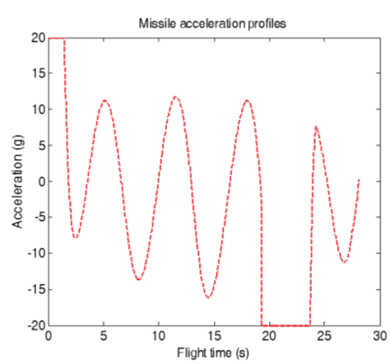

Fig. 4. Acceleration command

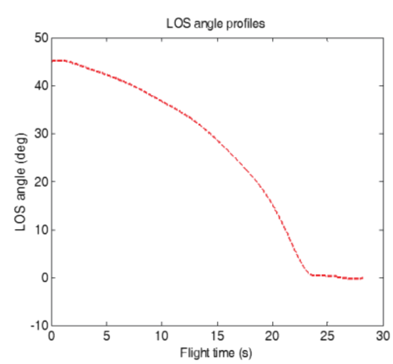

Fig. 5. LOS angle profile

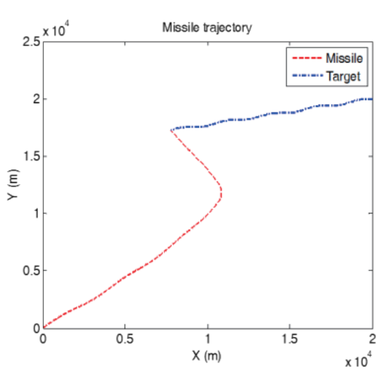

Fig. 6. Missile flight trajectory

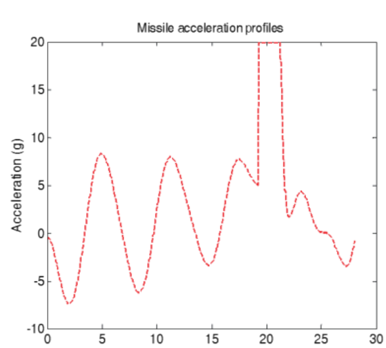

Fig. 7. Acceleration command

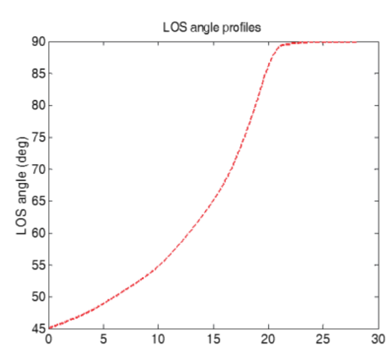

Fig. 8. LOS angle profile

corresponding desired values in finite time, and the design parameters mentioned above can be tuned to regulate the convergence rate. Furthermore, due to the use of boundary layer technique, no chattering occurs in the control channel.

5. Conclusion

This paper presented a new adaptive NTSM based guidance law with terminal angle constraints for maneuvering targets. By virtue of the new adaptive law, no information about the target maneuvering is required for implementation of the proposed guidance law. Using Lyapunov method, the finite-time convergence of the closed loop guidance system was guaranteed. Furthermore, this kind of guidance law can be used for target observability enhancement via the LOS angular rate shaping approach. Future works includ adding other constraints, such as impact time, into the proposed method.

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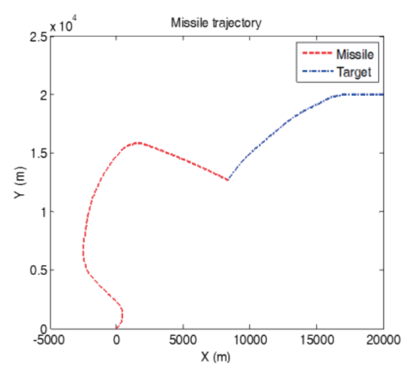

Fig. 9. Missile flight trajectory

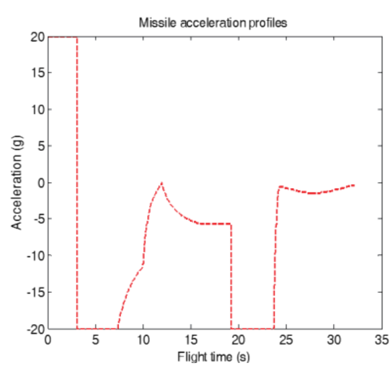

Fig. 10. Acceleration command

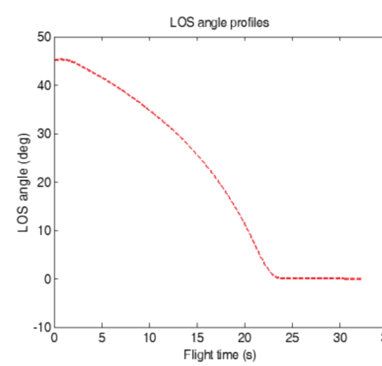

Fig. 11. LOS angle profile

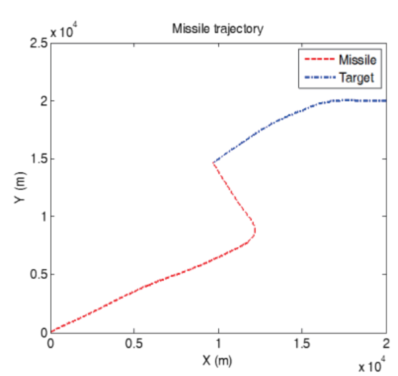

Fig. 12. Missile flight trajectory

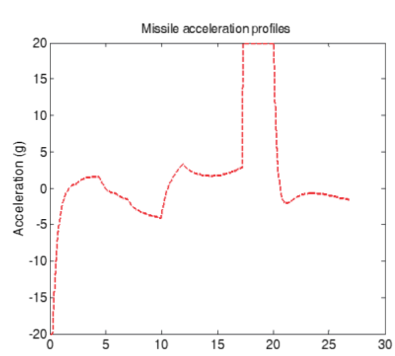

Fig. 13. Acceleration command

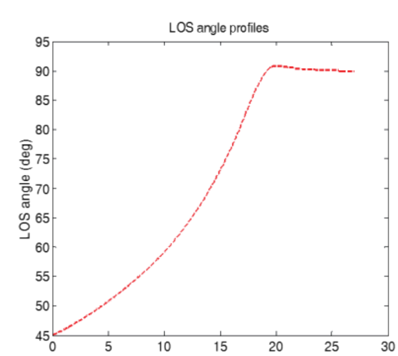

Fig. 14. LOS angle profile

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