Design Loads on Railway Substructure:
Sensitivity Analysis of the Influence of the Fastening Stiffness

Konstantinos GIANNAKOS[†]

Abstract

The superstructure of the railway track undertakes the forces that develop during train passage and distributes them towards its seating. The track panel plays a key role in terms of load distribution, while at the same time it maintains the geometrical distance between the rails. The substructure and ballast undergo residual deformations under high stresses that contribute to the deterioration of the so-called geometry of the track. The track stiffness is the primary contributing factor to the amount of the stresses that develop on the substructure and is directly influenced by the fastening resilience. Four methods from the international literature are used in this paper to calculate the loads and stresses on the track substructure and the results are compared and discussed. A parametric investigation of the stresses that develop on the substructure of different types of railway tracks (i.e. balastless vs ballasted) is performed and the results are presented as a function of the total static track stiffness.

*Keywords: Ballasted Track, Ballastless Track, Static and Dynamic Stiffness Coefficients, Actions, Mean Pressure*

1. Introduction

The railway track superstructure is the equivalent of the road pavement and as in the case of pavements there are flexible and rigid track superstructures. The main difference between the two types of infrastructure is that the loads in railways are applied in two discrete locations along the rails, whereas in road pavements the location of the load application is random. The track superstructure is a multilayered system that consists of the rails, which support and guide the train wheels and distribute the wheel loads, the ties with their fastenings which distribute the loads applied by the rails and maintain the rail gauge. In the case of the classic ballasted track the superstructure also includes the ballast (Fig. 1), the equivalent of a flexible pavement, and the blanket layer (sub-ballast) consisting of compacted sand and gravel, which further distributes the loads and protects the substructure from the penetration of crashed ballast particles, mud ascent and pumping. In the case of the more recently developed Ballastless or Slab Track (Fig. 2) the superstructure also includes the concrete slab or Continuously Reinforced Concrete Pavement (CRCP) which is the equivalent of a rigid pavement. The concrete slab seats on a series of successive bearing layers with a gradually decreasing modulus of elasticity: the Cement Treated Base (CTB), underlain by the Frost Protection Layer (FPL) and the foundation or prepared subgrade. The three layers under the concrete slab (i.e. CTB, FPL and foundation) constitute the substructure of the Slab Track. The Slab Track is typically used in High Speed lines (V>200 km/h) of mixed passenger and freight traffic with maximum axle load of 22.5 t.

Another type of Ballastless Track is the Embedded Track, which is similar to the Slab Track and is typically used in terminal port stations and maintenance facilities of railway vehicles to minimize the maintenance needs of the track. In these applications there is a need to replace the ballast-bed with a concrete floor for functional reasons (i.e. washing of vehicles and flowing out of the waste water and oils, maintenance pits between the two rails, cir-
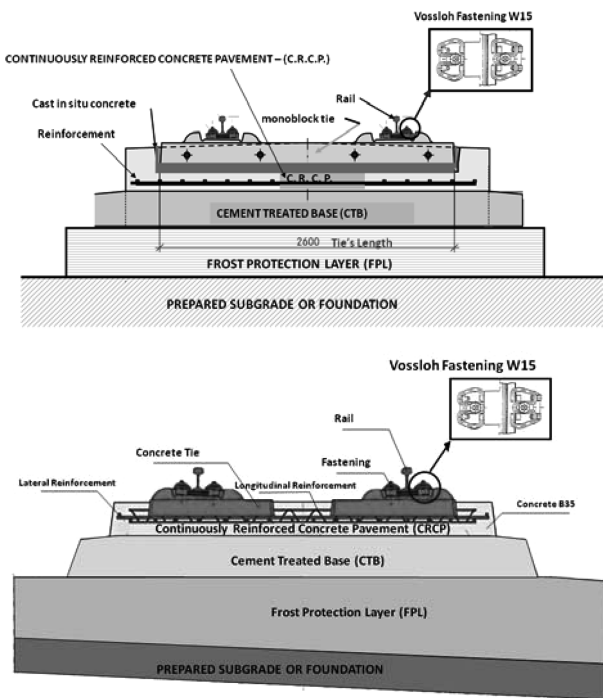
† Corresponding author: F. ASCE, M. TRB-AR050 & AR060 Comm., AREMA, fib., Visiting Professor of Design and Operation of Railway Systems, Dpt. Civil Engineering, University of Thessaly, Greece.
E-mail : k.giannakos@on.gr, kyannak@civ.uth.gr.
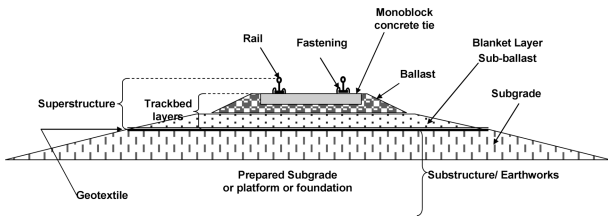
Fig. 1 Ballasted Track's Cross-section, Terminology according to U.I.C.
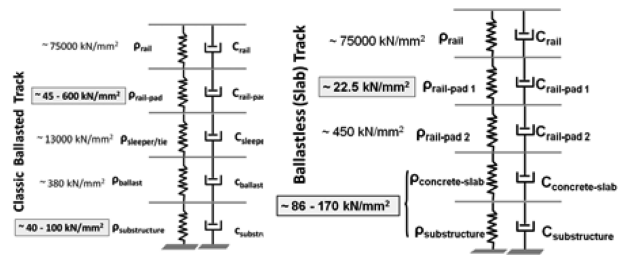
Fig. 2b The track structure as an ensemble of springs and dashpots no dampers (left) classic ballasted track (right) Slab Track, with characteristic values of ρ_i in kN/mm.

Fig. 2a Cross section of Rheda type Slab Track systems: (upper illustration) Rheda Classic with monoblock concrete ties (lower illustration) Rheda 2000 with prefabricated "concrete semi-ties" and reinforcement. Both systems have been applied in the Greek Railway Network (Tsoukantas *et al.* 2006, Tsoukantas, 1999).

and ballast-bed, resulting in a smooth variation of the forces that act on the track (see Giannakos *et al.*, 2010c).

In order to adopt the Slab Track technology, the Greek Railways performed an extensive investigation program that studied Slab Track systems laid in High-Speed tracks ($V_{max} > 200$ km/h) under operation, mainly in Germany. Two types of Slab Track systems were considered: Rheda Classic, which was the first application of Slab Track that took place in 1972, and Rheda 2000, which is the most up-to-date, modern and most technologically advanced type of the Rheda Slab Track "family". Based on the findings of the research program these two types of Slab Track were selected for the Greek railway network and applied in the High-Speed Line between Patras - Athens/Piraeus – Thessaloniki, co-funded by the European Union and the Greek Government.

The stress on the substructure plays a key role in the design and maintenance of High Speed railway tracks and its magnitude mainly depends on the track stiffness coefficient (Giannakos, 2011). However, there is a lack of data in the international literature correlating the magnitude of stress on the track substructure and the track stiffness coefient of High Speed lines under operation. The research performed for the Greek Railway network (Giannakos, 2008, Tsoukantas, 1999) addressed this issue. Its findings, highlighting the interaction between superstructure and substructure of a railway track, are presented in this paper. A method for the calculation of loads and stresses on a railway track was developed as a result of this research (Giannakos, 2004). This method together with three methods found in the international literature are used to calculate the stresses on the track substructure and the results are compared and discussed. The results are presented as a function of the total static track stiffness, for the two systems applied in High-speed lines under operation in the Greek network: (i) Rheda type Slab Track (Fig. 2a), and (ii) ballasted track with 2.60 m monoblock prestressed concrete ties and W14 fastening(Fig. 1).

culation of road vehicles on the tracks, transshipment of cargo etc.). Its main difference from the Slab Track is the low speed of train circulation and, consequently, the small magnitude of dynamic loads. In both cases of Ballastless Track systems presented above the role of ballast-bed and blanket layer is undertaken by a concrete slab.

The application of Slab Track and Embedded Track technology in a railway network creates the need for Transition Zones, which serve as interfaces between the ballastless and ballasted track sections, where significant and abrupt change in stiffness occurs. The Transition Zones guarantee a smooth stiffness transition between slab track

2. Methods for the Calculation of Loads on a Railway Track

2.1 General

The four methods used for the calculation of the stresses on track are:

• The method presented in the French literature (Alias, 1984, Prud'homme *et al.*, 1976, RGCF, 1973),

• The method presented in the German Literature (Fastenrath, 1981, Eisenmann, 2004),

• The method presented in the American literature (AREMA, 2005, Hay, 1982)

• The method proposed by Giannakos (2004, Giannakos *et al.*, 2009d) combining an analytical framework, experimental results and field observations form the Greek Railway Network.

All methods model the railway track as a continuous beam on elastic foundation and use different assumptions in their approximation of the random nature of load application on a railway track. In all methods the total static stiffness coefficient of the railway track, total, plays a key role in the load distribution. This coefficient, is given by:

$$\frac{1}{\rho_{total}} = \sum_{i=1}^{v} \frac{1}{\rho_i} \tag{1}$$

where i is each individual layer of the multilayered structure that constitutes the track as depicted in Fig. 2b).

Due to the random nature of the moving loads, a probabilistic approach is more appropriate for the calculation of the loads acting on each point along the track, the resulting actions on each tie, and stresses and strains on the different layers of the track. According to this approach, the increase of the mean value of the vertical wheel load is estimated for the statistically desirable safety level. In the aforementioned methods this is considered either through a dynamic load coefficient, which essentially increases the value of the static load, or through a probabilistic framework, where the mean load is increased by an appropriate number of standard deviations to cover a certain probability of occurrence.

2.2 Method cited in the french literature

The action on the tie is calculated through the following equation (2):

$$R_{total} = (Q_{wheel} + Q_\alpha + 2 \cdot \sqrt{[\sigma^2(\Delta Q_{NSM})] + [\sigma^2(\Delta Q_{SM})]})$$
$$\cdot \overline{A}_{stat} \cdot 1.35 \tag{2}$$

where: R_{total} = the total action on the tie after the distribution of the acting load, Q_{wheel} = the static load of the wheel

(half the axle load), Q_α = load due to cant deficiency (or superelevation), $\sigma(\Delta Q_{NSM})$ = standard deviation of the Non-Suspended (or unsprung) Masses of vehicle, $\sigma(\Delta Q_{SM})$ = standard deviation of the Suspended (or sprung) Masses of the vehicle. The factor of 2 in the equation above covers a 95.5 % probability of occurrence.

Moreover, $\overline{A}_{stat}$ is the static reaction coefficient of the tie which is equal to:

$$\overline{A}_{stat} = \frac{1}{2\cdot\sqrt{2}} \cdot 4\sqrt{\frac{\rho_{total}\cdot\ell^3}{E\cdot J}} \tag{3}$$

ρ_{total} = coefficient of total static stiffness of track in kN/mm, ℓ = distance among the ties in mm, J = modulus of elasticity and moment of inertia of the rail.

2.3 Method cited in the german literature

The action on the tie is estimated from the following:

$$R_{max} = S = \frac{Q_{total}\cdot\ell}{2\cdot L} \Rightarrow R_{max} = \frac{Q_{total}}{2}\cdot 4\sqrt{\frac{\rho_{total}\cdot\ell^4}{4\cdot E\cdot J\cdot\ell}} \Rightarrow$$

$$\Rightarrow R_{max} = Q_{total}\cdot\frac{1}{2\cdot\sqrt{2}}\cdot 4\sqrt{\frac{\rho_{total}\cdot\ell^3}{E\cdot J}} = \overline{A}_{stat}\cdot Q_{total} \tag{4a}$$

where L the elastic length of the track given by:

$$L = 4\sqrt{\frac{4\cdot E\cdot J\cdot\ell}{\rho_{total}}} \tag{5}$$

and $\overline{A}_{stat}$ is calculated through equation (3).

Furthermore:

$$Q_{total} = Q_{wheel}\cdot(1 + t\cdot\overline{s}) \tag{6}$$

where $\overline{s}$ ranges between 0.1φ and 0.3φ depending on the condition of the track according to the following:

$\overline{s}$ = 0.1·φ for excellent track condition, $\overline{s}$ = 0.2·φ for good track condition, $\overline{s}$ = 0.3·φ for poor track condition and φ is a function of the speed as shown below: in the whole paragr.

For V < 60 km/h: φ = 1.

For 60 < V < 200 km/h:

$$\varphi = 1 + \frac{V - 60}{140} \tag{7}$$

where V the maximum speed on a section of track and t a coefficient that depends on the probability of occurrence (t=1 for P=68.3%, t=2 for P=95.5% and t=3 for P=99.7%).

Finally equation (4a) is transformed to:

$$R_{max} = \left(1 + 0.9\cdot\left(1 + \frac{V_{max} - 60}{140}\right)\right)\cdot\overline{A}_{stat}\cdot Q_{wheel} \tag{4b}$$

for $V_{max} \leq 200$ km/h, with probability of occurrence P= 99.7%, where, Q_{wheel} = the static load of the wheel (half

the axle load), $\overline{A}_{stat}$ is calculated through equation (3). Prof. Eisenmann for speeds above 200 km/h proposed a reduced factor of dynamic component:

$$R_{max} = \left(1 + 0.9 \cdot \left(1 + \frac{V-60}{380}\right)\right) \cdot \overline{A}_{stat} \cdot Q_{wheel} \qquad (4c)$$

Equation (4c) leads to even greater under-estimation of the acting loads on track -than equation (4b)- with possible consequences to the dimensioning of track elements - like e.g. sleepers- as the literature describe ([1], [2], [15]), thus equation (4b) should be preferred for the sleepers' dimensioning.

2.4 Method cited in the american literature

This method is described in AREMA (2005, p. 16-10-26 to 16-10-32 and Chapter 30) and in Hay (1982, p. 247-273). The total load (static and dynamic) acting on the track depends on an impact factor (AREMA, 2003, p. 16-10-9, Hay, 1982, p. 256):

$$\theta = \frac{D_{33} \cdot V}{D_{wheel} \cdot 100} \qquad (8)$$

where D_{33} is the diameter of a 33-inch reference wheel, D_{wheel} the wheel diameter of the vehicle examined in inches, and V the speed in miles/hour

The total load is:

$$Q_{total} = Q_{wheel} \cdot (1 + \theta) \qquad (9)$$

U in psi is the rail support modulus derived by the relation p=Uy, U=ρ/ℓ, and

$$\beta = \sqrt[4]{\frac{U}{4 \cdot E \cdot J}} = \sqrt[4]{\frac{\rho}{4 \cdot E \cdot J \cdot \ell}} = \frac{1}{L} \qquad (10)$$

The influence curve for the deflection, y, is used to determine the maximum value of pressure $\overline{p}_{max}$ and the maximum rail seat load R_{max} on an individual tie, which is given by the following equation:

$$R_{max} = p_{max} \cdot \ell = U \cdot y_{max} \cdot \ell = U \cdot y_{max} \cdot \ell$$

$$= U \cdot \frac{\beta \cdot Q_{total}}{2 \cdot U} \cdot \ell = \sqrt[4]{\frac{\rho_{total}}{4EJ\ell}} \cdot \frac{Q_{total} \cdot \ell}{2}$$

$$\Rightarrow R_{max} = \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{\frac{\rho_{total} \cdot \ell^3}{E \cdot J}} \cdot Q_{total} = \overline{A}_{stat} \cdot Q_{total} \qquad (11a)$$

where Q_{total} is calculated from equation (9) and $\overline{A}_{stat}$ from equation (3). Note that this method does not account for the probability of occurrence.

Finally equation (11a) is transformed to:

$$R_{max} = \overline{A}_{stat} \cdot \left(1 + \frac{D_{33} \cdot V}{D_{wheel} \cdot 100}\right) \cdot Q_{wheel} \qquad (11b)$$

2.5 Giannakos (2004) method

The actions on the track panel are calculated through the following equation (12a):

$$R'_{max} = (Q_{wh} + Q_{\alpha}) \cdot \overline{A}_{dyn} + \mu \cdot \sqrt{\sigma(\Delta Q_{NSM})^2 + \sigma(\Delta Q_{SM})^2} \qquad (12a)$$

where Q_{wh} = the static wheel load, Q_{α} = the load due to cant deficiency, $\overline{A}_{dyn}$ = dynamic coefficient of the tie reaction, μ = coefficient of dynamic load (3 for a probability 99.7 % and 5 for 99.9 %), $\sigma(\Delta Q_{NSM})$ = the standard deviation of the dynamic load due to non suspended masses, (Q_{SM}) = the standard deviation of the dynamic load due to suspended masses and :

$$\overline{A}_{dynam} = \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{\frac{\ell^3 \cdot h_{TR}}{E \cdot J}} \qquad (13)$$

where h_{TR} the total dynamic stiffness of the track:

$$h_{TR} = \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{E \cdot J \cdot \frac{\rho_{total}}{\ell}} \qquad (14)$$

Giannakos (2004, 2010a) and Giannakos *et al.* (2009d) describe in detail the determination of parameters $\overline{A}_{dynam}$ and h_{TR}. The main differences of this method compared to the methods presented above are: (i) the use of the dynamic coefficient $\overline{A}_{dynam}$ (instead of $\overline{A}_{stat}$) for the distribution of the static and semi-static components of the load, and (ii) the dynamic component of the load is not distributed to the adjacent ties (see Giannakos *et al.*, 2009d).

Finally equation (12a) is transformed to equation (12b) [Giannakos, 2014]:

$$R_{max} = \frac{1}{2^{8}\sqrt{2}} \cdot \sqrt[16]{\left(\frac{\rho_{total} \cdot \ell^3}{E \cdot J}\right)} \cdot (Q_{wheel} + Q_{\alpha}) \qquad (12b)$$

$$+ (3 \sqrt{\underbrace{\left(k_{\alpha} \cdot V \cdot \sqrt[8]{2^6 \cdot (m_{NSM-vehicle} + m_{TRACK})^4 \cdot E \cdot J \cdot \left(\frac{\rho_{total}}{\ell}\right)^3}\right)^2}_{\sigma(\Delta Q_{NSM})} + \underbrace{\left(\frac{V-40}{1000} \cdot N_L \cdot Q_{wheel}\right)^2}_{\sigma(\Delta Q_{SM})}}$$

2.6 Comparison of theoretical calculations of the loads with observations on track

In Greece between 1972 and 1999, twin-block concrete ties of French technology were exclusively used, namely Vagneux U2, U3 with RN fastenings, for tracks designed for V_{max} = 200 km/h and temporary operational speed V_{oper} = 120-140 km/h. Extended cracking was observed at a percentage of more than 60 % of the U2/U3 ties laid on track. The methods cited in the international literature at that time (French, German, American) did not provide any satisfactory justification for the appearance of the cracks, resulting in much lower values of actions on ties than the cracking threshold, thus predicting no cracking at all. After
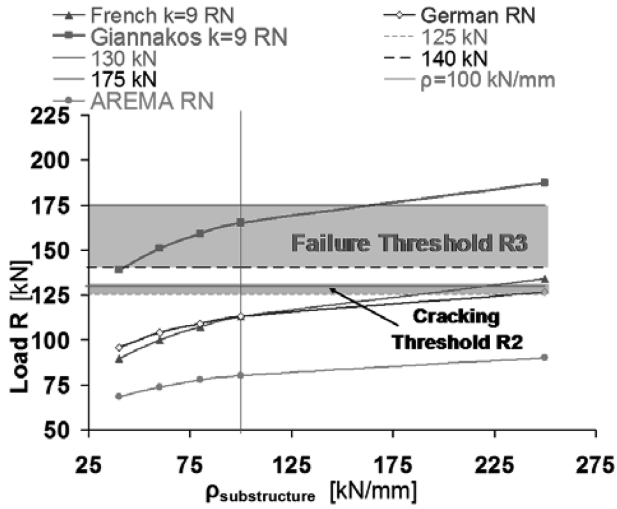
Fig. 3 Calculation of actions on U2/U3 twin-block ties with the four methods

Table 1 Relation between ballast coefficient C and stiffness coefficient ρ in a track equipped with UIC60 rails, prestressed monoblock concrete sleepers B70, and concrete plate/ slab

	Bearing Capacity of Substructure					
	Ballasted Track			Ballastless Track		
	poor	good	very good	poor	good	very good
C [N/mm³]	0.05	0.10	0.15	0.30	0.35	0.40
ρ [kN/mm]	14	29	43	86	100	114

an extensive research that included collaboration among various universities and railway organizations in Europe, the Giannakos (2004) method was developed whose results successfully predicted the extended cracking of the U2/U3 ties (Giannakos, 2004, 2010c, 2011; Giannakos & Loizos, 2009d), calculating actions over the cracking threshold and in some cases over the failure threshold. This method was derived from theoretical analyses, measurements from laboratory tests performed in Greece, Austria, France, Belgium and other European countries and observations from real on-track experience. The results of the four methods for a sensitivity analysis of the influence of the subgrade's and the pad's static stiffness coefficient on the acting loads are depicted in Fig. 3. The method was calibrated to for the calculation of loads on ballastless track systems and the results were verified against measurements on Slab Track (Giannakos, 2010a). The results of the method were also presented for lines with Heavy Haul traffic that are typical in the United States (Giannakos, 2011). Moreover, the International Federation of Concrete (fib) has adopted this method for precast concrete railway track systems (fib, 2006).

3. Calculation of Track's Total Static Stiffness Coefficient - Verification against Measurements

As shown in the above equations, the load that is applied on the track is dependent on the total track stiffness ρ_{total}. A range of ballasted track total stiffness values ρ_{total}, is used for different combinations of railway track layers/elements, and their corresponding parameters derived from measurement data of the German Railways (Alias, 1984, Giannakos, 2004). The following static stiffness coefficients per layer/element have been measured on track:

ρ_{rail} ranging from 50,000 to 100,000 kN/mm with an average of 75,000 kN/mm

$\rho_{sleeper}$ ranging from 500 to 800 kN/mm for oak wooden tie and 12,000 to 15,000 kN/mm with an average of 13,500 kN/mm for a concrete tie

$\rho_{ballast}$ in the order of 380 kN/mm for "polluted" ballast 2 years after laying

$\rho_{substructure}$ ranging from: (i) 20 to 60 kN/mm for pebbly subgrade with an average of 40 kN/mm, (ii) 80 to 100 kN/ mm in the case of frozen ballast and ground, and (iii) on the order of 250 kN/mm for the case of a ballast-bed of small thickness laid on the concrete base of a tunnel or a bridge deck .

The tie-pad stiffness, pad, plays a key role in the loading of the tie, and its value is typically estimated from the load-deflection curve that is provided by the manufacturer using a trial-and-error method (Giannakos, 2004).

For the Slab Track, Table 1 can be used to determine the total track stiffness, ρ_{total}. On the same table total stiffness values are also provided for the ballasted track for different ballast conditions. The values were derived from measurements in the German railway network (Leykauf *et al*., 1990). For the ballastless case the classic Rheda type slab track was used (Fig. 2a – upper illustration).

The data on Table 1 verify the linear relation between track stiffness, ρ_{total}, and ballast coefficient, C, which is $\rho_{total}=CF/2$ (Giannakos, 2004), where F is the active seating surface of the tie (for a B70 tie F=5700 cm²). Based on the ballast coefficient values listed on Table 1, the following stiffness values (Giannakos *et al*., 2009a) are calculated (equations 15 a, b, c, d, e, f):

$$\rho_{total} = C \cdot \frac{F}{2} = 0.05 \frac{\frac{1}{1000}kN}{mm^3} \cdot \frac{5700 \cdot 100mm^2}{2} = 14.25 \approx 14kN/mm$$

(15a)

$$\rho_{total} = C \cdot \frac{F}{2} = 0.10 \frac{\frac{1}{1000} kN}{mm^3} \cdot \frac{5700 \cdot 100 mm^2}{2} = 28.50 \approx 29 kN/mm$$

$$(15b)$$

$$\rho_{total} = C \cdot \frac{F}{2} = 0.15 \frac{\frac{1}{1000} kN}{mm^3} \cdot \frac{5700 \cdot 100 mm^2}{2} = 42.75 \approx 43 kN/mm$$

$$(15c)$$

$$\rho_{total} = C \cdot \frac{F}{2} = 0.30 \frac{\frac{1}{1000} kN}{mm^3} \cdot \frac{5700 \cdot 100 mm^2}{2} = 85.5 \approx 86 kN/mm$$

$$(15d)$$

$$\rho_{total} = C \cdot \frac{F}{2} = 0.35 \frac{\frac{1}{1000} kN}{mm^3} \cdot \frac{5700 \cdot 100 mm^2}{2} = 99.75 \approx 100 kN/mm$$

$$(15e)$$

$$\rho_{total} = C \cdot \frac{F}{2} = 0.40 \frac{\frac{1}{1000} kN}{mm^3} \cdot \frac{5700 \cdot 100 mm^2}{2} = 114.0 \approx 114 kN/mm$$

$$(15f)$$

The sleepers in the classic Rheda type system are quite similar to the B70 prestresed concrete ties with active seating surface of F=5700 cm² as well. In the case of the Slab Track, the total track stiffness can be calculated through Equation (1). It is noted here that the value of slab stiffness (CRCP and the underlying layers) is similar to the stiffness of a substructure consisting of ballast and frozen soil as cited in Giannakos (2004, 2009b).

The methodology described above models both the concrete bearing slab (CRCP) and the underlying layers (CTP, FPL, foundation). According to Eisenmann (1994), in situ measurements of the ballast coefficient in the Newly-Constructed Lines in Germany (NBS-Neubaustrecke) indicate values in the order of 0.60 N/mm³, which would correspond to a total track stiffness of 171 kN/mm. Eisenmann (1979) also mentions that the mean value of concrete slab subsidence y_{slab} is 0.23 mm (fluctuating between 0.17 and 0.31 mm). In the Slab Track case (with embedded concrete sleepers) Equation (1) is analytically written in the following form:

$$\frac{1}{\rho_{total}} = \frac{1}{\rho_{rail}} + \frac{1}{\rho_{pad1}} + \frac{1}{\underbrace{\rho_{pad2}}_{if-it-exists}} + \frac{1}{\rho_{sleeper}} + \frac{1}{\rho_{concrete-slab}} \quad (16)$$

Once the total static track stiffness is calculated, the total dynamic track stiffness, $\rho_{total-dynam}=h_{TRACK}$, is estimated through Equation (14). Three out of the five layers that constitute the track structure, namely the rail, the tie, and the ballast, contribute only 6 to 10% to the total track stiffness. The total track stiffness is mainly affected by the
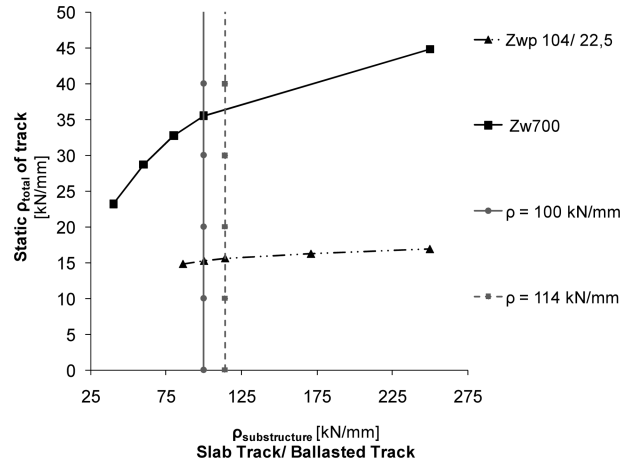
Fig. 4 Relation between static stiffness coefficient of substructure substruct and total static stiffness coefficient total in ballasted and ballastless (slab) track, for resilient fastening (W14 and pad Zw700) with $\rho_{pad} \approx 50$ kN/mm in the ballasted track case and W15 fastening (with pad Zwp104) with $\rho_{pad} \approx 22.5$ kN/mm in the slab track case.
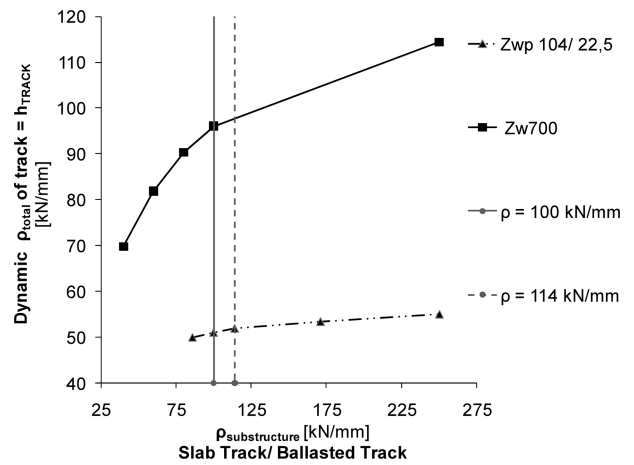
Fig. 5 Relation between static stiffness coefficient of substructure substruct and total dynamic stiffness coefficient h_{TRACK} in ballasted and ballastless (slab) track, for resilient fastening (W14 and pad Zw700) with $\rho_{pad} \approx 50$ kN/mm in the ballasted track case and W15 fastening (with pad Zwp104) with $\rho_{pad} \approx 22.5$ kN/mm in the slab track case.

static stiffness coefficients of the pad, ρ_{pad}, and of the substructure, $\rho_{substruct}$. Fig. 4 depicts the relation between the static stiffness coefficient of the substructure and the total static stiffness coefficient of the railway track for both ballasted and ballastless tracks. The relation between the static stiffness coefficient of the substructure and the total dynamic stiffness coefficient of the railway track is shown

in Fig. 5 for both cases considered.

The results in Figs. 4 and 5 were calculated through Equations (1) and (16) for axle load 22.5 tonnes, maximum speed 200 km/h, a very resilient fastening with $\rho_{pad} \approx 50$ kN/mm (i.e. W14 with Zw700 pad) in the case of ballasted track, and a W15 fastening (with Zwp104 pad) with pad ≈ 22.5 kN/mm in the case of slab track. For the rest of the stiffness coefficients, average values have been used: rail $\approx 75,000$ kN/mm, concrete-tie 13,500 kN/mm and ballast ≈ 380 kN/mm.

As shown in Fig. 4 the total static stiffness coefficient for the case of ballasted track equipped with very resilient fastening varies significantly, ranging from 23.20 to 44.84 kN/mm. On the contrary, the total static stiffness coefficient of the ballastless track equipped with a very resilient fastening obtains values within a very narrow range between 14.83 and 16.90 kN/mm. It is important to note that although the slab track is a much more rigid structure than the ballasted track, the total static track stiffness of slab track is 36% to 62% lower than the corresponding ballasted track stiffness. This is attributed to the use of the much more resilient fastening W15 in the case of the slab track.

4. Influence of the Total Static Stiffness Coefficient of Track on the Actions

The four methods for the estimation of loads on railway track presented above, were programmed in a computer code and parametric investigations were performed by varying the total static track stiffness, ρ_{total}. Analyses were performed for both ballasted and ballastless track. The same methodology was applied for both track systems assuming maximum axle load 225 kN, maximum speed 250 km/h (assuming mixed traffic of passenger and freight trains), Non-Suspended Masses 7.5 kN/wheel, UIC 60 rail (60 kg/m), maximum superelevation deficiency 160 mm, distance of the vehicle's center of gravity from the rail running table 1 m, average condition of the rail running table with corresponding coefficient $k_1=9$ (for more details see Giannakos *et al.*, 2009d, and Giannakos, 2010c,), and wheel diameter 33.86 in. For the slab track W15 fastenings were considered consisting of a combination of two rail pads: Zwp104 ($\rho_{pad} \sim 22.5$ kN/mm) and Zw687 ($\rho_{pad} \sim 450$ kN/mm). For the ballasted track W14 fastening was considered with rail pad Zw700 Saargummi ($\rho_{pad} \sim 50$ kN/mm). The stiffness coefficients were calculated from the corresponding load-deflection curve of each pad, assuming equilibrium between the five "springs" constituting the track's structure and using the trial-and-error method (Fig. 3).

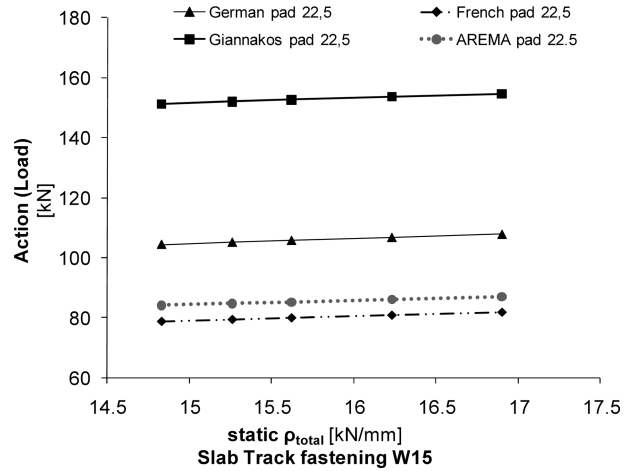Fig. 6 presents the actions on the track superstructure for

Fig. 6 Actions on the track panel as a function of the static total track stiffness ρ_{total} for the case of ballastless track equipped with W15 fastening and Zw104 pad (ρ_{pad}=22.5 kN/mm).
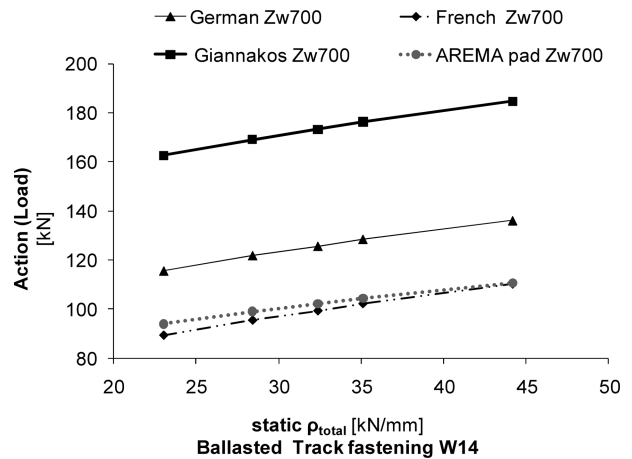

Fig. 7 Actions on the track panel as a function of the static total track stiffness ρ_{total} for the case of classic ballasted track equipped with W14 fastening and Zw700 pad (ρ_{pad}=50 kN/mm).

varying static track stiffness for the case of ballastless track with W15 fastening and Zw104 elastic pad. The actions on the track superstructure as a function of total track stiffness for the case of ballasted track with W14 fastening and Zw700 Saargummi elastic pad are depicted in Fig. 7. The four methods for the calculation of loads on track described above have been used to estimate the actions on track and the results are plotted as separate lines on Figs. 6 and 7.

The following main points are noteworthy:

• In both cases, Giannakos (2004) predicts higher loads on the superstructure than the other three methods used in this study. This is because the static component of the total
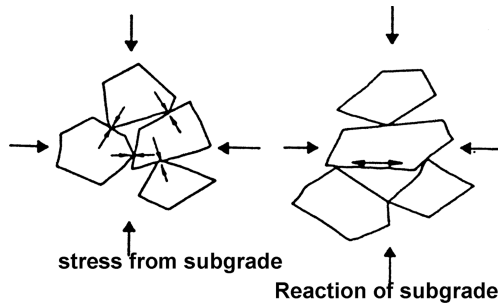
Fig. 8 Ballast grains in the ballast bed and transmission of stresses and actions.

load acting on the tie is derived from a distribution through the $\overline{A}_{dyn}$ and three standard deviations are used (99.7% probability of occurrence) for the increase of the dynamic component of the load which is not distributed to the adjacent sleepers.

- The loads on the superstructure of the ballastless track are lower than in the case of the ballasted track even though the former is a more rigid system. This is attributed to the use of the very resilient W15 fastening in the case of the ballastless track. In particular, using Giannakos (2004) method we estimate loads on the order of 150 kN acting on a ballastless track with total static stiffness ρ_{total} between 15 kN/mm to 17 kN/mm (corresponding to substructure stiffness $\rho_{substructure}$ varying from 86 kN/mm to 250 kN/mm) compared to 170 kN for a ballasted track with total static stiffness between 23 kN/mm to 44 kN/mm (substructure stiffness of pebbly subgrade or rocky tunnel base).

5. Sensitivity Analysis of the Mean Stress on the Ballast and Substructure

In reality, in a railway line under operation, the seating surface of the tie is supported on discrete points (points of contact between the tie and the grains of the ballast) as shown in Fig. 8. However, the "acting" stress per grain of ballast (or per point of contact) cannot be meaningfully calculated. Instead, the mean value of pressure in combination with a confidence level can be used to estimate the stresses under a tie (see also Giannakos, 2010b).

The average stress, $\overline{p}_{ballast}$, under the tie seating surface can be used qualitatively as a "decision criterion" and not quantitatively, since in practice there is no uniform support of the tie on the ballast, or uniform compaction of the ballast and the ground, there are faults on the rail running table, imperfections on the wheels etc. The simplest way

to calculate the average stress is to divide the action, R, on the tie by the "effective" tie seating surface:

$$\overline{p}_{ballast} = \frac{R_{max}}{F_{eff-tie}} = \frac{R_{max}}{L_{eff-tie} \cdot b_{tie}} = \frac{R_{max}}{(L_{tie}-e) \cdot b_{tie}} \qquad (17)$$

where R_{max} the maximum action on the tie derived from each method, F_{tie}= effective tie seating surface, L_{tie} = tie length, e = track gauge, $L_{eff-tie}$ calculated from Equation (18) with the assumption that the center of the tie is unsupported, b_{tie} = tie width at the seating surface,

$$L_{eff-tie} = (L_{tie}-e) \qquad (18)$$

The main issue is the determination of the confidence level (possibility of occurrence) in calculation of the stresses that yields results close to the measured values on track. Eisenmann (1988) proposes a probability of 68.3% for the substructure (t=1) and 68.3% ÷ 99.7% for the stress on ballast depending on the speed and the level and quality of maintenance of each line. A similar conclusion has been derived from the research performed by the collaboration of the Greek and French railways (Giannakos 2004): for the cases of the ballast and substructure the stresses should be calculated based on design loads corresponding to a probability of occurrence of 68.3% to 95.5% instead of 99.7% (see also Giannakos, 2010b, Giannakos *et al.*, 2009c). It is noted here that the AREMA method does not account for the probability of occurrence.

Consequently, for the estimation of the actions on the seating surface of the sleepers on the ballast the probability of occurrence used by each method should be considered as follows:

- *Method cited in the French literature:* Probability 68.3% in Equation (2).
- *Method cited in the German literature:* Probability 68.3 % to 95.5 %, (t=1 to 2 respectively) in Equations (4).
- *Method in the American literature (AREMA):* There is no probabilistic approach. Equations (11) are used in any case for the calculation of the action on the tie.
- *The Giannakos (2004) Method:* Probability 68.3% to 95.5% (=1 to 2 respectively) in Equations (12).

The stress on the ballast is calculated through Equation (17).

The aforementioned actions and stresses are used in the dimensioning of the track panel as well as the various layers that constitute the track, for both Ballastless and Ballasted Track. According to Esveld (1989) the permissible mean stress $\overline{p}_{ballast}$ should be:

$$\overline{p}_{ballast} \leq 0.30 \text{ N/mm}^2 \text{ or } 43.5 \text{ psi} \qquad (20)$$

For the mean stress on the substructure, a 45° angle of
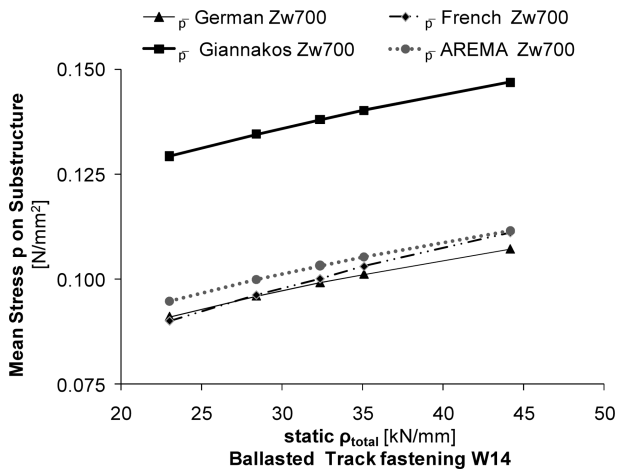
Fig. 9 Mean stress on substructure in the case of classic ballasted track equipped with W14 Fastening with pad Zw700 (ρ_{pad}=50 kN/mm) in relation to the static total track stiffness $_{total}$ according to the methods cited in the German, French, American Literature and Giannakos (2004) method.
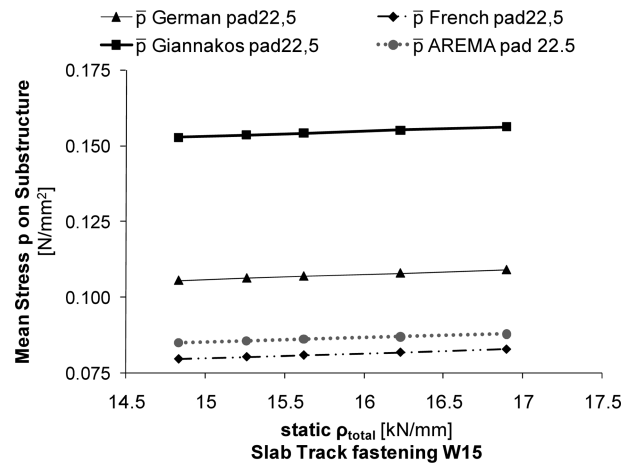
Fig. 10 Mean stress on substructure in the case of ballastless (Slab) track equipped with W15 Fastening with pad Zw104 (ρ_{pad}=22.5 kN/mm or 128.5 kips/in) in relation to the static total track stiffness total according to the methods cited in the German, French, American Literature and Giannakos (2004) method.

load distribution is assumed, so at a depth of 350 mm under the tie's lower (seating) surface the loaded area is:

Length of loaded area=Length of semi-tie 1300 mm + 350 mm = 1650 mm, Width of loaded area = distance between the ties 600 mm, Surface of loaded area = $1650 \times 600 = 990,000$ mm^2 [equation (21)].

The Sensitivity Analysis of the mean stress on the substructure in relation to the total static track stiffness ρ_{total} was calculated using the four methods described above. The results are depicted in Figs. 9 and 10 for the cases of ballasted and ballastless track, respectively. Again, very resilient fastenings were considered: W14 fastening with Zw700 Saargummi pad in the case of ballasted track and W15 fastening with Zwp104 pad in the case of ballasted track. A 95.5% probability of appearance is used for the French, German and Giannakos (2004) methods. A surface of 990,000 mm^2 is used for the application of the load (Equation (21)), derived from an average height of ballast or CRCP under the seating surface of the tie of 350 mm.

The Sensitivity Analysis of the mean stress on the substructure in relation to the total static track stiffness ρ_{total} in the case of ballasted track (Fig. 1), varies between 0.090 MPa to 0.112 MPa according to the AREMA, German and French methods and between 0.129 MPa to 0.147 MPa according to Giannakos (2004) method. The mean stress on the substructure in the case of ballastless track (Fig. 2a), varies between 0.079 MPa to 0.088 MPa as predicted from the AREMA and French methods. The German method yields estimates of mean stress between 0.105

MPa and 0.109 MPa, and Giannakos(2004) method between 0.153 MPa to 0.156 MPa.

It is noteworthy that in the case of slab track with the use of very resilient fastenings (ρ_{pad}=22.5±2.5 kN/mm) the stress level is independent of the ρ_{total} due to the characteristics of the load-deflection curves of the pads and the clips of the fastenings (Fig. 10). On the contrary in the case of the ballasted track the stress on the substructure increases proportionally to the increase of the total static stiffness coefficient of the track (Fig. 9). The use of very resilient fastenings in combination with high quality substructure (i.e. 100% Modified Proctor or 105% Proctor normal), results in acceptable levels of stresses for the substructure life-cycle and, consequently, for the track behaviour under circulation.

Eisenmann & Kaess (1980) report measured stress values in High Speed ballasted tracks between 0.083 and 0.118 N/mm^2. There is no reference for stress measurements on the substructure of the slab track since in this case stresses are negligible, as is also illustrated by the theoretical results. Instead, the shear stress between the slab track layers is examined (Eisenmann *et al*., 1979). Leykauf & Mattner (1990) report that the average theoretically calculated values of deflection and reaction are verified by measurements on track (for both ballasted and ballastless track). This means that the calculated stresses are also in agreement with in situ values, since they are derived by dividing the reaction R_{max} by the seating surface of tie.

6. Conclusions

In this paper the loads and stresses acting on the sub-structure of ballasted and ballastless tracks equipped with very resilient fastenings are analyzed. Four existing methods, the AREMA, French, German and Giannakos (2004) method, are used for the estimation of the loads on the track. These actions are then used to calculate the mean stress on the substructure with appropriate confidence levels. A parametric investigation is performed for various track stiffnesses and the results from the four methods are compared. The sensitivity analysis was performed for a fluctuation of the static stiffness coefficient of the subgrade between 40 kN/mm and 250 kN/mm and the relevant ρ_{total} for fastenings W14 for ballasted track and W15 for Slab Track. The mean stress, derived by this analysis, acting on the substructure of the railway track, is one of the main factors influencing the dimensioning of the layers the track and its behavior over time.

References

1. Alias J., (1984). "La voie ferree:", IIème edition, Eyrolles, Paris.
2. AREMA, (2005). "Manual for Railway Engineering".
3. Eisenmann J., (2004). "Die Schiene als Tragbalken", Eisenbahningenieur 5/2004.
4. Eisenmann J., (1988). "Schotteroberbau – Moglichkeiten und Perspektiven fur dieModerne Bahn".
5. Eisenmann J., Kaess G., (1980). "Das Verhalten des Schotters unter Belastung", ETR (29) 3, Darmstadt.
6. Eisenmann J., Duwe B., Lempe Ul., Leykauf G., Steinbeisser L., (1979). "Entwinklung, Bemessung und Erforschung des schotterlosen Oberbaues "Rheda", AET (34).
7. Eisenmann J., Leykauf G., Mattner L., (1994). "Vorsläge zur Erhöhung der Oberbauelastizität", ETR 43, H 7/8.
8. Esveld C., (1989). "Modern Railway Track", MRT-Productions, The Netherlands.
9. Fastenrath Fritz, (1981). "Railroad Track - Theory and Practice", Frederic Ungar Pub.Co., New York, part 2, "The Rail as support and Roadway, theoretical principles and practical examples", by J. Eisenmann.
10. fib (International Federation of Concrete), (2006). "Precast concrete railway track systems".
11. Giannakos K., (2014). "Actions on a Railway Track, due to an Isolated Defect", International Journal of Control and Automation (IJCA), Vol.7, No.3, p.195-212.
12. Giannakos K., (2011). "Heavy Haul Railway Track Maintenance and Use of Resilient versus Stiff Fastenings", approved to be published in TRR/2011 ("Railway Maintenance" Committee), Washington D.C., U.S.A., January 23-27, TRB 90th Annual Meeting proceedings.
13. Giannakos K., (2010a). "The Use of Strain Attenuating Tie Pads and its Influence on the Rail Seat Load in Heavy-Haul Railroads", ASCE – ASME – AREMA, Joint Rail Conference JRC-2010, Urbana – Champaign IL, USA, April 27-29.
14. Giannakos K., (2010b). "Interaction between Superstructure and Substructure in Railways", in the Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in honor of professor I. M. Idriss, San Diego, CA - USA, May 24-29, proceedings.
15. Giannakos K., (2010c). "Loads on track, Ballast Fouling and Life-cycle under Dynamic Loading in Railways", Journal of Transportation Engineering, ASCE, approved to be published in the Journal, electronically published DOI: 10.1061/(ASCE)TE.1943-5436.0000182, Vol. 136, Issue 12, December, pp. 1075-1084.
16. Giannakos K., Tsoukantas S., (2010d). "Requirements for Stiffness Variation between Slab and Ballasted Railway Track and Resulting Design Loads", ICASTOR (Indian Centre For Advanced Scientific and Technological Research) Journal for Engineering, January 2010c, Vol. 3, No. 1, pp. 21-35.
17. Giannakos K., Tsoukantas S., (2009a). "Transition Zone between Slab Track and Ballasted - Variation of Elasticity and Influence on the Acting Forces", Greek Department of Concrete (Member of FIB – RILEM), Conference, Limassol Cyprus, 24-27 October, Proceedings.
18. Giannakos K., (2009b) "Selected Topics on Railways", University of Thessaly Greece, Department of Civil Engineering, Volos.
19. Giannakos A., Loizos A., 2009c, "Actions on Railway Track Panel and Ballast - Behavior of the Hellenic limestone ballast", 8th International Conference on the Bearing Capacity of Roads, Railways and Airfields, 29 June–2 July, Champaign-Urbana, Illinois, USA, Proceedings.
20. Giannakos K., Loizos A., (2009d). "Evaluation of actions on concrete sleepers as design loads–Influence of fastenings", International Journal of Pavement Engineering (IJPE), November 2009, DOI: 10.1080/10298430903402161, Vol. 11, Issue 3 June 2010, pp. 197-213.
21. Giannakos K., Obermeyer Planen + Beraten Gmbh, (2008). "Calculation of Stiffness coefficients, in the Transition Zone between Slab Track and Ballasted Track – Slab Track Rheda 2000 type Design of the New Railway Track of Normal Gauge in the Kiato – km 2+000 position (Rododaphne)", for ERGA-OSE, Athens.
22. Giannakos K., (2004). "Actions on Railway Track", Papazissis publications, Athens, English edition, www.papazisi.gr.
23. Hay W., (1982). "Railroad Engineering", John Wiley & Sons.
24. Leykauf G., Mattner L., (1990). "Elastisches Verformungsverhalten des Eisenbahnoberbaus – Eigenschaften und Anforderungen", Eisenbahningenieur 41 (1990) 3.
25. ORE/UIC, Question D117, Rp2, (1973). "Study of the change in the track level as a function of the traffic and of

the track components (First results of laboratory and site tests)", April 1.

26. ORE/UIC, Question D117, Rp2, (1974). "Rheological properties of the track", April 1.

27. Prud' homme A., Erieau, J., (1976). "Les nouvelles traverses en beton de la SNCF", RGCF-2.

28. R.G.C.F. (Revue Generale des Chemins de Fer), Comite de redaction, (1973). "Sollicitations de la Voie, du Ballast et de la Plate-forme", Mai.

29. Tsoukantas S., Giannakos K., Topintzis T., Zois H., (2006). "System Rheda 2000 from the point of view of a Structural Engineer – the most modern evolution of superstructure technology in Railway projects", Greek Department of Concrete, Alexandroupolis, 24-27 October, Proceedings.

30. Tsoukantas S., 1999, "Investigation of Problems concerning the Slab Track application in Greece in tunnels, plain line and bridges", (Research Program of OSE-ERGOSE)", Athens.

31. U.I.C. (International Union of Railways), fiche UIC (Code) 719R, 1994, "Earthworks and track-bed layers for railway lines", 1-1-1994.