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On Soft Topological Space via Semiopen and Semiclosed Soft Sets

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Abstract. This paper introduces semiopen and semiclosed soft sets in soft topological spaces and then these are used to generalize the notions of interior and closure. Further, we study the properties of semiopen soft sets, semiclosed soft sets, semi interior and semi closure of soft set in soft topological spaces. Various forms of soft functions, like semicontinuous, irresolute, semiopen and semiclosed soft functions are introduced and characterized including those of soft semicompactness, soft semiconnectedness. Besides, soft semiseparation axioms are also introduced and studied.

1. Introduction and Preliminaries

Molodtsov [3] introduced soft set in the year 1999 to deal with problems of incomplete information. The concept of soft sets enhanced the application potential of the different generalizations of crisp sets due to the additional advantage of parametrization tools. The notion of topological space for soft sets was formulated by Shabir *et al.* [5] and Çağman *et al.* [1] separately in 2011. But one group studied different aspects of topology like soft interior, soft closure, soft neighborhood of a point, soft limit point of a soft set and soft boundary whereas the other group defined soft topology consisting of a parameterized family of soft topologies on a set and studied soft closure, interior, neighborhood including the separation axioms. Introduction of soft point alongwith the study on interior point, neighborhood system, continuity and compactness is found in [6].

Here are some definitions and results required in the sequel. Throughout this study, U and V denote universal sets, $E, E^{'}$ denote two sets of parameters, and $A, B, C, D, A_i, B_i, D_i, F, B^*, D^* \subseteq E$ or $E^{'}$, where $i \in \mathbb{N}$.

Definition 1.1. [5] Let τ be a collection of soft sets over a universe U with a fixed

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set E of parameters, then $\tau \subseteq SS(U)_E$ is called a soft topology on U with a fixed set E if
(i) Φ_E, U_E belongs to τ;

(ii) the union of any number of soft sets in τ belongs to τ;

(iii) the intersection of any two soft sets in τ belongs to τ.

Then (U_E, τ) is called a soft topological space over U. Members of τ are called open soft sets and their complements are called closed soft sets.

Definition 1.2. [1] A soft basis of a soft topological space (U_E, τ) is a subcollection $\mathcal{B}$ of τ such that every element of τ can be expressed as the union of elements of $\mathcal{B}$.

Definition 1.3. [1] Let (U_E, τ) be a soft topological space and $U_B \mathrel{\tilde{\subseteq}} U_E$ where $B \subseteq E$. Then the collection $\tau_{U_B} = \{U_{A_i} \widetilde{\bigcap} U_B \mid U_{A_i} \in \tau, i \in I \subseteq \mathbb{N}\}$ is called a soft subspace topology on U_B.

Definition 1.4. [6] A soft set $F_A \in SS(U)_E$ is called a soft point in U_E, denoted by e_F, if for the element $e \in A$, $F(e) \neq \Phi$ and $F(e^{'}) = \Phi$ for all $e^{'} \in A - \{e\}$.

Definition 1.5. [6] A soft point e_F in said to be in the soft set G_C, denoted by $e_F \mathrel{\tilde{\in}} G_C$, if for the element $e \in A \cap C$ and $F(e) \subseteq G(e)$.

Definition 1.6. [6] A family Ψ of soft sets is a cover of a soft set F_A if $F_A \mathrel{\tilde{\subseteq}} \widetilde{\bigcup}\{(F_A)_\lambda \mid (F_A)_\lambda \in \Psi, \lambda \in \Lambda\}$.

A subcover of Ψ is a subfamily of Ψ which is also a cover.

Definition 1.7. [6] A family Ψ of soft sets has the finite intersection property (FIP) if the intersection of the members of each finite subfamily of Ψ is not a null soft set.

2. Semiopen and Semiclosed Soft Sets

Various generalization of closed and open sets in topological spaces and fuzzy topological spaces are of recent developments, but for soft topological spaces such generalization have not been studied so far. In this section, we move one step forward to introduce semiopen and semiclosed soft sets and study various properties and notions related to these structures.

Definition 2.1. In a soft topological space (U_E, τ), a soft set
(i) G_C is said to be semiopen soft set if $\exists$ an open soft set H_B such that
$H_B \mathrel{\tilde{\subseteq}} G_C \mathrel{\tilde{\subseteq}} clH_B$;

(ii) L_A is said to be semiclosed soft set if $\exists$ a closed soft set K_D such that
$intK_D \mathrel{\tilde{\subseteq}} L_A \mathrel{\tilde{\subseteq}} K_D$.

Example 2.2. Consider a soft topological spaces (U_E, τ), where $U = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi_E, U_E, L_{1A}, L_{2A}, L_{3A}\}$. The soft sets are defined as follows $L_1(e_1) = \{h_2\}$, $L_1(e_2) = \{h_1\}$, $L_2(e_1) = \{h_2\}$, $L_2(e_2) = U$, $L_3(e_1) = U$, $L_3(e_2) = U$.

Then the soft set G_C given by

$$G(e_1) = U, G(e_2) = \{h_1\}$$

is a semiopen soft set, as L_{1A} is a open soft set such that $L_{1A} \tilde{\subseteq} G_C \tilde{\subseteq} clL_{1A} = U_E$.

Again the soft set K_D given by

$$K(e_1) = \Phi, K(e_2) = \{h_2\}$$

is a semiclosed soft set, as $(L_{1A})^c$ is a closed soft set such that $int(L_{1A})^c = \Phi_E \tilde{\subseteq} K_D \tilde{\subseteq} (L_{1A})^c$.

Remark 2.3. From definition of semiopen (semiclosed) soft sets and Example 8.2.2 it is clear that every open (closed) soft set is a semiopen (semiclosed) soft set but not conversely.

Remark 2.4. Φ_E and U_E are always semiclosed and semiopen.

From now onwards, we shall denote the family of all semiopen soft sets (semiclosed soft sets) of a soft topological space (U_E, τ) by $SOSS(U)_E$ ($SCSS(U)_E$).

Theorem 2.5. Arbitrary union of semiopen soft sets is a semiopen soft set.

Proof. Let $\{(G_C)_\lambda \mid \lambda \in \Lambda\}$ be a collection of semiopen soft sets of a soft topological space (U_E, τ). Then $\exists$ an open soft sets $(H_B)_\lambda$ such that $(H_B)_\lambda \tilde{\subseteq} (G_C)_\lambda \tilde{\subseteq} cl(H_B)_\lambda$ for each λ; hence $\tilde{\bigcup}(H_B)_\lambda \tilde{\subseteq} \tilde{\bigcup}(G_C)_\lambda \tilde{\subseteq} \tilde{\bigcup} cl(H_B)_\lambda$ and $\tilde{\bigcup}(H_B)_\lambda$ is open soft set. Hence $\tilde{\bigcup}(G_C)_\lambda$ is a semiopen soft set. □

Remark 2.6. Arbitrary intersection of semiclosed soft sets is a semiclosed soft set.

Theorem 2.7. If a semiopen soft set G_C is such that $G_C \tilde{\subseteq} K_D \tilde{\subseteq} clG_C$, then K_D is also semiopen.

Proof. As G_C is semiopen soft set $\exists$ an open soft set H_B such that $H_B \tilde{\subseteq} G_C \tilde{\subseteq} clH_B$; then by hypothesis $H_B \tilde{\subseteq} K_D$ and $clG_C \tilde{\subseteq} clH_B \Rightarrow K_D \tilde{\subseteq} clG_C \tilde{\subseteq} clH_B$ i.e., $H_B \tilde{\subseteq} K_D \tilde{\subseteq} clH_B$, hence K_D is a semiopen soft set. □

Theorem 2.8. If a semiclosed soft set L_A is such that $intL_A \tilde{\subseteq} K_D \tilde{\subseteq} L_A$, then K_D is also semiclosed.

Theorem 2.9. A soft set $G_C \in SOSS(U)_E \Leftrightarrow$ for every soft point $e_G \tilde{\in} G_C$, $\exists$ a soft set $H_B \in SOSS(U)_E$ such that $e_G \tilde{\in} H_B \tilde{\subseteq} G_C$.

Proof. ($\Rightarrow$) Take $H_B = G_C$.

($\Leftarrow$) $G_C = \tilde{\bigcup}_{e_G \tilde{\in} G_C} (e_G) \tilde{\subseteq} \tilde{\bigcup}_{e_G \tilde{\in} G_C} H_B \tilde{\subseteq} G_C$. □

Definition 2.10. Let (U_E, τ) be a soft topological space and G_C be a soft set over U. Then

(i) The soft semi closure of G_C, $ssclG_C = \tilde{\bigcap}\{S_F \mid G_C \tilde{\subseteq} S_F \text{ and } S_F \in SCSS(U)_E\}$ is a soft set.

(ii) The soft semi interior of G_C, $ssintG_C = \tilde{\bigcup}\{S_F \mid S_F \tilde{\subseteq} G_C \text{ and } S_F \in SOSS(U)_E\}$ is a soft set.

Thus $ssclG_C$ is the smallest semiclosed soft set containing G_C and $ssintG_C$ is the largest semiopen soft set contained in G_C.

Theorem 2.11. Let (U_E, τ) be a soft topological space and G_C and K_D be two soft sets over U, then

(i) $G_C \in SCSS(U)_E \Leftrightarrow G_C = ssclG_C$;

(ii) $G_C \in SOSS(U)_E \Leftrightarrow G_C = ssintG_C$;

(iii) $(ssclG_C)^c = ssint(G_C^c)$;

(iv) $(ssintG_C)^c = sscl(G_C^c)$;

(v) $G_C \tilde{\subseteq} K_D \Rightarrow ssintG_C \tilde{\subseteq} ssintK_D$;

(vi) $G_C \tilde{\subseteq} K_D \Rightarrow ssclG_C \tilde{\subseteq} ssclK_D$;

(vii) $sscl\Phi_E = \Phi_E$ and $ssclU_E = U_E$;

(viii) $ssint\Phi_E = \Phi_E$ and $ssintU_E = U_E$;

(ix) $sscl(G_C \tilde{\cup} K_D) = ssclG_C \tilde{\cup} ssclK_D$;

(x) $ssint(G_C \tilde{\cap} K_D) = ssintG_C \tilde{\cap} ssintK_D$;

(xi) $sscl(G_C \tilde{\cap} K_D) \tilde{\subset} ssclG_C \tilde{\cap} ssclK_D$;

(xii) $ssint(G_C \tilde{\cup} K_D) \tilde{\subset} ssintG_C \tilde{\cup} ssintK_D$;

(xiii) $sscl(ssclG_C) = ssclG_C$;

(xiv) $ssint(ssintG_C) = ssintG_C$.

Proof. Let G_C and K_D be two soft sets over U.

(i) Let G_C be a semiclosed soft set. Then it is the smallest semiclosed set containing itself and hence $G_C = ssclG_C$.
On the other hand, let $G_C = ssclG_C$ and $ssclG_C \in SCSS(U)_E \Rightarrow G_C \in SCSS(U)_E$.

(ii) Similar to (i).

(iii)

$$\begin{aligned}(ssclG_C)^c &= (\widetilde{\bigcap}\{S_F \mid G_C \,\widetilde{\subseteq}\, S_F \textit{ and } S_F \in SCSS(U)_E\})^c \\ &= \widetilde{\bigcup}\{S_F^c \mid G_C \,\widetilde{\subseteq}\, S_F \textit{ and } S_F \in SCSS(U)_E\} \\ &= \widetilde{\bigcup}\{S_F^c \mid S_F^c \,\widetilde{\subseteq}\, G_C^c \textit{ and } S_F^c \in SOSS(U)_E\} \\ &= ssint(G_C^c).\end{aligned}$$

(iv) Similar to (iii).

(v) Follows from definition.

(vi) Follows from definition.

(vii) Since Φ_E and U_E are semiclosed soft sets so $sscl\Phi_E = \Phi_E$ and $ssclU_E = U_E$.

(viii) Since Φ_E and U_E are semiopen soft sets so $ssint\Phi_E = \Phi_E$ and $ssintU_E = U_E$.

(ix) We have $G_C \,\widetilde{\subset}\, G_C\widetilde{\bigcup}K_D$ and $K_D \,\widetilde{\subset}\, G_C\widetilde{\bigcup}K_D$. Then by $(vi), ssclG_C \,\widetilde{\subset}\, sscl(G_C\widetilde{\bigcup}K_D)$ and $ssclK_D \,\widetilde{\subset}\, sscl(G_C\widetilde{\bigcup}K_D) \Rightarrow ssclK_D\widetilde{\bigcup}ssclG_C \,\widetilde{\subset}\, sscl(G_C\widetilde{\bigcup}K_D)$.
Now, $ssclG_C, ssclK_D \in SCSS(U)_A \Rightarrow ssclG_C\widetilde{\bigcup}ssclK_D \in SCSS(U)_A$.
Then $G_C \,\widetilde{\subset}\, ssclG_C$ and $K_D \,\widetilde{\subset}\, ssclK_D$ imply $G_C\widetilde{\bigcup}K_D \,\widetilde{\subset}\, ssclG_C\widetilde{\bigcup}ssclK_D$, i.e., $ssclG_C\widetilde{\bigcup}ssclK_D$ is a semiclosed set containing $G_C\widetilde{\bigcup}K_D$. But $sscl(G_C\widetilde{\bigcup}K_D)$ is the smallest semiclosed soft set containing $G_C\widetilde{\bigcup}K_D$. Hence $sscl(G_C\widetilde{\bigcup}K_D) \,\widetilde{\subset}\, ssclG_C\widetilde{\bigcup}ssclK_D$. So, $sscl(G_C \,\widetilde{\cup}\, K_D) = ssclG_C \,\widetilde{\cup}\, ssclK_D$.

(x) Similar to (ix).

(xi) We have $G_C\widetilde{\bigcap}K_D \,\widetilde{\subset}\, G_C$ and $G_C\widetilde{\bigcap}K_D \,\widetilde{\subset}\, K_D$
$\Rightarrow sscl(G_C\widetilde{\bigcap}K_D) \,\widetilde{\subset}\, ssclG_C$ and $sscl(G_C\widetilde{\bigcap}K_D) \,\widetilde{\subset}\, ssclK_D$
$\Rightarrow sscl(G_C\widetilde{\bigcap}K_D) \,\widetilde{\subset}\, ssclG_C\widetilde{\bigcap}ssclK_D$.

(xii) Similar to (xi).

(xiii) Since $ssclG_C \in SCSS(U)_E$ so by (i), $sscl(ssclG_C) = ssclG_C$.

(xiv) Since $ssintG_C \in SOSS(U)_E$ so by (ii), $ssint(ssintG_C) = ssintG_C$. □

Theorem 2.12. If G_C is any soft set in a soft topological space (U_E, τ) then following are equivalent:

(i) G_C is semiclosed soft set;

(ii) $int(clG_C) \tilde{\subseteq} G_C$;

(iii) $cl(intG^c_C) \tilde{\supseteq} G^c_C$;

(iv) G^c_C is semiopen soft set.

Proof. (i) $\Rightarrow$ (ii)
If G_C is semiclosed soft set, then $\exists$ closed soft set H_B such that $intH_B \tilde{\subseteq} G_C \tilde{\subseteq} H_B \Rightarrow intH_B \tilde{\subseteq} G_C \tilde{\subseteq} clG_C \tilde{\subseteq} H_B$. By the property of interior we then have $int(clG_C) \tilde{\subseteq} intH_B \tilde{\subseteq} G_C$.
(ii) $\Rightarrow$ (iii)
$int(clG_C) \tilde{\subseteq} G_C \Rightarrow G^c_C \tilde{\subseteq} (int(clG_C))^c = cl(intG^c_C) \tilde{\supseteq} G^c_C$.
(iii) $\Rightarrow$(iv)
$H_B = intG^c_C$ is an open soft set such that $intG^c_C \tilde{\subseteq} G^c_C \tilde{\subseteq} cl(intG^c_C)$, hence G^c_C is semiopen.
(iv) $\Rightarrow$ (i)
As G^c_C is semiopen $\exists$ an open soft set H_B such that $H_B \tilde{\subseteq} G^c_C \tilde{\subseteq} clH_B \Rightarrow H^c_B$ is a closed soft set such that $G_C \tilde{\subseteq} H^c_B$ and $G^c_C \tilde{\subseteq} clH_B \Rightarrow intH^c_B \tilde{\subseteq} G_C$, hence G_C is semiclosed soft set. □

3. Soft Functions

Kharal *et al*. [2] introduced soft function over classes of soft sets. The authors also defined and studied the properties of soft images and soft inverse images of soft sets, and used these notions to the problem of medical diagnosis in medical expert systems.

Definition 3.1.[2] Let U be a universe and E a set of parameters. Then the collection $SS(U)_E$ of all soft sets over U with parameters from E is called a soft class.

Definition 3.2.[2] Let $SS(U)_E$ and $SS(V)_{E'}$ be two soft classes. Then $u : U \to V$ and $p : E \to E^{'}$ be two functions. Then a functions $f : SS(U)_E \to SS(V)_{E'}$ and its inverse are defined as

- Let L_A be a soft set in $SS(U)_E$, where $A \subseteq E$. The image of L_A under f is a soft set in $SS(V)_{E'}$ such that

$$f(L_A)(\beta) = \left(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} L(\alpha) \right)$$

for $\beta \in B = p(A) \subseteq E^{'}$.

- Let G_C be a soft set in $SS(V)_{E'}$, where $C \subseteq E^{'}$. Then the inverse image of G_C under f is a soft set in $SS(U)_E$ such that

$$f^{-1}(G_C)(\alpha) = u^{-1}(G(p(\alpha)))$$

for $\alpha \in D = p^{-1}(C) \subseteq E$.

Using semiopen and semiclosed soft sets, now we introduce different generalizations of soft functions in soft topological spaces and investigate their properties.

Definition 3.3. Let (U_E, τ) and $(V_{E'}, \delta)$ be two soft topological spaces. A soft function $f : SS(U)_E \to SS(V)_{E'}$ is said to be

(i) soft semicontinuous if for each soft open set G_C of $V_{E'}$, the inverse image $f^{-1}(G_C)$ is a semiopen soft set of U_E;

(ii) soft irresolute if for each semiopen soft set G_C of $V_{E'}$, the inverse image $f^{-1}(G_C)$ is a semiopen soft set of U_E;

(iii) soft semiopen function if for each open soft set L_A of U_E, the image $f(L_A)$ is a semiopen soft set of $V_{E'}$;

(iv) soft semiclosed function if for each closed soft set K_D of U_E, the image $f(K_D)$ is semiclosed soft set of $V_{E'}$.

Remark 3.4.
(i) A soft function $f : SS(U)_E \to SS(V)_{E'}$ is soft semicontinuous if for each closed soft set $K_{D'}$ of $V_{E'}$, the inverse image $f^{-1}(K_{D'})$ is a semiclosed soft set of U_E.

(ii) A soft semicontinuous function is soft irresolute.

Theorem 3.5. A soft function $f : SS(U)_E \to SS(V)_{E'}$ is soft semicontinuous iff $f(ssclL_A) \mathrel{\widetilde{\subseteq}} cl(f(L_A))$ for every soft set L_A of U_E.

Proof. Let $f : SS(U)_E \to SS(V)_{E'}$ be a soft semicontinuous function. Now $cl(f(L_A))$ is a soft closed set of $V_{E'}$, so by soft semicontinuity of f, $f^{-1}(cl(f(L_A)))$ is soft semiclosed and $L_A \mathrel{\widetilde{\subseteq}} f^{-1}(cl(f(L_A)))$. But $ssclL_A$ is the smallest semiclosed set containing L_A, hence $ssclL_A \mathrel{\widetilde{\subseteq}} f^{-1}(cl(f(L_A))) \Rightarrow f(ssclL_A) \mathrel{\widetilde{\subseteq}} cl(f(L_A))$.

Conversely, let G_C be any soft closed set of $V_{E'} \Rightarrow f^{-1}(G_C) \mathrel{\widetilde{\in}} U_E \Rightarrow f(sscl(f^{-1}(G_C)))$
$\widetilde{\subseteq}$ $cl(f(f^{-1}(G_C))) \Rightarrow f(sscl(f^{-1}(G_C))) \mathrel{\widetilde{\subseteq}} clG_C = G_C \Rightarrow sscl(f^{-1}(G_C)) = f^{-1}(G_C)$. Hence $f^{-1}(G_C)$ is semiclosed. □

Theorem 3.6. A soft function $f : SS(U)_E \to SS(V)_{E'}$ is soft semicontinuous iff $f^{-1}(intG_C) \mathrel{\widetilde{\subseteq}} ssint(f^{-1}(G_C))$ for every soft set G_C of $V_{E'}$.

Proof. Let $f : SS(U)_E \to SS(V)_{E'}$ is soft semicontinuous. Now $int(f(G_C))$ is a soft open set of $V_{E'}$, so by soft semicontinuity of f, $f^{-1}(int(f(G_C)))$ is soft semiopen and $f^{-1}(int(f(G_C))) \tilde{\subseteq} G_C$. As $ssintG_C$ is the largest soft semiopen set contained in G_C, $f^{-1}(int(f(G_C))) \tilde{\subseteq} ssintG_C$.

Conversely, take a soft open set $G_C \Rightarrow f^{-1}(intG_C) \tilde{\subseteq} ssint(f^{-1}(G_C)) \Rightarrow f^{-1}(G_C) \tilde{\subseteq} ssint(f^{-1}(G_C)) \Rightarrow f^{-1}(G_C)$ is soft semiopen. □

Theorem 3.7. A soft function $f : SS(U)_E \to SS(V)_{E'}$ is soft semiopen iff $f(intL_A) \tilde{\subseteq} ssint(f(L_A))$ for every soft set L_A of U_E.

Proof. If $f : SS(U)_E \to SS(V)_{E'}$ is soft semiopen, then $f(intL_A) = ssintf(intL_A) \tilde{\subseteq} ssintf(L_A)$.

On the other hand, take a soft open set L_A of U_E. Then by hypothesis, $f(L_A) = f(intL_A) \tilde{\subseteq} ssint(f(L_A)) \Rightarrow f(L_A)$ is soft semiopen in $V_{E'}$. □

Theorem 3.8. Let $f : SS(U)_E \to SS(V)_{E'}$ be soft semiopen. If $K_{D'}$ is a soft set in $V_{E'}$ and L_C is closed soft set containing $f^{-1}(K_{D'})$ then $\exists$ a semiclosed soft set H_B such that $K_{D'} \tilde{\subseteq} H_B$ and $f^{-1}(H_B) \tilde{\subseteq} L_C$.

Proof. Take $H_B = (f(L_C^c))^c$. Now $f^{-1}(K_{D'}) \tilde{\subseteq} L_C \Rightarrow f(L_C^c) \tilde{\subseteq} K_{D'}^c$. Then L_C^c is soft open $\Rightarrow f(L_C^c)$ is semiopen, so H_B is semiclosed and $K_{D'} \tilde{\subseteq} H_B$ and $f^{-1}(H_B) \tilde{\subseteq} L_C$. □

Theorem 3.9. A soft function $f : SS(U)_E \to SS(V)_{E'}$ is soft semiclosed iff $ssclf(L_A) \tilde{\subseteq} f(clL_A)$ for every soft set L_A of U_E.

4. Soft Semicompactness

The study on compactness (which depends on open sets) for a soft topological space was initiated by Zorlutuna *et al.* in [6]. Generalization of open sets to semiopen sets in soft topological spaces also demands generalization of compactness. This section is devoted to introduce semicompactness in soft topological spaces along with its characterization.

Definition 4.1. A cover of a soft set is said to be a semiopen soft cover if every member of the cover is a semiopen soft set.

Definition 4.2. A soft toplogical space (U_E, τ) is said to be semicompact if each semiopen soft cover of U_E has a finite subcover.

Remark 4.3. Every compact soft topological space is also semicompact.

Theorem 4.4. A soft topological space (U_E, τ) is semicompact $\Leftrightarrow$ each family of semiclosed soft sets in U_E with the FIP has a nonempty intersection.

Proof. Let $\{(L_A)_\lambda \mid \lambda \in \Lambda\}$ be a collection of semiclosed soft sets with the FIP. If possible, assume $\widetilde{\bigcap}_{\lambda\in\Lambda} (L_A)_\lambda = \Phi_E \Rightarrow \widetilde{\bigcup}_{\lambda\in\Lambda} ((L_A)_\lambda)^c = U_E$. So, the collection $\{((L_A)_\lambda)^c \mid \lambda \in \Lambda\}$ forms a soft semiopen cover of U_E, which is semicompact. So, $\exists$ a finite subcollection Δ of Λ which also covers U_E. i.e., $\widetilde{\bigcup}_{\lambda\in\Delta} ((L_A)_\lambda)^c = U_E \Rightarrow \widetilde{\bigcap}_{\lambda\in\Delta} (L_A)_\lambda = \Phi_E$, a contradiction.

For the converse, if possible, let (U_E, τ) be not semicompact. Then $\exists$ a semiopen cover $\{(G_C)_\lambda \mid \lambda \in \Lambda\}$ of U_E, such that for every finite subcollection Δ of Λ we have $\widetilde{\bigcup}_{\lambda\in\Delta} (G_C)_\lambda \neq U_E \Rightarrow \widetilde{\bigcap}_{\lambda\in\Delta} ((G_C)_\lambda)^c \neq \Phi_E$. Hence $\{((G_C)_\lambda)^c \mid \lambda \in \Lambda\}$ has the FIP. So, by hypothesis $\widetilde{\bigcap}_{\lambda\in\Lambda} ((G_C)_\lambda)^c \neq \Phi_E \Rightarrow \widetilde{\bigcup}_{\lambda\in\Lambda} (G_C)_\lambda \neq U_E$, a contradiction. □

Theorem 4.5. A soft topological space (U_E, τ) is semicompact iff for every family Ψ of soft sets with FIP, $\widetilde{\bigcap}_{G_C\in\Psi} ssclG_C \neq \Phi_E$.

Proof. Let (U_E, τ) be semicompact and if possible, let $\widetilde{\bigcap}_{G_C\in\Psi} ssclG_C = \Phi_E$ for some family Ψ of soft sets with the FIP. So, $\widetilde{\bigcup}_{G_C\in\Psi} (ssclG_C)^c = U_E \Rightarrow \Upsilon = \{(ssclG_C)^c \mid G_C \in \Psi\}$ is a semiopen cover of U_E. Then by semicompactness of U_E, $\exists$ a finite subcover ω of Υ. i.e., $\widetilde{\bigcup}_{G_C\in\omega} (ssclG_C)^c = U_E \Rightarrow \widetilde{\bigcup}_{G_C\in\omega} G_C^c = U_E \Rightarrow \widetilde{\bigcap}_{G_C\in\omega} G_C = \Phi_E$, a contradiction. Hence $\widetilde{\bigcap}_{G_C\in\Psi} ssclG_C \neq \Phi_E$.

Conversely, we have $\widetilde{\bigcap}_{G_C\in\Psi} ssclG_C \neq \Phi_E$, for every family Ψ of soft sets with FIP. Assume (U_E, τ) is not semicompact. Then $\exists$ a family Υ of semiopen soft sets covering U without a finite subcover. So, for every finite subfamily ω of Υ we have $\widetilde{\bigcup}_{G_C\in\omega} G_C \neq U_E \Rightarrow \widetilde{\bigcap}_{G_C\in\omega} G_C^c \neq \Phi_E \Rightarrow \{G_C^c \mid G_C \in \Upsilon\}$ is a family of soft sets with FIP. Now $\widetilde{\bigcup}_{G_C\in\Upsilon} G_C = U_E \Rightarrow \widetilde{\bigcap}_{G_C\in\Upsilon} G_C^c = \Phi_E \Rightarrow \widetilde{\bigcap}_{G_C\in\Upsilon} sscl(G_C^c) = \Phi_E$, a contradiction. □

Theorem 4.6. Semicontinuous image of a soft semicompact space is soft compact.

Proof. Let $f : SS(U)_E \to SS(V)_{E'}$ be a semicontinuous function where (U_E, τ) is a semicompact soft topological space and $(V_{E'}, \delta)$ is another soft topological space. Take a soft open cover $\{(G_C)_\lambda \mid \lambda \in \Lambda\}$ of $V_{E'} \Rightarrow \{f^{-1}((G_C)_\lambda) \mid \lambda \in \Lambda\}$ forms a soft semiopen cover of $U_E \Rightarrow \exists$ a finite subset Δ of Λ such that $\{f^{-1}((G_C)_\lambda) \mid \lambda \in \Delta\}$

forms a semiopen cover of $U_E \Rightarrow \{(G_C)_\lambda) \mid \lambda \in \Delta\}$ forms a finite soft opencover of $V_{E'}$. □

Theorem 4.7. Semiclosed subspace of a semicompact soft topological space is soft semicompact.

Proof. Let V_B be a semiclosed subspace of a semicompact soft topological space (U_A, τ) and $\{(G_C)_\lambda \mid \lambda \in \Lambda\}$ be a semiopen cover of V_B. Then the family $\{(G_C)_\lambda \mid \lambda \in \Lambda\}\widetilde{\bigcup}(U_A - V_B)$ is a soft semi open cover of U_A, which has a finite subcover, as U_A is soft semicompact. So, $\{(G_C)_\lambda \mid \lambda \in \Lambda\}$ has a finite subfamily to cover V_B. Hence V_B is semicompact. □

5. Soft Semiconnectedness

Connectedness is one of the important notions of topology. Peyghan *et al.* introduce and study the notions of soft connected topological spaces in [4]. In this section, we introduce semiconnectedness in a soft topological space and examine its properties.

Definition 5.1. Two soft sets L_A and H_B are said to be disjoint if $L_A(a) \cap H_B(b) = \phi, \forall\ a \in A, b \in B$.

Definition 5.2. A soft semiseparation of soft topological space (U_E, τ) is a pair L_A, H_B of disjoint nonnull semiopen sets whose union is U_E.

If there doesn't exist a soft semiseparation of U_E, then the soft topological space is said to be soft semiconnected, otherwise soft semidisconnected.

Example 5.3. In the soft topological space of Example 8.2.2, the semiopen soft sets are L_{1A}, L_{2A}, L_{3A} and G_C, whose none of the pairs is disjoint. So there doesn't exist a soft semiseparation of U_E, and hence is soft semiconnected.

Theorem 5.4. If the soft sets L_A and G_C form a soft semiseparation of U_E, and if V_B is a soft semiconnected subspace of U_E, then $V_B \tilde{\subset} L_A$ or $V_B \tilde{\subset} G_C$.

Proof. Since L_A and G_C are disjoint semiopen soft sets, so are $L_A\widetilde{\bigcap}V_B$ and $G_C\widetilde{\bigcap}V_B$ and their soft union gives V_B, i.e., they would constitute a soft semiseparation of V_B, a contradiction. Hence, one of $L_A\widetilde{\bigcap}V_B$ and $G_C\widetilde{\bigcap}V_B$ is empty and so V_B is entirely contained in one of them. □

Theorem 5.5. Let V_B be a soft semiconnected subspace of U_E and K_D be a soft set in U_E such that $V_B \tilde{\subset} K_D \tilde{\subset} cl(V_B)$, then K_D is also soft semiconnected.

Proof. Let the soft set K_D satisfies the hypothesis. If possible, let F_A and G_C form a soft semiseparation of K_D. Then, by theorem 8.5.4, $V_B \tilde{\subset} F_A$ or $V_B \tilde{\subset} G_C$. Let $V_B \tilde{\subset} F_A \Rightarrow sscl(V_B) \tilde{\subset} ssclF_A$; since $ssclF_A$ and G_C are disjoint, V_B cannot intersect G_C. This contradicts the fact that G_C is a nonempty subset of $V_B \Rightarrow \nexists$ a soft semiseparation of K_D and hence is soft semiconnected. $\square$

Theorem 5.6. A soft topological space (U_E, τ) is soft semidisconnected $\Leftrightarrow \exists$ a nonnull proper soft subset of U_E which is both soft semiopen and soft semiclosed.

Proof. Let K_D be a nonnull proper soft subset of U_E which is both semiopen and semiclosed. Now $H_C = (K_D)^c$ is nonnull proper subset of U_E which is also both semiopen and semiclosed $\Rightarrow sscl K_D = K_D$ and $sscl H_C = H_C \Rightarrow U_E$ can be expressed as the soft union of two semiseparated soft sets K_D, H_C and so, is semidisconnected.

Conversely, let U_E be semidisconnected $\Rightarrow \exists$ nonnull soft subsets K_D and H_C such that $sscl K_D \widetilde{\bigcap} H_C = \Phi_E, K_D \widetilde{\bigcap} sscl H_C = \Phi_E$ and $K_D \widetilde{\bigcup} H_C = U_E$. Now $K_D \widetilde{\subseteq} sscl K_D$ and $sscl K_D \widetilde{\bigcap} H_C = \Phi_E \Rightarrow K_D \widetilde{\bigcap} H_C = \Phi_E \Rightarrow H_C = (K_D)^c$. Then $K_D \widetilde{\bigcup} sscl H_C = U_E$ and $K_D \widetilde{\bigcap} sscl H_C = \Phi_E \Rightarrow K_D = (sscl H_C)^c$ and similarly $H_C = (sscl K_D)^c \Rightarrow K_D, H_C$ are semiopen sets being the complements of semiclosed soft sets. Also $H_C = (K_D)^c \Rightarrow$ they are also semiclosed. $\square$

Theorem 5.7. Semicontinuous image of a soft semiconnected soft topological space is soft connected.

Proof. Let $f : SS(U)_E \to SS(V)_{E'}$ be a semicontinuous function where (U_E, τ) a semiconnected soft topological space and $(V_{E'}, \delta)$ is a soft topological space . We wish to show $f(U_E)$ is soft connected. Suppose $f(U_E) = K_D \widetilde{\bigcup} H_C$ be a soft separation. i.e., K_D and H_C are disjoint soft open sets whose union is $f(U_E) \Rightarrow f^{-1}(K_D)$ and $f^{-1}(H_C)$ are disjoint soft semiopen sets whose union is U_E. So, $f^{-1}(K_D)$ and $f^{-1}(H_C)$ form a soft semiseparation of U_E, a contradiction. $\square$

Theorem 5.8. Irresolute image of a soft semiconnected soft topological space is soft semiconnected.

Proof. Similar to that of Theorem 5.7. $\square$

6. Soft Semiseparation Axioms

Soft separation axioms for soft topological space were studied by Shabir *et al.* [5]. Here we consider separation axioms for soft topological spaces using semiopen and semiclosed soft sets.

Definition 6.1. Two soft sets G_C and H_B are said to be distinct if $G(e) \bigcap H(e) = \Phi, \ \forall \ e \in B \cap C$.

Definition 6.2. A soft topological space (U_E, τ) is said to be a soft semi T_0-space if for two disjoint soft points e_G and e_F, $\exists$ a semiopen set containing one but not the other.

Example 6.3.
(i) A discrete soft topological space is a soft semi T_0-space since every $e_F \in U$ is a semiopen soft set in the discrete space.

(ii) Consider a soft topological spaces (U_E, τ), where $U = \{h_1, h_2\}, E = \{e_1, e_2\}$ and $\tau = \{\Phi_E, U_E, L_{1A}, L_{2A}\}$. The soft sets are defined as follows:
$L_1(e_1) = U$, $L_1(e_2) = \Phi$,
$L_2(e_1) = \Phi$, $L_2(e_2) = U$.

Then (U_E, τ) is a soft semi T_0-space since L_{1A} is a semiopen soft set which contains all the soft points with respect to the parameter e_1 but does not contain the soft points with respect to the parameter e_2, i.e., for all disjoint pairs of soft points in (U_E, τ), we can always find a semiopen soft set containing one but not the other.

Theorem 6.4. A soft topological space is a soft semi T_0-space if the soft semiclosures of distinct soft points are distinct.

Proof. Let e_F and e_H be two distinct soft points with distinct soft semiclosures in a soft topological space (U_E, τ).
If possible, suppose we had $e_F \tilde{\in} sscl e_H$, then $sscl e_F \tilde{\in} sscl e_H$, a contradiction.
So $e_F \tilde{\notin} sscl e_H \Rightarrow (sscl e_H)^c$ is a soft semi open set containing e_F but not e_H. Hence (U_E, τ) is a soft semi T_0-space. □

Theorem 6.5. A soft subspace of a soft semi T_0-space is soft semi T_0.

Proof. Let V_B be a soft subspace of a soft semi T_0-space U_E and let e_F, e_G be two distinct soft points of V_B. Then these soft points are also in $U_E \Rightarrow \exists$ a semiopen soft set H_C containing one of these soft points, say e_F, but not the other $\Rightarrow H_C \tilde{\bigcap} V_B$ is a semiopen soft set containing e_F but not the other. □

Definition 6.6. A soft topological space (U_E, τ) is said to be a soft semi T_1-space if for two distinct soft points e_F, e_G of $U_E, \exists$ soft semiopen sets H_B and G_C such that
$e_F \tilde{\in} H_B$ and $e_G \tilde{\notin} H_B$;
$e_G \tilde{\in} G_C$ and $e_F \tilde{\notin} G_C$.

Example 6.7. The soft topological space in Example 8.6.3.(ii). is also a soft semi T_1-space, since L_{1A} is a semiopen soft set which contains all the soft points with respect to the parameter e_1 but does not contain the soft points with respect to the parameter e_2 and L_{2A} is a semiopen soft set which contains all the soft points with respect to the parameter e_2 but does not contain the soft points with respect to the parameter e_1.

Theorem 6.8. If every soft point of a soft topological space (U_E, τ) is a semiclosed soft set then (U_E, τ) is a soft semi T_1-space.

Proof. Let e_F and e_G be two distinct soft points of $U_E \Rightarrow (e_F)^c$, $(e_G)^c$ are semiopen soft sets such that $e_G \tilde{\in} (e_F)^c$ and $e_G \tilde{\notin} e_F$; $e_F \tilde{\in} (e_G)^c$ and $e_F \tilde{\notin} e_G$. □

Theorem 6.9. A soft subspace of a soft semi T_1-space is soft semi T_1.

Definition 6.10. A soft topological space (U_E, τ) is said to be a soft semi T_2-space if and only if for distinct soft points e_F, e_G of $U_E, \exists$ disjoint soft semiopen sets H_B and G_C such that $e_F \tilde{\in} H_B$ and $e_G \tilde{\in} G_C$.

Example 6.11. The soft topological space in Example 8.6.3.(ii). is also a soft semi T_2-space, since L_{1A} and L_{2A} are disjoint semiopen soft set.

Theorem 6.12. A soft subspace of a soft semi T_2-space is soft semi T_2.

Proof. Let (U_E, τ) be a soft semi T_2-space and V_A be a soft subspace of U_E, where $A \subseteq E$ and $V \subseteq U$. Let e_F and e_G be two distinct soft points of V_A. U_E is soft semi $T_2 \Rightarrow \exists$ two disjoint soft semiopen sets H_D and G_C such that $e_F \tilde{\in} H_D$, $e_G \tilde{\in} G_C$. Then $H_D \tilde{\bigcap} V_A$ and $G_C \tilde{\bigcap} V_A$ are semiopen softs sets satisfying the requirements for V_A to be a soft semi T_2-space. □

Theorem 6.13. A soft topological space (U_E, τ) is soft semi T_2 if and only if for distinct soft points e_G, e_K of U_E, $\exists$ a semiopen soft set S_D containing e_G but not e_K such that $e_K \tilde{\notin} sscl S_D$.

Proof. Let e_G, e_K be distinct soft points in a soft semi T_2 space (U_E, τ).
$(\Rightarrow)$ $\exists$ distinct semiopen soft sets H_A and W_D such that $e_K \tilde{\in} H_A$, $e_G \tilde{\in} W_D$. This implies $e_G \tilde{\in} H_A^c$. So, H_A^c is a semiclosed soft set containing e_G but not e_K and $sscl H_A^c = H_A^c$.
$(\Leftarrow)$ Take a pair of distinct soft points e_G and e_K of $U_E, \exists$ a semiopen soft set S_D containing e_G but not e_K such that $e_K \tilde{\notin} sscl S_D \Rightarrow e_K \tilde{\in} (sscl S_D)^c \Rightarrow S_D$ and $(sscl S_D)^c$ are disjoint soft open set containing e_G and e_K respectively. □

Definition 6.14. A soft topological space (U_E, τ) is said to be a soft semiregular space if for every soft point e_K and semiclosed soft set L_A not containing $e_K, \exists$ disjoint soft semiopen sets G_{A_1}, G_{A_2} such that $e_K \tilde{\in} G_{A_1}$ and $L_A \tilde{\subseteq} G_{A_2}$, where $A_1, A_2 \subseteq E$.
A soft semiregular semi T_1-space is called a soft semi T_3-space.

Example 6.15. The soft topological space in Example 8.6.3.(ii). is a soft semiregular space which is also soft semi T_1 and hence is a soft semi T_3-space.

Remark 6.16. It can be shown that the property of being soft semi T_3 is hereditary.

Remark 6.17. Soft semi $T_3 \Rightarrow$ soft semi $T_2 \Rightarrow$ soft semi $T_1 \Rightarrow$ soft semi T_0.

Theorem 6.18. A soft topological space which is both soft semicompact and soft semi T_2 is soft semi T_3.

Proof. It suffices to show every semicompact soft topological space is semiregular. Let e_L be a soft point and H_A be a semiclosed soft set not containing the point $\Rightarrow e_L \tilde{\in} (H_A)^c$. Now for each soft point $e_F, \exists$ disjoint semiopen soft sets K_{D_1} and H_{B_1} such that $e_F \tilde{\in} K_{D_1}$ and $e_L \tilde{\in} H_{B_1}$.

So, the collection $\{K_{D_\lambda} \mid \lambda \in \Lambda\}$ forms a semiopen cover of H_A. Now H_A is a semiclosed soft set $\Rightarrow H_A$ is soft semicompact. Hence $\exists$ a finite subfamily Δ of Λ such that $H_A \tilde{\subseteq} \tilde{\bigcup}_{\lambda \in \Delta} K_{D_\lambda}$.

Take $H_B = \bigcap_{i=1}^{n} H_{B_i}$ and $K_D = \bigcup_{i=1}^{n} K_{D_i}$. Then H_B, K_D are disjoint semiopen sets such that e_L is a soft point of H_B and $F_A \tilde{\subseteq} K_D$. □

Definition 6.19. A soft topological space (U_E, τ) is said to be a soft seminormal space if for every pair of disjoint semiclosed soft sets L_A and $K_D, \exists$ two disjoint soft semiopen sets H_{A_1}, H_{A_2} such that $L_A \tilde{\subseteq} H_{A_1}$ and $K_D \tilde{\subseteq} H_{A_2}$.
A soft seminormal T_1-space is called a soft semi T_4-space.

Example 6.20. It can be verified that the soft topological space in Example 8.6.3.(ii). is a soft semi T_4-space.

Remark 6.21. Every soft semi T_4-space is soft semi T_3.

Theorem 6.22. A soft topological space (U_E, τ) is seminormal iff for any semiclosed soft set L_A and semiopen soft set G_C containing L_A, there exists an soft semiopen set H_B such that $L_A \tilde{\subset} H_B$ and $sscl(H_B) \tilde{\subset} G_C$.

Proof. Let (U_E, τ) be seminormal space and L_A be a semiclosed soft set and G_C be a semiopen soft set containing $L_A \Rightarrow L_A$ and $(G_C)^c$ are disjoint semiclosed soft sets $\Rightarrow \exists$ two disjoint semiopen soft sets H_{A_1}, H_{A_2} such that $L_A \tilde{\subset} H_{A_1}$ and $(G_C)^c \tilde{\subseteq} H_{A_2}$. Now $H_{A_1} \tilde{\subset} (H_{A_2})^c \Rightarrow ssclH_{A_1} \tilde{\subset} sscl(H_{A_2})^c = (H_{A_2})^c$ Also, $(G_C)^c \tilde{\subset} H_{A_2} \Rightarrow (H_{A_2})^c \tilde{\subset} G_C \Rightarrow ssclH_{A_1} \tilde{\subset} (G_C)$.

Conversely, let S_A and K_D be any disjoint pair semiclosed soft sets $\Rightarrow S_A \tilde{\subset} (K_D)^c$, then by hypothesis there exists an semiopen soft set H_B such that $S_A \tilde{\subset} H_B$ and $ssclH_B \tilde{\subset} (K_D)^c \Rightarrow (K_D) \tilde{\subset} (ssclH_B)^c \Rightarrow (H_B)$ and $(ssclH_B)^c$ are disjoint semiopen soft sets such that $S_A \tilde{\subset} H_B$ and $K_D \tilde{\subset} (ssclH_B)^c$. □

Theorem 6.23. Let $f : SS(U)_E \to SS(V)_{E'}$ be a soft surjective function which is both irresolute and soft semiopen where (U_E, τ) and $(V_{E'}, \delta)$ are soft topological spaces. If U_E is soft seminormal space then so is $V_{E'}$.

Proof. Take a pair of disjoint semiclosed soft sets L_A and K_D of $V_{E'} \Rightarrow f^{-1}(L_A)$ and $f^{-1}(K_D)$ are disjoint semiclosed soft sets of $U_E \Rightarrow \exists$ disjoint semiopen soft sets G_C and H_B such that $f^{-1}(L_A) \tilde{\subset} G_C$ and $f^{-1}(K_D) \tilde{\subset} H_B \Rightarrow L_A \tilde{\subset} f(G_C)$ and $K_D \tilde{\subset} f(H_B) \Rightarrow f(G_C)$ and $f(H_B)$ are disjoint open soft sets of $V_{E'}$ containing L_A and K_D respectively. Hence the result. $\square$

Theorem 6.24. A semiclosed soft subspace of a soft seminormal space is soft seminormal.

Proof. Let $V_{E'}$ be a semiclosed soft subspace of a soft seminormal space U_E. Take a disjoint pair L_A and G_C of semiclosed sets of $V_{E'} \Rightarrow \exists$ disjoint semiclosed soft sets K_D and H_B such that $L_A = K_D \tilde{\bigcap} V_{E'}, G_C = H_B \tilde{\bigcap} V_{E'}$. Now by soft seminormality of U_E, $\exists$ disjoint semiopen soft sets K_{D^*} and H_{B^*} such that $K_D \tilde{\subset} K_{D^*}$ and $H_B \tilde{\subset} H_{B^*} \Rightarrow L_A \tilde{\subset} K_{D^*} \tilde{\bigcap} V_{E'}$ and $G_C \tilde{\subset} H_{B^*} \tilde{\bigcap} V_{E'}$. $\square$

Theorem 6.25. Every soft semicompact semi T_2-space is seminormal.

Proof. Let (U_E, τ) be a semicompact semi T_2-space. Take a disjoint pair L_F and M_A of semiclosed sets. By Theorem 8.6.18, for each soft point e_L, $\exists$ disjoint semiopen soft sets G_{e_L} and H_{e_L} such that $e_L \tilde{\subset} G_{e_L}$ and $M_A \tilde{\subset} H_{e_L}$. So the collection $\{G_{e_{Li}} \mid e_L \tilde{\subset} G_{e_L}, i \in \Lambda\}$ is a semiopen cover of G_{e_L}. Then by Theorem 8.4.7, $\exists$ a finite subfamily $\{G_{e_{Li}} \mid i = 1, 2, \ldots, n\}$ such that $G_{e_L} \tilde{\subset} \tilde{\bigcup}\{G_{e_{Li}} \mid i = 1, 2, \ldots, n\}$. Take $G_C = \tilde{\bigcap}\{G_{e_{Li}} \mid i = 1, 2, \ldots, n\}$ and $H_B = \tilde{\bigcap}\{H_{e_{Li}} \mid i = 1, 2, \ldots, n\}$. Then G_C and H_B are disjoint semiopen soft sets such that $l_F \tilde{\subset} G_C$ and $M_A \tilde{\subset} H_B$. Hence U_E is seminormal. $\square$

7. Conclusion

In this paper, we continued the work of Shabir *et al.* [5] which is a step forward to further enlarge the theoretical base of soft topological spaces. The notions of semiclosed soft and semiopen soft set are introduced to study various topological structures like soft semi continuous function, soft semi open and closed function, soft semi irresolute function, soft semicompactness, soft semiconnectedness and soft semi separation axioms of soft topological spaces via these sets.

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