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# 시변지연 및 임의 발생 외란이 존재하는 선형 동적 시스템의 신뢰성 제어

# Reliable Control for Linear Dynamic Systems with Time-varying Delays and Randomly Occurring Disturbances

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**Abstract** - In this paper, the problem of reliable control of linear systems with time-varying delays, randomly occurring disturbances, and actuator failures is investigated. It is assumed that actuator failures occur when disturbances affect to the systems. Firstly, by using a suitable Lyapunov-Krasovskii functional and some recent techniques such as Wirtinger-based integral inequality and reciprocally convex approach, stabilization criterion for nominal systems with time-varying delays is derived. Secondly, the proposed method is extended to the reliable $H_\infty$ controller design for linear dynamic systems with time-varying delays, randomly occurring disturbances, and actuator failures. Since nonlinear matrix inequalities (NLMIs) are involved in proposed results, the cone complementarity algorithm will be introduced. Finally, two numerical examples are included to show the effectiveness of the proposed criteria.

**Key Words** : Reliable control, Time-delays, Randomly occurring disturbances, Actuator failures, Lyapunov method

## 1. Introduction

The stabilization of linear systems with time-delays is an important issue since time-delays occurs in various systems such as physical and chemical systems, industrial and engineering systems, and so on. It is well known that time-delays can lead to oscillation, poor performance or even instability. Therefore, the problem of delay-dependent stability and stabilization criteria for systems with time-delays have been received a great deal of efforts by many researchers [1-4].

One of the objectives of delay-dependent stabilization for systems with time-delays is to find maximum upper-bounds of time-delays which guarantee the asymptotic stability of the concerned. In order to reduce the conservatism of stabilization criteria for systems with time-delays, many researchers have focused on delay-dependent criteria than delay-independent ones since delay-dependent ones are less conservative than delay-independent ones especially when the sizes of time-delays are small. While delay independent once do not have information about time-delays, delay-dependent criteria have ones such as low-bound, upper-bounds and bounds of differential of delays.

In the last decade, the Jensen inequality has been intensively used for analysis of systems with time-delays since it plays key roles to derive a stability condition when estimating the time-derivative of Lyapunov-Krasovskii functional. Very recently, in order to reduce the conservatism of stability criteria obtained by utilizing the Jensen Inequality, Wirtinger-based integral inequality [5] is introduced for stability analysis based on Fourier analysis. It can lead to less conservative results than Jensen inequality for integral terms since Wirtinger-based integral inequality allows considering a more accurate integral inequality. In this paper, Wirtinger-based integral inequality is used to obtain stabilization criteria.

On the other hand, recently, the problem of designing reliable control systems has been attracted since practical systems often have actuator failures [6-7]. It has been known that the class of reliable control systems is to stabilize the systems against actuator failures or to design fault-tolerant control systems. In this paper, actuator failure model which consists of a scaling factor with upper and lower bounds to the signal to be measured or to the control action is introduced.

In line with this thinking, disturbances can have an adverse effect on the stability of systems. Thus, to design a controller for the systems considering disturbances is another important issue in control society. For instance, disturbances such as earthquake and typhoon, controllers are required to minimize the effect of

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disturbances on building or structure systems. The $H_\infty$ control has objective that is to design the controllers such that the closed-loop systems are stable and its $H_\infty$-norm of the transfer function between the controlled output and the disturbances will not exceed a prescribed level of performance. Therefore, since $H_\infty$ control [8] was introduced firstly, a number of research results on $H_\infty$ control have been utilized for various systems [9-13].

Recently, a variety of stochastic systems have been researched [13-15]. Systems with time-delays and stochastic sampling were considered in [13]. Also, Systems with randomly occurring uncertainties have introduced in [14-15]. From the idea of randomly occurring concept, it can be extended to reliable control problem since disturbances can bring out the actuator failures. In other words, when randomly occurring disturbances affect to the system, actuator failures occur simultaneously.

With motivations for the above discussions, this paper focused on the problem of the reliable $H_\infty$ controller design for linear systems with time-delays. Firstly, in Theorem 1, stabilization criterion will be proposed by using the appropriate Lyapunov-Krasovskii functional with Wirtinger-based integral inequality [5] and reciprocally convex approach [16]. Secondly, based on the results of Theorem 1, a reliable $H_\infty$ controller design method for the systems with time delays, randomly occurring disturbances, and actuator failures will be proposed in Theorem 2. Since results in Theorem 1 and Theorem 2 have developed in terms of NLMIs, the cone complementarity algorithm will be introduced which developed solve the NLMIs [12,17]. Two numerical examples are included to show the effectiveness of the proposed theorems.

**Notations:** $R^n$ denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices. $diag\{\cdots\}$ denotes the block diagonal matrix. $L_2$ is the space of square integrable functions on $[0,\infty)$. For two symmetric matrices $A$ and $B$, $A>(\geq)B$ means that $A-B$ is (semi-) positive definite. $A^T$ denotes the transpose of $A$. $I_n$ denotes the $n\times n$ identity matrix. $0_n$ and $0_{n\times m}$ are denote the $n\times n$ zero matrix and $n\times m$ zero matrix, respectively. If the context allows it, the dimensions of these matrices are often omitted. $L_2[0,\infty)$ is the space of square integrable vector. $\mathrm{E}\{x\}$ and $\mathrm{E}\{x|y\}$ will, respectively, mean the expectation of $x$ and the expectation of $x$ condition on $y$. $X_{[f(t)]}\in R^{m\times n}$ means that the elements of the matrix $X_{[f(t)]}$ includes the value of $f(t)$; e.g., $X_{[f_0]}\equiv X_{[f(t)=f_0]}$. $\Pr\{\cdot\}$ means the occurrence probability of the event "$\cdot$".

## 2. Problem Statements

Consider the following linear system with time-varying delay:

$$\begin{cases}\dot{x}(t)=Ax(t)+A_dx(t-h(t))+Bu^F(t)+B_ww(t),\\ z(t)=Cx(t),\end{cases}\tag{1}$$

where $x(t)\in\mathrm{R}^n$ is the state vector, $u^F(t)\in\mathrm{R}^m$ is the vector of controlled input with actuator failures, $z(t)\in\mathrm{R}^p$ is the vector of controlled output, $w(t)\in\mathrm{R}^l$ is the disturbance input which belongs to $L_2[0,\infty)$. $A\in\mathrm{R}^{n\times n}$, $A_d\in\mathrm{R}^{n\times n}$, $B_w\in\mathrm{R}^{n\times l}$, $B\in\mathrm{R}^{n\times m}$ and $C\in\mathrm{R}^{q\times n}$ are known real constant matrices.

Also, $h(t)$ is a time-delay satisfying time-varying continuous function as follows:

$$0\leq h(t)\leq h_M \text{ and } \dot{h}(t)\leq h_d\,,\tag{2}$$

where $h_M$ is a positive scalar and $h_d$ is any constant value.

In this paper, it is concerned that actuator has behaviour of faulty. The control input of actuator fault can be described as

$$u^F(t)=Ru(t),\tag{3}$$

where $u(t)\in R^m$ is the vector of controlled input and $R$ is the actuator fault matrix with

$$R=diag\{r_1,r_2,\ldots,r_m\},\ 0\leq \underline{r_i}\leq r_i\leq \overline{r_i},\ \overline{r_i}\geq 1,\\ (i=1,2,\ldots,m),\tag{4}$$

where $\underline{r_i}$ and $\overline{r_i}$ $(i=1,2,\ldots,m)$, are given constants. When $r_i=0$, it means the complete failure of $i$th actuator. If $r_i=1$, then $i$th actuator is normal.

Let us define

$$R_0=diag\{r_{10},r_{20},\ldots,r_{m0}\},\ r_{i0}=\frac{\overline{r_i}+\underline{r_i}}{2},\\ R_1=diag\{r_{11},r_{21},\ldots,r_{m1}\},\ r_{i1}=\frac{\overline{r_i}-\underline{r_i}}{2}.\tag{5}$$

Then, the actuator fault matrix $R$ can be rewritten as

$$R=R_0+R_1\Delta J,\tag{6}$$

where $\Delta J=diag\{j_1,j_2,\ldots,j_m\}$, $-1\leq j_i\leq 1$.

It is assumed that actuator failure and disturbances occur randomly. In details, if disturbances occur, then it affects to the system and leads to actuator failures. So, it can be seen that disturbances and actuator failures occur simultaneously.

In order to describe the random occurrence, let us define $\rho(t)$ as a stochastic variable which satisfy a Bernoulli distribution as follows:

$$\rho(t)=\begin{cases}1, \text{if disturbances and actuator failures occur}, \\ 0, \text{otherwise}.\end{cases} \quad (7)$$

Also, $\rho(t)$ satisfies the following condition

$$\Pr\{\rho(\mathrm{t})=1\}=\mathrm{E}\{\rho(t)\}=\rho_0, \quad (8)$$

where $0 \le \rho_0 \le 1$ is a given constant scalar. $\rho_0$ is the expectation of $\rho(t)$ and reflects the occurrence probability of disturbances and actuator failures.

With the concept introduced at Eqs. (3)-(8), let us consider the following linear system with time-varying delay with randomly occurring disturbances and actuator failures given by

$$\begin{cases}\dot{x}(t)=Ax(t)+A_d x(t-h(t))+Bu^F(t)+\rho(t)B_w w(t), \\ z(t)=Cx(t).\end{cases} \quad (9)$$

Also, actuator failure model with randomly occurrence can be described as

$$u^F(t)=(\rho(t)R+(1-\rho(t))I_m)u(t), \quad (10)$$

where term of $(1-\rho(t))I_m$ reflects normal actuator when $\rho(t)$ is 0.

The problem under consideration is to design a memoryless state feedback controller of the following form:

$$u(t)=Kx(t), \quad (11)$$

where $K \in \mathrm{R}^{m\times n}$ is a gain matrix of the feedback controller.

To develop a delay-dependent reliable $H_\infty$ controller for the system (9) satisfying following conditions:

(i) With zero disturbance, the closed loop system (9) with control input $u(t)$ is asymptotically stable.

(ii) With zero condition and a given constant $\gamma>0$, the following condition holds:

$$J=\mathrm{E}\left\{\int_0^\infty z^T(t)z(t)-\gamma^2 w^T(t)w(t)dt\right\}\le 0, \quad (12)$$
$$\left(i.e. \sup_{w\neq 0, w\in L_2[0,\infty]}\frac{\|z(t)\|_2}{\|w(t)\|_2}\le\gamma\right),$$

where $\gamma\ge 0$ is a prescribed scalar. The objective of this paper is to design a state feedback controller (11) such that system (9) is asymptotically stable and an disturbance attenuation level $\gamma$ is minimize. If the above objective is achieved, controller (11) is said to be a reliable $H_\infty$ controller.

Before deriving main results, the following lemmas are introduced.

**Lemma 1.** [5] *For a given matrix $R>0$, the following inequality holds for all continuously differentiable function $\omega$ in $[a,b]\to\mathrm{R}^n$*

$$\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du \ge \frac{1}{b-a}(\omega(b)-\omega(a))^T R(\omega(b)-\omega(a)) + \frac{3}{(b-a)}\Omega^T R\Omega,$$

*where* $\Omega=\omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b\omega(u)du.$

**Lemma 2.** [16] *For a scalar $\alpha$ in the interval $(0,1)$, a given matrix $R\in\mathrm{R}^{n\times n}>0$, two matrices $W_1\in\mathrm{R}^{n\times m}$ and $W_2\in\mathrm{R}^{n\times m}$, for all vector $\zeta\in R^m$, let us the function $\Theta(\alpha,R)$ given by:*

$$\Theta(\alpha,R)=\frac{1}{\alpha}\zeta^T W_1^T RW_1\zeta+\frac{1}{1-\alpha}\zeta^T W_2^T RW_2\zeta.$$

*Then, if there exists a matrix $X\in\mathrm{R}^{n\times m}$ such that $\begin{bmatrix}R & X\\ * & R\end{bmatrix}>0$, then the following inequality holds*

$$\min_{\alpha\in(0,1)}\Theta(\alpha,R)\ge\begin{bmatrix}W_1\zeta\\ W_1\zeta\end{bmatrix}^T\begin{bmatrix}R & X\\ * & R\end{bmatrix}\begin{bmatrix}W_1\zeta\\ W_1\zeta\end{bmatrix}.$$

**Lemma 3.** [18] Let $E$, $H$, and $F(t)$ be real matrices of appropriate dimensions, and let $F(t)$ satisfy $F^T(t)F(t)\le I$. Then, for any scalar $\epsilon>0$, the following matrix inequality holds:

$$EF(t)H+H^TF^T(t)E^T\le\epsilon H^TH+\epsilon^{-1}EE^T.$$

## 3. Main Results

This section consists of two subsections. The goal of first subsection is to design a controller which stabilize the nominal system. Second subsection will introduce a design method of a reliable $H_\infty$ controller for linear systems with time-varying delays, randomly occurring disturbances, and actuator failures.

### 3.1 Controller design for nominal system

In this subsection, a delay-dependent stabilization criterion for the nominal system of (9) without disturbances and actuator failures will be introduced. Here, the following nominal system with control input $u(t)$ is given by

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Bu(t), \tag{13}$$

where $h(t)$ is satisfied with (2) and $u(t)$ is defined in (11). Now, for simplicity of matrix and vector representation, $e_i (i=1,...,5) \in \mathbb{R}^{5n \times n}$ are defined as block entry matrices which will be used. For example, $e_1 = [I_n, 0_n, 0_n, 0_n, 0_n]^T$ and $e_3 = [0_n, 0_n, I_n, 0_n, 0_n]^T$. The other notations are defined as

$$\zeta(t) = \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds \\ \frac{1}{h_M - h(t)}\int_{t-h_M}^{t-h(t)} x(s)ds \end{bmatrix},$$

$$\tilde{N} = \begin{bmatrix} N & 0_n \\ * & 3N \end{bmatrix},\ \hat{N} = \begin{bmatrix} \overline{N} & 0_n \\ * & 3\overline{N} \end{bmatrix},$$

$$\Xi_1 = \left((A+BK)e_1^T + A_d e_2^T\right),$$

$$\Xi_2 = \begin{bmatrix} e_1^T \\ e_3^T \\ e_1^T \\ e_2^T \end{bmatrix}^T,\ \Xi_3 = \begin{bmatrix} e_1^T - e_2^T \\ e_1^T + e_2^T - 2e_4^T \\ e_2^T - e_3^T \\ e_2^T + e_3^T - 2e_5^T \end{bmatrix}^T,$$

$$\overline{\Xi}_1 = \left((AX+BY)e_1^T + A_d X e_2^T\right),$$

$$\Lambda_1 = diag\{Q_1, -Q_1, Q_2, -(1-h_d)Q_2\},$$

$$\Lambda_2 = \begin{bmatrix} \tilde{N} & M \\ * & \tilde{N} \end{bmatrix},$$

$$\overline{\Lambda}_1 = diag\{\overline{Q}_1, -\overline{Q}_1, \overline{Q}_2, -(1-h_d)\overline{Q}_2\},$$

$$\overline{\Lambda}_2 = \begin{bmatrix} \hat{N} & \overline{M} \\ * & \hat{N} \end{bmatrix},$$

$$\Phi_1 = e_1 P \Xi_1 + \Xi_1^T P e_1^T,\ \Phi_2 = \Xi_2 \Lambda_1 \Xi_2^T,\ \Phi_3 = -\Xi_3 \Lambda_2 \Xi_3^T,$$

$$\overline{\Phi}_1 = e_1 \overline{\Xi}_1 + \overline{\Xi}_1^T e_1^T,\ \overline{\Phi}_2 = \Xi_2 \overline{\Lambda}_1 \Xi_2^T,\ \overline{\Phi}_3 = -\Xi_3 \overline{\Lambda}_2 \Xi_3^T,$$

$$\overline{\Phi} = \begin{bmatrix} \overline{\Phi}_1 + \overline{\Phi}_2 + \overline{\Phi}_3 & h_M \overline{\Xi}_1^T \\ * & -X\overline{N}^{-1}X \end{bmatrix}. \tag{14}$$

Now, the following theorem is given as a stabilization criterion for the system (13).

**Theorem 1.** *For given scalars $h_M > 0$, $h_d$, the system (13) is asymptotically stable for $0 \le h(t) \le h_M$ and $\dot{h}(t) \le h_d$, if there exist positive definite matrices $X \in \mathbb{R}^{n \times n}$, $\overline{Q}_1 \in \mathbb{R}^{n \times n}$, $\overline{Q}_2 \in \mathbb{R}^{n \times n}$, $\overline{N} \in \mathbb{R}^{n \times n}$, any matrices $\overline{M} \in \mathbb{R}^{2n \times 2n}$ and $Y \in \mathbb{R}^{m \times n}$, satisfying the following conditions hold:*

$$\overline{\Phi} < 0, \tag{15}$$

$$\overline{\Lambda}_2 \ge 0, \tag{16}$$

*where $\overline{\Phi}$ and $\overline{\Lambda}_2$ are defined in (14). If the above conditions are feasible, a desired controller gain matrix is obtained by $K = YX^{-1}$.*

**Proof.** For positive definite matrices $P$, $Q_1$, $Q_2$ and $N$, let us consider the following the Lyapunov-Krasovskii functional candidate as:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{17}$$

where

$$V_1(t) = x^T(t)Px(t),$$
$$V_2(t) = \int_{t-h_M}^{t} x^T(s)Q_1 x(s)ds + \int_{t-h(t)}^{t} x^T(s)Q_2 x(s)ds,$$
$$V_3(t) = h_M \int_{t-h_M}^{t}\int_{s}^{t} \dot{x}^T(u)N\dot{x}(u)duds.$$

Then, the time-derivative of $V_1(x_t)$ is

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t)P\dot{x}(t) \\ &= 2x^T(t)P\left((A+BK)x(t) + A_d x(t)\right) \\ &= \zeta^T(t)\Phi_1\zeta(t). \end{aligned} \tag{18}$$

The upper-bound of $\dot{V}_2(t)$ can be given as follows:

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)Q_1x(t) - x^T(t-h_M)Q_2x(t-h_M) \\ &\quad + x^T(t)Q_2x(t) \\ &\quad - (1-\dot{h}(t))x^T(t-h(t))Q_2x(t-h(t)), \\ &\le x^T(t)Q_1x(t) - x^T(t-h_M)Q_2x(t-h_M) \\ &\quad + x^T(t)Q_2x(t) \\ &\quad - (1-h_d)x^T(t-h(t))Q_2x(t-h(t)), \\ &= \zeta^T(t)\Phi_2\zeta(t). \end{aligned} \tag{19}$$

$\dot{V}_3(t)$ is calculated as

$$\begin{aligned} \dot{V}_3(t) &= h_M^2\dot{x}^T(t)N\dot{x}(t) - h_M\int_{t-h(t)}^{t}\dot{x}^T(s)N\dot{x}(s)ds \\ &\quad - h_M\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)N\dot{x}(s)ds. \end{aligned} \tag{20}$$

By using Lemma 1, an upper-bound of $\dot{V}_3(t)$ can be obtained as

$$\begin{aligned} \dot{V}_3(t) &\le h_M^2\dot{x}^T(t)R\dot{x}(t) \\ &\quad - \frac{h_M}{h(t)}(x(t)-x(t-h(t)))^T N(x(t)-x(t-h(t))) \\ &\quad - \frac{h_M}{h(t)}(x(t-h(t))-x(t-h_M))^T N \\ &\quad \times (x(t-h(t))-x(t-h_M)) \end{aligned}$$

$$
\begin{aligned}
&-\frac{3h_M}{h(t)}\Omega_1^T N\Omega_1-\frac{3h_M}{h(t)+h_M}\Omega_2^T N\Omega_2\\
&=h_M^2\dot{x}^T(t)N\dot{x}(t)\\
&-\frac{1}{\phi(t)}\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\end{bmatrix}^T\hat{N}\\
&\times\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\end{bmatrix}\zeta(t)\\
&-\frac{1}{1-\phi(t)}\begin{bmatrix}x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}^T\\
&\times\hat{N}\begin{bmatrix}x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}\zeta(t),
\end{aligned}\tag{21}
$$

where

$$
\begin{aligned}
&\phi(t)=\frac{h(t)}{h_M},\ \tilde{N}=\begin{bmatrix}N & 0_n\\ 0_n & 3N\end{bmatrix},\\
&\Omega_1=x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds,\\
&\Omega_2=x(t-h(t))+x(t-h_M)-\frac{2}{h_M-h(t)}\int_{t-h_M}^{t-h(t)}x(s)ds.
\end{aligned}
$$

From Lemma 2, if the inequality for any matrix $M\in\mathbb{R}^{2n\times 2n}$ holds

$$
\Lambda=\begin{bmatrix}\tilde{N} & M\\ * & \tilde{N}\end{bmatrix}\geq 0,\tag{22}
$$

then, a new upper-bound of (21) is can be obtained as

$$
\begin{aligned}
\dot{V}_3(t)&\leq h_M^2\dot{x}^T(t)N\dot{x}(t)\\
&-\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\\ x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}^T\\
&\times\begin{bmatrix}\frac{1}{\phi(t)}\tilde{N} & 0_{2n}\\ * & \frac{1}{1-\phi(t)}\tilde{N}\end{bmatrix}\\
&\times\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\\ x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}\\
&\leq h_M^2\dot{x}^T(t)N\dot{x}(t)\\
&-\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\\ x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}^T\\
&\times\begin{bmatrix}\tilde{N} & M\\ * & \tilde{N}\end{bmatrix}\begin{bmatrix}x^T(t)-x^T(t-h(t))\\ \Omega_1\\ x^T(t-h(t))-x^T(t-h_M)\\ \Omega_2\end{bmatrix}\\
&=\zeta^T(t)\left(h_M^2\Xi_1 N\Xi_1^T+\Phi_3\right)\zeta(t).
\end{aligned}\tag{23}
$$

Note that $\phi(t)$ satisfies $0\leq\phi(t)\leq 1$. When $h(t)=0$, $x^T(t)-x^T(t-h(t))=0$ and $\Omega_1=0$ are obtained and when $h(t)=h_M$, $x^T(t-h(t))-x^T(t-h_M)=0$ and $\Omega_2=0$ are obtained. Thus, relation (23) still holds.

By combining (18)-(23), an upper-bound of $\dot{V}(t)$ is obtained as follows:

$$
\dot{V}(t)\leq\zeta^T(t)\left(\Phi_1+\Phi_2+\Phi_3+h_M^2\Xi_1 N\Xi_1^T\right)\zeta(t).\tag{24}
$$

By using Schur complement, stabilization criterion for the system (24) is equivalent to the following

$$
\begin{bmatrix}\Phi_1+\Phi_2+\Phi_3 & \Xi_1^T\\ * & -N^{-1}\end{bmatrix}<0.\tag{25}
$$

Let us define $X=P^{-1}$, $\overline{Q}_1=X^TQ_1X$, $\overline{Q}_2=X^TQ_2X$, $\overline{N}=X^TNX$, $\overline{M}=\begin{bmatrix}X & 0_n\\ * & X\end{bmatrix}^T M\begin{bmatrix}X & 0_n\\ * & X\end{bmatrix}$, and $Y=KX$. Then, following inequalities can be obtained by pre- and post-multiplying (25) and (22) by $diag\{X,X,X,X,X,I_n\}$ and $diag\{X,X,X,X\}$, respectively

$$
\overline{\Phi}<0,\tag{26}
$$

$$
\overline{\Lambda}_2\geq 0.\tag{27}
$$

where $\overline{\Phi}$ and $\overline{\Lambda}_2$ are defined in (14). This proof is completed. □

It should be note that the stabilization condition (15) have the nonlinear term $X\overline{N}X$. A simple way to solve it is to set $\overline{N}=\alpha X$, where $\alpha>0$ is a tuning parameter. However, this method is too conservative. To obtain better results, the cone complementarity algorithm can be used with computational effort.

In order to solve NLMIs, the cone complementarity algorithm in [12,17] is used which involves iteratively solving linear matrix inequalities (LMIs). Let us define a new variable matrix $L>0$ satisfying

$$
L\leq X\overline{N}^{-1}X,\tag{28}
$$

which is equivalent $X^{-1}\overline{N}X^{-1}\leq L^{-1}$. Letting $H=L^{-1}$, $G=X^{-1}$, $F=\overline{N}^{-1}$ and following a similar method in [12,17], the problem of finding a feasible solution of (15) and (16) can be converted to a minimization problem involving LMIs:

Minimize Trace $(LH+XG+\overline{N}F)$
Subject to

$$
\begin{bmatrix}X & I_n\\ * & G\end{bmatrix}\geq 0,\ \begin{bmatrix}H & G\\ * & F\end{bmatrix}\geq 0,\ \begin{bmatrix}L & I_n\\ * & H\end{bmatrix}\geq 0,\ \begin{bmatrix}\overline{N} & I_n\\ * & F\end{bmatrix}\geq 0,
$$

$$\begin{bmatrix} \overline{\Phi}_1 + \overline{\Phi}_2 + \overline{\Phi}_3 & \Xi_1 \\ * & -L \end{bmatrix} < 0,$$
$$\overline{\Lambda}_2 \geq 0. \qquad (29)$$

The above minimization problem can be solved using the cone complementarity algorithm in [12,17].

**Algorithm**

Let us define $\Gamma^k$ as the set of the variables of $\{X^k, \overline{Q}_1^k, \overline{Q}_2^k, \overline{N}^k, M^k, H^k, G^k, F^k\}$ and $k_{\max}$ as the number of iterations. Then, following Figure 1 is the flow chart of algorithm for Theorem 1.

### 3.2 Reliable H∞ controller design for randomly occurring disturbances and actuator failures

In this subsection, the reliable $H_\infty$ controller design for the system (9) will be derived based on Theorem 1. Now, for simplicity of matrix and vector representation, $\overline{e}_i (i=1,\ldots,6) \in \mathrm{R}^{(5n+l)\times n}$ are defined as block entry matrices which will be used. For an example,

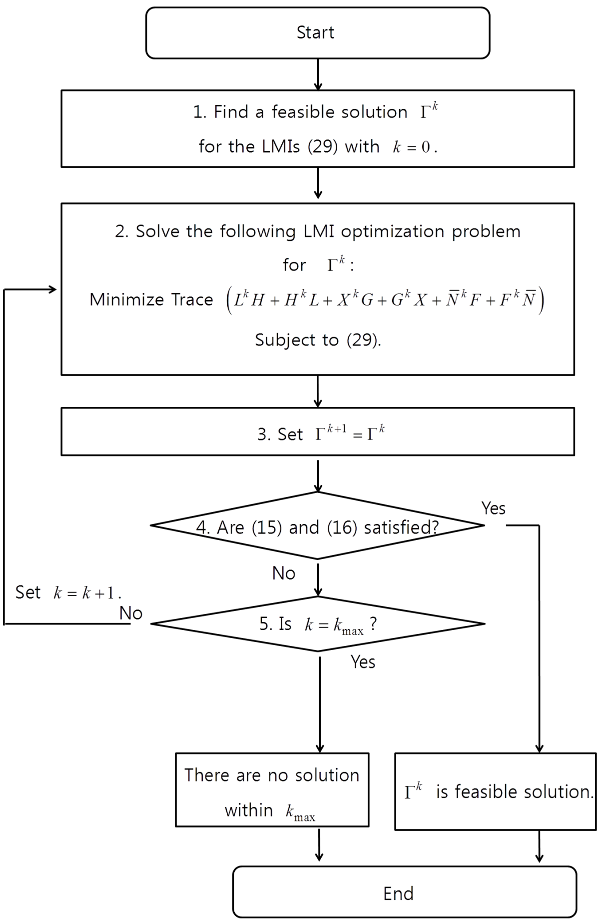



**그림** 1 Theorem 1을 위한 알고리듬 흐름선도

**Fig.** 1 Flow chart of Algorithm for Theorem 1

$\overline{e}_5 = [0_n, 0_n, 0_n, 0_n, I_n, 0_{n\times l}]^T$. The following notations are defined for simplicity:

$$H_1 = R_1^T B^T P \overline{e}_1^T,\ H_2 = R_1^T B^T,$$
$$E_{1[\rho(t)]} = \rho(t)\overline{e}_1 K^T,\ E_{2[\rho(t)]} = h_M \rho(t)\overline{e}_1 K^T,$$
$$\Pi_{1[\rho(t)]} = (A+BK)\overline{e}_1^T + A_d \overline{e}_2^T + \rho(t) B_w \overline{e}_6 + B\big(\rho(t)R_0 + (1-\rho(t))\big)K\overline{e}_1^T,$$
$$\Delta\Pi_{1[\rho(t)]} = \rho(t) B R_1 \Delta J K \overline{e}_1^T,$$
$$\Psi_{1[\rho(t)]} = \overline{e}_1 P \Pi_{1[\rho(t)]} + \Pi_{1[\rho(t)]}^T P \overline{e}_1^T,$$
$$\Psi_{2[\rho(t)]} = h_M \Pi_{1[\rho(t)]},$$
$$\Psi_{[\rho(t)]} = \Psi_{1[\rho(t)]} + \Phi_2 + \Phi_3 - \gamma^2 I \overline{e}_6 \overline{e}_6^T,$$
$$\Delta\Psi_{1[\rho(t)]} = \overline{e}_1 P \Delta\Pi_{1[\rho(t)]} + \Delta\Pi_{1[\rho(t)]}^T P \overline{e}_1^T,$$
$$\Delta\Psi_{2[\rho(t)]} = h_M \Delta\Pi_{1[\rho(t)]},$$
$$\Sigma_{1[\rho(t)]} = \begin{bmatrix} \Psi_{[\rho(t)]} & \Psi_{2[\rho(t)]} & \overline{e}_1 C^T \\ * & -R^{-1} + \epsilon_2 H_2^T H_2 & 0_{n\times q} \\ * & * & -I_q \end{bmatrix},$$
$$\Sigma_{2[\rho(t)]} = \begin{bmatrix} E_{1[\rho(t)]} & E_{2[\rho(t)]} \\ 0_{n\times m} & 0_{n\times m} \\ 0_{q\times m} & 0_{q\times m} \end{bmatrix},\ \Sigma_3 = diag\{-\epsilon_1 I_m, -\epsilon_2 I_m\},$$
$$\Sigma_{[\rho(t)]} = \begin{bmatrix} \Sigma_{1[\rho(t)]} & \Sigma_{2[\rho(t)]} \\ * & \Sigma_3 \end{bmatrix},$$
$$\overline{H}_1 = R_1^T B^T \overline{e}_1^T,\ \overline{H}_2 = R_1^T B^T,$$
$$\overline{E}_{1[\rho(t)]} = \rho(t)\overline{e}_1 Y^T,\ \overline{E}_{2[\rho(t)]} = h_M \rho(t)\overline{e}_1 Y^T,$$
$$\overline{\Pi}_{1[\rho(t)]} = (AX+BY)\overline{e}_1^T + A_d X \overline{e}_2^T + \rho(t) B_w \overline{e}_6 + B\big(\rho(t)R_0 + (1-\rho(t))\big) Y \overline{e}_1^T,$$
$$\overline{\Psi}_{1[\rho(t)]} = \overline{e}_1 \overline{\Pi}_{1[\rho(t)]} + \overline{\Pi}_{1[\rho(t)]}^T \overline{e}_1^T,$$
$$\overline{\Psi}_{2[\rho(t)]} = h_M \overline{\Pi}_{1[\rho(t)]},$$
$$\overline{\Psi}_3 = -X\overline{N}^{-1}X + \epsilon_2 H_2^T H_2,$$
$$\overline{\Psi}_{[\rho(t)]} = \overline{\Psi}_{1[\rho(t)]} + \overline{\Phi}_2 + \overline{\Phi}_3 + \epsilon_1 \overline{H}_1^T \overline{H}_1 - \gamma^2 \overline{e}_6 \overline{e}_6^T,$$
$$\overline{\Sigma}_{[\rho(t)]} = \begin{bmatrix} \overline{\Psi}_{[\rho(t)]} & \overline{\Psi}_{2[\rho(t)]} & \overline{e}_1 X C^T & \overline{E}_{1[\rho(t)]} & \overline{E}_{1[\rho(t)]} \\ * & \overline{\Psi}_3 & 0_{n\times q} & 0_{n\times m} & 0_{n\times m} \\ * & * & -I_q & 0_{q\times m} & 0_{q\times m} \\ * & * & * & -\epsilon_1 I_m & 0_m \\ * & * & * & * & -\epsilon_2 I_m \end{bmatrix}, \qquad (30)$$

where $\Phi_2$, $\Phi_3$, $\overline{\Phi}_2$ and $\overline{\Phi}_3$ are defined in (14).

Now, we have the following theorem.

**Theorem 2.** *For given scalars* $\overline{r_i}$, $\underline{r_i}$ $(i=1,\ldots,m)$, $h_M > 0$, $h_d$, *the system (9) is asymptotically stabilized by reliable* $H_\infty$ *control (11) with disturbance attenuation* $\gamma > 0$ *for* $0 \leq h(t) \leq h_M$ *and* $\dot{h}(t) \leq h_d$, *if there exist positive definite matrices* $X \in \mathrm{R}^{n\times n}$, $\overline{Q}_1 \in \mathrm{R}^{n\times n}$, $\overline{Q}_2 \in \mathrm{R}^{n\times n}$, $\overline{N} \in \mathrm{R}^{n\times n}$, *any matrices* $\overline{M} \in \mathrm{R}^{2n\times 2n}$, $Y \in \mathrm{R}^{m\times n}$, *positive scalars* $\epsilon_1$ *and* $\epsilon_2$, *satisfying the following conditions hold:*

$$\overline{\Sigma}_{[\rho_0]} < 0, \qquad (31)$$
$$\overline{\Lambda}_2 \geq 0, \qquad (32)$$

*where $\overline{\Sigma}_{[\rho_0]}$ and $\overline{\Lambda}_2$ are defined in (30). If the above conditions are feasible, a desired reliable $H_\infty$ controller gain matrix is obtained by $K=YX^{-1}$.*

**Proof.** Let us consider the same Lyapunov-Krasovskii candidate functional in (17). By infinitesimal operator $L$ in [13], a new upper-bound of $LV(t)$ is obtained by

$$LV(t)\le 2x^T(t)P\dot{x}(t)+\dot{x}^T(t)N\dot{x}(t) \\ +\zeta^T(t)(\Phi_2+\Phi_3)\zeta(t), \quad (33)$$

$$\Lambda_2\ge 0. \quad (34)$$

From the system (9), replacing $\dot{x}(t)=(\Pi_{1[\rho(t)]}+\Delta\Pi_{1[\rho(t)]})\zeta(t)$ leads to following inequality

$$LV(t)\le 2\zeta^T(t)\Psi_{1[\rho(t)]}\zeta(t) \\ +\zeta^T(t)(\Pi_{1[\rho(t)]}+\Delta\Pi_{1[\rho(t)]})^TN \\ \times(\Pi_{1[\rho(t)]}+\Delta\Pi_{1[\rho(t)]})\zeta(t)\} \\ +\zeta^T(t)(\Phi_2+\Phi_3)\zeta(t). \quad (35)$$

Now, $H_\infty$ performance for the system (9), let us consider the following inequality under the zero initial condition satisfying $V(0)=0$ and $V(\infty)\ge 0$

$$\begin{aligned} J=&\ \mathrm{E}\left\{\int_0^\infty z^T(t)z(t)-\gamma^2w^T(t)w(t)dt\right\} \\ =&\ \mathrm{E}\left\{\int_0^\infty z^T(t)z(t)-\gamma^2w^T(t)w(t)+LV(t)dt\right\} \\ &-\mathrm{E}\left\{\int_0^\infty LV(t)dt\right\} \\ =&\ \mathrm{E}\left\{\int_0^\infty z^T(t)z(t)-\gamma^2w^T(t)w(t)+LV(t)dt\right\} \\ &-\mathrm{E}\{V(\infty)\}+\mathrm{E}\{V(0)\} \\ \le&\ \mathrm{E}\left\{\int_0^\infty z^T(t)z(t)-\gamma^2w^T(t)w(t)+LV(t)dt\right\}<0. \end{aligned} \quad (36)$$

When the inequality (36) is satisfied, the system (9) is stable with $H_\infty$ performance level $\gamma$ under the obtained controller (11). Inequality (36) is equivalent to

$$\mathrm{E}\{z^T(t)z(t)-\gamma^2w^T(t)w(t)+LV(t)\}<0. \quad (37)$$

Replacing $z(t)=Cx(t)$ and using Schur complement, following inequality can be obtained as

$$\mathrm{E}\left\{\begin{bmatrix}\Psi_{[\rho(t)]} & \Psi_{2[\rho(t)]} & \bar{e}_1C^T \\ * & -R^{-1} & 0_{n\times q} \\ * & * & -I_q\end{bmatrix}+\begin{bmatrix}\Delta\Psi_{1[\rho(t)]} & \Delta\Psi_{2[\rho(t)]} & 0_{(5n+l)\times q} \\ * & 0_n & 0_{n\times q} \\ * & * & 0_q\end{bmatrix}\right\} \\ <0. \quad (38)$$

Since $\Delta\Psi_{1[\rho(t)]}=E_1\Delta J^TH_1+H_1\Delta JE_1$ and $\Delta\Psi_{2[\rho(t)]}=E_2\Delta J^TH_2+H_2\Delta JE_2$, using Lemma 3 leads a new upper-bound of (38) as follows:

$$\mathrm{E}\left\{\begin{bmatrix}\Psi_{[\rho(t)]}+\epsilon_1H_1^TH_1 & \Psi_{2[\rho(t)]} & \bar{e}_1C^T \\ * & -R^{-1}+\epsilon_2H_2^TH_2 & 0_{n\times q} \\ * & * & -I_q\end{bmatrix}\right. \\ \left.+\begin{bmatrix}\epsilon_1^{-1}E_{1[\rho(t)]}E_{1[\rho(t)]}^T+\epsilon_2^{-1}E_{2[\rho(t)]}E_{2[\rho(t)]}^T & 0_{(5n+l)\times n} & 0_{(5n+l)\times q} \\ * & 0_n & 0_{n\times q} \\ * & * & 0_q\end{bmatrix}\right\} \\ <0. \quad (39)$$

By using Schur complement, inequality (35) is equivalent to

$$E\{\Sigma_{[\rho(t)]}\}<0. \quad (40)$$

Let us define $X=P^{-1}$, $\overline{Q}_1=X^TQ_1X$, $\overline{Q}_2=X^TQ_2X$, $\overline{N}=X^TNX$, $\overline{M}=\begin{bmatrix}X & 0_n \\ * & X\end{bmatrix}^TM\begin{bmatrix}X & 0_n \\ * & X\end{bmatrix}$, $Y=KX$. Then, following inequalities can be obtained by pre- and post-multiplying (40) and (34) by $diag\{X,X,X,X,X,I_l,I_n,I_q,I_m,I_m\}$ and $diag\{X,X,X,X\}$, respectively

$$\mathrm{E}\{\overline{\Sigma}_{[\rho(t)]}\}<0, \quad (41)$$

$$\overline{\Lambda}\ge 0. \quad (42)$$

With (8), inequality $\mathrm{E}\{\overline{\Sigma}_{[\rho(t)]}\}<0$ is equivalent to $\overline{\Sigma}_{[\rho_0]}<0$. This proof is completed. □

It should be noted that the stabilization condition (31) is not LMIs due to the presence of the nonlinear term $X\overline{N}X$. With similar way in Theorem 1, better results can be obtained by using the cone complementarity algorithm. Let us define a new variable matrix $L>0$ satisfying

$$L\le X\overline{N}^{-1}X, \quad (43)$$

which is equivalent $X^{-1}\overline{N}X^{-1}\le L^{-1}$. Letting $H=L^{-1}$, $G=X^{-1}$, $F=\overline{N}^{-1}$ and following a similar method in [12,17], the problem of finding a feasible solution of (31) and (32) can be converted to a minimization problem involving LMIs:

Minimize Trace $(LH+XG+\overline{N}F)$

Subject to

$$\begin{bmatrix}X & I_n \\ * & G\end{bmatrix}\ge 0,\ \begin{bmatrix}H & G \\ * & F\end{bmatrix}\ge 0,\ \begin{bmatrix}L & I_n \\ * & H\end{bmatrix}\ge 0,\ \begin{bmatrix}\overline{N} & I_n \\ * & F\end{bmatrix}\ge 0,$$

$$\begin{bmatrix}\overline{\Psi}_{[\rho_0]} & \overline{\Psi}_{2[\rho_0]} & \bar{e}_1XC & \overline{E}_{1[\rho_0]} & \overline{E}_{1[\rho_0]} \\ * & -L+\epsilon_2H_2^TH_2 & 0_{n\times q} & 0_{n\times m} & 0_{n\times m} \\ * & * & -I_q & 0_{q\times m} & 0_{q\times m} \\ * & * & * & -\epsilon_1I_m & 0_m \\ * & * & * & * & -\epsilon_2I_m\end{bmatrix}<0,$$

$$\overline{\Lambda}_2\ge 0. \quad (44)$$

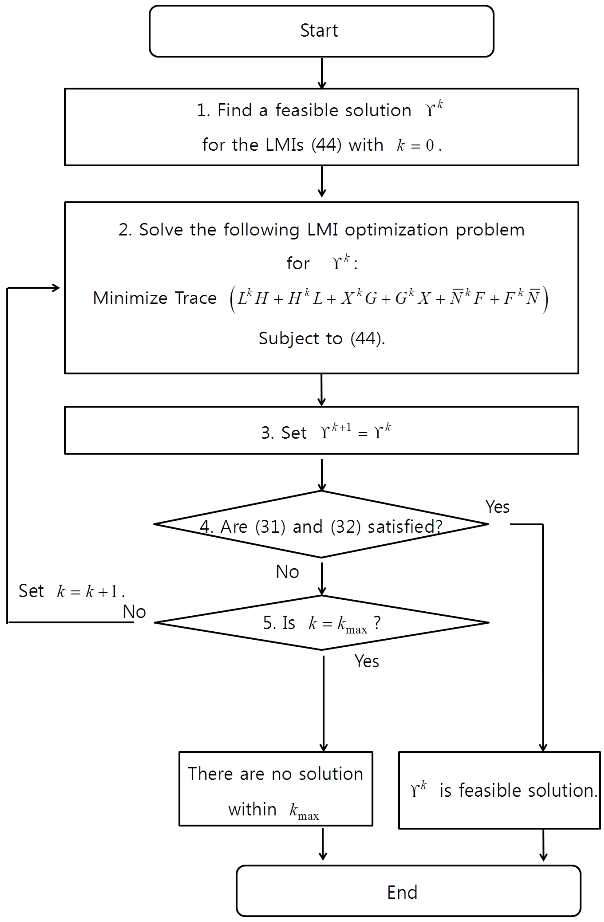



그림 2 Theorem 2을 위한 알고리듬 흐름선도

Fig. 2 Flow chart of Algorithm for Theorem 2

The above minimization problem can be solved using complementarity algorithm in [12,17].

### Algorithm

Let us define $\Upsilon^k$ as the set of the variables of $\{X^k, \overline{Q}_1^k, \overline{Q}_2^k, \overline{N}^k, M^k, H^k, G^k, F^k, \epsilon_1^k, \epsilon_2^k\}$ and $k_{\max}$ as the number of iterations. Then, following Figure 2 is the flow chart of algorithm for Theorem 2.

## 4. Numerical Examples

In this section, two numerical examples are introduced to demonstrate the effectiveness of the proposed criteria. In examples, MATLAB, YALMIP, SeDuMi 1.3 and Intel(R) Core(TM) i5-2500 CPU @ 3.30Ghz (4 CPUs) are used to solve LMI problems.

**Example 1.** Consider the system (13) with following parameters:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{45}$$

표 1 예제 1에서 $h_d=0$일 때 제어 이득 $K$를 고려한 최대의 $h_M$.

Table 1 Maximum $h_M$ with controller gain $K$ when $h_d=0$ in Example 1.

| Method | $h_M$ | Iterations | Computation time(sec) | $K$ |
|---|---|---|---|---|
| Fridman and Shaked[9] | 1.408 | - | - | - |
| Fridman and Shaked[1] | 1.51 | - | - | $[58.31, -294.9]$ |
| Gao and Wang[13] | 3.20 | 211 | - | $[-7.964, -14.77]$ |
| Theorem 1 | 2.0 | 11 | 3.31 | $[-0.748, -2.332]$ |
| | 3.0 | 74 | 22.79 | $[-4.180, -8.537]$ |
| | 3.2 | 114 | 35.17 | $[-5.808, -11.039]$ |
| | 4.0 | 603 | 217.96 | $[-15.489, -24.593]$ |
| | 4.3 | 920 | 326.93 | $[-19.076, -29.050]$ |

When $\dot{h}(t) \le h_d = 0$, Theorem 1 is used to obtain the feedback controller gain $K$ which stabilize the system (13) with upper-bounds of $h(t)$ and number of iterations. The maximum allowable upper-bound of $h(t)$ is 4.3 when the number of iterations is 920. Their results are listed in Table 1 with previous results in [1], [9] and [13]. Also, in order to confirm the results, the simulation results is illustrated in Figure 3 with time-delay $h(t)=4.3$ and feedback controller gain $K=[-19.076, -29.050]$.

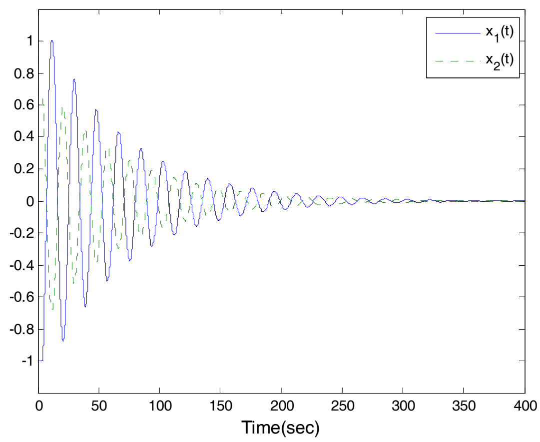



그림 3 제어 이득 $K=[-19.076, -29.050]$를 고려한 $h(t)=4.3$일 때 예제 1의 시뮬레이션

Fig. 3 Simulation for Example 1 with controller gain $K=[-19.076, -29.050]$ when $h(t)=4.3$

**Example 2.** Consider the system (9) with

$$A = \begin{bmatrix} 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \\ -0.3 & 0.3 & -0.0004 & 0.0004 \\ 0.3 & -0.3 & 0.0004 & -0.0004 \end{bmatrix},$$

$$A_d=\begin{bmatrix}0&0&0&0\\0&0&0&0\\-0.03&0.03&-0.00004&0.00004\\0.03&-0.03&0.00004&-0.00004\end{bmatrix},$$

$$B=\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\quad B_w=\begin{bmatrix}0.5&0&0&0\\0&0.5&0&0\\0&0&0.5&0\\0&0&0&0.5\end{bmatrix},$$

$$C=[1\,1\,0\,0],\quad \overline{r}_1=1,\quad \underline{r}_1=0.5. \tag{41}$$

Moreover, the disturbances are defined as follows:

$$w(t)=[w_1^T(t),w_2^T(t),w_3^T(t),w_4^T(t)]^T, \tag{42}$$

where

$$w_1(t)=\begin{cases}1.2, & 3\le t\le 6,\\ 0, & otherwise,\end{cases}$$
$$w_2(t)=\begin{cases}\sin(2\pi 10t)+1, & 4\le t\le 7,\\ 0, & otherwise,\end{cases}$$
$$w_3(t)=\begin{cases}0.8, & 2\le t\le 5,\\ 0, & otherwise,\end{cases}$$
$$w_4(t)=\begin{cases}\cos(2\pi 10t)+1, & 5\le t\le 8,\\ 0, & otherwise.\end{cases}$$

Time-delay is defined as

$$h(t)=0.5h_M(\sin((2h_d/h_M)t)+1), \tag{43}$$

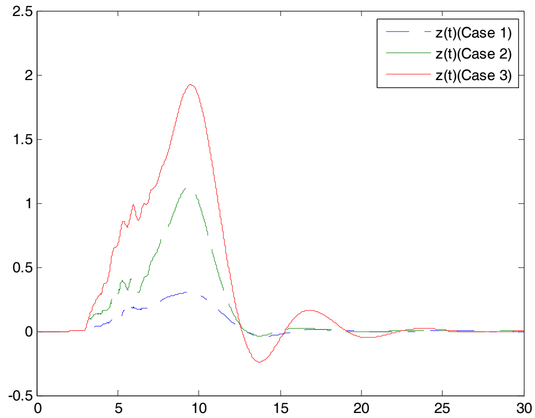
which satisfied with $0\le h(t)\le h_M$ and $\dot{h}(t)\le h_d$. By applying Theorem 2, minimum value of $\gamma$ and controller gain $K$ for system (9) when $h_M=0.2$, $h_d=3$, and $\rho_0$ are 0.1, 0.5 and 0.9 are listed in Table 2.

표 2 $\rho_0$에 따른 제어 이득 $K$, $\gamma_{\min}$, 그리고 반복횟수.

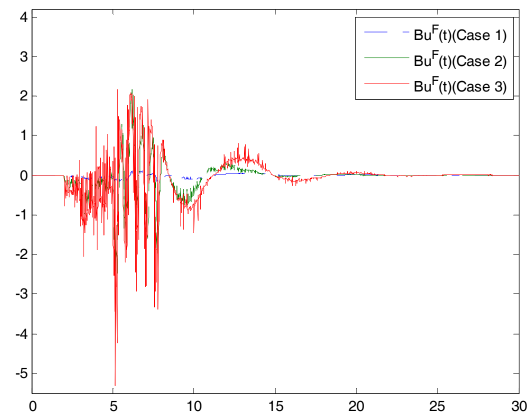
Table 2 Controller gain $K$, $\gamma_{\min}$ and iterations with $\rho_0$.

| Case | $\rho_0$ | $K$ | $\gamma_{\min}$ | Iterations | Computation time(sec) |
|---|---|---|---|---|---|
| 1 | 0.1 | $\begin{bmatrix}-4.2557\\-0.2345\\-3.8910\\-8.4241\end{bmatrix}^T$ | 0.3 | 6 | 3.14 |
| 2 | 0.5 | $\begin{bmatrix}-25.7573\\-9.0003\\-13.8908\\-71.9548\end{bmatrix}^T$ | 1.2 | 37 | 18.51 |
| 3 | 0.9 | $\begin{bmatrix}-242.4149\\-68.4737\\-201.6272\\-611.0830\end{bmatrix}^T$ | 2.3 | 128 | 106.22 |

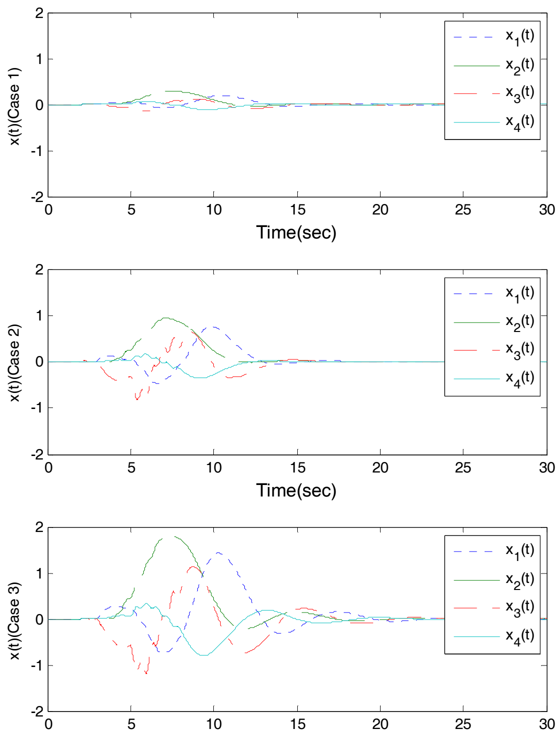
Results in Table 2 show that $\rho_0$ increases $\gamma_{\min}$ which is minimum of $H_\infty$ disturbance attenuation level $\gamma$. It can be shown that $\rho_0$ is increased, then disturbances and actuator failure occur more frequently. Therefore, when $\rho_0$ is 0.9, feedback controller gain $K$ is obtained as $[-242.4149,-68.4737,-201.6272,-611.0830]$ which is larger



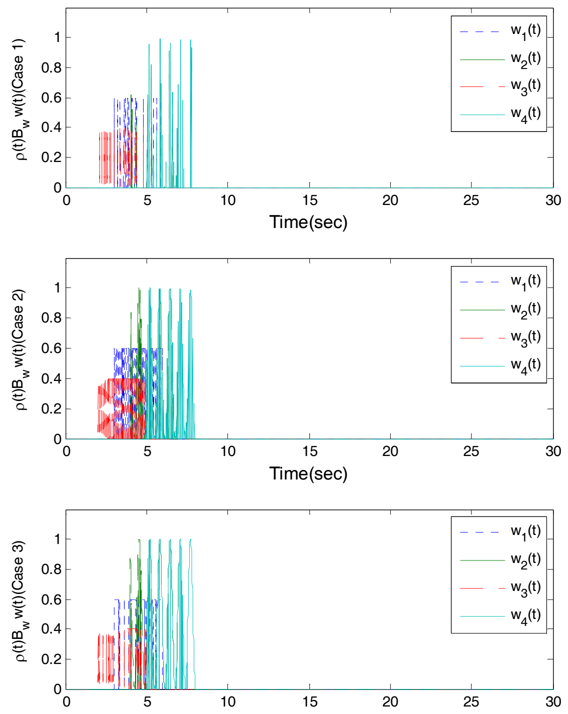

(a) Controlled output $z(t)$ for each case




(b) Trajectories of $Bu^F(t)$ for each case




(c) Responses of $x(t)$ for each case




(d) Disturbances $\rho(t)B_w w(t)$ for each case

그림 4 표 2의 각각의 상황을 고려한 시뮬레이션

Fig. 4 Simulations for each case in Table 2

than other ones. Figure 4 shows linear system responses for each case in Table 2. From Figure 4, the system (9) with the controller gain in Table 2 is asymptotically stable with $H_\infty$ disturbance attenuation level $\gamma$ for any time-varying delay $h(t)$ satisfying (2). Furthermore, trajectories of $Bu^F(t)$ show that actuator failure more frequently occur as the value of $\rho_0$ increases. It can be seen that the system and actuator are more influenced by the disturbances when the value of $\rho_0$ increases. As a result, when the effect of disturbances increases, the state responses and controlled output performances become worse.

## 5. Conclusions

In this paper, the reliable $H_\infty$ controller design for linear systems with time-delays was presented. Firstly, in Theorem 1, the stabilization criterion was proposed by constructing the appropriate Lyapunov-Krasovskii functional and utilizing Wirtinger-based integral inequality [5] and reciprocally convex approach [16]. Secondly, results of Theorem 1 was extended to design the reliable $H_\infty$ controller for the systems with time delays, randomly occurring disturbances, and actuator failures in Theorem 2. Since results have NLMIs in Theorem 1 and Theorem 2, the cone complementarity algorithm was used to solve the NLMIs [12,17] with computational effort. To show the effectiveness of the proposed results, two numerical examples were included.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## References


[1] E. Fridman, U. Shaked, "An improved stabilization method for linear time-delay systems," IEEE Transactions on Automatic Control, vol. 47, pp. 1931-1937, 2002.

[2] O.M. Kwon, J.H. Park, "Delay-range-dependent stabilization of uncertain dynamic systems with interval time-varying delays," Applied Mathematics and Computation, vol. 208, pp. 58-68, 2009.

[3] O.M. Kwon, J.H. Park, "On improved delay-dependent robust control for uncertain time-delay systems," Automatic Control, IEEE Transactions on, vol. 49, pp.1991-1995, 2004.

[4] Y. He, Q.G. Wang, C. Lin, M. Wu, "Delay-range-dependent stability for systems with time-varying delay," Automatica, vol. 43, pp.371-376, 2007.

[5] A. Seuret, F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," Automatica, vol. 49, pp. 2860-2866, 2013.

[6] C. Peng, T.C. Yang, E.G. Tian, "Brief paper: Robust fault-tolerant control of networked control systems with stochastic actuator failure," Control Theory and Applications, vol. 4, pp. 3003-3011, 2010.

[7] C.H. Lien, K.W. Yu, Y.F. Lin, Y.J. Chung, L.Y. Chung, "Robust reliable $H_\infty$ control for uncertain nonlinear systems via LMI approach," Applied Mathematics and Computation, vol. 198, pp. 453-462, 2008.

[8] G. Zames, "Feedback and optimal sensitivity: Model reference transformations, multiplicative semi norms, and approximate inverses," IEEE Transactions on Automatic Control, vol. 26, pp. 301-320, 1981.

[9] E. Fridman, U. Shaked, "Delay-dependent stability and $H_\infty$ control: constant and time-varying delays," International Journal of Control, vol. 76, pp. 48-60, 2003.

[10] H. Gao, C. Wang, "Comments and further results on A descriptor system approach to $H_\infty$ control of linear time-delay systems," IEEE Transactions on Automatic Control, vol. 48, pp. 520-525, 2003.

[11] O.M. Kwon, M.J. Park, J.H. Park, S.M. Lee, E.J. Cha, "Analysis on robust $H_\infty$ performance and stability for linear systems with interval

time-varying state delays via some new augmented Lyapunov - Krasovskii functional," Applied Mathematics and Computation, vol. 224, pp. 108-122, 2013.
[12] Y.S. Lee, Y.S. Moon, W.H. Kwon, P.G. Park, "Delay-dependent robust $H_\infty$ control for uncertain systems with a state-delay," Automatica, vol. 40, pp. 65-72, 2004.
[13] H. Gao, J. Wu, P. Shi, "Robust sampled-data $H_\infty$ control with stochastic sampling", Automatica, Vol. 45, pp. 1729-1736, 2009.
[14] Z. Wang, Y. Wang, Y. Liu , "Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays," IEEE Transactions on Neural Networks, vol. 21, pp. 11-25, 2010.
[15] J. Hu, Z. Wang, H. Gao, L.K. Stergioulas, "Robust sliding mode control for discretestochastic systems with mixed time delays, randomly occurring uncertainties, and randomly occurring nonlinearities," IEEE Transactions on Industrial Electronics, vol. 29, pp. 3008-3015, 2012.
[16] P.G. Park, J.W. Ko, C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays", Automatica, Vol. 47, pp. 235 - 238, 2011.
[17] L. El. Ghaoui, F. Oustry, M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems,"IEEE Transactions on Automatic Control, vol. 42, pp. 1171-1176, 1997.
[18] M.C. de Oliveira, R.E. Skelton, "Stability tests for Constrained Linear systems," SpringerVerlag, pp. 241-257, Berlin, 2001.

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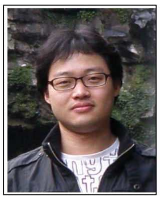

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