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An Integer Programming Formulation for Outpatient Scheduling with Patient Preference

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ABSTRACT

Patients' satisfaction while receiving medical service is affected by whether or not their preferences can be met, including time and physician preference. Due to scarcity of medical resource in China, efficient use of available resources is urgently required. To guarantee the utilization ratio, the scheduling decisions are made after all booking information is received. Two integer models with different objectives are formulated separately, maximizing the degree of satisfaction and revenue. The optimal value of the two models can be considered as the bound of corresponding objectives. However, it is improper to implement any of the extreme policies. Because revenue is a key element to keep the hospital running and satisfaction degree is related to the hospital's reputation, neither the revenue nor the satisfaction can be missed. Therefore, hospitals should make a balance. An integrated model is developed to find out the tradeoff between the two objectives. The whole degree of mismatching that is related to patient satisfaction and other separate mismatching degree are considered. Through a computational study, it is concluded that based on the proposed model hospitals can make their decisions according to service requirement.

Keywords: Appointment Scheduling, Integer Programming, Healthcare, Outpatient Department

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1. INTRODUCTION

The healthcare system is facing an increasing pressure to satisfy more and more demand especially when an aging society is coming. A good design of the appointment system can increase utilization of facilities, cut waiting time for patients, and then improve patients' satisfaction. However, there are a number of variables in patients' arrival and service processes, which make outpatient scheduling complicated. Gupta and Denton (2008) summarize appointment research in the healthcare system and discover some challenges that affect the efficiency of the system. One of the challenges is patient preferences. After that, there are some papers focusing on appointment scheduling with patient preference of choices, such as Gupta and Wang (2008), Wang and Gupta (2011), and Qu and Shi (2011). However, these researches focus on the western system which is comparatively well-developed, while little study is conducted on the healthcare system in China that has more pressure. An overview of healthcare system is as follows. According to the statistics from the Ministry of Health of China, the number of visits to health institutions in 2011 approached 6.3 billion, a 7.4% increase over 2010. Another figure from China Statistical Yearbook (2011) says that there are only 2.24 medical technical personnel in healthcare institutions per 1,000 persons. Therefore, the healthcare system in China extremely short of resources while demand for medical care keeps increasing. Due to the poor condition in healthcare, the medical quality goes down, which makes medical treatment disputes rise year by year. Since it is hard to improve the condition in a short time, it is essential to use healthcare resource effectively, promote medical quality and improve patient satisfaction. The objective of this paper is to develop a framework for outpatient scheduling considering patient preference to improve healthcare service in China

This work mainly focuses on appointment scheduling in outpatient department (OPD) in China. Because of the difference between the healthcare system in China and Western countries, OPD in China also faces a different situation. Gupta and Denton (2008) summarize three different healthcare environments in the Western primary care clinic, specialty care clinic, and hospital. In the West, most people go to a clinic near where they live when they get sick. If needed, they are introduced to a hospital. The clinics near the communities greatly alleviate the pressure for the hospital. However, things are so different in China where many people want to go to the hospital directly. As a result, the hospital becomes very crowded, especially for some prestigious hospitals. To some extent, OPD in China is like the primary care clinic referred in the paper. However, there are some points making the two different from each other. First, in China, most of physicians in OPD are also the physicians in inpatient department. Therefore, physicians of OPD are different every day. They are in OPD on a fixed day of a week. For example, a physician may be in OPD on Monday every week and in inpatient Department on other days. As a result, when a patient prefers a particular physician, he/she must wait for at least a week if he/she fails to see the physician. Second, in primary care clinic, people have a designated physician, so called primary clinic provider (PCP) (Gupta and Denton 2008). Patients have a degree of loyalty toward their own PCP. Patients go to their PCP whatever is wrong with them. However, in China, people believe that different hospitals are good at dealing with different diseases. They tend to go to a hospital which specializes in that disease they get. Third, in most cases, primary care clinic is closed to patients' home. Whereas people in China are willing to go to a hospital far from their home if the hospital is well-known. Therefore, in China, patient preferences are based mainly the reputation of a hospital or a physician, and scarcity of resource, not loyalty. Besides, under the healthcare environment in China, it is very important to satisfy patient preferences.

In China, the appointment system is not developed sound. Since the hospital faces more and more pressure in recent years, Ministry of Health required hospitals to build the appointment system in 2009. By now, patients who want to see experts (senior physicians) must make appointments, because experts are scarce. So this paper will focus on the appointment system for experts.

With the special healthcare environment as background, this paper mainly discusses the appointment scheduling problem in OPD. With revenue and preference satisfaction as objectives, static models based on integer programming are developed. The main contributions of this paper are as follows:

- This paper not only attaches importance to physician preference, but time slot preference. For some patients with a strict scheduling, they might want to finish a treatment within a particular time slot. Considering the two difference preferences, so-called two-dimensional preference, patients are classified into four categories.
- Two measurements for outpatient scheduling are developed, preference satisfaction and revenue.
- The relationship between the two measurements is studied. The tradeoff between the two objectives is illustrated through a computational study.

The remainder of this paper is organized as follows: in Section 2, we review the related literatures. In Section 3, appointment processes under the healthcare environment are introduced. Then appointment models based on integer programming are formulated. In Section 4, the relationship between revenue and different types of mismatching degree are discussed through computational study. Finally, conclusion and future work are provided in Section 5.

2. LITERATURE REVIEW

Appointment scheduling is to make patient get access to medical services efficiently and timely. It has become a hot research topic in recent years. The two review articles by Cayirli and Veral (2003) and Gupta and Denton (2008) provide a broad information and review of this research area. Gupta and Denton (2008) point out that how to meet patient preference will be a part of the next generation of appointment scheduling system.

Before studying patient preferences in healthcare industry, some researchers have studied preference when customers pick up commodities, such as Zhang and Cooper (2005), Talluri and Van Ryzin (2004) and Gosavi *et al*. (2002). A handful of researchers have begun to concentrate on appointment scheduling with patient preferences. Some authors apply some mature revenue management theory to appointment scheduling directly. However, Gupta and Denton (2008) point out that there are two kinds of differences. First, the patient choice is affected by various factors, such as provider, date, time of the day. Second, price fluctuation usually is not a feasible method to control patient choice. Patient preference is first modeled explicitly by Gupta and Wang (2008) To some extent, it is a significant progress in appointment scheduling research for healthcare service. Patient choice in their paper includes his/her preferred physician and the convenient time he/she would like to arrive. Patients are divided into two categories: regular patients who call with the lead time bigger than one day, and same-day patients who arrive at the start of the workday. Patients can switch their choice if the preferred time slot or physician is not available. Wang and Gupta (2011) develop an adaptive appointment system, which can dynamically learn and update patients' preferences. The patients who want to book a block have an acceptable set, in which scheduler should choice a block to appoint for the patient. It is claimed that the adaptive system will be a framework for the design of the next generation of appointment system. Vermeulen *et al.* (2009) combine patient preferences and the urgencies. Dynamic rules are proposed for urgent needs. Since it is hard to put values on preferences, a Boolean-type model is proposed, in which a patient is assigned either to a preferred time slot or non-preferred time slot. In Feldman *et al.* (2012), patient preference is referred to the preferred arrival time. The service provider dynamically makes decisions that which working days should be available for the appointments. Both static and dynamic models are developed, depending on whether the current state of the scheduled appointments is considered.

Besides, scheduling problem can be divided into two broad categories, online and offline problem. Online systems deal with sequential scheduling. Without complete information of the future, they estimate the situation of future system and build dynamic programming models. Offline systems deal with scheduling problems with complete information and can make decisions after all appointments are available. However, the static system cannot be considered as job-shop problem, because patients who have choices and preferences cannot be seen as parts. Godin and Wang (2010) proposed a linear agent-based scheduling system in which the available slots might be changed. There are several stages of iterations. Diagnostic services agent implements the first iteration after patients provide a bundle of acceptable slots. Some patients may not accept the given result and hope a better arrangement. Besides, some patients may cancel appointment, so some occupied slots become available. Therefore, iteration is needed. Zhu *et al.* (2012) considered outpatient scheduling problem as a group role assignment problem. They developed linear programming model to found an assignment scheme in order to maximize priority value. If the available slots change, patients will be asked to give bidding for bundles of time slots. Based on patients' responses a new rescheduling will be made. Burke *et al.* (2011) present a new integer programming model to resolve real-world radiotherapy treatment scheduling. Without attempting to predict the patients who will arrive in the future, a myopic approach is evaluated. In this paper, we focus on an offline scheduling problem, considering the special healthcare environment in China.

There are various methods to model appointment system states. Lin *et al.* (2011) discuss sequential clinical scheduling problem with no-shows. Patients are categorized according to their no-show rates. Physicians' working hours are divided into several time slots. It is possible that there are more than one patient in a slot. Different physicians are not distinguished. In Gupta and Wang (2008), system state is constituted by several physicians and some time slots. A two-dimensional system state is considered, which makes the problem more complicated. However, each time slot can only allow one physician. In this paper, the modelling in Gupta and Wang (2008) is used.

Another important point in appointment scheduling problem is how to evaluation the proposed appointment system. One main objective is to minimize waiting time. Kunnumkal and Topaloglu (2010) implement various scheduling rules to minimize waiting time of patients and idle time of service provide in a dynamic environment. Klassen and Yoogalingam (2013) design an appointment system considering service interruptions and physician lateness. The purpose is also to reduce waiting time of patients and increase utilization of physicians. Robinson and Chen (2003) propose an appointment system to balance the patients' waiting times and the doctors' idle time. Another main objective of appointment system is to maximize profit/revenue. The papers that try to maximize profit/revenue of a clinic often consider patient behaviors, such as patient preferences (Feldman *et al.*, 2012), patient choices (Gupta and Wang, 2008), no-shows (Liu and Ziya, 2014). To some extent, reducing waiting time of patients and idle time of physicians is equivalent to increasing revenue. In this paper, the objective is also to maximize revenue. However, the revenue is this paper is greatly related to patient preferences.

3. MODEL FORMULATION

3.1 Appointment Process

In our model, patients can make appointments through telephone or Web-based interface, which can record patient preferences. As with the work in Zhu *et al.* (2012), we assume that all the appointment scheduling decisions can be made after all the appointments have come and before service process. Patients with independent preference can arrive punctually or before the appointment time. In the system, there are I physicians, each of who has several blocks every day. Each block can be divided into several time slots. The length of slot in our system is fixed in advance. Physician can finish service within every time slot.

When patients make their appointments, they often have their preferences. It is assumed that each patient just has at most one preference. In this paper, preference of patient can be categorized into four kinds: time-dominated preference, physician-dominated preference, strong preference, and weak preference. If patients are very busy in their work or something else, they might pay attention to when they can see a doctor, who can be considered as time-dominated patients. They do not care which physician will serve them. In contrast, for physician-dominated patients, they just care which physician will treat them, rather than when they can see the physician. In practice, there are two main reasons why they prefer a particular physician. First, patients who are return visit for the treatment may see their previous physician according to doctor's advice. Second, a physician in the hospital may be very famous. Patients mostly believe that famous physician is more helpful for their recovery. As for patients with strong preference, they have time and physician preferences, while patients with weak preference they care neither time nor physician. Generally, the condition of patients who have weak preferences is comparatively less serious. In addition, since no emergency patients are considered in our model, non-arranged policy is permitted if the penalty of non-arranged patient is less than the expected revenue coming from empty blocks.

3.2 Revenue-Based Mathematical Model

In this subsection, we propose an integer programming model with maximal revenue as objective. Some notations are shown in Table 1. In our model, revenue is related to the efforts paid by physicians in the department. In our system, the efforts depend on preference of patient. For patients with physician preference, they mostly are return visitors. Apparently, patient–physician mismatching can decrease revenue according to O'Hare and Corlett (2004). One reason is that physicians may spend a lot of valuable time on reading and checking materials of unfamiliar patients (Gupta and Wang, 2008). As patients with time-slot preference, the department can get more revenue if the patients can be served after their preferred time, because patient can have more time to be familiar with materials of patients. To simplify our model, c_{nij} is used to describe the revenue if patient n is arranged into time slot j of physician i (physician-slot combination (i, j)).

Different kinds of patients have different revenue matrix C. For example, there are 2 physicians. And each physician has 3 slots a day. For time-dominated patient, the revenue matrix can be written as $\begin{pmatrix} 3 & 4 & 2 \\ 3 & 4 & 2 \end{pmatrix}$, which depends on time-slot, and is independent on physicians. Similarly, a revenue matrix for physician-dominated patient, strong preference patient and weak preference patient may be expressed as $\begin{pmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 6 & 4 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix}$.

A revenue-based integer programming model (Model I) is formulated as follows.

$$\max \sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{T_i} c_{nij}x_{nij} + \sum_{i=1}^{I}\sum_{m=1}^{S_i} r_{im}y_{im} - \sum_{n=1}^{N} l_n \left(1 - \sum_{i=1}^{I}\sum_{j=1}^{T_i} x_{nij}\right) \quad (1)$$

$s.t.$

$$\sum_{i=1}^{I}\sum_{j=1}^{T_i} x_{nij} \le 1, \quad n = 1, 2, \cdots, N \quad (2)$$

$$\sum_{n=1}^{N} x_{nij} \le 1, \quad i = 1, 2, \cdots, I,\ j = 1, 2, \cdots, T_i \quad (3)$$

$$1 - \frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk} x_{nij} \ge y_{im} \quad i = 1, 2, \cdots, I,\ m = 1, \cdots, k \quad (4)$$

$$x_{nij} \in \{0, 1\} \quad (5)$$

$$y_{im} \in \{0, 1\} \quad (6)$$

The objective of this model is to maximize revenue, including revenue of service, revenue of closed blocks, and the loss of refusing patient. Expression (2) means every patient should be arranged into a slot. Expression (3) means at most one patient can be arranged into a slot. Expressions (5) and (6) are binary constraints.

Table 1. Notation

Symbol	Description	Support
m	denotation of blocks	$m \in \{1, 2, \cdots, S_i\}$, S_i is the total number of blocks of physician i
i	denotation of physician	$i \in \{1, 2, \cdots, I\}$
j	denotation of slot in blocks, and each block has k slots	$m \in \{1, 2, \cdots, T_i\}$, T_i is the total number of slots of physician i
N_i	the number of type i patient	$\sum_{i=1}^{4} N_i = N$
n	denotation of patient	$n \in \{1, 2, \cdots, N\}$
k	total of number of slots in each block	$k \times S_i = T_i$
c_{nij}	c_{nij} the average revenue if patient n is arranged into slot (i, j)	$C = \{C_{nij}\}$
x_{nij}	is equal to 1 if patient n is arranged into slot (i, j), otherwise, is equal to 0	
y_{im}	is equal to 1 if the mth block of physician i is empty, otherwise 0	$m \in \{1, 2, \cdots, s_i\}$,
r_{im}	average revenue from closed block m of physician i	
l_n	average penalty if patient n is not arranged	

Expression (4) is used to judge whether a block is closed or not. If slots in a block are all closed, the physician can available for other service, such as, surgery, service for inpatients. Therefore, it is possible to close some blocks to save recourse, gaining more value revenue. Based on this idea, we have to judge which blocks are empty, that is, whether any x_{nij} in the block is zero or not. That is,

$$y_{im}=\begin{cases}1 & x_{nij}=0,\ for\ j=(m-1)k+1,\cdots,mk;\ n=1,2,\cdots,N\\ 0 & Otherwise\end{cases} \tag{7}$$

So a linear expression can be written as follows.

$$x_{nij}+y_{im}\le 1,\ j=(m-1)k+1,\cdots,mk;\ n=1,2,\cdots,N \tag{7.1}$$

Another method is to judge whether the sum of number of patients arranged into a block, $\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}$, is zero or not. That is,

$$y_{im}=\begin{cases}1 & \sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}=0\\ 0 & Otherwise\end{cases}.$$

Because $0\le\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}\le k$, so $0\le\frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}\le 1$, then get

$$y_{im}=\begin{cases}1 & \frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}=0\\ 0 & Otherwise\end{cases}.$$

So there are two different linear expressions to describe this relation. The first one is

$$1-\frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}x_{nij}\ge y_{im}. \tag{7.2}$$

Another expression is

$$\frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk}\left(1-x_{nij}\right)\ge y_{im}. \tag{7.3}$$

Though both of the two formulations can express the constraints, they have different efficiency. The choice of a formulation is crucial, which is greatly related to the number of variables and constraints. Notice that Formulation (7.1) has $kN\sum_i iS_i$ contraints, while Formulation (7.2) and (7.3) have $\sum_i iS_i$ constraints, respectively. To compare which formulation is stronger, a linear programming relaxation is needed, which replaces the integer constraints, $x_{nij}, y_{im}\in(0,1)$ by $0\le x_{nij},\ y_{im}\le 1$. By comparing the different polyhedrons defined by corresponding relaxed constraints (see Bertsimas and Tsitsiklis (1997), p. 461-463), Formulation (7.2) is more efficient.

3.3 Mismatching-Based Mathematical Model

Aside from revenue maximization, another objective is to maximize the degree of matching between patient preference and arrangements, i.e. minimize the degree of mismatching. We assume that the degree of matching is proportional to patient satisfaction. This degree includes two dimensions, time and physician. Let i_n be the physician preference of patient n, and j_n be the time slot preference of patient n. Patients with time slot preference are supposed to be convenient at that prospective time. Any deviation can result in being uncomfortable to some extent. As for patients who have physician preference, their satisfaction degree is associated with the degree of physician matching. It is rational to assume that the different physicians in the department have different attributes. For example, different physicians have different special talents. Take the OPD in a cardiovascular hospital for example. A physician may be good at coronary disease, while another one may specialize in cardiac failure. If patients cannot see their preferred physician, their satisfaction can decrease. It should be pointed out that this kind of inconvenience does not necessarily impact the actual quality of service.

An integer programming model (Model II) with mismatching-based measurement is formulated as follows.

$$\min\frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{I}\left|\sum_{i=1}^{I}iX_{ni}-i_n\right|+\frac{1}{T_{i_n}}\left|\sum_{j=1}^{T_{i_n}}jY_{nj}-j_n\right|\right) \tag{8}$$

$s.t.$

$$X_{ni}\ge x_{nij},\quad n=1,2,\cdots,N,\ i=1,2,\cdots,I,\ j=1,2,\cdots,T_i \tag{9}$$

$$\sum_{i=1}^{I}X_{ni}\le 1,\quad n=1,2,\cdots,N \tag{10}$$

$$Y_{nj}\ge x_{nij},\quad n=1,2,\cdots,N,\ i=1,2,\cdots,I,\ j=1,2,\cdots.T_i \tag{11}$$

$$\sum_{j=1}^{J}Y_{nj}\le 1,\quad n=1,2,\cdots,N \tag{12}$$

$$x_{nij}, X_{ni}, Y_{nj}\in\{0,\ 1\},\ n=1,2,\cdots,N,\ i=1,2,\cdots,I,\ j=1,2,,\cdots,T_i \tag{13}$$

In this model, X_{ni} equals to one if patient n is arranged to physician i and equals to zero otherwise. Y_{nj} equals to one if patient n is arranged to time slot j and equals to zero otherwise. Expression (8) is to minimize the average degree of mismatch, including time slot mismatching and physician mismatching. Expression (9) means that if patient n is arranged to a combination (i, j), we use X_{ni} describe that patient n is arrange to physician i. Expression (10) is to guarantee that one patient only can be arranged to one physician. Expressions (11) and

(12) have the similar function. Expressions (13) are binary constraints.

Let us name the result of Expression (8) is the total degree of mismatching. Similarly, we can get the expression of mismatching degree of each type of patient.

$$\text{Mismatching degree of type 1: } \frac{1}{N_1}\sum_{n=1}^{N_1}\frac{1}{T_{i_n}}\left|\sum_{j=1}^{T_{i_n}} jY_{nj} - j_n\right| \quad (14)$$

$$\text{Mismatching degree of type 2: } \frac{1}{N_2}\sum_{N_1+1=1}^{N_1+N_2}\frac{1}{I}\left|\sum_{i=1}^{I} iX_{ni} - i_n\right| \quad (15)$$

Mismatching degree of type 3:

$$\frac{1}{N_3}\sum_{N_1+N_2+1}^{N-N_4}\left(\frac{1}{I}\left|\sum_{i=1}^{I} iX_{ni} - i_n\right| + \frac{1}{T_{i_n}}\left|\sum_{j=1}^{T_{i_n}} jY_{nj} - j_n\right|\right) \quad (16)$$

4. COMPUTATIONAL STUDY

In order to evaluate our model and test the relationship between the two objectives, maximizing revenue and minimizing the degree of mismatching, experiments are run simulating all calling of appointments for a day. Some environmental parameters are set as follows. There are five physicians, each of which has six blocks in a day. Each block has four slots in it. So under this scenario, the capacity of the department is 120. That is, the department can serve 120 patients each day. Overtime is not permitted in our experiments. It is assumed that there are precisely 120 appointments, constituted by uniform number of each type of patients. Preferences are randomly generated. Revenues from each patient are normally distributed. IBM ILOG CPLEX Optimization Studio V12.4 is used to resolve all models. The computational environment is I5-2400 CPU 3.10 GHz, 4.00 GB RAM, Window 7 Enterprise Edition.

4.1 Bounds

If we just consider revenue objective, it just needs to achieve an optimal solution of Model I. This maximal value should be the upper bound of revenue of scheduling. Similarly, minimal solution of Model II should be lower bound of mismatching degree of scheduling.

From Model I, the maximal revenue we can get is 2470.3, and corresponding mismatching degrees of patients with different preferences are shown in Table 2. This model can be resolved within 220 ms. It shows that the mismatching degree of the patients with strong preference is largest. It is understandable because they have more preference than other patients and it is harder to satisfy the strong preference.

On the other hand, from Model II, the minimal mismatching degree is 0.0097, which should be the lower bound of mismatching degree, and related revenue is 2215.3. This revenue, compared to maximal revenue, is less almost 10%.

Table 2. Mismatching degree under maximal revenue

Type	Mismatching degree
Time-dominated	0.238
Physician-dominated	0.153
Strong	0.521
Total	0.304

Actually, we do not take the two extreme policies. If hospitals just want to maximize the revenue, neglecting patients' satisfaction, they will lose reputation and at last lose their revenue. Therefore, some policies considering both revenue and preference satisfaction should be taken. The two models with different objectives should be combined.

4.2 Revenue and Mismatching Degree

4.2.1 Revenue and total degree of mismatching

In this subsection, we will build an integer model which objective is to minimize revenue with total degree of mismatching as a constraint. The method we use is to add Model II into Model I. The first step is to let the objective of Model II be less than some threshold C, formulating a new model (Model III). We can adjust the threshold to see how the revenue changes.

$$\max \sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{T_i} c_{nij}x_{nij} + \sum_{i=1}^{I}\sum_{m=1}^{S_i} r_{im}y_{im} - \sum_{n=1}^{N} l_n\left(1-\sum_{i=1}^{I}\sum_{j=1}^{T_i} x_{nij}\right)$$

$$s.t.$$

$$\frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{I}\left|\sum_{i=1}^{I} iX_{ni} - i_n\right| + \frac{1}{T_i}\left|\sum_{j=1}^{T_i} jY_{nj} - j_n\right|\right) \le C$$

$$\sum_{i=1}^{I}\sum_{j=1}^{T_i} x_{nij} \le 1, \quad n = 1, 2, \cdots, N$$

$$\sum_{n=1}^{N} x_{nij} \le 1, \quad i = 1, 2, \cdots, I,\ j = 1, 2, \cdots, T_i$$

$$-\frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk} x_{nij} - a \le y_{im} \le 1 - \frac{1}{k}\sum_{n=1}^{N}\sum_{j=(m-1)k+1}^{mk} x_{nij}$$

$$X_{ni} \ge x_{nij}, \quad n = 1, 2, \cdots, N,\ i = 1, 2, \cdots, I,\ j = 1, 2, \cdots, T_i$$

$$\sum_{i=1}^{I} X_{ni} \le 1, \quad n = 1, 2, \cdots, N$$

$$Y_{nj} \ge x_{nij}, \quad n = 1, 2, \cdots, N,\ i = 1, 2, \cdots, I,\ j = 1, 2, \cdots, T_i$$

$$\sum_{j=1}^{J} Y_{nj} \le 1, \quad n = 1, 2, \cdots, N$$

$$x_{nij}, y_{im}, X_{ni}, Y_{nj} \in \{0, 1\},\ n = 1, 2, \cdots, N,\ i = 1, 2, \cdots, I,$$
$$j = 1, 2, \cdots, T_i$$

Actually, if C > 0.304, the first constraint of Model III will not work. The new constraint means the mismatching degree must meet some requirement. That is, the OPD wants to improve service level through satisfy more patient preference. On the other hand, sometimes, OPD cannot meet the patients' preference at a high level, even at the level where the maximal revenue can be gotten. For example, in our experiments, the degree of mismatching has to be more than 0.304. In this scenario, we should try to get maximal revenue under a high level of mismatching. Therefore, the model should be modified a little. That is, the first constraint of Model III should be rewritten as follows.

$$\frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{I}\left|\sum_{i=1}^{I} iX_{ni}-i_n\right|+\frac{1}{T_i}\left|\sum_{j=1}^{T_i} jY_{nj}-j_n\right|\right)\geq C$$

Combining the two different scenarios, revenues and related degree of mismatching can be achieved by changing the first constraint (See Table 3).

Based on the data of Table 3, the relationship between revenue and total degree of mismatching is shown in Figure 1. This figure tells that revenue increases in total mismatching degree first, and then decreases after going through extreme point. Besides, when we try to satisfy more preference, the revenue can decrease dramatically. The shape of the curve beside the extreme point is comparatively flat, which means that mismatching degree can decrease largely through decreasing revenue slightly. For example, when the degree of mismatching goes down from 0.401 to 0.2 (go down 50%), the revenue just changes within the range from 2465.778 to 2459.410 (0.3%).

Other information can also be gotten from data shown in Table 1. Figure 2 shows the relationship be-

Table 3. Revenues and related degree of mismatching by changing constraint of Model III

C	Revenue	Total degree of mismatching	Mismatching degree		
			Type 1	Type 2	Type 3
≤0.01	2215.253	0.010	0.007	0.000	0.022
≤0.05	2363.023	0.050	0.067	0.000	0.083
≤0.10	2415.944	0.100	0.103	0.040	0.157
≤0.15	2444.550	0.150	0.128	0.060	0.262
≤0.20	2459.410	0.200	0.153	0.080	0.367
≤0.25	2467.016	0.250	0.181	0.107	0.463
	2470.332	**0.304**	**0.238**	**0.153**	**0.521**
≥0.35	2469.295	0.351	0.258	0.227	0.569
≥0.40	2465.778	0.401	0.269	0.273	0.659
≥0.45	2458.967	0.451	0.319	0.307	0.726
≥0.50	2447.565	0.500	0.332	0.347	0.821
≥0.55	2433.011	0.550	0.369	0.393	0.888
≥0.60	2416.054	0.600	0.426	0.427	0.947
≥0.65	2392.670	0.650	0.501	0.453	0.996
≥0.70	2360.862	0.700	0.525	0.473	1.103

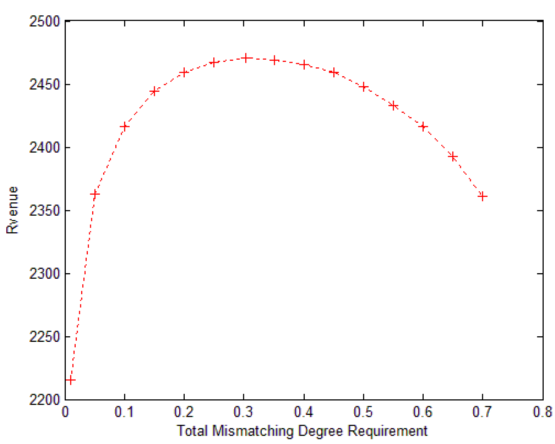

Figure 1. The relationship between revenue and total degree of mismatching.

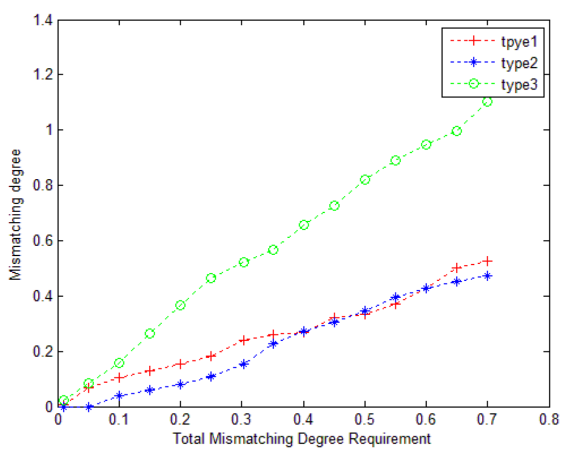

Figure 2. The relationship between total degree of mismatching and mismatching degree of each type of patient.

tween total degree of mismatching and mismatching degree of each type of patient. Mismatching degree of each type patient strictly increases in total mismatching degree. Mismatching degree of type 3 increases much faster than the other two types. That is reasonable because type 3 patient has both slot and physician preference, where is harder to satisfy.

4.2.2 Revenue and mismatching degree of patient with strong preference

Patients with strong preferences should be the most important customers of hospital, who have highest priority. Therefore, we need to study the relationship between revenue and the satisfaction of strong preference. The method is to replace the total degree of mismatching (the first constraint of Model III) by mismatching degree of type 3. Following the same step of last subsection, we can get some data about revenues and related degree of mismatching (shown in Table 4). The maximal revenue is still 2470.332, and corresponding total degree of mismatching degree is 0.304. Then the relationship between revenue and mismatching degree of type 3 is shown in Figure 3. In Figure 3, when preference type 3 patients are almost satisfied (That is, mismatching degree of type 3 patient is zero), it needs to decrease satisfaction of other types to increase total revenuee, which causes that the mismatching degree of types 1 and 2 gets maximal value. At the beginning (from 0 to 0.2), mismatching degree of types 1 and 2 decreases greatly, almost 20%. It is more interesting that the shape of neighborhood of extreme point (0.521) is comparatively flat. Meanwhile, the shape of neighborhood of extreme point in Figure 4 is also comparatively flat. Therefore, we can get the conclusion that preference satisfaction of type 3 can be increasing greatly without decreasing satisfaction of other patients and revenue.

5. CONCLUSION AND FUTURE WORK

In this paper, integer models are developed for the appointment scheduling in OPD under the background

Table 4. Revenues and related degree of mismatching by changing constraint of Model III

C	Revenue	Total degree of Mismatching	Mismatching degree of each type	Total degree of Mismatching	Mismatching degree of each type
≤0.01	2310.043	0.151	0.244	0.200	0.010
≤0.05	2397.621	0.161	0.232	0.200	0.050
≤0.10	2421.296	0.171	0.226	0.187	0.099
≤0.15	2436.693	0.186	0.222	0.187	0.150
≤0.20	2446.003	0.181	0.197	0.147	0.199
≤0.25	2453.175	0.204	0.217	0.147	0.250
≤0.30	2458.748	0.223	0.210	0.160	0.299
≤0.35	2462.765	0.236	0.199	0.160	0.349
≤0.40	2465.709	0.253	0.211	0.153	0.394
≤0.45	2467.927	0.277	0.232	0.153	0.446
≤0.50	2469.766	0.290	0.224	0.153	0.494
	2470.332	**0.304**	**0.238**	**0.153**	**0.521**
≥0.55	2470.046	0.321	0.236	0.173	0.553
≥0.60	2469.503	0.336	0.240	0.167	0.600
≥0.65	2468.227	0.352	0.232	0.173	0.651
≥0.70	2466.262	0.371	0.235	0.173	0.705
≥0.75	2463.665	0.389	0.225	0.193	0.750
≥0.80	2460.281	0.402	0.239	0.167	0.801
≥0.85	2456.237	0.423	0.225	0.193	0.851
≥0.90	2451.214	0.425	0.199	0.173	0.903
≥0.95	2445.510	0.440	0.204	0.167	0.950
≥1.00	2437.707	0.455	0.197	0.167	1.000
≥1.05	2428.540	0.477	0.208	0.173	1.050
≥1.10	2418.259	0.500	0.200	0.200	1.101
≥1.15	2405.484	0.519	0.214	0.193	1.150

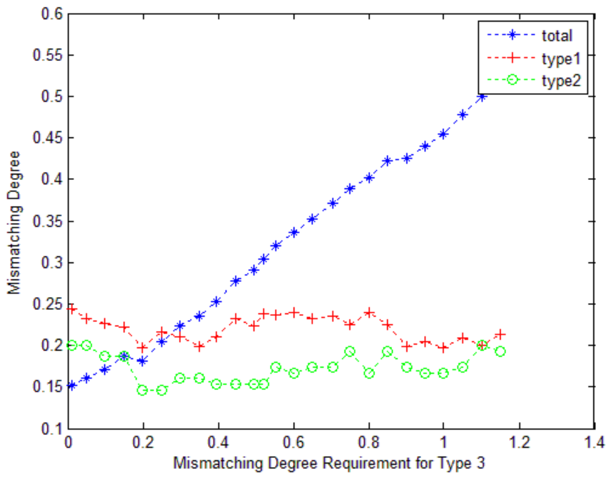

Figure 3. The relationship between mismatching degree of type 3 and mismatching degree of other type.

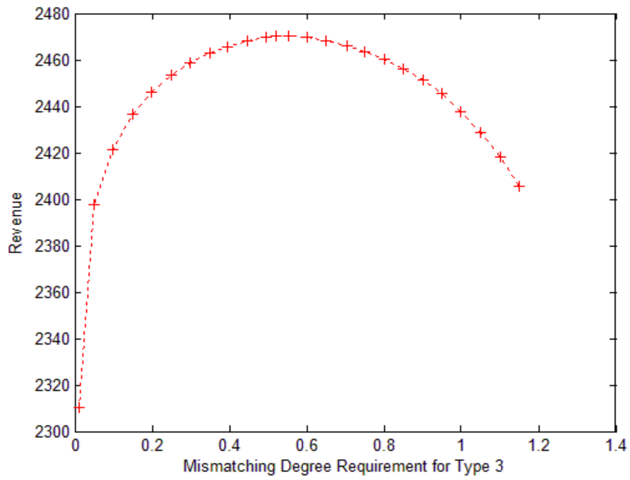

Figure 4. The relationship between revenue and mismatching degree of type 3.

of healthcare system in China. Patient preferences are considered to improve satisfaction during medical service. Integer models with different objectives are formulated separately, through which maximal revenue and minimal degree of mismatch can be achieved. However, in most cases, hospitals will not implement the two extreme policies. Therefore, the tradeoff between the two objectives is studied. How different kinds of mismatching degree affect revenue is simulated. According to the results of these simulations, the hospital can take the appropriate policy based on the requirement of revenue or patient satisfaction. It is important to note that this static model can be applied only when patients can accept the appointment mechanism that the appointment result can be given after a while. Because medical resource is scarce in China, it is very important for the healthcare system to work efficiently. Accordingly, patients in China care more about the service quality than the appointment process, the late reply policy can be suitable to China's healthcare system.

Some future work should be done to expend this work. First, the model presented in this paper is limited because it just considers the appointment problem in a separate workday. Though the maximal objective can be achieved through the booking process, it does not guarantee that the scheduling is optimal for a long run. For example, patients whose preferences cannot be satisfied on a day may be willing to book a time slot on another day. Therefore, a model dealing with appointment scheduling within a period should be developed in future. Second, a sequential appointment scheduling model with evaluation standard in this paper should be developed. Sequential model might not get the optimal solution. However, it can respond to patients' requirement quickly, which is comfortable for the booking process. Comparison between the sequential model and the static model should be made. Hospitals can make use of the results to decide which policy they can take. Third, in this paper, a time slot only can allow one patient. In this setting, if a patient is late for the healthcare service or does not go to the hospital according to the appointment time, the physician will waste time. Hence, some healthcare institutes let a time slot allow more than one patient, which, of course, is longer than that in this paper. However, if several patients arrive at the same time, waiting time will generate. Therefore, how to balance the waiting time and the service efficiency should be studied. Finally, how to evaluate the patient's satisfaction level should be studied. In this paper, we proposed an experimental method of evaluating it. However, it might be inappropriate for some circumstances.

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