

# Sequential Optimization for Subcarrier Pairing and Power Allocation in CP-SC Cognitive Relay Systems

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## **Abstract**

A sequential optimization algorithm (SOA) for resource allocation in a cyclic-prefixed single-carrier cognitive relay system is proposed in this study. Both subcarrier pairing (SP) and power allocation are performed subject to a primary user interference constraint to minimize the mean squared error of frequency-domain equalization at the secondary destination receiver. Under uniform power allocation at the secondary source and optimal power allocation at the secondary relay, the ordered SP is proven to be asymptotically optimal in maximizing the matched filter bound on the signal-to-interference-plus-noise ratio. SOA implements the ordered SP before power allocation optimization by decoupling the ordered SP from the power allocation. Simulation results show that SOA can optimize resource allocation efficiently by significantly reducing complexity.

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**Keywords:** Cyclic-prefixed single-carrier, cognitive radio relay system, power allocation, subcarrier pairing

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## 1. Introduction

Being combined with frequency domain equalization, cyclic-prefixed single-carrier (CP-SC) transmission exhibits performance similar to that of orthogonal frequency-division multiplex (OFDM) with essentially the same overall complexity [1]. In CP-SC-based systems, a cyclic prefix (CP) is prepended to each transmission symbol block to prevent inter-block symbol interference (IBSI) such that the convolutional channel becomes a right circulant matrix in the time domain after the removal of the signal part related to CP. Moreover, CP prepending allows CP-SC-based systems to achieve multipath diversity gain in the practical signal-to-noise ratio (SNR) region [2, 3]. Owing to its low peak-to-average power ratio and insensitivity to Doppler shift and carrier frequency offsets, CP-SC transmission has become a choice to implement many wireless systems, including future cooperative technology [2–6].

Resource allocation for CP-SC relay systems has recently elicited some attention although these systems are still in their infancy. In [4], an optimal power allocation (OPA) scheme across subcarriers for a dual hop CP-SC relay system was developed. [5] presented several power allocation schemes by assuming a dual hop CP-SC-based system with multiple relays and cooperative beamforming. Meanwhile, the relay that receives a message from a particular subcarrier in the first hop has an opportunity to forward the processed message to a different subcarrier in the second hop because of the independent fading in each subcarrier in each hop [7, 8]. Thus, subcarrier pairing (SP) has become a simple but effective method to enhance the transmission performance in broadband relay systems [7–9]. Although beamforming and equalization can be performed in the frequency-domain (FD) of a relay, SP has not been applied in CP-SC relay systems so far.

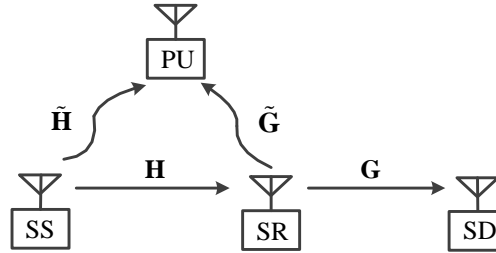
As an effective method to enhance the utilization of existing radio spectra, cognitive radio (CR) has elicited much attention from researchers [2, 3][6–8][10–12]. Particularly, the scarcity of the spectrum can be alleviated by allowing the secondary user (SU) to reuse the radio spectrum licensed to the primary user (PU). In underlay CR systems, SU is allowed to access the spectrum of PU only when the peak interference power constraint at PU is satisfied [2, 3]. One drawback of this approach is that the constrained transmission power of SU typically results in unstable transmission and restricted coverage. Cognitive relay was proposed as a powerful solution to extend communication coverage of the SU system and reduce interference at the PU system [2, 3][6–8]. Recent studies have shown that CP-SC transmission achieves good performance in cognitive relay systems [2, 3, 6]. Given that the SU system has to limit the generated interference toward the PU system in CP-SC cognitive relay systems, resource allocation becomes more challenging than that in non-cognitive relay systems.

A sequential optimization algorithm (SOA) for SP and power allocation in a dual hop CP-SC cognitive relay system is proposed in this study. The SU system operates in the underlay CR model [10, 11], and the secondary relay (SR) employs the amplify-and-forward protocol [6, 7]. Equalization or beamforming is not assumed at SR to maintain a simple operation. Furthermore, the system is designed such that no channel state information (CSI) of the source-to-relay link is fed back to the secondary source (SS); thus, uniform power allocation (UPA) is employed at SS. SR is assumed to contain the CSI of the source-to-relay and relay-to-destination links in this study [6, 7]. Therefore, both SP and OPA are adopted in SR to minimize the mean squared error (MSE) of the receiver at the secondary destination (SD). For the primary channel, this study assumes perfect CSI of the SS-to-PU and SR-to-PU

links, which can be obtained through direct feedback from PU or indirect feedback from a third party [12]. FD linear equalization (FD-LE), FD decision feedback equalization (FD-DFE), and an idealized matched-filter (MF) receiver are considered at SD, and the corresponding objective functions are specified with resource allocation subject to a pre-specified interference threshold at PU.

The equivalent Lagrange dual problem is decomposed into two sub-problems to solve the resource allocation optimization problem. One of the sub-problems is for power allocation, and the other one is for SP, which requires a joint iteration to optimize power allocation and SP. With UPA at SS and OPA at SR, the ordered SP is proven to be asymptotically optimal in maximizing the MF bound (MFB) on signal-to-interference-plus-noise ratio (SINR), which enables the ordered SP to be decoupled from the power allocation such that the ordered SP and power allocation can be solved in a sequential manner. Then, SOA is proposed with the ordered SP determined before power allocation optimization, which greatly improves the error performances of all considered receivers with low complexity.

*Notation:* The superscripts  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and conjugate transpose, respectively.  $\mathbf{0}_N$  denotes a zero vector with  $N$  elements,  $[\mathbf{A}]_{l,k}$  is the  $(l,k)$ -th entry of matrix  $\mathbf{A}$ ,  $\mathbf{I}_N$  is an  $N \times N$  identity matrix,  $\mathbf{F}$  is the  $N \times N$  Fourier transformation matrix, and  $\text{Tr}(\mathbf{A})$  is the trace of matrix  $\mathbf{A}$ .  $\mathbf{CN}(x, y)$  denotes the complex Gaussian distribution with mean  $x$  and variance  $y$ .  $\mathbf{E}\{\cdot\}$  is the expectation.



**Fig. 1.** CP-SC cognitive relay system.

## 2. System Model

We consider a dual hop CP-SC cognitive relay system with one SS, one amplify-and-forward SR, one SD, and a PU as shown in Fig. 1. In the SU system, SS and SR are assumed to transmit in the same primary licensed frequency band subject to interference constraints imposed by PU. It is assumed that SR operates in half-duplex mode and that no direct link exists between SS and SD because of the deep fading between them. With the help of SR, one period of relaying is accomplished within two hops: the first hop from SS to SR and the second hop from SR to SD. Similar to the model employed in [2, 3] and [7, 8], PU is assumed to be located far from the SU system; as such, interference from PU is negligible. Assuming that the number of subcarriers of CP-SC transmission is  $N$ , the channels of the two hops of the secondary system can be expressed by the  $N \times N$  right circulant matrices  $\mathbf{H}$  and  $\mathbf{G}$ , respectively, with their first columns provided by  $\mathbf{h} = [h_0, h_1, \dots, h_{N_f-1}, \mathbf{0}_{N-N_f}^T]^T$  and  $\mathbf{g} = [g_0, g_1, \dots, g_{N_f-1}, \mathbf{0}_{N-N_f}^T]^T$ , respectively. The power delay profiles of the channels satisfy  $\mathbf{E}\{|x_n|^2\} = ce^{-n}$  with  $n = 0, \dots, N_f - 1$ , where  $x_n \in \{h_n, g_n\}$  and the constant  $c$  is selected such that

$\sum_{n=0}^{N_f-1} \mathbf{E}\{|x_n|^2\} = 1$ . According to the properties of a right circulant matrix, channel matrices can be decomposed into  $\mathbf{H} = \mathbf{F}^H \mathbf{\Lambda} \mathbf{F}$  and  $\mathbf{G} = \mathbf{F}^H \mathbf{\Phi} \mathbf{F}$ , where  $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_N)$  and  $\mathbf{\Phi} = \text{diag}(\phi_1, \dots, \phi_N)$  are diagonal matrices [5, 6]. The SS-to-PU and SR-to-PU channels are denoted by  $\tilde{\mathbf{H}}$  and  $\tilde{\mathbf{G}}$ , respectively, which are similarly defined as  $\mathbf{H}$  and  $\mathbf{G}$ , respectively. Time-domain (TD) UPA is adopted at SS because we assume that SS has no CSI of the SS-to-SR channel. The transmit symbol block at SS is denoted by  $\sqrt{p_0} \mathbf{s}$ , where  $\mathbf{s}$  is an  $N \times 1$  vector that satisfies  $\mathbf{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_N$  and  $p_0$  is the UPA factor at SS. UPA factor  $p_0$  satisfies  $Np_0 \leq P_0$ , where  $P_0$  is the total power budget of each symbol block at SS. Furthermore,  $p_0$  is limited such that the interference introduced by SS at PU is under the pre-specified interference threshold. After appending a CP of  $N_g$  symbols in its front, the symbol block  $\sqrt{p_0} \mathbf{s}$  is transmitted from SS. To prevent IBSI, the length of CP is assumed to comprise the maximum path delay, namely,  $N_f < N_g$ .

After removing the CP-related part, the received signal at SR is provided by

$$\mathbf{r} = \sqrt{p_0} \mathbf{H} \mathbf{s} + \mathbf{n}_1, \quad (1)$$

where  $\mathbf{n}_1 \in \mathbf{CN}(\mathbf{0}_N, \sigma_1^2 \mathbf{I}_N)$  is the additive noise at SR. By using FFT,  $\mathbf{r}$  is transformed to the FD as

$$\mathbf{R} = \mathbf{F} \mathbf{r} = \sqrt{p_0} \mathbf{\Lambda} \mathbf{S} + \mathbf{N}_1, \quad (2)$$

where  $\mathbf{S} = \mathbf{F} \mathbf{s}$  and  $\mathbf{N}_1 = \mathbf{F} \mathbf{n}_1$ .  $\mathbf{R}$  is then normalized by an  $N \times N$  diagonal matrix  $\mathbf{B} = \text{diag}(B_1, B_2, \dots, B_N)$  with  $B_k = (p_0 \lambda_k^2 + \sigma_1^2)^{-1/2}$ . Aside from basic amplify-and-forward processing [5–7], SP and power allocation are also employed in SR. The power-normalized signal,  $\mathbf{B} \mathbf{R}$ , is multiplied with an  $N \times N$  row permutation matrix,  $\mathbf{M}$ , followed by an  $N \times N$  diagonal power allocation matrix,  $\mathbf{P} = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \dots, \sqrt{p_N})$ . The power constraints at SR is denoted by  $\sum_{l=1}^N p_l \leq P_1$ , where  $P_1$  is the total power budget of each symbol block at SR. Moreover, the transmission power of SR is limited such that the interference introduced by SR at PU is under the pre-specified interference threshold. With the help of the row permutation matrix, the signal received on the  $k$ -th subcarrier in the first hop will be transmitted on the  $l$ -th subcarrier in the second hop, namely, SP-aided relaying through the subcarrier pair  $(k, l)$ . The corresponding signal after SP and power allocation can be expressed by

$$\mathbf{T} = \sqrt{p_0} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{A} \mathbf{S} + \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{N}_1, \quad (3)$$

which will be transformed back to TD as  $\mathbf{t} = \mathbf{F}\mathbf{T}$ . After appending a CP of  $N_g$  symbols in its front,  $\mathbf{t}$  is transmitted from SR to SD. At the end of the second hop transmission, the received signal at SD (after removing the CP-related signal) is

$$\begin{aligned}\mathbf{y} &= \mathbf{G}\mathbf{t} + \mathbf{n}_2 \\ &= \sqrt{p_0} \mathbf{F}^H \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{\Lambda} \mathbf{F} \mathbf{s} + \mathbf{F}^H \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{F} \mathbf{n}_1 + \mathbf{n}_2 \\ &= \mathbf{H}_t \mathbf{s} + \mathbf{n}_t,\end{aligned}\quad (4)$$

where  $\mathbf{H}_t = \sqrt{p_0} \mathbf{F}^H \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{\Lambda} \mathbf{F}$  is the equivalent channel and  $\mathbf{n}_t = \mathbf{F}^H \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{F} \mathbf{n}_1 + \mathbf{n}_2$  is the equivalent noise, with  $\mathbf{n}_2 \sim \mathbf{CN}(\mathbf{0}_N, \sigma_2^2 \mathbf{I}_N)$  being the additive noise at SD.

### 3. Receiver Processing at SD

We consider three different receivers at SD, namely FD-LE, FD-DFE, and idealized MF receiver, to detect the transmitted signal. At SD, the received TD signal is transformed to FD. Then, the received FD signal is filtered by  $N \times N$  feed-forward filtering matrix  $\mathbf{W}$ . For FD-LE,  $\tilde{\mathbf{y}} = \mathbf{F}^H \mathbf{W} \mathbf{F} \mathbf{y}$  is utilized to obtain the estimation of the transmitted signal. For FD-DFE,  $\tilde{\mathbf{y}}$  is fed into a symbol-by-symbol decision feedback module, which is described by  $N \times N$  right circulant matrix  $\mathbf{D}$ . The first column of  $\mathbf{D}$  is provided by the  $N \times 1$  vector  $\mathbf{d} = [\tilde{\mathbf{d}}, \mathbf{0}_{N-N_d}]^T$ , where  $N_d$  is the number of taps of the feedback filter. Assuming that the decision feedback processing is error-free, the output of the feedback filter is  $\hat{\mathbf{y}} = \tilde{\mathbf{y}} - (\mathbf{D} - \mathbf{I}_N) \mathbf{s}$ . When  $N_d = 1$ ,  $\mathbf{D}$  becomes an identity matrix and FD-DFE degenerates into FD-LE. The error vector between the filtered received signal and the transmitted signal of both FD-LE and FD-DFE can be expressed by

$$\begin{aligned}\mathbf{e} &= \hat{\mathbf{y}} - \mathbf{s} \\ &= \mathbf{F}^H (\mathbf{W} \mathbf{F} \mathbf{y} - \mathbf{\Gamma} \mathbf{F} \mathbf{s}).\end{aligned}\quad (5)$$

In Eq. (5), the property  $\mathbf{D} = \mathbf{F}^H \mathbf{\Gamma} \mathbf{F}$  is applied, where  $\mathbf{\Gamma} = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_N\}$  with  $\gamma_k = \sum_{l=1}^{N_d} d_l e^{-j2\pi(k-1)(l-1)/N}$ . Then, the error covariance matrix can be written as

$$\begin{aligned}\mathbf{E} &= \mathbf{E}\{\mathbf{e} \mathbf{e}^H\} \\ &= \mathbf{F}^H (\mathbf{W} \mathbf{F} \mathbf{y} \mathbf{y}^H \mathbf{F}^H \mathbf{W}^H - \mathbf{W} \mathbf{F} \mathbf{y} \mathbf{s}^H \mathbf{F}^H \mathbf{\Gamma}^H - \mathbf{\Gamma} \mathbf{F} \mathbf{s} \mathbf{y}^H \mathbf{F}^H \mathbf{W}^H + \mathbf{\Gamma} \mathbf{\Gamma}^H) \mathbf{F}.\end{aligned}\quad (6)$$

With the error covariance matrix, the MSE at SD is provided by  $\text{Tr}\{\mathbf{E}\}$ . By differentiating  $\text{Tr}\{\mathbf{E}\}$  with respect to  $\mathbf{W}$  and setting the result to zero, the optimal feed-forward filter is obtained by

$$\mathbf{W} = \mathbf{\Gamma} \mathbf{H}_f^H (\mathbf{H}_f \mathbf{H}_f^H + \mathbf{C})^{-1}, \quad (7)$$

where  $\mathbf{H}_f = \sqrt{p_0} \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{\Lambda}$  and  $\mathbf{C} = \mathbf{E}\{\tilde{\mathbf{F}} \tilde{\mathbf{n}}_f \tilde{\mathbf{n}}_f^H \mathbf{F}^H\} = \sigma_1^2 \mathbf{\Phi} \mathbf{P} \mathbf{M} \mathbf{B} \mathbf{B}^T \mathbf{M}^T \mathbf{P}^T \mathbf{\Phi}^H + \sigma_2^2 \mathbf{I}_N$ . By substituting Eq. (7) into Eq. (6) and using the matrix inversion lemma, the error covariance matrix can be rewritten as

$$\mathbf{E} = \mathbf{F}^H \mathbf{\Gamma} \mathbf{\Psi}^{-1} \mathbf{\Gamma}^H \mathbf{F}, \quad (8)$$

where  $\mathbf{\Psi} = \mathbf{I}_N + \mathbf{H}_f^H \mathbf{C}^{-1} \mathbf{H}_f$  is an  $N \times N$  diagonal matrix. Eq. (8) indicates that  $\mathbf{E}$  has a circulant form with the all diagonal elements being identical. Given that the  $k$ -th diagonal element of  $\mathbf{E}$  stands for the MSE of the  $k$ -th transmitted symbol, the circulant form of  $\mathbf{E}$  indicates that the MSEs of all transmitted symbols in each symbol block are identical; this case is different from the case of OFDM-based transmission, where the MSEs of all the symbols are different [5].

For FD-LE, by substituting  $\mathbf{\Gamma} = \mathbf{I}_N$  into Eq. (7), the optimal equalizer is provided by  $\mathbf{W} = \mathbf{H}_f^H (\mathbf{H}_f \mathbf{H}_f^H + \mathbf{C})^{-1}$ . The MSE of FD-LE can be expressed by

$$\text{MSE}_{\text{FD-LE}} = \frac{1}{N} \text{Tr}\{\mathbf{F}^H \mathbf{\Psi}^{-1} \mathbf{F}\} = \frac{1}{N} \text{Tr}\{\mathbf{\Psi}^{-1}\} = \frac{1}{N} \sum_{l=1}^N \psi_l^{-1}, \quad (9)$$

where  $\psi_l$  is the  $l$ -th diagonal element of  $\mathbf{\Psi}$ , which is provided by

$$\psi_l = [\mathbf{\Psi}]_{l,l} = 1 + \frac{p_0 p_l \alpha_k \beta_l}{p_0 \alpha_k + p_l \beta_l + 1} \quad (10)$$

with  $\alpha_k = \lambda_k^2 / \sigma_1^2$  and  $\beta_l = \phi_l^2 / \sigma_2^2$ .  $\psi_l$  corresponds to the SP-aided relaying transmission on the subcarrier pair  $(k, l)$ .

Similarly, the MSE of FD-DFE can be expressed by

$$\text{MSE}_{\text{FD-DFE}} = \frac{1}{N} \text{Tr}\{\mathbf{\Gamma} \mathbf{\Psi}^{-1} \mathbf{\Gamma}^H\} = \frac{1}{N} \sum_{l=1}^N \gamma_l \psi_l^{-1} \gamma_l^*. \quad (11)$$

The first tap of the feedback filter is set to  $d_1 = 1$  to ensure the causal cancellation of inter-symbol interference (ISI). The optimal feedback filter is provided by [13]

$$\tilde{\mathbf{d}} = (\boldsymbol{\eta}^T \mathbf{A}^{-1} \boldsymbol{\eta})^{-1} \boldsymbol{\eta}^H \mathbf{A}^{-1}, \quad (12)$$

where  $\boldsymbol{\eta} = [1, \mathbf{0}_{N_d-1}]^T$  and  $\mathbf{A}$  is an  $N_d \times N_d$  Hermitian matrix, with its  $(m, n)$ -th entry being  $\mathbf{A}_{m,n} = \sum_{l=1}^N \psi_l^{-1} e^{-j2\pi(m-n)(l-1)/N}$ . To obtain a tractable objective function for FD-DFE, the following asymptotic MSE expression is adopted for FD-DFE [5, 13].

$$\text{MSE}_{\text{FD-DFE}} = \det\{\Psi\}^{-\frac{1}{N}} = \prod_{l=1}^N \psi_l^{-\frac{1}{N}}. \quad (13)$$

In Eq. (13), asymptotic optimality is achieved when both  $N$  and  $N_d$  approach infinity [13].

For an idealized MF receiver, we assume that perfect CSI is available at the receiver and that the receiver is ideally synchronized to the received signal [5, 14]. The so-called MFB achieved by the idealized MF receiver describes the performance of uncoded and ISI-free signaling over additive white Gaussian noise [14]. In general, MFB is a theoretical bound that cannot be achieved by practical equalizers because of several factors, such as ISI and the inaccuracies of channel estimation. One-to-one mapping exists between minimum MSE and maximum SINR [15]. The MFB on SINR is selected in this study as the goal of resource allocation optimization for the idealized MF receiver. Considering that the effective noise in Eq. (4) is colored, the pre-whitened equivalent channel matrix required by idealized MF processing is provided by  $\tilde{\mathbf{H}}_t = \mathbf{C}^{-1/2} \mathbf{H}_t$ . Then, the MFB on SINR in the output of the idealized MF receiver can be expressed by

$$\text{SINR}_{\text{MF}} = \frac{1}{N} \text{Tr}\{\tilde{\mathbf{H}}_t \tilde{\mathbf{H}}_t^H\} = \frac{1}{N} \text{Tr}\{\Psi - \mathbf{I}_N\} = \frac{1}{N} \sum_{l=1}^N (\psi_l - 1). \quad (14)$$

The goal of resource allocation is to minimize the MSE (or equivalently maximize the MFB on SINR) given that the performance of FD equalization is directly influenced by the MSE of the SD receiver. Considering the power allocation and SP and based on Eqs. (12), (13), and (14), the objective functions to be minimized can be compactly expressed by

$$f_{\text{Rx}}(\psi_l) = \begin{cases} \sum_{l=1}^N \psi_l^{-1}, & \text{Rx=FD-LE} \\ -\sum_{l=1}^N \log \psi_l, & \text{Rx=FD-DFE} \\ -\sum_{l=1}^N (\psi_l - 1), & \text{Rx=MF} \end{cases} \quad (15)$$

In Eq. (15), the logarithm of the MSE is substituted in the objective function of FD-DFE and has no effect on the optimal solution because of the monotonicity of the logarithm. Considering that the objective functions in Eq. (15) have the summation forms over all the subcarrier pairs, the minimization of Eq. (15) is equal to the sum of the minimization of the objective functions of all the subcarrier pairs. To this end, the objective functions over a given subcarrier pair  $(k, l)$  can be written as

$$f_{\text{Rx}}^{\text{sub}}(\psi_l) = \begin{cases} \psi_l^{-1}, & \text{Rx=FD-LE} \\ -\log \psi_l, & \text{Rx=FD-DFE} \\ -(\psi_l - 1), & \text{Rx=MF} \end{cases} \quad (16)$$

Eq. (16) shows that the all objective functions over any given subcarrier pair are monotonically decreasing functions of  $\psi_l$ . Therefore, for the all considered receivers, a unified framework of resource allocation optimization is implemented.

#### 4. SP and Power Allocation

The problem of joint optimization of SP and power allocation is formulated in this section, and SOA is proposed to optimize SP and power allocation.

According to the principles of the underlay CR model [10, 11], the SU system must limit the generated interference toward PU to coexist with the PU system. Thus, the following interference constraints are considered.

$$\frac{P_0}{N} \sum_{k=1}^N \tilde{\lambda}_k^2 S_k^2 \leq I_{\text{th}} \quad \text{and} \quad \frac{1}{N} \sum_{l=1}^N p_l \tilde{\phi}_l^2 \leq I_{\text{th}}, \quad (17)$$

where  $S_k$  is the  $k$ -th element of  $\mathbf{S}$ ,  $I_{\text{th}}$  is the pre-specified interference threshold at PU, and  $\tilde{\lambda}_k$  ( $\tilde{\phi}_l$ ) is the FD channel response on the  $k$ -th ( $l$ -th) subcarrier of  $\tilde{\mathbf{H}}$  ( $\tilde{\mathbf{G}}$ ). The optimization problem of interest can now be formulated to minimize the objective functions, with resource allocation optimization subject to individual power constraints and the pre-specified interference threshold. The optimal transmit power at SS is obviously provided by  $p_0^\circ = \min\left(P/N, NI_{\text{th}} / \sum_{k=1}^N \tilde{\lambda}_k^2 S_k^2\right)$ . Considering that each and every subcarrier in the first hop can only be paired with a unique subcarrier in the second hop, the SP constraint with respect to permutation matrix  $\mathbf{M}$  can be written as

$$\sum_{k=1}^N [\mathbf{M}]_{l,k} = 1, \forall l \quad \text{and} \quad \sum_{l=1}^N [\mathbf{M}]_{l,k} = 1, \forall k. \quad (18)$$

We let  $p_{lk}$  denote the value of  $p_l$  to be optimized with a given subcarrier pair  $(k, l)$ . We introduce an  $N \times N$  matrix  $\tilde{\mathbf{P}}$  with its  $(l, k)$ -th element being  $p_{lk}$ . Then, with the obtained optimal solution  $p_0^\circ$ , the optimizing problem can be reformulated as

$$\min_{\mathbf{M}, \tilde{\mathbf{P}}} \sum_{l=1}^N \sum_{k=1}^N [\mathbf{M}]_{l,k} f_{\text{RX}}^{\text{sub}}(\tilde{\psi}_{lk}) \quad \text{s.t.} \quad \begin{cases} \text{Eq. (18)}, \sum_{l=1}^N p_{lk} \leq P_1, \\ \text{and } \frac{1}{N} \sum_{l=1}^N p_{lk} \tilde{\phi}_l^2 \leq I_{\text{th}}, \end{cases} \quad (19)$$

where  $\tilde{\psi}_{lk} = 1 + \frac{p_0^\circ p_{lk} \alpha_k \beta_l}{p_0^\circ \alpha_k + p_{lk} \beta_l + 1}$ . The minimization in Eq. (19) with respect to  $\tilde{\mathbf{P}}$  and  $\mathbf{M}$

is a mixed integer programming problem.  $N!$  possible combinations of subcarrier pairs exist, a condition that makes Eq. (19) computationally prohibitive even for a small number of subcarriers. The solution to the dual problem is asymptotically optimal because the duality gap between the optimal solution of Eq. (19) and that of the corresponding dual problem



approaches zero for sufficiently large  $N$  [16]. The corresponding dual Lagrangian is provided by

$$\mathbf{L}(\eta_1, \eta_2, \mathbf{M}, \tilde{\mathbf{P}}) = \sum_{l=1}^N \sum_{k=1}^N [\mathbf{M}]_{l,k} f_{\text{RX}}^{\text{sub}}(\tilde{\psi}_{lk}) + \eta_1 \left( \sum_{l=1}^N p_{lk} - P_1 \right) + \eta_2 \left( \frac{1}{N} \sum_{l=1}^N p_{lk} \tilde{\phi}_l^2 - I_{\text{th}} \right), \quad (20)$$

where  $\eta_1$  and  $\eta_2$  are the dual variables associated with the power constraint and the interference constraint, respectively. By recomposing  $\mathbf{L}(\eta_1, \eta_2, \mathbf{M}, \tilde{\mathbf{P}})$  with respect to the SP constraint, the dual function can be written as

$$\mathbf{G}(\eta_1, \eta_2) = \min_{\mathbf{M}} \sum_{l=1}^N \sum_{k=1}^N [\mathbf{M}]_{l,k} \min_{\mathbf{P}} \left( f_{\text{RX}}^{\text{sub}}(\tilde{\psi}_{lk}) + p_{lk} \left( \eta_1 + \frac{1}{N} \eta_2 \tilde{\phi}_l^2 \right) \right) - \eta_1 P_1 - \eta_2 I_{\text{th}}, \quad (21)$$

s.t. Eq. (18).

As can be seen in Eq. (21), the dual function can be decomposed into two sub-problems: power allocation for any subcarrier pair  $(k, l)$  and SP for a known power allocation.

OPA for any given subcarrier pair  $(k, l)$  is first determined. With the subcarrier pair  $(k, l)$ , the OPA solutions of  $p_{lk}$  can be obtained from

$$\min_{p_{lk} \in \mathcal{D}} f_{\text{RX}}^{\text{sub}}(\tilde{\psi}_{lk}) + p_{lk} \left( \eta_1 + \frac{1}{N} \eta_2 \tilde{\phi}_l^2 \right) - \eta_1 P_1 - \eta_2 I_{\text{th}}. \quad (22)$$

Given that Eq. (22) is a standard convex problem, the KKT conditions provide OPA solutions for FD-LE, FD-DFE, and an idealized MF receiver as follows:

$$p_{lk, \text{FD-LE}}^{\circ} = \left[ \frac{\alpha_k p_0^{\circ}}{\sqrt{\beta_l \chi_l}} - \frac{\alpha_k p_0^{\circ}}{\beta_l (1 + \alpha_k p_0^{\circ})} \right]^+,$$

$$p_{lk, \text{FD-DFE}}^{\circ} = \left[ \sqrt{\frac{p_0^{\circ} \alpha_k}{\beta_l \chi_l} + \frac{p_0^{\circ 2} \alpha_k^2}{4 \beta_l^2}} - \frac{2 + p_0^{\circ} \alpha_k}{2 \beta_l} \right]^+,$$

$$p_{lk, \text{MF}}^{\circ} = \left[ \sqrt{\frac{(1 + p_0^{\circ} \alpha_k) p_0^{\circ} \alpha_k}{\beta_l \chi_l}} - \frac{1 + p_0^{\circ} \alpha_k}{\beta_l} \right]^+,$$

where  $[x]^+ = \max\{0, x\}$  and  $\chi_l = (\eta_1 + \frac{1}{N} \eta_2 \tilde{\phi}_l^2)$ . The above solutions are not only determined by the power budget constraints but also by the maximum allowed interference to PU. By substituting UPA solution  $p_0^{\circ}$  and the OPA solutions into Eq. (21), the dual function becomes

$$\mathbf{G}(\eta_1, \eta_2) = \min_{\mathbf{M}} \sum_{k=1}^N \sum_{l=1}^N [\mathbf{M}]_{l,k} \tilde{f}_{\text{Rx}}^{\text{sub}} - \eta_1 P_1 - \eta_2 I_{\text{th}} \quad \text{s.t. Eq. (18)}, \quad (23)$$

where  $\tilde{f}_{\text{Rx}}^{\text{sub}} = f_{\text{Rx}}^{\text{sub}}(\tilde{\psi}_{lk}^\circ) + p_{lk, \text{Rx}}^\circ (\eta_1 + \frac{1}{N} \eta_2 \tilde{\phi}_l^2)$  with  $\tilde{\psi}_{lk}^\circ = 1 + \frac{p_0^\circ \alpha_k p_{lk, \text{Rx}}^\circ \beta_l}{p_0^\circ \alpha_k + p_{lk, \text{Rx}}^\circ \beta_l + 1}$ .

Once OPA is determined for each and every subcarrier pair, optimal SP can be obtained by solving the following:

$$\mathbf{M}^\circ = \min_{\mathbf{M}} \sum_{k=1}^N \sum_{l=1}^N [\mathbf{M}]_{l,k} \tilde{f}_{\text{Rx}}^{\text{sub}} - \eta_1 P_1 - \eta_2 I_{\text{th}} \quad \text{s.t. Eq. (18)}. \quad (24)$$

The optimal permutation matrix,  $\mathbf{M}^\circ$ , can be obtained by the well-known Hungarian algorithm because the minimization in Eq. (24) is a linear assignment problem [17]. However, the complexity of the Hungarian algorithm is  $\mathcal{O}(N^3)$ , which is too large to be implemented in a real-time system. With the obtained OPA expressions, the complexity of resource allocation is mainly determined by computing the optimal SP. To reduce the complexity of resource allocation, we establish a simplified method of SP. Generally, two simple SP schemes exist, namely, ordered SP and inverse SP, which have both been applied in OFDM-based relay systems to maximize the sum-rate and the minimum SNR, respectively (see [9] and the references therein). Unlike OFDM-based relay systems, the goal of resource allocation in the CP-SC cognitive relay system is to minimize the MSE (or maximize the MFB on SINR). Moreover, SP should be optimized with this objective. The optimality of the ordered SP in maximizing the MFB on SINR for the CP-SC cognitive relay system is provided as Theorem 1.

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**Algorithm 1.** Sequential Optimization Algorithm

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- 1: **Initialization:** Define the maximum number of iterations  $t_{\text{max}}$ ; set the iteration number to  $t = 0$ ; set the initialization values for  $\eta_1$  and  $\eta_2$
  - 2: Determine  $\mathbf{M}^\circ$  using the ordered SP
  - 3: Compute  $p_0^\circ$
  - 4: **while**  $t \leq t_{\text{max}}$  **do**
  - 5:     Compute  $p_{lk}^\circ$  for  $N$  subcarrier pairs  $(k, l)$  according to  $\mathbf{M}^\circ$
  - 6:     Update  $\eta_i^{(t+1)} = \eta_i^{(t)} - \mu^{(t)} \Delta \eta_i^{(t)}$  for  $i=1, 2$
  - 7: **end while**
- 

*Theorem 1:* In a CP-SC cognitive relay system that adopts UPA at SS and OPA at SR, the ordered SP is asymptotically optimal in maximizing the MFB on SINR for a sufficiently large  $N$ .

*Proof:* See Appendix.

According to Theorem 1, regardless of the values of UPA at SS and OPA at SR, the MFB on SINR is asymptotically maximized with the ordered SP. Thus, the permutation matrix

$\mathbf{M}^\circ$  achieved by the ordered SP can be judged as asymptotical optimal beforehand to maximize the MFB on SINR with UPA at SS and OPA at SR.

Sub-gradient method is applied with the obtained  $p_0^\circ$ ,  $\mathbf{M}^\circ$ , and  $\tilde{\mathbf{P}}^\circ$  to update the dual variables by  $\eta_i^{(t+1)} = \eta_i^{(t)} - \mu^{(t)} \Delta \eta_i^{(t)}$  for  $i=1, 2$ , where the sub-gradients of  $\eta_1$  and  $\eta_2$  are provided by  $\Delta \eta_1 = P_1 - \sum_{l=1}^N p_{l|k}^\circ$  and  $\Delta \eta_2 = I_{\text{th}} - \frac{1}{N} \sum_{l=1}^N p_{l|k}^\circ \tilde{\phi}_l^2$ , respectively, and  $\mu^{(t)}$  is a diminishing step size at the  $t$ -th iteration. With the diminishing step size rule, sub-gradient method is guaranteed to converge to the optimal value [16]. To avoid computing all the elements of  $\tilde{\mathbf{P}}^\circ$  and to avoid executing the Hungarian algorithm per iteration, which is required by the joint optimization algorithm (JOA) [7], SOA is proposed in Algorithm 1. In SOA, the ordered SP is decoupled from the power allocation. SOA determines the ordered SP before the power allocation given that implementing the ordered SP before and after the sub-gradient iterations requires  $N$  and  $N^2$  computations of the OPA solution per iteration, respectively. Thus, a significant reduction in complexity is observed with the use of SOA as shown in Table 1.

**Table 1.** Comparison of the Complexity of Different Algorithms

Algorithm Name	OPA Computation Number	SP Computational Complexity
Exhaustive Search	$tN^2$	$\mathcal{O}(tN!)$
JOA	$tN^2$	$\mathcal{O}(tN^3)$
SOA	$tN$	$\mathcal{O}(N^2)$

## 5. Simulation Results

The performances of the proposed SOA are evaluated by simulations in this section. We assume that quadrature phase shift keying (QPSK) modulation is adopted at SS. The number of subcarriers of CP-SC transmission is  $N = 64$ , and the channel length is  $N_f = 16$ . The number of the feedback filter taps is assumed to be  $N_d = N_f$ , and the first tap of the feedback filter is set to 1. The normalized transmit SINRs are defined by  $\text{SINR}_1 = \frac{P_1}{N\sigma_1^2}$  and  $\text{SINR}_2 = \frac{P_2}{N\sigma_2^2}$  for the first and second hops, respectively. The normalized transmission power budgets are set to  $P_0/N = 1$  and  $P_1/N = 1$ , respectively. For simplicity,  $\sigma_1^2 = \sigma_2^2$  is assumed such that  $\text{SINR} = \text{SINR}_1 = \text{SINR}_2$ . In the all simulations, the optimal UPA is employed at SS, i.e.,  $p_0^\circ = \min\left(P/N, NI_{\text{th}} / \sum_{k=1}^N \tilde{\lambda}_k^2 s_k^2\right)$ . For the purpose of comparison, the following resource allocation schemes are considered at SR: (1) Only UPA, which employs UPA at SR without SP; (2) SP+UPA, which employs both the ordered SP and UPA at SR; (3) Only OPA, which employs OPA at SR without SP; (4) JOA of [7], which optimizes SP and OPA at SR in an iterative manner; and (5) SOA, which is the proposed SOA adopted at SR to implement the ordered SP and OPA.

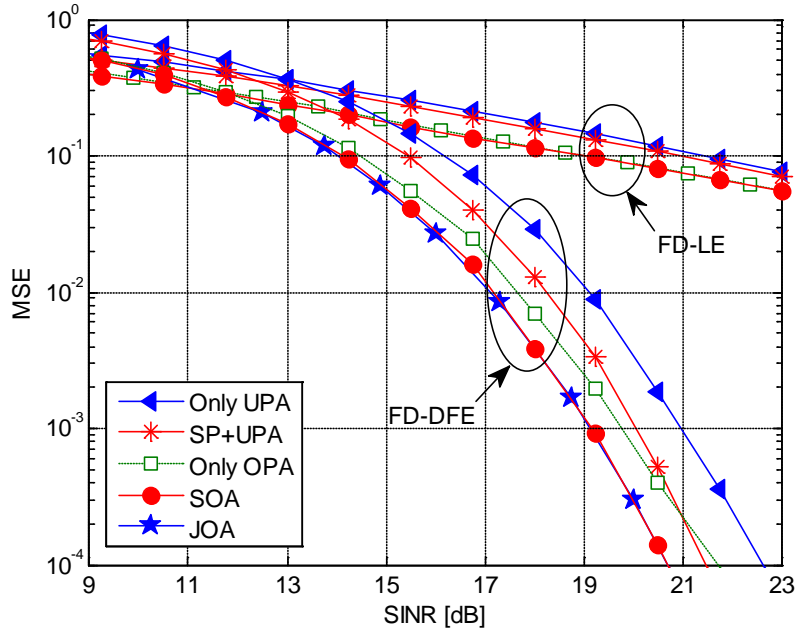


Fig. 2. MSE vs. SINR with  $I_{th} = -3$  dB.

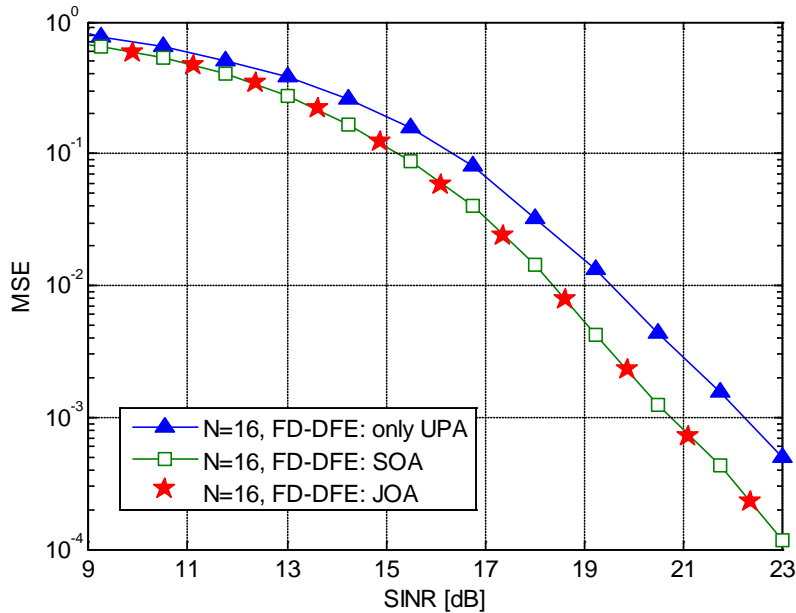


Fig. 3. MSE vs. SINR with  $I_{th} = -3$ dB and  $N = 16$ .

Fig. 2 shows MSE versus SINR for the different resource allocation schemes, with  $I_{th} = -3$ dB. For FD-LE and FD-DFE, when the ordered SP and UPA are employed at SR (denoted by SP+UPA), the MSEs decrease compared with when only UPA is employed. However, for FD-LE, the marginal gain achieved by the ordered SP is trivial because of the large MSE (approximately  $10^{-1}$ ). For FD-LE and FD-DFE, the only OPA scheme decreases

the MSEs. For FD-LE, the MSE achieved by SOA is almost similar to that of only OPA because of the trivial marginal gain achieved by the ordered SP; this result verifies that the MSE reduction by SOA is mainly achieved by power allocation. The proposed SOA achieves the lowest MSEs for the FD-LE and FD-DFE receivers. For FD-DFE with the MSE of  $10^{-3}$ , SOA achieves 2dB gain compared with only UPA. Furthermore, for FD-DFE, the MSE achieved by SOA is almost coincident with that of JOA. This result indicates that SOA can achieve almost the same MSE performance as that of JOA. A similar situation is observed in the case of FD-LE, which is not plotted in Fig. 2 to conserve space.

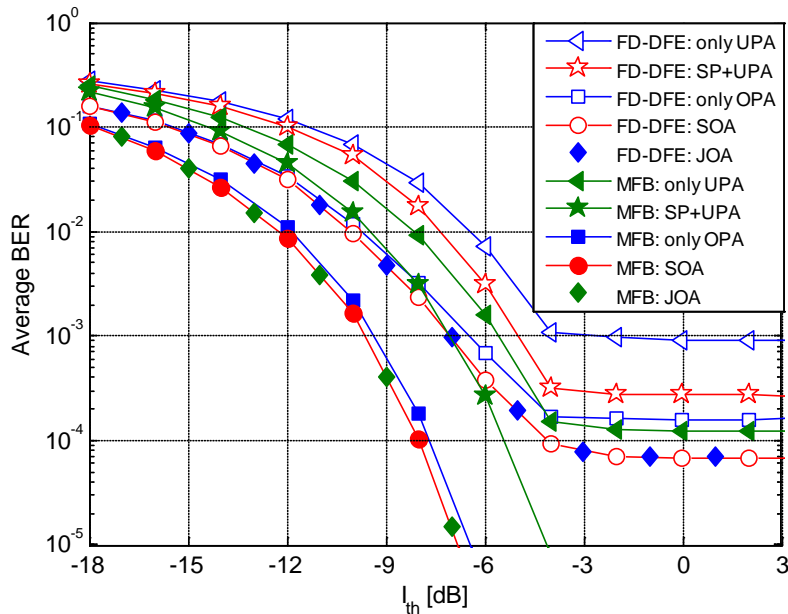


Fig. 4. Average BER vs.  $I_{th}$  with the SINR = 20 dB.

The simulation results of MSE versus SINR for FD-DFE in the case of ( $N = 16, N_q = N_f = 4$ ) are presented in Fig. 3. Compared with the scheme of only UPA, SOA achieves approximately 1.5 dB gain with the MSE of  $10^{-3}$ . SOA achieves almost the same MSE as that of JOA, a result that verifies not only the effectiveness of SOA but also the effectiveness of the ordered SP in minimizing MSE even with a small  $N$ .

Fig. 4 shows the average BER versus the interference threshold for the different schemes with SINR fixed at 20 dB. For FD-DFE and MFB, the marginal gains achieved by the ordered SP and OPA are verified by the simulation results, respectively. Both SOA and JOA achieve the lowest average BER for FD-DFE and the lowest MFB, respectively. Once  $I_{th}$  reaches a large value ( $> -4$ dB in this case), the average BER of all the schemes cannot decrease anymore. An error floor occurs, which corresponds to the scenario in which the allowed interference to PU is sufficiently large and the power budget must be allocated fully to minimize the MSE of the SD receiver.

## 6. Conclusion

SOA was developed for the optimization of SP and power allocation in a dual hop CP-SC cognitive relay system. Resource allocation optimization was transformed to its equivalent Lagrange dual problem to minimize the MSE of the SD receiver. The Lagrange dual problem was then decomposed into two sub-problems of power allocation and SP. The ordered SP was proven to be asymptotically optimal in maximizing the MFB on SINR. The resource allocation problem was effectively addressed by SOA in a sequential optimizing manner with a significant reduction in complexity.

### Appendix: Proof of Theorem 1

For the scheme of the ordered SP, the signal received on the subcarrier with the largest effective SINR on the SS-to-SR link should be forwarded to the subcarrier with the largest effective SINR on the SR-to-SD link. The signal received on the second-best subcarrier on the SS-to-SR link should be forwarded to the second-best subcarrier on the SR-to-SD link and so on. We first consider two subcarrier pairs with  $\alpha_1, \alpha_2$  and  $\beta_1, \beta_2$ . Using an idealized MF receiver, the MFB on SINR achieved with the ordered subcarrier pairs  $((\beta_1, \alpha_1)$  and  $(\beta_2, \alpha_2))$  for any given UPA solution  $p_0^\circ$  and OPA solution  $p_{1|k, MF}^\circ$  is provided by

$$\text{SINR}_{MF}^\circ = \frac{1}{2} \left( \frac{p_0^\circ p_{1|1, MF}^\circ \alpha_1 \beta_1}{p_0^\circ \alpha_1 + p_{1|1, MF}^\circ \beta_1 + 1} + \frac{p_0^\circ p_{2|2, MF}^\circ \alpha_2 \beta_2}{p_0^\circ \alpha_2 + p_{2|2, MF}^\circ \beta_2 + 1} \right). \quad (\text{A.1})$$

By substituting  $p_{1|k, MF}^\circ = \left[ \sqrt{\frac{(1+p_0^\circ \alpha_k) p_0^\circ \alpha_k}{\beta_1 \chi_1}} - \frac{1+p_0^\circ \alpha_k}{\beta_1} \right]^+$  into Eq. (A.1),  $\text{SINR}_{MF}^\circ = \frac{(\alpha_1 + \alpha_2) p_0^\circ}{2} - \frac{\sqrt{\alpha_1 \beta_1 p_0^\circ (1 + \alpha_1 p_0^\circ)} \chi_1}{2 \beta_1} - \frac{\sqrt{\alpha_2 \beta_2 p_0^\circ (1 + \alpha_2 p_0^\circ)} \chi_2}{2 \beta_2}$ . Similarly, the MFB on SINR achieved with the subcarrier pairs  $((\beta_2, \alpha_1)$  and  $(\beta_1, \alpha_2))$  is provided by

$$\text{SINR}_{MF}^\circ = \frac{(\alpha_1 + \alpha_2) p_0^\circ}{2} - \frac{\sqrt{\alpha_2 \beta_1 p_0^\circ (1 + \alpha_2 p_0^\circ)} \chi_1}{2 \beta_1} - \frac{\sqrt{\alpha_1 \beta_2 p_0^\circ (1 + \alpha_1 p_0^\circ)} \chi_2}{2 \beta_2}. \quad (\text{A.2})$$

Thus, we have

$$\begin{aligned}
\Delta_{\text{SINR}} &= \text{SINR}_{\text{MF}}^{\circ} - \text{SINR}_{\text{MF}} \\
&= \frac{\sqrt{\beta_1 \chi_1 p_0^{\circ}} \left( \sqrt{\alpha_2 (1 + \alpha_2 p_0^{\circ})} - \sqrt{\alpha_1 (1 + \alpha_1 p_0^{\circ})} \right)}{2\beta_1} \\
&\quad + \frac{\sqrt{\beta_2 \chi_2 p_0^{\circ}} \left( \sqrt{\alpha_1 (1 + \alpha_1 p_0^{\circ})} - \sqrt{\alpha_2 (1 + \alpha_2 p_0^{\circ})} \right)}{2\beta_2} \tag{A.3} \\
&\stackrel{(a)}{\approx} \frac{\sqrt{\beta_1 \chi_1 p_0^{\circ}} (\alpha_2 - \alpha_1)}{2\beta_1} + \frac{\sqrt{\beta_2 \chi_2 p_0^{\circ}} (\alpha_1 - \alpha_2)}{2\beta_2} \\
&= \frac{p_0^{\circ} (\alpha_1 - \alpha_2) (\beta_1 \sqrt{\beta_2 \chi_2} - \beta_2 \sqrt{\beta_1 \chi_1})}{2\beta_1 \beta_2},
\end{aligned}$$

where an approximation of  $1 + \alpha_k p_0^{\circ} \approx \alpha_k p_0^{\circ}$  is applied in the high SINR region in (a).  $\chi_l = (\eta_1 + \frac{1}{N} \eta_2 \tilde{\phi}_l^2)$ , and we have  $\chi_1 \approx \chi_2$  for a sufficiently large  $N$ . Thus,  $\Delta_{\text{SINR}} \rightarrow 0$  for a sufficiently large  $N$  with an equality when  $\alpha_1 = \alpha_2$  or  $\beta_1 = \beta_2$ . This condition proves the case of two subcarrier pairs. Similarly, we can readily prove Theorem 1 for more than two subcarrier pairs.

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