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연속과 이산시간 폴리토픽 불확실성을 가지는 특이시스템의 산일성 필터링

Robust Dissipative Filtering for Polytopic Uncertain Singular Systems in Continuous & Discrete Time

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Abstract - This paper considers the problem of continuous and discrete time dissipative filter design method for the singular systems with polytopic uncertainties. Two bounded real lemmas (BRL) for the robust filters of dissipative singular systems along with the polytopic uncertainties are proposed on the basis of Lyapunov criterion. The sufficient conditions for both continuous and discrete time dissipative filter design methods are derived using the obtained BRL by linear matrix inequality (LMI) approach. Finally, the validities of the proposed methods are shown by numerical examples.

Key Words : Dissipativity, Singular systems, Robust filter, Polytopic uncertainty

1. 서 론

Dissipative System takes on an important role when considering a flow of any entity. Matter such as liquid, gas, or energy could be considered as a dissipative system [1], as described and modeled by many scientists. In the early 70's, one of the vikings for dissipative system, J. C. Willems [1], wrote the general theory about dissipativeness. J. C. Willems [1] used state space model and dissipativeness and defined an inequality involving storage function and the supply function [2]. However, the concept of dissipativeness steered in a different direction. Ilya Prigogine, a well known pioneer in dissipative system was the first to establish the dissipative structure for thermodynamics [3]. Though Prigogine's fundamental concept is based on chemistry and physics, nevertheless, the theory of dissipative structure is treated as an open system model with extended capacity to transformation and revolutionize based on the same concept, the flow of entity. Progogine's dissipative structure concluded that the order in an open system can be maintained only in a non-equilibrium condition [4], which regards to the need of a continuous exchange of energy and resources in an open system with the environment in order to be able to renew itself. However, dissipative system under control theory has developed to be somewhat opposite to thermodynamic's model; a model in which the dissipative structure is considered to be operated under an equilibrium state. The extensive study of this topic redefines its significants, as mechanical and electrical engineers continue to strive deeper and expand its role in various applications.

Nevertheless, dissipative system is not only being researched under the bases of control theory, other branches of study such as Physics, specifically quantum mechanics, model the system as quantum dissipative systems, where the dissipative systems in quantum mechanics studies the system in microscopic level that focuses on the behaviour of the system [5].

The system dynamics of dissipative system is quite diverse ever since the model was introduced by Prigogine [4]. The topic of study for dissipative system branches of into a wide range of theories and contexts, the branches of study also becomes narrow, complex, and specific. In terms of control theory, it is important to continue to explore and pursue the impact of dissipative system, for its contribution meets no bound nor a single form which can describes its essence.

Regardless with the perspective of dissipative system, the dissipative property, for which it can be considered when dealing with a flow of any entity, is handle uniquely by the system modeller as desired. Thus, there is no generalization nor a "right way" to model dissipative system, it act as an add-on toward a

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pre-existing model, which one can only judge the model's efficiency, simplicity and performance without the lost of generality.

The models and considering systems of dissipative systems usually deals with unique cases, such as Yan and Liu [6] or Kaneko et al. [7]. Though the cases are different, modelling and the system dynamics are unlikely to be the same, it is difficult to compare or judge one's model with another. However, even with a similar objective and considering systems, by using different methods and techniques, it would still be difficult to compare. A similar paper back in 2010 [8], with similar objective and considering system, it is noticeable that their method is basic and lack of conservativeness.

In this paper, we will present a more efficient and compact method, to retain assurance of the system performance in spite of model inaccuracies and changes [9], and the notion of dissipativity being the boundedness of free supply that can be extracted from a system [1] is expressed in a compact filter form. The advantage of having dissipative properties in such filter form allows us to relate and handle many mechanical and electrical engineering applications [10], where energy is of interest. Hence, the search for a compacted approach to combine these properties into one useful tool begins here.

2. Continuous-time filter design

Consider the continuous-time singular systems,

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bw(t) \\ y(t) &= Cx(t) + Dw(t) \\ z(t) &= Lx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measurement vector, $z(t) \in R^p$ is the signal to be estimated, $w(t) \in R^q$ is the input signal, and E is singular matrix that satisfies the $rank(E) = r \le n$. All systems matrices have appropriate dimensions.

Definition 1. [9-10] *Linear uncertain system is a convex polytopic set defined as* $\delta(x) = A(\alpha)x$ *, and is said to be quadratically stable in the uncertain box of* $A(\alpha)x = \sum_{i=1}^{N} \alpha_i A_i$ *, given that* $\sum \alpha_i = 1$ *and* $\alpha_i \ge 0$*, if and only if there exists a matrix* $P = P^T > 0$ *such that* $A_i^T P + PA_i < 0$.

By Definition 1, the considering system (1) will now be represented with polytopic uncertainty properties in the following form,

$$\begin{aligned} E\dot{x}(t) &= A_i x(t) + B_i w(t) \\ y(t) &= C_i x(t) + D_i w(t) \\ z(t) &= L_i x(t) \end{aligned} \tag{2}$$

where i is the natural number index.

The objective is to design the full-order linear filter with polytopic uncertainty, defined in Definition 1, in the following is the state-space realization form

$$\begin{aligned} \dot{\hat{x}}(t) &= A_f \hat{x}(t) + B_f w(t) \\ \hat{z}(t) &= C_f \hat{x}(t) + D_f w(t) \end{aligned} \tag{3}$$

where $A_f \in R^{n \times n}, B_f \in R^{n \times m}, C_f \in R^{p \times n}$, and $D_f \in R^{p \times m}$. In the following sections, time t will omit the for simplicity.

2.1 Robust filtering for continuous-time

By combining the considering system (2) with the full-order linear filter (3), where the augmented state vector is defined as $\tilde{x} = \left[x^T \; \hat{x}^T\right]^T$ and the estimation error as $\tilde{z} = z - \hat{z}$, we can obtain the following filtering error singular system as

$$\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ B_f C_i & A_f \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B_i \\ B_f Di \end{bmatrix} w \quad . \tag{4}$$

Taking the difference between the estimated signal with the linear filter signal, the following solution is obtained:

$$\tilde{z} = [L_i - D_f C_i \quad -C_f] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} - D_f D_i w \tag{5}$$

By defining some terms as follows

$$\begin{aligned} &\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \dot{\tilde{x}} = \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix}, \quad \tilde{x} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \\ &\tilde{z} = z - \hat{z}, \quad \tilde{A} = \begin{bmatrix} A_i & 0 \\ B_f C & A_f \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_i \\ B_f D_i \end{bmatrix}, \\ &\tilde{C} = \left[L_i - D_f C_i \; - C_f\right], \quad \tilde{D} = -D_f D_i \end{aligned}$$

thus, the filtering error dynamics is expressed by

$$\begin{aligned} \tilde{E}\dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}w \\ \tilde{z} &= \tilde{C}\tilde{x} + \tilde{D}w \end{aligned} \tag{6}$$

Using the Lyapunov stability criterion, a sufficient condition for the stability of an equilibrium system can be determined without solving the state equation. Let $V(\tilde{x})$ be a continuously differentiable vector function. The system is asymptotically stable if there exists a positive-definite function $V(\tilde{x}) : P \rightarrow R^n$ [11], where

$$V(\tilde{x}) = \tilde{x}^T \tilde{E}^T P \tilde{E} \tilde{x} \tag{7}$$

such that $\frac{dV(\tilde{x})}{dt} < 0$,

$$\frac{dV(\tilde{x})}{dt} = \dot{\tilde{x}}^T \tilde{E}^T P \tilde{E} \tilde{x} + \tilde{x}^T \tilde{E}^T P \tilde{E} \dot{\tilde{x}} \tag{8}$$

Therefore (8) is said to be negative definite, where $\frac{dV(\tilde{x})}{dt}$ is the derivative along the trajectory of filtering error dynamics (6).

Definition 2. [12] *The filtering error dynamic system expressed by (5) is said to be passive with a dissipation* η *if* $\int_0^t (w^T \tilde{z} - \eta w^T w)dt \geq 0$ *holds all trajectories with* $x(0) = 0$ *and all* $t \geq 0$.

To add on the dissipativity characteristics to this linear filter, the descriptor system must satisfy the dissipation performance inequality by Definition 2, expressed as J_p. η is the dissipative constant that controls the overall performance of the system, $w^T \tilde{z}$ is the supply rate of the system, and $w^T w$ is the weighting factor of the dissipation rate.

Theorem 1. *Consider the continuous-time system (2). For a given positive constant* η *that is less than 1, if there exist a positive-definite matrix of* P *and* $R \in R^{nxn}$ *matrix satisfying* $E^T R = 0$ *such that*

$$\begin{bmatrix} \Theta_1 & \Theta_2 & \Theta_3 \\ * & \Theta_4 & \Theta_5 \\ * & * & \Theta_6 \end{bmatrix} < 0 \tag{9}$$

$$\begin{aligned}
\Theta_1 &= \langle A_i^T P_1 E \rangle + \langle E^T P_2 \overline{B}_f C_i \rangle + \langle A_i^T R Z^T \rangle \\
\Theta_2 &= \overline{A_f} + \overline{B_f}^T C_i^T P_2^T + E^T \overline{A_f} \\
\Theta_3 &= E^T P_1 B_i + E^T P_2 \overline{B}_f D_i + Z R^T B_i - L_i^T + \overline{D}_f^T C_i^T \\
\Theta_4 &= \langle \overline{A}_f \rangle \\
\Theta_5 &= F E B_i + \overline{B}_f D_i + \overline{C}_f^T \\
\Theta_6 &= 2\eta I + \langle \overline{D}_f D_i \rangle
\end{aligned}$$

is quadratically stable with optimum system performance and dissipativeness. The robust filter variables are defined as the following

$$A_f = \overline{A}_f F^{-1}, \; B_f = \overline{B_f}, \; C_f = \overline{C_f} F^{-1}, \; D_f = \overline{D}_f .$$

Proof. An auxiliary term introduced to (8) to prevent any rank deficiency, and to maintain the system to be negative definite in the final solution as shown in the following [13]

$$2\dot{\tilde{x}}^T \tilde{E}^T \tilde{R} \tilde{Z}^T \tilde{x} = 0 . \tag{10}$$

$\tilde{Z}$ is a constant matrix yet to be determined by the LMI condition simulator, and $\tilde{R}$ through R that satisfies

$$E^T R = 0 . \tag{11}$$

The robust filter variables can be determined from the following inequality,

$$\frac{dV(\tilde{x})}{dt} + J_p + 2\dot{\tilde{x}}^T \tilde{E}^T \tilde{R} \tilde{Z}^T \tilde{x} < 0 . \tag{12}$$

With the appropriate manipulation of (12), the following condition is derived,

$$\zeta^T \Lambda \zeta < 0 . \tag{13}$$

$$\zeta = [x^T w^T]^T$$

$$\Lambda = \begin{bmatrix} \tilde{A}^T P \tilde{E} + \tilde{E}^T P \tilde{A} + \tilde{A}^T \tilde{R} \tilde{Z}^T + \tilde{A} \tilde{R}^T \tilde{Z} & \tilde{E}^T P \tilde{B} - \tilde{C}^T + \tilde{B} \tilde{R}^T \tilde{Z} \\ \tilde{B}^T P \tilde{E} - \tilde{C} + \tilde{B}^T \tilde{R} \tilde{Z}^T & -\tilde{D} - \tilde{D}^T + 2\eta I \end{bmatrix}$$

To further derive a sufficient condition, some terms are defined as follows:

$$\tilde{E}^T = \begin{bmatrix} E^T & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{Z} = \begin{bmatrix} Z & I \\ 0 & I \end{bmatrix} \tag{14}$$
$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad \tilde{E}^T \tilde{R} = 0.$$

In order to separate out the variables within the same term in Λ to construct the LMI, similarity transformation technique [14] will be performed, and the following terms are defined as

$$J = \begin{bmatrix} I & 0 \\ 0 & P_2 P_3^{-1} \end{bmatrix}, \quad \Gamma = diag(J, I) . \tag{15}$$

The result of $\Gamma \Lambda \Gamma^T < 0$ can be changed to the following form

$$\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 \\ * & \Pi_4 & \Pi_5 \\ * & * & \Pi_6 \end{bmatrix} < 0 . \tag{16}$$

$$\begin{aligned}
\Pi_1 &= \langle A_i^T P_1 E \rangle + \langle E^T P_2 B_f C_i \rangle + \langle A_i^T R Z^T \rangle \\
\Pi_2 &= A_i^T P_2 P_3^{-1} P_2^T + B_f^T C_i^T P_2^T + E^T P_2 P_3^{-1} P_2^T A_f \\
\Pi_3 &= E^T P_1 B_i + z R^T B_i + E^T P_2 B_f D_i - L_i^T + D_f^T C_i^T \\
\Pi_4 &= \langle P_2 A_f P_3^{-1} P_2^T \rangle \\
\Pi_5 &= P_2 P_3^{-1} P_2^T E B_i + P_2 B_{fi} D_i + P_2 P_3^{-1} C_f^T \\
\Pi_6 &= 2\eta I + \langle D_f D_i \rangle
\end{aligned}$$

Where $*$ in (16) denotes the symmetric terms in the matrix, and for simplicity, terms such as $\Phi+\Phi^T$ are represented as $<\Phi>$. We define the following new variables as

$$\overline{A_f} = P_2A_fP_3^{-1}P_2^T,\quad \overline{B_f} = B_f,\quad \overline{C_f} = C_fP_2^TP_3^{-T},\quad \overline{D_f} = D_f,\quad F = P_2P_3^{-T}P_2^T \tag{17}$$

in order to avoid nonlinear terms within the LMI of (16). Therefore, by substituting in (17), we can obtained the LMI condition of (9).

Example 1. Consider the following continuous-time systems matrices of (2) with polytopic uncertainty of two vertices that satisfies Theorem 1:

$$A_1 = \begin{bmatrix} 0.5 & 0.3 \\ 0.5 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.4 & -0.1 \\ 0.4 & 0.9 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.4 & -0.1 \\ 0.4 & 0.9 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix},\ D = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ R = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix},\ \eta = 0.8$$

All four possible configurations of the two vertices are implemented for simulation. Using the command "*[tmin,aopt]=feasp(LMISYS);*", a feasible solution of the system is obtained, with the feasibility radius saturation at 0.005%. the system variables are as follows:

$$P_1 = 5021.3,$$
$$F = \begin{bmatrix} 1002 & 2272 \\ 2272 & 21579 \end{bmatrix},\qquad Z = \begin{bmatrix} 0.0 & -24664 \\ -24664 & 3954 \end{bmatrix}$$
$$\overline{A_f} = \begin{bmatrix} -7277.5 & -508.2 \\ -508.2 & -9025.1 \end{bmatrix}\quad \overline{B_f} = \begin{bmatrix} 13038 & -6599 \\ -6599 & -18155 \end{bmatrix}$$
$$\overline{C_f} = \begin{bmatrix} -10743 & 5646 \\ 5646 & 18494 \end{bmatrix}.\quad \overline{D_f} = \begin{bmatrix} -15849 & 6454 \\ 6454 & -5611 \end{bmatrix}$$

By using the transfer matrix identity [15], the relationship between $[\overline{A_f}, \overline{B_f}, \overline{C_f}, \overline{D_f}]$ and $[A_f, B_f, C_f, D_f]$ is derived to be

$$A_f = \overline{A_f}F^{-1},\quad B_f = \overline{B_f},\quad C_f = \overline{C_f}F^{-1},\quad D_f = \overline{D_f}$$

Therefore,

$$A_f = \begin{bmatrix} -9.4714 & 0.9738 \\ 0.5797 & -0.4793 \end{bmatrix},\quad B_f = \begin{bmatrix} 13038 & -6599 \\ -6599 & -18155 \end{bmatrix},$$
$$C_f = \begin{bmatrix} -14.8653 & 1.8270 \\ 4.8499 & 0.3463 \end{bmatrix},\quad D_f = \begin{bmatrix} -15849 & 6454 \\ 6454 & -5611 \end{bmatrix}.$$

Thus, the resulting filter guarantees the system's stability and optimal performance with the given dissipative variable $\eta = 0.8$.

3. Discrete-time filter design

Consider the discrete-time singular systems,

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bw(k) \\ y(k) &= Cx(k) + Dw(k) \\ z(k) &= Lx(k) \end{aligned} \tag{18}$$

where $x(k) \in R^n$ is the state vector, $y(k) \in R^m$ is the measurement vector, $z(k) \in R^p$ is the signal to be estimated, $w(k) \in R^q$ is the input signal, and E is singular matrix that satisfies the $rank(E) = r \le n$. All systems matrices have appropriate dimensions.

By Definition 1, the considering system (18) will now be represented with polytopic uncertainty properties in the following form,

$$\begin{aligned} Ex(k+1) &= A_ix(k) + B_iw(k) \\ y(k) &= C_ix(k) + D_iw(k) \\ z(k) &= L_ix(k) \end{aligned} \tag{19}$$

where i is the natural number index.

With the objective same as continuous-time, the design the full-order linear filter with polytopic uncertainty, defined by Definition 1, is in the following state-space realization form

$$\begin{aligned} E\hat{x}(k+1) &= A_f\hat{x}(k) + B_fw(k) \\ \hat{z}(k) &= C_f\hat{x}(k) + D_fw(k) \end{aligned} \tag{20}$$

where $A_f \in R^{n\times n}$, $B_f \in R^{n\times m}$, $C_f \in R^{p\times n}$, $D_f \in R^{p\times m}$.

3.1 Robust filtering for discrete-time

Similar to continuous-time, system (19) and (20) are combined, where the augmented state vector is defined as $\tilde{x}(k) = [x^T(k)\ \hat{x}^T(k)]^T$ and the estimation error as $\tilde{z}(k) = z(k) - \hat{z}(k)$, we can obtain the following filtering error descriptor system as

$$\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ B_fC_i & A_f \end{bmatrix}\begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} + \begin{bmatrix} B_i \\ B_fDi \end{bmatrix}w \tag{21}$$

and by taking the difference between the estimated signal with the linear filter signal we will obtain the following

$$\tilde{z}(k) = [L_i - D_fC_i \quad -C_f]\begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} - D_fD_iw. \tag{22}$$

Once again, some terms will be defined as follows

$$\tilde{E}=\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix},\quad \tilde{x}(k+1)=\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix},\quad \tilde{x}(k)=\begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix},$$
$$\tilde{z}(k)=z(k)-\hat{z}(k),\quad \tilde{A}=\begin{bmatrix} A_i & 0 \\ B_f C & A_f \end{bmatrix},\quad \tilde{B}=\begin{bmatrix} B_i \\ B_f D_i \end{bmatrix},$$
$$\tilde{C}=[L_i-D_f C_i-C_f],\quad \tilde{D}=-D_f D_i$$

then the filtering error dynamics is expressed by

$$\begin{aligned} \tilde{E}\tilde{x}(k+1) &= \tilde{A}\tilde{x}(k)+\tilde{B}w(k) \\ \tilde{z}(k) &= \tilde{C}\tilde{x}(k)+\tilde{D}w(k) \end{aligned} \tag{23}$$

By Lyapunov stability criterion, $V(\tilde{x})$ is defined as the infinite sum vector function, where

$$V(k)=\tilde{x}^T(k)\tilde{E}^T P\tilde{E}\tilde{x}(k) \tag{24}$$

such that $\Delta V<0$. Therefore

$$\begin{aligned} \Delta V &= V(k+1)-V(k) \\ &= \tilde{x}^T(k+1)\tilde{E}^T P\tilde{E}\tilde{x}(k+1)-\tilde{x}^T(k)\tilde{E}^T P\tilde{E}\tilde{x}(k). \end{aligned} \tag{25}$$

Similar to continuous-time, definition 2 is extended to discrete-time, the filtering error dynamic system expressed by (23) is said to be passive with a dissipation η if $\sum_{k=0}^{n}(w(k)^T\tilde{z}(k)-\eta w(k)^T w(k)\geq 0$ holds for all trajectories with $x(0)=0$ and $k\geq 0$ [12].

To join the dissipative characteristics with the linear filter, the singular system must satisfy the dissipation performance inequality defined by definition 2, and it is expressed below as J_p.

$$\begin{aligned} J_p &= -2w(k)^T\tilde{z}+2nw(k)^T w(k)\geq 0 \\ &= -w(k)^T\tilde{z}-w(k)\tilde{z}^T+2nw(k)^T w(k)\geq 0 \end{aligned} \tag{26}$$

where η is the dissipation variable that controls the overall performance of the system, $w(k)^T\tilde{z}$ is the supply rate of the system, and $w(k)^T w(k)$ is the weighting factor of the dissipation rate.

Theorem 2. *Consider the discrete-time system (19). For a given positive constant η that is less than 1, if there exist a positive-definite matrix of P and $R\in R^{n\times n}$ matrix satisfying $E^T R=0$ such that*

$$\Pi=\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 \\ * & -P_3 & \overline{C_f}^T & \overline{A_f}^T & \overline{A_f}^T \\ * & * & \Pi_6 & \Pi_7 & \Pi_8 \\ * & * & * & \Pi_9 & \Pi_{10} \\ * & * & * & * & \Pi_{11} \end{bmatrix}<0 \tag{27}$$

$$\begin{aligned}
\Pi_1 &= -E^T P_1 E+< A_i^T R Z^T > \\
\Pi_2 &= -E^T P_2 \\
\Pi_3 &= -L_i^T+C_i^T D_f^T+ZR^T B_i \\
\Pi_4 &= A_i^T X_1^T+C_i^T\overline{B_f}^T \\
\Pi_5 &= A_i^T F_1^T+C_i^T\overline{B_f}^T \\
\Pi_6 &= -< D_f D_i >+2\eta I \\
\Pi_7 &= B_i^T X_1^T+D_i^T\overline{B_f}^T \\
\Pi_8 &= B_i^T F_1^T+D_i^T\overline{B_f}^T \\
\Pi_9 &= P_1-< X_1 > \\
\Pi_{10} &= P_2-F_2-F_1^T \\
\Pi_{11} &= P_3-< F_2 >
\end{aligned}$$

is quadratically stable with optimum system performance and dissipativeness. The robust filter variables are defined as the following

$$A_f=\overline{A_f}F_2^{-1},\quad B_f=\overline{B_f},\quad C_f=\overline{C_f}X_4^T X_2^{-T},\quad D_f=\overline{D_f}.$$

Proof. Since the approach is different than continuous-time, minor steps are highlighted to show the manipulation difference for both cases. Likewise to continuous-time, an auxiliary term is introduced to prevent any rank deficiency, and to maintain the system negative definite properties in the final solution as shown in the following [13]

$$2\tilde{x}(k+1)^T\tilde{E}^T\tilde{R}\tilde{Z}^T\tilde{x}(k)=0 \tag{28}$$

where $\tilde{Z}$ is the constant matrix to be determined and $\tilde{R}$ can be selected satisfy $\tilde{E}^T\tilde{R}=0$. The linear filter variables can be determined by combining all three properties matrix inequality: filtering error dynamic system, dissipativity, and auxiliary term.

$$\Delta V+J_p+2\tilde{x}(k+1)^T\tilde{E}^T\tilde{R}\tilde{Z}^T\tilde{x}(k)<0 \tag{29}$$

With the appropriate manipulation and substitution of (29), the matrix inequality can be expressed as

$$\begin{aligned} &(\tilde{A}\tilde{x}+\tilde{B}w)^T P(\tilde{A}\tilde{x}+\tilde{B}w)-\tilde{x}^T\tilde{E}^T P\tilde{E}\tilde{x} \\ &-w^T(\tilde{C}\tilde{x}+\tilde{D}w)-w(\tilde{C}\tilde{x}+\tilde{D}w)^T+2\eta w^T w \\ &+2(\tilde{A}\tilde{x}+\tilde{B}w)^T R\tilde{Z}^T\tilde{x}<0 \end{aligned} \tag{30}$$

Note: all function of k is omitted from here on for simplicity.

By reforming (30) in matrix inequality form and apply the first technique, Schur complement [16], we will obtain the following matrix inequality,

$$\zeta^T\Lambda\zeta<0 \tag{31}$$

where (31) is defined as follows

$$\zeta = \begin{bmatrix} x \\ w \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \tilde{E}^T P \tilde{E} + \tilde{A}^T \tilde{R} \tilde{Z}^T + \tilde{Z} \tilde{R}^T \tilde{A} & -\tilde{C}^T + \tilde{Z} \tilde{R}^T \tilde{B} & \tilde{A}^T H^T \\ -\tilde{C} + \tilde{B}^T \tilde{R} \tilde{Z}^T & -\tilde{D} - \tilde{D}^T + 2\eta I & \tilde{B}^T H^T \\ \tilde{A} H & \tilde{B} H & P - H - H^T \end{bmatrix}$$

Since the matrix inequality (31) becomes nonlinear from simply matrix substitutions and expansions, the stability check using LMI solver in Matlab cannot be performed and will lead to an unsolvable matrix inequality if we proceed further. Therefore, a second technique is introduced, called the slack variable denote by H [16-17]. To derive the sufficient condition from (30), we will define the following as

$$\tilde{E}^T = \begin{bmatrix} E^T & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{Z} = \begin{bmatrix} Z & I \\ 0 & I \end{bmatrix},$$
$$JPJ^T = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad \tilde{E}^T \tilde{R} = 0, \quad H = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix},$$
$$F_1 = X_2 X_4^{-1} X_3, \quad F_2 = X_2 X_4^{-T} X_2^T$$

There exist matrix H where $H + H^T > 0$, thus, $X_4 + X_4^T > 0$, which implies X_4 is invertible.

$$\tilde{\Lambda} =$$
$$\begin{bmatrix} -E^T P_1 E + < A_i^T R Z^T > & -E^T P_2 & -L_i^T + D_f^T C_i^T + Z R^T B_i & A_i^T X_1^T + C_i^T B_f^T X_2^T & A_i^T X_3^T X_4^{-T} X_2^T \; C_i^T B_f^T X_2^T \\ * & -P_3 & C_i^T & X_2 X_4^{-1} A_f^T X_2^T & X_2 X_4^{-1} A_f^T X_2^T \\ * & * & -< D_f D_i > + 2\eta I & B_i X_1^T + B_f D_i X_2^T & B_i X_3^T \; B_f D_i X_4^T \\ * & * & * & P_1 - < X_1 > & P_2 - X_2 - X_3^T \\ * & * & * & * & P_3 - < X_4 > \end{bmatrix}$$

The $*$ denotes the symmetric term in a symmetric matrix. For simplicity, terms such as $\Phi + \Phi^T$ are represented as $< \Phi >$.

Although the slack variable allows us to proceed on with the matrix manipulation, however, a similar nonlinear result will arise as illustrated above, $\tilde{\Lambda}$. The LMI solver still cannot be used until a third technique is applied. Similar transformation will be performing on Λ to separate out multiple variables in one term.

Let $J = \begin{bmatrix} I & 0 \\ 0 & X_2 X_4^{-1} \end{bmatrix}, \quad \Gamma = diag(J, I, J)$

$\overline{A_f} = A_f F_2, \quad \overline{B_f} = B_f, \quad \overline{C_f} = C_f X_4^{-T} X_2^T, \quad \overline{D_f} = D_f.$

The results of $\Gamma \Lambda \Gamma^T < 0$, along with its internal matrices expansion, the final form is obtained as (27).

Example 2. Consider the following discrete-time system:

$$A_1 = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.4 & -0.1 \\ 0.4 & 0.9 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}, \; L_1 = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}, \; L_2 = \begin{bmatrix} 0.9 & -0.1 \\ 0.4 & 0.9 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad \eta = 0.8$$

that satisfies Theorem 2. Once again, all possible configurations of the two vertices are used for this simulation. Thus, by using the same Matlab command *"[tmin,aopt]=feasp(LMISYS);"*, a feasible solution is obtained for this system, with the feasibility radius saturation at 0%. The variables are obtained as follows:

$$P_1 = 0.8353, \; P_2 = 0.8353, \; P_3 = 1.3973,$$
$$F_1 = \begin{bmatrix} -0.6200 & 0.2268 \\ 0.2268 & 0.0544 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1.6087 & -0.0640 \\ -0.0640 & 1.4342 \end{bmatrix},$$
$$Z = \begin{bmatrix} 0.0 & 0.5129 \\ 0.5129 & 1.3068 \end{bmatrix},$$

$$\overline{A_f} = \begin{bmatrix} 0.0223 & -0.0060 \\ -0.0060 & -0.0018 \end{bmatrix}, \; \overline{B_f} = \begin{bmatrix} -0.2235 & -0.0556 \\ -0.0556 & -0.5549 \end{bmatrix},$$
$$\overline{C_f} = \begin{bmatrix} -0.1514 & -0.1343 \\ -0.1343 & 0.0002 \end{bmatrix}, \; \overline{D_f} = \begin{bmatrix} 2.2862 & -0.5041 \\ -0.5041 & 2.3316 \end{bmatrix}.$$

By using the transfer matrix identity [16], the relationship between $[\overline{A_f}, \overline{B_f}, \overline{C_f}, \overline{D_f}]$ and $[A_f, B_f, C_f, D_f]$ is derived to be

$$A_f = \overline{A_f} F_2^{-1}, \quad B_f = \overline{B_f}, \quad C_f = \overline{C_f} F_2^{-1}, \quad D_f = \overline{D_f}.$$

The resulting filter guarantees the system's stability and optimal performance with the given dissipative variable $\eta = 0.8$.

4. Conclusion

A sufficient condition has been satisfied for this robust dissipative filtering system. Formulated as a compact set of linear matrix inequality, the filtering variable has been determined in continuous and discrete time case. The filter is constructed with properties of robustness and dissipativity. Filtering variables can be easily obtained by using a LMI solver.

Hence, this dissipative filtering technique raises significant improvement and uses for application, in both continuous and discrete time, that deals with the flow of energy due to its compactness and robustness. The dissipative filtering technique guarantees maximum performance of flow rate and stability of the system. Though this may not be the most generic form, however, the proposed method offers multi-purpose and flexibility within its design. Future study, research, and extension may continue along this path for deeper and wider use of dissipativeness.

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