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Proofs of Utkin's Theorem for MIMO Uncertain Integral Linear Systems

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Abstract

The uncertain integral linear system is the integral-augmented uncertain system to improve the resultant performance. In this note, for a MI(Multi Input) uncertain integral linear case, Utkin's theorem is proved clearly and comparatively. With respect to the two transformations(diagonalizations), the equation of the sliding mode is invariant. By using the results of this note, in the SMC for MIMO uncertain integral linear systems, the existence condition of the sliding mode on the predetermined sliding surface is easily proved. The effectiveness of the main results is verified through an illustrative example and simulation study.

Key words: Utkin's theorem, Sliding mode control, Variable structure system, Diagonalization method, Transformation method

I. Introduction

Recently the integral action is augmented to the variable structure system or sliding mode control to improve the output performance[1]-[12]. Concerning introduction of an integral action to the VSS so-called the *integral variable structure system(IVSS)*, the performance of the zero steady state error and/or no reaching phase can be obtained[1]-[12], while it may exists in the digital implementation of the conventional VSS. For canonical SISO systems, the output error is integrated only to improve the steady state performance against external disturbances in [1] and [2]. In the cases of Choi and Chang, the sliding surfaces are integrated itself for control of multi-input systems[4][7]. In [8], the integral action as function of the low pass filter is employed to reduce the chattering in the control input. In [9], the time varying sliding surfaces are presented to remove the reaching phase. In [3] and [5], the integral state with a special initial condition is augmented to the conventional VSS in order to completely remove the reaching phase. The integral-augmented uncertain linear system is called as the uncertain integral linear system to improve the resultant performance.

To take the advantages of the sliding mode on the predetermined sliding surface in the VSS or SMC, the existence condition of the sliding mode, $s_i \cdot \dot{s}_i < 0,\ i=1,2,...m$ for the MI(multi input) linear case is satisfied. Therefore the existence condition of the sliding mode must be proved. For the linear MI case, a few control design method was studied, those are hierarchical control methodology[13][15], diagonalization methods[13][14][22][23], simplex algorithm[28], Lyapunov approach[23][29] and etc. Only the results of the derivative of the Lyapunov function is negative, i,e. $\dot{V}<0$ is obtained when $V=1/2\boldsymbol{s}^T\boldsymbol{s}$. The two methodologies to prove the existence condition of the sliding mode on the sliding surface were presented for MI uncertain

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nonlinear systems by Utkin[13]. Those two methodologies are the control input transformation and sliding surface transformation. The proof of Utkin's theroem is necessary for proving existence condition of the sliding mode. the But the proof is not sufficient. DeCarlo, Zak, and Matthews reviewed and tried to prove Utkin's invariance theorem. But, the proof is also not clear. The control input transformation without uncertainties and disturbance is used by Zak and Hui In [22]. But, they did not prove for the explicit control input transformation under uncertainties and disturbance. The sliding surface transformation was mentioned by Su, Drakunov, and Ozguner in [29]. In the case of the MI uncertain integral linear system, the proof of Utkin's theorem is necessary to prove the existence condition of the sliding mode on the predetermined sliding surface.

In this paper, a proof of Utkin's theorem is presented for MI uncertain integral linear systems. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods are proved clearly and comparatively for MI uncertain integral linear systems. By using the results of this note, in the SMC for a MIMO uncertain integral linear system, the existence condition of the sliding mode on the predetermined sliding surface is easily proved. A design example and simulation study shows usefulness of the main results.

II. Main Results of Proofs of Utkin's Theorem for MI Uncertain Integral Linear Systems

The invariant theorem of Utkin's for MI systems is as follows[13][14]:

Theorem 1: The equation of the sliding mode is invariant with respect to the two nonlinear transformations, i.e. the control input transformation and sliding surface transformation:

$$s^*(x) = H_s(x,t) \cdot s(x) \qquad u^*(x) = H_u(x,t) \cdot u(x) \tag{1}$$

for $\det H_s \neq 0$ and $\det H_u \neq 0$.

The theorem means that the sliding mode equation is governed by the original (1) if the components of the controlled vector undergo discontinuity on the new surface $s^*(X_0, X) = 0$ or the components of the new control vector $U^*(X)$ undergo discontinuity on the already chosen surface $s(X_0, X) = 0$, that is $s^*(X_0, X) = 0 \Leftrightarrow s(X_0, X) = 0$. Thus the performances designed in (1) can be guaranteed by the sliding mode on the new surface $s^*(X_0, X) = 0$.

For a MI uncertain linear system:

$$\dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u + \Delta D(t) \tag{2}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $A_0 \in R^{n \times n}$ is the nominal system matrix, $B_0 \in R^{n \times 1}$ is the nominal input matrix, ΔA and ΔB are the system matrix uncertainty and input matrix uncertainty, those are bounded, and $\Delta D(t)$ is bounded external disturbance, respectively.

For use later, an integral state for the integral linear systems is augmented as follows:

$$x_0(t) = \int_0^t x(\tau)d\tau + \int_{-\infty}^0 x(\tau)d\tau = \int_0^t x(\tau)d\tau + x_0(0) \tag{3}$$

where for non zero $C_0 = [c_{0_i}]$ and $C = [c_i]$

$$x_i(0) = -c_i / c_{0_i} x_i(0), \quad i = 1,2,...,n \tag{4}$$

Then, the integral sliding surface becomes

$$s = C_0 x_0 + Cx = [C_0 \quad C]\bar{x}, \qquad \bar{x} = \begin{bmatrix} x_0 \\ x \end{bmatrix} \tag{5}$$

For the coefficient matrix of the sliding surface C, the following assumptions are made.

Assumption 1:

CB_0 has the full rank and its inverse

Assumption 2:

$\Delta I = C\Delta B(CB_0)^{-1}$. ΔI is diagonal and $|\Delta I_i| \le \rho < 1, \quad i = 1,2,...,m$

Assumption 3:

$\Delta I' = (CB_0)^{-1} C\Delta B$. $\Delta I'$ is diagonal and $|\Delta I_i'| \le \rho < 1, \quad i = 1,2,...,m$

The VSS control input is as follows:

$$u_1 = -K \cdot x - \Delta K_1 \cdot x - \Delta K_2 \cdot sign(s) \tag{6}$$

where K is a constant gain, $\Delta K_1 = [\Delta k_{1_{ij}}]$ is a state dependent switching gain, $\Delta K_2 = [\Delta K_{2_i}]$ is a switching gain.

$$K = C_0 + CA_0 \tag{7}$$

$$\triangle k_{1_{ij}} = \begin{cases} \geq \dfrac{\max\{C\triangle A - C\triangle IK\}_{ij}}{\min\{I+\triangle I\}_i} & sign(s_i x_j) > 0 \\ \leq \dfrac{\min\{C\triangle A - C\triangle IK\}_{ij}}{\min\{I+\triangle I\}_i} & sign(s_i x_j) < 0 \end{cases}$$

$$i=1,2,\ldots,m, \quad j=1,2,\ldots,n \tag{8}$$

$$\triangle K_{2_i} = \begin{cases} \geq \dfrac{\max\{C\triangle D(t)\}_i}{\min\{I+\triangle I\}_i} & sign(s_i) > 0 \\ \leq \dfrac{\min\{C\triangle D(t)\}_i}{\min\{I+\triangle I\}_i} & sign(s_i) < 0 \end{cases}$$

$$i=1,2,\ldots,m \tag{9}$$

where min{ · } means the minimum value function and max{ · } implies the maximum value function.

2.1 control input transformation[15][17][23]

The control input is transformed as

$$\begin{aligned} u^* &= H_u \cdot u_1 \\ &= -H_u \cdot [Kx + \triangle K_1 x + \triangle K_2 sign(s)], \quad H_u = (CB_0)^{-1} \\ &= -(CB_0)^{-1} \cdot [Kx + \triangle K_1 x + \triangle K_2 sign(s)] \end{aligned} \tag{10}$$

Then, the real dynamics of s, i.e. the time derivative of s is as follows:

$$\begin{aligned} \dot{s} &= C\dot{x} + C_0 \dot{x}_0 \\ &= C(A_0+\triangle A)x + C(B_0+\triangle B)u^* + C\triangle D(t) + C_0 x, \\ &= C(A_0+\triangle A)x + (I+\triangle I)(-Kx - \triangle K_1 x \\ &\qquad - \triangle K_2 sign(s)) + C\triangle D(t) + C_0 x \\ &= CA_0 x + C_0 x - Kx + C\triangle Ax - C\triangle I Kx \\ &\qquad -(I+\triangle I)\triangle K_1 x + C\triangle D(t) - (I+\triangle I)\triangle K_2 sign(s) \end{aligned} \tag{11}$$

By (7), the real dynamics of s becomes

$$\begin{aligned} \dot{s} &= [C\triangle A - C\triangle IK]x - (I+\triangle I)\triangle K_1 x \\ &\qquad + C\triangle D(t) - (I+\triangle I)\triangle K_2 sign(s) \end{aligned} \tag{12}$$

By (8) and (9), then one can obtain the following equation

$$s_i \cdot \dot{s}_i < 0, \quad i=1,2,\ldots,m \tag{13}$$

The existence condition of the sliding mode is proved. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation

2.2 sliding surface transformation[15][17][24]

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(s,t) = (CB_0)^{-1} \tag{14}$$

The transformation matrix is selected as $H_s(x,t) = (CB_0)^{-1}$. In [14], its proof is not sufficient. Now, the VSS control input is taken as follows:

$$u_2 = -G \cdot x - \triangle G_1 \cdot x - \triangle G_2 \cdot sign(s^*) \tag{15}$$

where

$$G = (CB_0)^{-1} CA_0 + (CB_0)^{-1} C_0 \tag{16}$$

$$\triangle G_{1_{ij}} = \begin{cases} \geq \dfrac{\max\left\{(CB_0)^{-1} C\triangle A - \triangle I'(CB_0)^{-1} CA_0\right\}_{ij}}{\min\{I+\triangle I'\}_i} & sign(s_i^* x_j) > 0 \\ \leq \dfrac{\min\left\{(CB_0)^{-1} C\triangle A - \triangle I'(CB_0)^{-1} CA_0\right\}_{ij}}{\min\{I+\triangle I'\}} & sign(s_i^* x_j) < 0 \end{cases}$$

$$i=1,2,\ldots,m, \quad j=1,2,\ldots,n \tag{17}$$

$$\triangle G_{2_i} = \begin{cases} \geq \dfrac{\max\left\{(CB_0)^{-1} C\triangle D(t)\right\}_i}{\min\{I+\triangle I'\}_i} & sign(s_i^*) > 0 \\ \leq \dfrac{\min\left\{(CB_0)^{-1} C\triangle D(t)\right\}}{\min\{I+\triangle I'\}_i} & sign(s_i^*) < 0 \end{cases}$$

$$i=1,2,\ldots,m \tag{18}$$

The real dynamics of the sliding surface, i.e. the time derivative of s^* becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1}\dot{s} = (CB_0)^{-1} C\dot{x} + (CB_0)^{-1} C_0 \dot{x}_0 \\ &= (CB_0)^{-1} C(A_0+\triangle A)x + (CB_0)^{-1} C(B_0+\triangle B)u_2 \\ &\qquad + (CB_0)^{-1} C\triangle D(t) + (CB_0)^{-1} C_0 x \\ &= (CB_0)^{-1} C(A_0+\triangle A)x + (I+\triangle I')u_2 \\ &\qquad + (CB_0)^{-1} C\triangle D(t) + (CB_0)^{-1} C_0 x \\ &= (CB_0)^{-1} C(A_0+\triangle A)x + (I+\triangle I')(-Gx - \triangle G_1 x \\ &\qquad - \triangle G_2 sign(s^*)) + (CB_0)^{-1} C\triangle D(t) + (CB_0)^{-1} C_0 x \\ &= (CB_0)^{-1} CA_0 x + (CB_0)^{-1} C_0 x - Gx + (CB_0)^{-1} C\triangle Ax \\ &\qquad - \triangle I' Gx - (I+\triangle I')\triangle G_1 x \\ &\qquad + (CB_0)^{-1} C\triangle D(t) - (I+\triangle I') G_2 sign(s^*) \end{aligned} \tag{19}$$

By (16), then the real dynamics of s^* becomes

$$\begin{aligned} \dot{s}^* &= [(CB_0)^{-1} C\triangle A - \triangle I' G]x - (I+\triangle I')\triangle G_1 x \\ &\qquad + (CB_0)^{-1} C\triangle D(t) - (I+\triangle I')\triangle G_2 sign(s^*) \end{aligned} \tag{20}$$

In [23], without uncertainty and disturbance, it is mentioned that the sliding surface transformation would diagonalize the control coefficient matrix to the dynamics for s and only the $\dot{V}(x) < 0$ is proved when $V(x) = x^T P x > 0$.

From (20) and by (17) and (18), the following equation is obtained as

$$s_i^* \cdot \dot{s}_i^* < 0, \quad i=1,2,\ldots,m \tag{21}$$

If the sliding mode equation $s_i^* = 0$, then $s_i = 0$

since $CB_0 \neq 0$. The inverse augment also holds, therefore the both sliding surfaces are equal i.e., $s_i = s_i^* = 0$, which completes the proof of Theorem 1.

The sliding mode equation i.e. the sliding surface $s_i = 0$ is the same as that of $s_i^* = 0$. To compare the control inputs, u_1 and u_2, the form is the same but the gains of u_1 are multiplied by $(CB_0)^{-1}$. The both methods equivalently diagonalize the system, so those are called the diagonalization methods. By using the results of this note, in the SMC for MIMO uncertain integral linear systems, the existence condition of the sliding mode on the predetermined sliding surface is easily proved.

Ⅲ. Design Example and Simulation Studies

3.1 Plant

Consider a fifth-order system described by the state equation which is slightly modified from that in [30]

$$\dot{x} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 3.20\pm0.3 & 1.98\pm0.15 \\ 0.0 & 0.0 & 1.0 & -14.72\pm1.5 & 0.49 \\ -8.86 & 8.0\pm1.2 & 9.36 & -7.92 & 36.01\pm2. \\ 1.69 & 1.26 & 0.08 & 0.0 & 1.0 \\ -7.52 & -5.23\pm0.8 & -0.45 & 32.32 & -1.36 \end{bmatrix} x + \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 2.0\pm0.1 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 2.0\pm0.2 \end{bmatrix} u + \begin{bmatrix} 0.0 & .0.0 \\ 0.0 & 0.0 \\ \pm3.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & \pm5.0 \end{bmatrix} \tag{22}$$

where the nominal parameter A_0 and B_0, matched uncertainties ΔA and ΔB, and disturbance $\Delta D(t)$ are

$$A_0 = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 3.20 & 1.98 \\ 0.0 & 0.0 & 1.0 & -14.72 & 0.49 \\ -8.86 & 8.0 & 9.36 & -7.92 & 36.01 \\ 1.69 & 1.26 & 0.08 & 0.0 & 1.0 \\ -7.52 & -5.23 & -0.45 & 32.32 & -1.36 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 2.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 2.0 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0.0 & 0.0 & 0.0 & \pm0.3 & \pm0.15 \\ 0.0 & 0.0 & 0.0 & \pm1.5 & 0.0 \\ 0.0 & \pm1.2 & 0.0 & 0.0 & \pm2.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & \pm0.8 & 0.0 & 0.0 & 0.0 \end{bmatrix},$$

$$\Delta B = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ \pm0.1 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & \pm0.2 \end{bmatrix} \quad \Delta D(t) = \begin{bmatrix} 0.0 & .0.0 \\ 0.0 & 0.0 \\ \pm3.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & \pm5.0 \end{bmatrix} \tag{23}$$

3.2 An Example of Integral Case

The integral state is augmented as follows:

$$x_0(t) = \int_0^t x(\tau)d\tau + x_0(0) \tag{24}$$

The coefficient of the sliding surface is determined as

$$C = \begin{bmatrix} -0.436 & 1.802 & 1.0 & -14.568 & 0.0 \\ 1.01 & 0.505 & 0.0 & 1.616 & 0.5 \end{bmatrix} \quad \text{and}$$

$$C_0 = \begin{bmatrix} -12.8801 & 26.5174 & 31.0025 & -80.5224 & -4.6022 \\ 32.6521 & 4.6890 & 8.0205 & 82.7724 & 18.3361 \end{bmatrix} \tag{25}$$

1) control input transformation

$$H_u = (CB_0)^{-1} = \begin{bmatrix} 2.0 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \tag{26}$$

$$u^* = -\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}[Kx + \Delta K_1 x + \Delta K_2 sign(s)]$$

Then, the real dynamics of s, i.e. the time derivative of s is as follows:

$$\begin{aligned} \dot{s} &= C\dot{x} + C_0\dot{x}_0 \\ &= CA_0x + C_0x - Kx + C\Delta Ax - C\Delta IKx \\ &\quad -(I+\Delta I)\Delta K_1x + C\Delta D(t) - (I+\Delta I)\Delta K_2 sign(s) \end{aligned} \tag{27}$$

where

$$\Delta I = C\Delta B(CB_0)^{-1} = \begin{bmatrix} \pm0.05 & 0 \\ 0 & \pm0.1 \end{bmatrix} \tag{28}$$

Thus, Assumption A1 is satisfied. By letting the constant gain

$$\begin{aligned} K &= CA_0 + C_0 \\ &= \begin{bmatrix} -46.3600 & 15.7257 & 40.9991 & -116.3630 & 16.8595 \\ 31.6231 & 5.1202 & 8.4297 & 94.7302 & 21.5193 \end{bmatrix} \end{aligned} \tag{29}$$

then the real dynamics of s becomes

$$\begin{aligned} \dot{s} &= [C\Delta A - C\Delta IK]x - (I+\Delta I)\Delta Kx \\ &\quad + C\Delta D(t) - (I+\Delta I)\Delta K_2 sign(s) \end{aligned} \tag{30}$$

If one takes the switching gain as design parameters

$$\Delta k_{11} = \begin{cases} 1.2 & \text{if } s_1x_1 > 0 \\ -1.2 & \text{if } s_1x_1 < 0 \end{cases},$$

$$\Delta k_{12} = \begin{cases} 4.2 & \text{if } s_1x_2 > 0 \\ -4.2 & \text{if } s_1x_2 < 0 \end{cases}, \quad \Delta k_{13} = \begin{cases} 6.5 & \text{if } s_1x_3 > 0 \\ -6.5 & \text{if } s_1x_3 < 0 \end{cases}$$

$$\Delta k_{14} = \begin{cases} 8.5 & \text{if } s_1x_4 > 0 \\ -8.5 & \text{if } s_1x_4 < 0 \end{cases},$$

$$\Delta k_{15} = \begin{cases} 16.5 \ \text{if}\, s_1 x_5 > 0 \\ -16.5 \ \text{if}\, s_1 x_5 < 0 \end{cases}, \quad \Delta K_{21} = \begin{cases} 3.8 \ \text{if}\, s_1 > 0 \\ -3.8 \ \text{if}\, s_1 < 0 \end{cases}$$

$$\Delta k_{21} = \begin{cases} 5.2 \ \text{if}\, s_2 x_1 > 0 \\ -5.2 \ \text{if}\, s_2 x_1 < 0 \end{cases}, \quad \Delta k_{22} = \begin{cases} 4.2 \ \text{if}\, s_2 x_2 > 0 \\ -4.2 \ \text{if}\, s_2 x_2 < 0 \end{cases},$$

$$\Delta k_{23} = \begin{cases} 5.5 \ \text{if}\, s_2 x_3 > 0 \\ -5.5 \ \text{if}\, s_2 x_3 < 0 \end{cases} \quad \Delta k_{24} = \begin{cases} 2.5 \ \text{if}\, s_2 x_4 > 0 \\ -2.5 \ \text{if}\, s_2 x_4 < 0 \end{cases},$$

$$\Delta k_{25} = \begin{cases} 3.5 \ \text{if}\, s_2 x_5 > 0 \\ -3.5 \ \text{if}\, s_2 x_5 < 0 \end{cases}, \quad \Delta K_{22} = \begin{cases} 3.8 \ \text{if}\, s_2 > 0 \\ -3.8 \ \text{if}\, s_2 < 0 \end{cases}$$

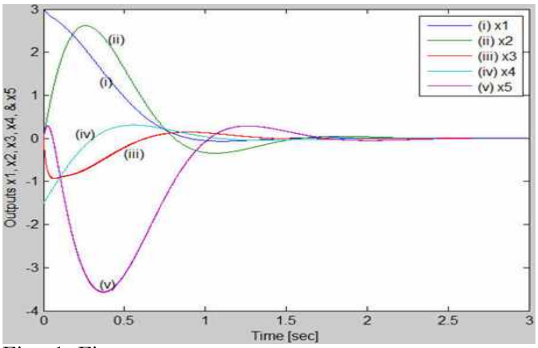

Fig. 1 Five state output responses, x_1, x_2, x_3, x_4, and x_5

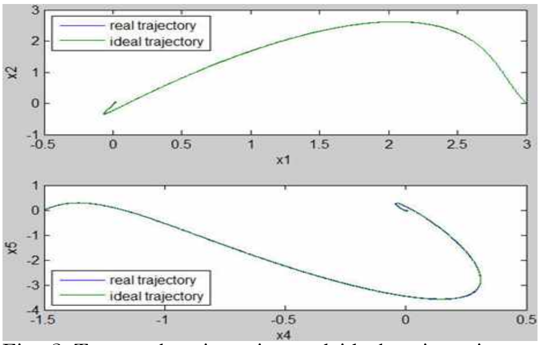

Fig. 2 Two real trajectories and ideal trajectories on x_1-x_2 plane(upper) and x_4-x_5 plane(below)

(31)

then one can obtain the following equation

$$s_i \cdot \dot{s}_i < 0, \ \ i = 1 \ 2 \tag{32}$$

The existence condition of the sliding mode is proved. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation. The simulation is carried out under 0.1[msec] sampling time and with $x(0) = [3 \ \ 0 \ \ 0 \ -1.5 \ \ 0]^T$ initial state, by means of Fortran language. Fig. 1 shows the five state output responses, x_1, x_2, x_3, x_4, and x_5. Fig. 2 shows the two real trajectories and ideal trajectories on on x_1-x_2 plane(upper) and x_4-x_5 plane(below). The controlled system slides from the beginning as shown in these figures. The two sliding surfaces and two control inputs are depicted in Fig. 3 and Fig. 4, respectively.

2) sliding surface transformation

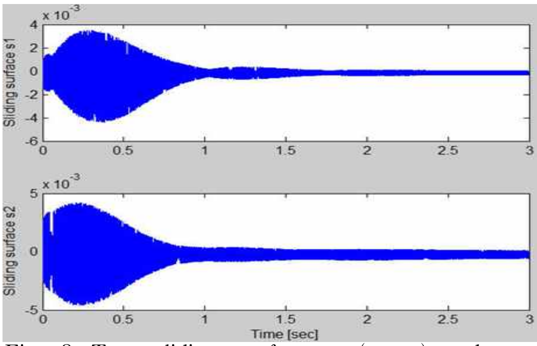

Fig. 3 Two sliding surfaces s_1(upper) and s_2 (below)

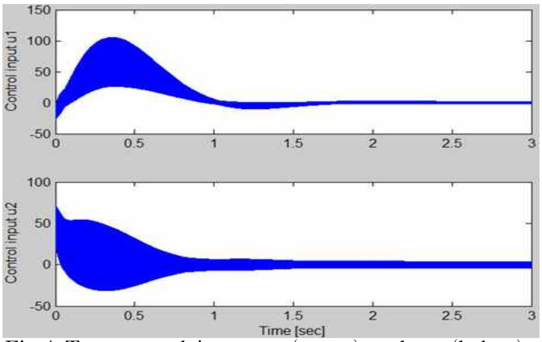

Fig.4 Two control inputs u_1(upper) and u_2(below)

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(x,t) = (CB_0)^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.0 \end{bmatrix} \tag{33}$$

Now, the VSS control input are taken as follows:

$$u_2 = -G \cdot x - \Delta G_1 \cdot x - \Delta G_2 \cdot sign(s^*) \tag{34}$$

The real dynamics of the sliding surface, i.e. the time derivative of s^* becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1} \dot{s} = (CB_0)^{-1} C\dot{x} \\ &= (CB_0)^{-1} CA_0 x - Gx + (CB_0)^{-1} C\Delta A x \\ &\quad - \Delta I' Gx - (I + \Delta I')\Delta G_1 x + (CB_0)^{-1} C\Delta D(t) \\ &\quad - (I + \Delta I')\Delta G_2 sign(s^*) \end{aligned} \tag{35}$$

By letting gain

$$G=(CB_0)^{-1}CA_0+(CB_0)^{-1}C_0$$
$$=\begin{bmatrix}-23.1800 & 7.8628 & 20.4955 & -58.1815 & 8.4298\\ 31.6231 & 5.1202 & 8.4297 & 94.7302 & 21.5193\end{bmatrix} \tag{36}$$

then the real dynamics of s^* becomes

$$\dot{s}^*=[(CB_0)^{-1}C\Delta A-\Delta I K]x-(I+\Delta I)\Delta G_1 x + (CB_0)^{-1}C\Delta D(t)-(I+\Delta I)\Delta G_2 sign(s^*) \tag{37}$$

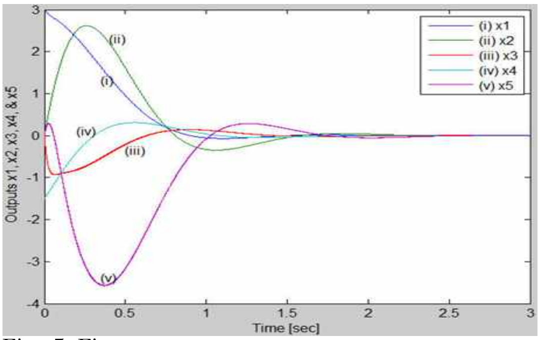

Fig. 5 Five state output responses, x_1, x_2, x_3, x_4, and x_5

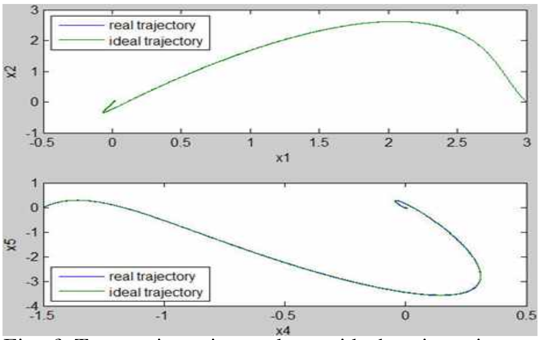

Fig. 6 Two trajectories and two ideal trajectories on x_1-x_2 plane(upper) and x_4-x_5 plane(below)

$$\Delta I=(CB_0)^{-1}C\Delta B=\begin{bmatrix}\pm 0.05 & 0\\ 0 & \pm 0.1\end{bmatrix} \tag{38}$$

Thus, Assumption A2 is satisfied. If one takes the switching gains as follows

$$\Delta g_{11}=\begin{cases}0.6 & \text{if } s_1^*x_1>0\\ -0.6 & \text{if } s_1^*x_1<0\end{cases},\quad \Delta g_{12}=\begin{cases}2.1 & \text{if } s_1^*x_2>0\\ -2.1 & \text{if } s_1^*x_2<0\end{cases}$$

$$\Delta g_{13}=\begin{cases}3.25 & \text{if } s_1^*x_3>0\\ -3.25 & \text{if } s_1^*x_3<0\end{cases}$$

$$\Delta g_{14}=\begin{cases}4.25 & \text{if } s_1^*x_4>0\\ -4.25 & \text{if } s_1^*x_4<0\end{cases}$$

$$\Delta g_{15}=\begin{cases}8.25 & \text{if } s_1^*x_5>0\\ -8.25 & \text{if } s_1^*x_5<0\end{cases},\quad \Delta G_{21}=\begin{cases}1.9 & \text{if } s_1^*>0\\ -1.9 & \text{if } s_1^*<0\end{cases}$$

$$\Delta g_{21}=\begin{cases}5.2 & \text{if } s_2^*x_1>0\\ -5.2 & \text{if } s_2^*x_1<0\end{cases}\quad \Delta g_{22}=\begin{cases}4.2 & \text{if } s_2^*x_2>0\\ -4.2 & \text{if } s_2^*x_2<0\end{cases},$$

$$\Delta g_{23}=\begin{cases}5.5 & \text{if } s_2^*x_3>0\\ -5.5 & \text{if } s_2^*x_3<0\end{cases}$$

$$\Delta g_{24}=\begin{cases}2.5 & \text{if } s_2^*x_4>0\\ -2.5 & \text{if } s_2^*x_4<0\end{cases}\quad \Delta g_{25}=\begin{cases}3.5 & \text{if } s_2^*x_5>0\\ -3.5 & \text{if } s_2^*x_5<0\end{cases},$$

$$\Delta G_{22}=\begin{cases}3.8 & \text{if } s_2^*>0\\ -3.8 & \text{if } s_2^*<0\end{cases} \tag{39}$$

then

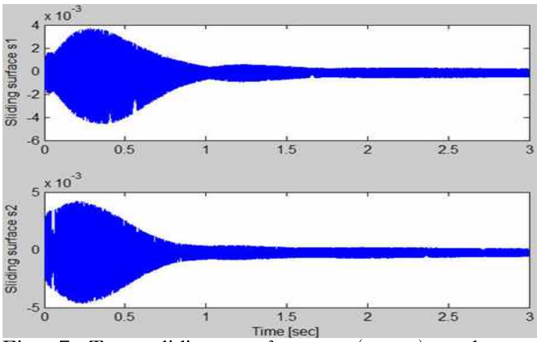

Fig. 7 Two sliding surfaces s_1(upper) and s_2 (below)

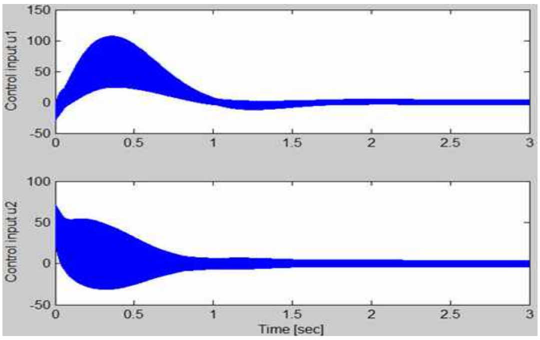

Fig.8 Two control inputs u_1(upper) and u_2(below)

$$s_i^*\cdot\dot{s}_i^*<0,\quad i=1\ 2 \tag{40}$$

If $s_i^*=0$, then $s_i=0$. The inverse augment also holds. The switching gains in (63) can be obtained also from (28) by multiplying $(CB_0)^{-1}=\begin{bmatrix}0.5 & 0\\ 0 & 1.0\end{bmatrix}$. The simulation is carried out under 1[msec] sampling time and with $x(0)=[3\ \ 0\ \ 0\ -1.5\ \ 0]^T$ initial condition. Fig. 5 shows the five state output responses, x_1, x_2, x_3, x_4, and x_5. Those are almost identical to Fig. 1 because the sliding surface $\boldsymbol{s}=0=\boldsymbol{s}^*$ is equal and the continuous gains and discontinuous gains of the both controls, $\boldsymbol{u}^*$ and $\boldsymbol{u}_2$

are equal. Fig. 6 shows the two real trajectories and ideal trajectories on on x_1-x_2 plane(upper) and x_4-x_5 plane(below). The controlled system slides from the beginning as shown in these figures. The two sliding surfaces and two control inputs are depicted in Fig. 7 and Fig. 8, respectively.

Ⅳ. Conclusions

In this note, the invariant theorem of Utkin is rigorously proved for MI uncertain integral linear systems. The invariance theorem of the two diagonal methods i.e., the control input transformation and sliding surface transformation is proved clearly and comparatively. Therefore, the equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods. These two methods diagonalize the input system of the real sliding dynamics of the sliding surface s or s^* so that the existence condition of the sliding mode on the predetermined sliding surface is easily proved. During the proof of Utkin's theorem for MI uncertain integral linear systems, the design rules of both control inputs are proposed. Through an illustrative example and simulation study, the effectiveness of the proposed main results is verified. The same results of the outputs by the two diagonalization methods are obtained. The equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods for MI uncertain integral linear systems. By using the results of this note, in the SMC for a MIMO uncertain integral linear system, the existence condition of the sliding mode on the predetermined sliding surface is easily proved

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