

A Hierarchical Hybrid Meta-Heuristic Approach to Coping with Large Practical Multi-Depot VRP

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ABSTRACT

Under amazing increase in markets and certain demand on qualified service in the delivery system, global logistic optimization is becoming a keen interest to provide an essential infrastructure coping with modern competitive prospects. As a key technology for such deployment, we have been engaged in the practical studies on vehicle routing problem (VRP) in terms of Weber model, and developed a hybrid approach of meta-heuristic methods and the graph algorithm of minimum cost flow problem. This paper extends such idea to multi-depot VRP so that we can give a more general framework available for various real world applications including those in green or low carbon logistics. We show the developed procedure can handle various types of problem, i.e., delivery, direct pickup, and drop by pickup problems in a common framework. Numerical experiments have been carried out to validate the effectiveness of the proposed method. Moreover, to enhance usability of the method, Google Maps API is applied to retrieve real distance data and visualize the numerical result on the map.

Keywords: Hybrid Meta-Heuristic Approach, Multi-Depot VRP, Weber Model, Modified Saving Method, Google Maps API

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1. INTRODUCTION

Under amazing increase in markets and certain demand on qualified service in delivery system, global logistic optimization is becoming a keen interest to provide an essential infrastructure coping with modern competitive prospects. As a key technology for such deployment, we have been engaged in the practical studies on vehicle routing problems (VRP) noticing that the transportation cost and/or CO₂ emission actually depend not only on distance but also on loading weight. This cost accounting is known as the Weber model and has been popularly applied in a strategic planning like allocation/location problems (O'Kelly, 1986). To the best of our knowledge, however, this idea has never been used in the VRP studies. Giving a general solution procedure under this new cost accounting, we developed a hybrid procedure to practically cope with various types of VRP, i.e.,

delivery, direct pickup, and drop by pickup for single depot problems (Shimizu, 2011, 2012). Through a family of these hybrid methods (Shimizu *et al.*, 2008; Shimizu and Fujikura, 2010; Shimizu and Wada, 2004; Wada and Shimizu, 2006), we validated the effectiveness of such approaches in a few ways, i.e., comparison with other hybrid methods, such as genetic algorithm and tabu search, and exact solutions for small instances.

This paper extends our approach for a single-depot problem to multi-depot problems in order to provide a more general framework available for some important issues in modern logistics including those of green or low carbon logistics. Finally, numerical experiments are carried out to validate the effectiveness of the proposed method. Moreover, to enhance the usability of the method, Google Maps API is applied to obtain the real distance data and visualize the routes derived from the proposed method on the map.

The rest of the paper is organized as follows. In Section 2, we briefly describe the problem statement. Then, we formulate the problem in Section 3. Section 4 outlines the proposed solution procedure in a hybrid manner. Numerical experiments are provided in Section 5. Finally, we give some conclusions.

2. PROBLEM STATEMENT

Let us consider such a logistic network design problem known as VRP. The VRP is an NP-hard combinatorial optimization problem that tries to minimize total distance traveled by a fleet of vehicles under various constraints. This transportation from depot to their client customers must take a circular route making the depot as its starting point and destination at the same time. When we consider only one depot, the problem is called single-depot problem (Figure 1(a)). Meanwhile, it is called multi-depot problem when we consider multiple depots (Figure 1(b)).

Recent studies of VRP can be roughly classified in the following ways. The first one is an extension from the basic conditions such as customer demand satisfaction and vehicle payload to more practical issues such as customer availability or time windows (Mester *et al.*, 2007), pickups (Gribkovskaia *et al.*, 2007) and split and mixed deliveries (Mota *et al.*, 2007). These extensions are considered both separately and in combination (Zhong and Cole, 2005). The second one is known as the multi-depot problem, in which deliveries can originate from multiple depots (Chen *et al.*, 2005; Crevier *et al.*, 2007). The third type concerns multi-objective formulations for single depot and multi-depot problems (Jozefowicz *et al.*, 2008). Recently, some researchers have been interested in VRP with varying pickup and delivery configurations

because this is the most practical and suitable way to consider reverse logistics (Goksal *et al.*, 2013).

Except for the literature (Kytojoki *et al.*, 2007), however, those studies solved only small problems in numerical experiments due to the difficulty of solution. Moreover, though the transportation cost actually depends not only travelling distance but also loading weight (Weber model or Ton-Kilo basis), all of them accounted it only by the distance (Kilo basis). With a practical viewpoint, therefore, it is significant to remake all studies on the Ton-Kilo basis and increase solution ability for larger problems. This is a major motivation of this study.

3. PROBLEM FORMULATION OF MULTI-DEPOT PROBLEM

The multi-depot VRP for delivery is formulated by the following combinatorial optimization problem. It is easy to formulate the problem for pickup in a similar manner.

$$\begin{aligned}
 \text{(p.1) min } & \sum_{j \in J} H_j \sum_{v \in V} \sum_{k \in K} g_{jkv} z_{jkv} \\
 & + \sum_{v \in V} \sum_{p \in P} \sum_{p' \in P} c_v d_{pp'} (g_{pp'v} + w_v) z_{pp'v} + \sum_{j \in I} F1_j x_j + \sum_{v \in V} F2_v y_v \\
 \text{subject to } & \\
 & \sum_{p \in P} z_{kp v} \leq 1, \quad \forall k \in K; \forall v \in V \quad (1) \\
 & \sum_{p' \in P} z_{pp'v} - \sum_{p \in P} z_{p'pv} = 0, \quad \forall p \in P; \forall v \in V \quad (2) \\
 & \sum_{j \in J} z_{jj'v} = 0, \quad \forall j' \in J; \forall v \in V \quad (3) \\
 & \sum_{v \in V} \sum_{k \in K} g_{jkv} z_{jkv} \leq U_j x_j, \quad \forall j \in J \quad (4) \\
 & g_{pp'v} \leq W_u z_{pp'v}, \quad \forall p \in P; \forall p' \in P; \forall v \in V \quad (5) \\
 & \sum_{p \in P} \sum_{p' \in P} z_{pp'v} \leq M y_v, \quad \forall v \in V \quad (6) \\
 & \sum_{k \in K} g_{k j v} = 0, \quad \forall j \in J; \forall v \in V \quad (7) \\
 & \sum_{v \in V} \sum_{p \in P} g_{p k v} - \sum_{v \in V} \sum_{p \in P} g_{k p v} = q_k, \quad \forall k \in K \quad (8) \\
 & \sum_{p' \in P} (g_{k p v} - q_k z_{p k v}) = \sum_{p \in P} g_{k p v}, \quad \forall k \in K, \forall v \in V \quad (9) \\
 & \sum_{j \in J} \sum_{k \in K} z_{j k v} = y_v, \quad \forall v \in V \quad (10) \\
 & \sum_{j \in J} \sum_{k \in K} z_{k j v} = y_v, \quad \forall v \in V \quad (11) \\
 & \sum_{p \in \Omega} \sum_{p' \in \Omega} z_{pp'v} \leq |\Omega| - 1, \quad \forall \Omega \subseteq P \setminus \{v\}, |\Omega| \geq 2, \forall v \in V \quad (12) \\
 & x_j \in \{0, 1\}, \quad \forall j \in J; \quad y_v \in \{0, 1\}, \quad \forall v \in V \\
 & z_{pp'v} \in \{0, 1\}, \quad \forall p \in P, \forall p' \in P, \forall v \in V \\
 & g_{pp'v} \geq 0, \quad \forall p \in P; \forall p' \in P; \forall v \in V
 \end{aligned}$$

Notations of this problem are shown below.

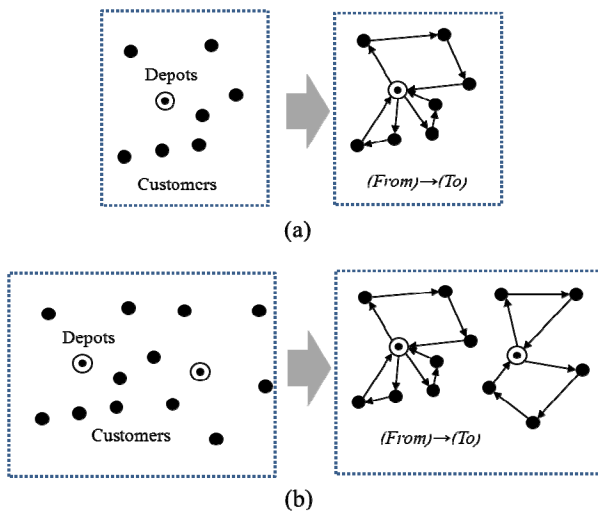


Figure 1. Logistic network design problem concerned here. (a) Single-depot vehicle routing problem (VRP), (b) multi-depot VRP.

Variables

$g_{pp'v}$: load of vehicle v on the path from $p \in P$ to $p' \in P$ [ton]
 $x_j = 1$ if candidate depot j is opened; otherwise 0 [-]
 $y_v = 1$ if vehicle v is used; otherwise 0 [-]
 $z_{pp'v} = 1$ if vehicle v travels on the path from $p \in P$ to $p' \in P$; otherwise 0 [-]

Parameters

c_v : transportation cost per unit load per unit distance of vehicle v [cost unit /ton/km]
 q_k : demand of customer k [ton]
 $d_{pp'}$: path distance between $p \in P$ and $p' \in P$ [km]
 $F1_j$: fixed charge of depot j [cost unit]
 $F2_v$: fixed charge of vehicle v [cost unit]
 H_j : handling cost of depot j [cost unit /ton]
 M : auxiliary constant (Large real number) [-]
 U_j : maximum capacity of depot j [ton]
 w_v : unladen weight of vehicle v [ton]
 W_v : maximum capacity of vehicle v [ton]

Index set

J: depot, K: customer, V: vehicle, $P = J \cup K$, Ω : sub-tour

Here, objective function is composed of handling cost at depot, routing transportation cost and fixed charges of vehicles and opening depots. On the other hand, each constraint means that each vehicle cannot visit the customer twice by Eq. (1); that coming in vehicle must leave out by Eq. (2); avoiding to make a path between depots by Eq. (3); upper holding capacity at depot by Eq. (4); upper bound load capacity for vehicle by Eq. (5); that

vehicle must travel on a certain path by Eq. (6); that load must be empty for returning vehicle by Eq. (7); demand satisfaction at depot by Eq. (8); decreasing the load just by the amount from the foregoing visit by Eq. (9); that each vehicle leaves only one depot and return there by Eqs. (10) and (11); sub-tour elimination by Eq. (12).

Integrity conditions and positive conditions are imposed on the respective variables.

It is well known that this kind of problem belongs to an NP-hard class, and becomes extremely difficult to obtain an exact optimal solution for large problems. Hence, it is meaningful for applications to provide a practical method that can derive a near optimum solution with acceptable computational efforts.

4. HYBRID APPROACH FOR PRACTICAL SOLUTION

To practically work with the above problem, we used a hierarchical procedure outlined as a flow chart with each solution image in Figure 2. In this procedure, we apply three major components, i.e., graph algorithm to solve minimum cost flow (MCF) problems, a modified saving method and a modified tabu search. The graph algorithm is used to allocate the client customers for each depot on the Ton-Kilo basis. Then, the modified saving method is used to derive the initial solution of VRP in the inner loop search. In addition, the modified tabu search is applied to improve the tentative solution both in the inner and outer loop searches. In the below, each component will be explained in detail.

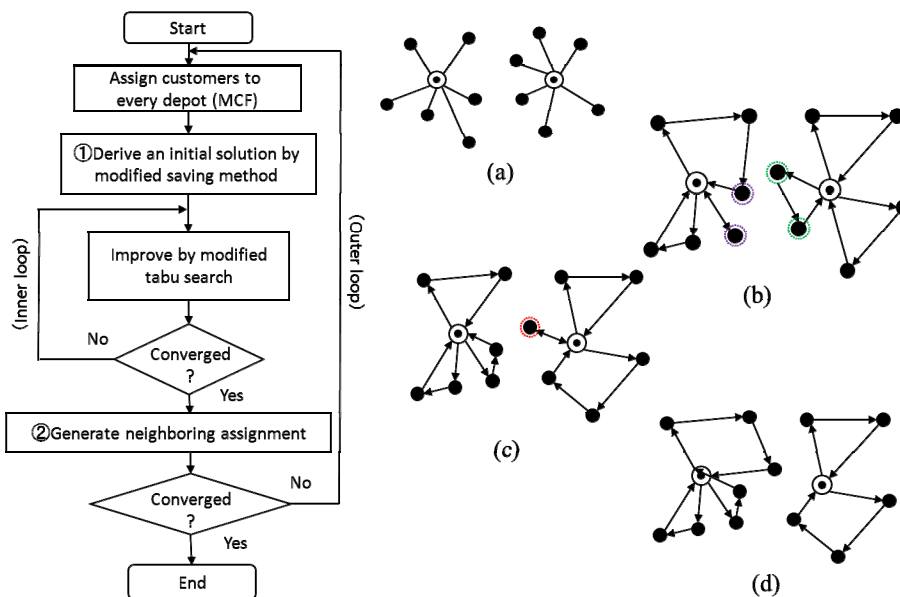


Figure 2. Idea of proposed procedure for multi-depot problem with solution images. (a) Assign (MCF problem), (b) inner loop search (modified saving method), (c) inner loop search (modified tabu search), and (d) outer loop search (modified tabu search).

4.1 Allocation Problem of Customers

If we can decide the client customers to each depot, the problem tentatively refers to multiple single-depot problems. For this purpose, we give an allocation problem formulated below.

$$(p.2) \min \sum_{j \in J} \sum_{k \in K} c_v d_{jk} g_{jk} + \sum_{j \in J} H_j \sum_{k \in K} g_{jk}$$

subject to

$$\sum_{k \in K} g_{jk} \leq U_j, \quad \forall j \in J \tag{13}$$

$$\sum_{j \in J} g_{jk} = q_k, \quad \forall k \in K \tag{14}$$

$$g_{jk} \geq 0, \quad \forall j \in J, \forall j \in J$$

where g_{jk} denotes the amount allocated from depot k to customer j .

Actually, we solve the above linear programming problem using a graph algorithm of MCF problem to enhance the solution ability. Here, the depot with no inflow from the source in the MCF graph will not be opened. We show the graph structure and its numeric information in Figure 3 and Table 1, respectively. Presently, this approach is suitable compared with the conventional Kilo basis methods as discussed in the later. After all, we can allocate every customer to each depot efficiently.

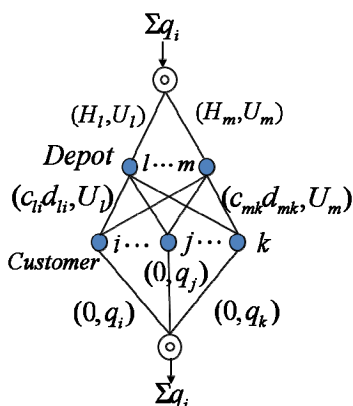


Figure 3. Minimum cost flow graph for allocation.

Table 1. Specification of minimum cost flow graph

Edge	Cost	Capacity
Inlet	-	Σq_j
Source-Depot i	H_i	U_i
Depot i -Customer j	1: $c_v d_{ij}$, 2: $c_v \gamma q_j^{\alpha-1} d_{ij}^\beta$	U_j
Customer j -Sink	0	q_j
Outlet	-	Σq_j

C_{ij} (in Figure 3) = {1: Weber, 2: generalized Weber}.

4.2 Inner Loop Search

4.2.1 Modified saving method

Saving method (Kobayashi, 1999) is a popularly known heuristic method for the conventional VRP. The great, saving value that is the reward from integrating the return paths plays a key role. It is conventionally calculated on Kilo basis. On the other hand, for the Weber model whose transportation cost between customer i and j is described by Eq. (15), the saving value $s_{i,j}$ is given by Eq. (16) in the delivery case (refer to Figure 4).

$$C_{trans} = c_v d_{i,j} (w + q_j) \tag{15}$$

$$s_{i,j} / c_v = (d_{0,j} - d_{0,i} - d_{i,j}) q_j + (d_{0,j} + d_{i,0} - d_{i,j}) w$$

$$i, j = 1, 2, \dots, n, i \neq j \tag{16}$$

where n is a total number of customers, and let the suffix for depot be 0 and $s_{i,i} = 0$. In order to evaluate the problem from a more practical viewpoint, the above idea is possible to extend to the generalized Weber model. Then, the foregoing Eqs. (15) and (16) are modified as Eqs. (17) and (18), respectively.

$$C'_{trans} = c_v \gamma d_{i,j}^\alpha (w_v + q_j)^\beta \tag{17}$$

$$s_{i,j} / (c_v \gamma) = (w_v + q_i)^\alpha d_{0,i}^\beta + w_v^\alpha d_{i,0}^\beta + (w_v + q_j)^\alpha d_{0,j}^\beta - (w_v + q_i + q_j)^\alpha d_{0,i}^\beta - (w_v + q_j)^\alpha d_{i,j}^\beta, \tag{18}$$

where α and β denote the elastic coefficients for the distance and weight, respectively and γ is a constant.

Moreover, for a practical evaluation of economy, it makes sense to account the fixed operating cost of vehicle, $F2_v$ beside the transportation cost. In this case, it becomes more economical to visit a new customer even if its saving cost ($\gamma c_v s_{i,j}$) would become negative as long as its absolute value stays within the fixed operating cost of vehicle.

Applying these ideas to the original saving method, we developed the modified saving method in the previous studies mentioned already. Through this method, we can derive the initial routes more practically compared with the conventional method that ignores the loading and unladen weights and the fixed operating cost of vehicles. This procedure is outlined as follows.

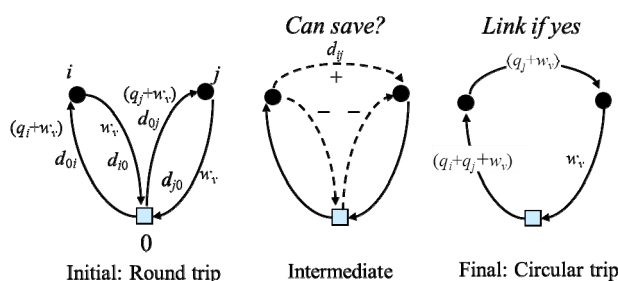


Figure 4. Scheme to compute the saving value (delivery).

- Step 1: Create round trip routes from the depot to its client customers. Compute the savings value in terms of Eq. (16) or Eq. (18).
 Step 2: Order these pairs in descending order of savings value.
 Step 3: Merge the path, following the order obtained from Step 2 as long as it is feasible and the savings value is greater than $-F2_v/(\gamma c_v)$.

After all, we can evaluate the cost TC by Eq. (19).

$$TC = \sum_{i=1}^L TR_i + L \cdot F2_v \quad (19)$$

where TR_i denotes travelling cost of route i , and L total number of routes (necessary vehicle number).

4.2.2 Modified tabu search

Since the (modified) saving method derives only an approximated solution, we try to improve it by applying the modified tabu search. The tabu search (Glover, 1989) is a simple but powerful heuristic method that refers to a local search with certain memory structure. In the present local search, we generate a neighbor solution from either of insert, swap or 2-opt operation selected randomly. In particular, we allow even a degraded solution to be a new tentative solution as long as it would not be involved in the tabu lists. This decision is made in terms of the probability p whose distribution obeys the following Maxwell-Boltzmann function and used in simulated annealing (Kirkpatrick *et al.*, 1983).

$$p = \begin{cases} 1 & \text{if } \Delta e \leq 0 \\ \exp(-\Delta e/T) & \text{if } \Delta e < \varepsilon. \\ 0 & \text{if } \Delta e > \varepsilon \end{cases} \quad (20)$$

where Δe denotes difference of objective function from the present best value, and ε a small positive number. Moreover, T is the temperature that will decrease along with the iteration k geometrically, i.e., $T^k = \lambda T^{k-1}$, $\lambda < 1$. We successfully used this method in the previous studies mentioned already.

4.3 Outer Loop Search

To improve the initial allocation decided from the MCF problem, we also applied the above modified tabu search by generating the neighbor solution in such a way as outlined below (refer to Figure 5).

- Step 1: Select randomly two depots, i.e., an outgoing depot i and an incoming depot j .
 Step 2: Select randomly emigrant customers in depot i , and immigrant ones in depot j . To reserve the near optimum structure decided from MCF problem while keeping the simplicity of the algorithm, numbers are restricted within narrow ranges, i.e., $n \in \{1, 2\}$ and $m \in \{0, 1\}$ where n and m denote the

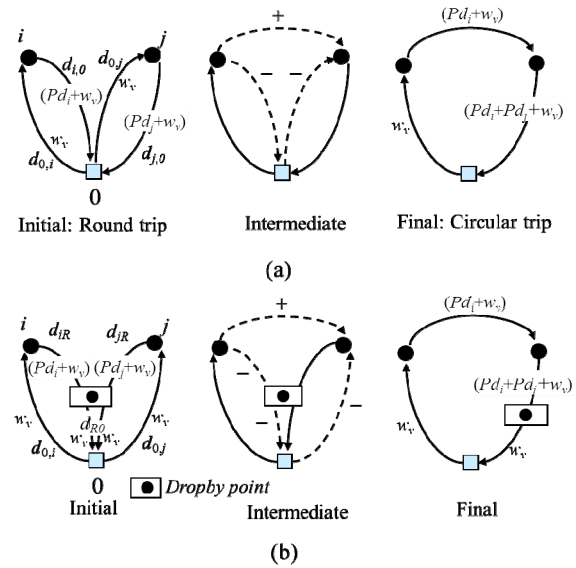


Figure 5. Procedure on neighboring in outer loop search. (a) Direct pickup, (b) drop-by pickup.

numbers of emigrants and immigrant, respectively. Moreover, if the conditions mentioned below on the increments regarding distance or capacity constraints of depot are not satisfied, go back to Step 1 after cancelling this migration. Otherwise go to the next step.

- Step 3: If the above selection in Step 1 and 2 is not involved in the tabu list, go to the next step while adding this in the tabu list. Otherwise go back to Step 1.
 Step 4: Replace the costs on the edges of MCF graph concerned here as shown in Figure 3. Then, we can induce the flow to the edge with $-M$ while prevent it to that with M . Here, M denotes the large number. Through such re-labeling on a few edges of the MCH graph, we can realize the intended migration substantially while keeping solution efficiency in terms of sensitivity analysis of the graph algorithm.

In Step 2, we can calculate the increments of distance and capacity accompanying the migration as follows.

1. when $n = 1, m = 0$:
 $\Delta d = d(j, i_1) - d(i, i_1), \Delta Q_j = q_{i_1}$
2. when $n = 1, m = 1$:
 $\Delta d = d(j, i_1) + d(i, j_1) - (d(i, i_1) + d(j, j_1))$
 $\Delta Q_i = q_{i_1} - q_i, \Delta Q_j = -\Delta Q_i$
3. when $n = 2, m = 1$:
 $\Delta d = d(j, i_1) + d(i, j_1) + d(j, i_2) - (d(i, i_1) + d(i, i_2) + d(j, j_1))$
 $\Delta Q_i = q_{j_1} - (q_i + q_{i_2}), \Delta Q_j = -\Delta Q_i$

If we might ignore a slight possibility missing the

better solution in Step 2, it makes sense not to take such a migration that anyone of the following relation holds, i.e., $\Delta d \geq d_{max}$, $\Delta Q_i \geq \bar{Q}_i$, $\Delta Q_j \geq \bar{Q}_j$. Here, d_{max} denotes the threshold of allowable increase in distance. On the other hand, \bar{Q}_i and \bar{Q}_j represent the margin of capacity at depot i and j , respectively.

4.4 Variants of VRP

We can easily cope with the variants of VRP that are increasing their significance associated with interests in green and/or reverse logistics. We can apply the same framework shown in Figure 2 only by replacing the saving value used in the modified saving method as shown below. This feature presents various prospects for future studies in this field.

4.4.1 In case of direct pickup

Being different from the conventional formulations or delivery type VRP, it is common that the reverse logistic collects garbage and/or spent products from distributed pickup points. Here, each vehicle visits every customer for pickup and return the depot directly. Then, referring to Figure 6(a), the generalized saving value in this case is obtained as Eq. (21).

$$s_{ij}/(\gamma c_r) = w_v^\alpha d_{0i}^\beta + (w_v + Pd_i)^\alpha d_{i0}^\beta + w_v^\alpha d_{0j}^\beta + (w_v + Pd_j)^\alpha d_{j0}^\beta - w_v^\alpha d_{0i}^\beta - (w_v + Pd_i)^\alpha d_{ij}^\beta - (w_v + Pd_i + Pd_j)^\alpha d_{j0}^\beta \quad (21)$$

$i, j = 1, 2, \dots, n, i \neq j$

where Pd_j denotes the pick up demand of customer j .

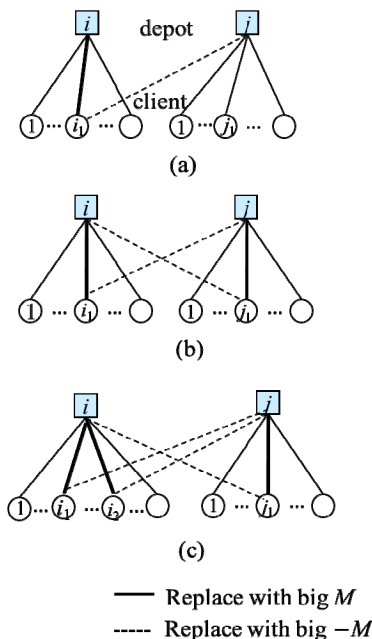


Figure 6. Scheme to compute the saving value. (a) $n = 1$, $m = 0$, (b) $n = 1$, $m = 1$, and (c) $n = 2$, $m = 1$.

4.4.2 In case of drop by pickup

In order to evaluate the scenario from another viewpoint, the foregoing idea is possible to extend to the case where vehicle drops by an intermediate destination before returning to the depot. This is the case where vehicle visits a disposal site to dump the debris, for example. Referring to Figure 6(b), we can similarly derive the generalized saving value as follows.

$$s_{ij}/(\gamma c_r) = w_v^\alpha d_{0i}^\beta + (w_v + Pd_i)^\alpha d_{iR}^\beta + w_v^\alpha d_{R0}^\beta + w_v^\alpha d_{0j}^\beta + (w_v + Pd_j)^\alpha d_{jR}^\beta + w_v^\alpha d_{R0}^\beta - w_v^\alpha d_{0i}^\beta - (w_v + Pd_i)^\alpha d_{ij}^\beta - (w_v + Pd_i + Pd_j)^\alpha d_{jR}^\beta - w_v^\alpha d_{R0}^\beta \quad (22)$$

where subscript R denotes the intermediate destination. When $\alpha = \beta = \gamma = 1$, these equations (Eqs. (21) and (22)) refer to those of the Weber model.

5. NUMERICAL EXPERIMENT

Numerical experiments were carried out to validate the effectiveness of the proposed method. We randomly generated the prescribed numbers of customers within the entire rectangular region while the depots within the smaller region involved in it (refer to Figure 7). The distances between depot and customers and also between the customers are given by Euclidian basis for the benchmark problems. Moreover, the other parameters including demand of each customer are randomly given within certain prescribed ranges. Size of tabu list is changed in the range (3, 25) depending on the problem size. We used PC with CPU: Intel(R) Xeon(R) W3565 3.20 GHz, and 8 GB RAM. The following discussions are made by averaging the results over 10 samples.

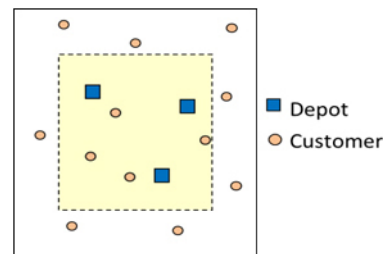


Figure 7. A scheme to generate location of members.

Table 2. Result under various problem sizes (Weber/delivery)

Size*	Cost [unit]	Rate [-]	Rate2 [-]	CPU [s]	Iteration [-]
(5, 50)	0.1725E+07	0.0603	0.0666	11.32	0.151E+05
(5, 100)	0.2872E+07	0.0392	0.0568	46.76	0.570E+05
(10, 500)	0.1340E+08	0.0103	0.0198	722.39	0.722E+06
(10, 1000)	0.2381E+08	0.0105	0.1157	5953.18	0.294E+07

* (Depot#, Customer#).

In Tables 2–4, we summarize the results of four different benchmark problems each of which size is shown in ‘Size’ column. Thereat, figures in the parenthesis denote the number of depot and customer, respectively. It involves such large problems that have not been solved elsewhere.

The ‘Rate’ and ‘Rate2’ columns denote the improved rate of the final solution from the converged solution and the initial one (result from the modified saving method) in the inner loop search, respectively. Accordingly, it always satisfy the relation such that Rate2 > Rate.

Table 2 shows the results for the delivery Weber problems. Thereat, we know those rates roughly stay at small values since the search begins from the nearly optimized state attained by the modified saving method after solving the MCF problem. In other words, the proposed method can provide the superior initial solution for the outer loop search.

As known from the ‘CPU’ column, CPU time expands rapidly according to the increase in problem size. This is a common property of the combinatorial optimization. Even for the largest problems, however, we can solve them within several hours. Moreover, their CPUs are possibly overestimated by from ten to twenty times if we notice the convergence profiles in Figure 8. On the other hand, the ‘iteration’ column shows the total number of iteration required for the inner and outer loop searches. Its upper bound is varied according to the problem sizes (number of depots and number of customers).

Convergence profile of the outer loop search is shown for a small problem ($|J| = 3$ and $|K| = 30$) in Figure 8(a). From this, we know the sufficient convergence

is attained at the earlier stage. We also observe the similar profiles for the other larger problems.

Regarding the direct pickup problems, there are a few upset results between the problem sizes and computation times in Table 3 due to a nature of evolutionary methods. As a whole, we have the similar performance and convergence profile (Figure 8(b)) to the delivery case. After the big improvement at the earlier stage, there is no significant improvement.

Finally, in the case of drop by pickup problem, we can observe a bit different results known both from Table 4 and Figure 8(c). Thereat, we can realize somewhat better improvement compared with the foregoing cases. This is because the MCF problem ignores the drop by point and cannot derive a solution as good as those of the foregoing two cases.

As a nature of evolutionary method, it is unable to guarantee the optimality of the solution obtained here. Moreover, we cannot compare the performance with other methods since any method relied on the Ton-Kilo basis has never been known elsewhere. Together with the easy deployment to the variants shown in Section 4.4, this practical cost accounting has a great advantage in real world applications. Finally, evaluating these results totally, we can claim the proposed method derives a well-approximated solution within acceptable computation time even for large problems.

To deploy the proposed method for system development, we can effectively utilize some software of the Google Maps API. We have developed the following stepwise procedure to collect real distance data and visualize the results by using JavaScript and appropriate free software.

- Step 1: Collect the addresses of locations in an Excel spreadsheet or text file.
- Step 2: Add longitude and latitude information for every

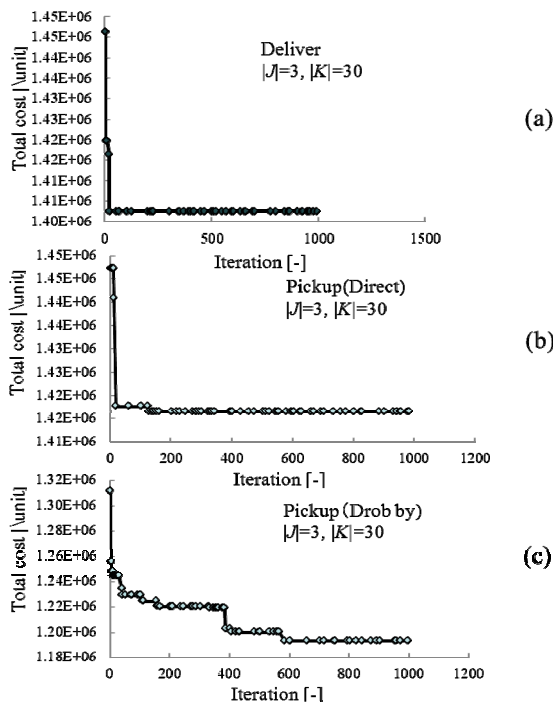


Figure 8. Profiles of convergence. (a) Delivery, (b) pickup (direct), and (c) pickup (drop by).

Table 3. Result under various problem sizes (Weber/pickup:direct)

Size	Cost [unit]	Rate [-]	Rate2 [-]	CPU [s]	Iteration [-]
(5, 50)	0.2192E+07	0.0616	0.0882	15.06	0.208E+05
(5, 100)	0.3406E+07	0.0244	0.0289	48.31	0.580E+05
(10, 500)	0.1326E+08	0.0114	0.0204	4972.36	0.718E+06
(10, 1000)	0.2254E+08	0.0080	0.0156	2992.77	0.277E+07

Table 4. Result under various problem sizes (Weber/pickup:drop by)

Size	Cost [unit]	Rate [-]	Rate2 [-]	CPU [s]	Iteration [-]
(5, 50)	0.1871E+07	0.0589	0.2688	19.52	0.297E+05
(5, 100)	0.2613E+07	0.0364	0.3138	71.55	0.107E+06
(10, 500)	0.1294E+08	0.0378	0.1904	805.77	0.998E+06
(10, 1000)	0.2354E+08	0.0240	0.3279	15411.30	0.415E+07

location in the sheet.

- Step 3: Calculate the distance between every pair of locations by using the Google geocoding API.
- Step 4: Solve the optimization problem by the proposed method.
- Step 5: Display the routes obtained from Step 4 in Google Maps.

Figure 9 shows a result for an illustrative small problem. Taking Nagoya city area in Japan, we set three depots and 17 customers on the actual map ($|J| = 3$ and $|K| = 17$). Then, we retrieve the real distance data from the above procedure and obtained the final result in which every depot has a single route. For simplicity, the routing path from only Depot 1 is shown here. Marks for locations (A (Depot 1) – B – ... – K – A) and dotted arrows are superimposed to help clarify the actual circular route. We can see this kind of visual information is very helpful for some tasks at an operational level. However, there still remain some possibilities to add more valuable service information from GIS applications and

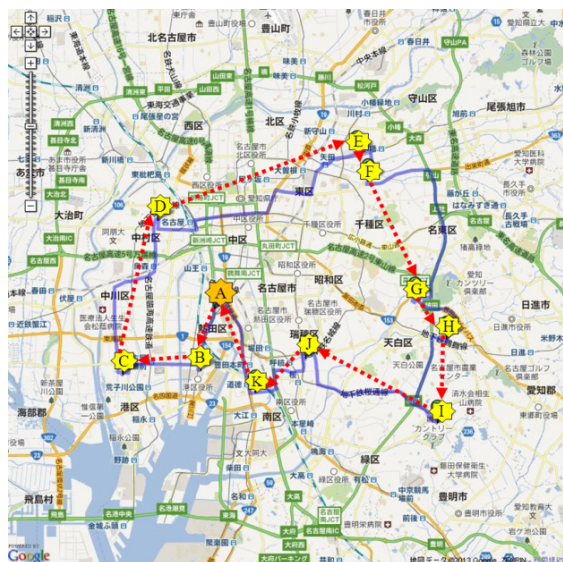


Figure 9. A part of display of result (route from depot 1 marked by A).

Table 5. Comparison in terms of initial allocation methods

	Member	Amount	Limit
Proposed (by MCF)			
Depot 1	{1, 2, 4, 5, 7, 9, 11, 12, 13, 14}	400	400
Depot 2	{3, 6, 9, 10, 15}	270	300
Depot 3	{8, 16, 17}	145	400
By Voronoi			
Depot 1	{1, 2, 11, 12, 13, 14}	250	400
Depot 2	{3, 4, 6, 9, 10, 15}	385	300
Depot 3	{5, 7, 8, 16, 17}	180	400

the Google Maps API.

Using the same model, we also evaluated the effectiveness of the proposed allocation method mentioned in Section 4.1 through comparison. For this comparison, we applied a Kilo basis method that relies on the Voronoi division and has been used popularly. As shown in Table 5, the allocation from the Voronoi method violates the capacity constraint at Depot 2 while the proposed allocation method can satisfy everyone certainly (Eq. (13)). Moreover, for this small problem, this initial allocation has not updated any more by the outer loop search in the proposed procedure. This also claims the efficiency of the present allocation method in another aspect.

6. CONCLUSION

As a key technology for logistics under global manufacturing and certain demand on qualified service in delivery system, we have proposed a general method of multi-depot vehicle routing problems for various real world applications. The idea can provide a framework naturally extensible for variants of VRP. It is developed by taking a practical cost accounting known as the usual and generalized Weber models. This is a novel attempt taken place nowhere in VRP studies. Finally, the algorithm is implemented by making the best use of the modified saving method and the modified tabu search in a hybrid manner by virtue of the graph algorithm of MCF problem.

Numerical experiments have been carried out to validate the effectiveness of the proposed method. Moreover, to enhance the usability of the method, Google Maps API is applied to retrieve the real distance data and visualize the numerical result on the map.

In future studies, we aim at extending the idea so that we can cope with the problems more in general framework and consider various practical conditions. It is meaningful to turn our interests toward multi-objective optimizations that manage the trade-off among economics, risks, services, environment issues and so on.

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