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Optimal Operation for Green Supply Chain Considering Demand Information, Collection Incentive and Quality of Recycling Parts

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ABSTRACT

This study proposes an optimal operational policy for a green supply chain (GSC) where a retailer pays an incentive for collection of used products from customers and determines the optimal order quantity of a single product under uncertainty in product demand. A manufacturer produces the optimal order quantity of product using recyclable parts with acceptable quality levels and covers a part of the retailer's incentive from the recycled parts. Here, two scenarios for the product demand are assumed as: the distribution of product demand is known, and only both mean and variance are known. This paper develops mathematical models to find how order quantity, collection incentive of used products and lower limit of quality level for recycling affect the expected profits of each member and the whole supply chain under both a decentralized GSC (DGSC) and an integrated GSC (IGSC). The analysis numerically compares the results under DGSC with those under IGSC for each scenario of product demand. Also, the effect of the quality of the recyclable parts on the optimal decisions is shown. Moreover, supply chain coordination to shift the optimal decisions of IGSC is discussed based on: I) profit ratio, II) Nash bargaining solution, and III) Combination of (I) and (II).

Keywords: Green Supply Chain, Demand Information, Distribution-Free Approach, Collection Incentive, Uncertainty Quality of Recyclable Parts, Game Theory

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1. INTRODUCTION

Due to the recent rise of social concern about the environment problem, the concept of a new supply chain management has been important in optimally controlling a supply chain including traditional forward chains/logistics and reverse chains/logistics. The traditional forward chains/logistics include the flows from procurement of new materials through production of new products to selling them. The reverse chains/logistics include the flows from collection of used products through recycling parts from the used products to reuse the recycled parts (Aras *et al*., 2004; Behret and Korugan, 2009; Ferguson *et al*., 2009; Fleischman *et al*., 1997; Guide and Wassenhove, 2001; Inderfurth, 2005; Konstantaras *et al*., 2010; Mukhopadhyay and Ma, 2009; Nenes *et al*., 2010; Pokharel and Liang, 2012; Teunter and Flapper, 2011; Wei *et al*., 2011; Wu, 2012).

Also, a supply chain including the forward chains and the reverse chains has been called a closed-supply chain, reverse supply chain or a green supply chain (GSC) (Bakal and Akcali, 2006; Fleischman *et al*., 1997; Guide *et al*., 2003; Inderfurth, 2005; Kaya, 2010; Lee *et al*., 2011; Shi *et al*., 2010, 2011; Tagaras and Zikopoulos, 2008; Thierry *et al*., 1995; Van Wassenhove and Zikopoulos, 2010; Wei *et al*., 2012; Yan and Sun, 2012; Zikopoulos and Tagaras, 2007, 2008). In this study, the supply chain that has the forward chains and the reverse chains is called a GSC. The manufacturing to reuse recycled parts is called the remanufacturing. It is necessary to take some measures and policies in order to promote 3R activities (Reuse-Recycle-Reduce) in the GSC.

Several previous papers have dealt with the optimal operations for GSC, and the uncertainty in remanufacturing has been attracting more attention in recent papers.

The incorporation of the uncertainty in demands of products/parts and collection quantity of used products into GSC have been discussed by Inderfurth (2005), Lee *et al.* (2011), Mukhopadhyay and Ma (2009), Shi *et al.* (2010, 2011), and Wei *et al.* (2011).

The incorporation of the price-sensitivity in collection quantity of used products and demands of products/ parts into the optimal tactical production planning GSC have been discussed by Bakal and Akcali (2006), Pokharel and Liang (2012), Shi *et al.* (2010), Teunter and Flapper (2011), Wei *et al.* (2012), and Yan and Sun (2012).

Also, the effects of inspection and sorting of used products on the optimal tactical production planning in GSC have been discussed by Aras *et al.* (2004), Behret and Korugan (2009), Ferguson *et al.* (2009), Guide *et al.* (2003), Konstantaras *et al.* (2010), Nenes *et al.* (2010), Tagaras and Zikopoulos (2008), Van Wassenhove and Zikopoulos (2010), and Zikopoulos and Tagaras (2007, 2008).

In dealing with the GSC, it is necessary to consider a variety of qualities of used products collected from the market. Some authors have discussed the optimal tactical production planning by incorporating uncertainty in the quality of used products into the GSC. Aras *et al.* (2004) investigated the issue of the stochastic nature of product returns and found conditions under which quality-based categorization was most cost effective. Zikopoulos and Tagaras (2007) investigated how the profitability of reuse activities was affected by uncertainty regarding the quality of returned products in two collection sites and determined the unique optimal solution (procurement and production quantities). In Guide *et al.* (2003) and Ferguson *et al.* (2009), returned products were assumed to have N quality categories, and the procurement prices and the remanufacturing costs were different based on the corresponding quality level. Behret and Korugan (2009) discussed a remanufacturing stage with uncertainties in the quality of remanufacturing products, return rates, and return times of returned products. After returned products were classified by considering quality uncertainties, remanufacturing processing times, material recovery rates, the remanufacturing costs, and disposal costs were determined by using the ARENA simulation program. Mukhopadhyay and Ma (2009) discussed a GSC consisting of a retailer who sold a single product and a manufacturer who collected used products from the market, remanufactured parts from the used products and then produced products. They assumed two situations for the remanufacturing ratio between reuse parts and used products: a constant situation and an uncertain situation. Under each situation, they proposed the optimal production strategy for the procurement quantity of used products, the remanufacturing quantity of parts from used products and the production quantity of new parts from new materials. Nenes *et al.* (2010) observed that both quality and quantity of returns (used products) were unfortunately highly stochastic, and investigated the optimal policies for ordering of new products and remanufacturing of products so as to maximize the companies' performance, such as minimizing their expected cost or maximizing their expected profit. Teunter and Flapper (2011) discussed how quality of cores (i.e., products supplied for remanufacturing) could vary significantly, affecting the cost of remanufacturing, and derived the optimal policies regarding acquisition and remanufacturing for both deterministic and uncertain demand.

Kaya (2010) discussed a GSC consisting of a retailer who collected used products from customers and sold a single product and a manufacturer who remanufactures parts from the used products and produced the products. They proposed the optimal decisions for collection incentive of used products and production quantities of both remanufacturing parts and new parts.

Also, it is necessary to determine the optimal operations to establish a GSC to obtain its profitability. In a decentralized GSC, all members in the GSC determine the optimal operations so as to maximize their profits. As one of the optimal decision-making approaches under a decentralized GSC, the Stackelberg game has been adopted in several previous papers. In the Stackelberg game, there is a single leader of the decision-making and a single (multiple) follower(s) of the decision-making of the leader. The leader of the decision-making determines the optimal strategy so as to maximize the leader's (expected) profit. The follower(s) of the decision-making determine(s) the optimal strategy so as to maximize the follower(s)'s (expected) profit under the optimal strategy determined by the leader of the decision-making (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al.*, 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al.*, 2011; Lee *et al.*, 2011; Leng and Parlar, 2009; Liu *et al.*, 2012; Mukhopadhyay *et al.*, 2011; Xu *et al.*, 2012; Yan and Sun, 2012).

Also, in a supply chain management, the optimal decisions under an integrated supply chain maximizing the whole supply chain's expected profit can bring the more expected profit to the whole supply chain than those under a decentralized supply chain maximizing the expected profit of each member in a supply chain. So, from the aspect of the total optimization in supply chain management, it is preferable for all members in supply chain to shift the optimal decisions under the integrated supply chain. In this case, it is the absolute requirement for all members under the integrated supply chain to obtain the more expected profits than those under the decentralized supply chain. In order to achieve the increases in profits of all members under the integrated supply chain, a variety of supply chain coordination approaches between all members have been discussed by Cachon and Netessine (2004), Du *et al*. (2011), Kaya (2010), Tsay *et al*. (1999), Wei *et al*. (2012), Wu (2012), Yan and Sun (2012), and Yano and Gilbert (2004).

The incorporation of the game theory into not only the optimal pricing strategies, but also the supply chain coordination in a GSC have been discussed by Kaya (2010), Wei *et al*. (2012), Wu (2012), Yan and Sun (2012), and Du *et al*. (2011).

From the previous papers regarding GSC, product recovery, recycling, remanufacturing and reverse logistics, the lower level of quality levels of used products were not considered for the optimal decision for the remanufacturing ratio. Also, in the previous papers above, the relation between a collection incentive of used products and the collection quantity of used products was not described clearly. In addition, the cost for recycling used products has not been considered as profits in GSC in the above previous papers.

Regarding these discussions mentioned above, Watanabe *et al*. (2013) discussed the following optimal production policy for two types of GSCs: optimal decisions for the product quantity, the unit collection incentive of used products and the lower limit of quality level for recycling of used products under both the decentralized GSC (DGSC) and the integrated GSC (IGSC).

In GSCs mentioned above, a demand of a single product assumes a random variable, and the probability distribution of the demand is known. This implies that it is possible to obtain the full information of the product demand. In a real situation for the GSC, it may be possible to get the limited information, such as mean and variance of the product demand. Under such a situation, Gallego and Moon (1993), Moon and Gallego (1994), Moon and Choi (1995), Alfares and Elmorra (2005) discussed the distribution-free newsboy problem for a single product in a single period.

This paper focuses on the optimal operation for a GSC to encourage to collect and recycle a single of used products, such as consumer electronics (mobile phone, personal computer), semiconductor and electronic component (Daniel *et al*., 2000; Guide and Van Wassenhove, 2001; Guide *et al*., 2003; Ferguson *et al*., 2009) under the uncertainties in product demand and quality of used products collected from customers.

When the GSC is operated to collect used products from customers, recycle them and sell a single product reusing the recycled parts under the uncertainties in product demand and quality of used products collected from customers, practitioners and academics may have the following questions to discuss the operation of a GSC: 1) how much a retailer should pay for an incentive to encourage to collect used product from customers, 2) how the quality of recyclable parts after disassembly of used products affect the recycling activity of a manufacturer and the profit, 3) how a retailer determines the optimal order quantity of the product under the uncertainty in demand of product. This study tries to answer the above questions to operate a GSC optimally and profitably, and to make the following contributions for academic researchers and real-world policymakers regarding operations in a GSC:

- Presentation of theoretical analysis to encourage the collection and the recycling of used products by incorporating a collection incentive of used products into a GSC.
- Presentation of theoretical analysis to evaluate the profitability obtained from the optimal operations for a product quantity of a single product, a collection incentive of used products and a lower limit of used product under DGSC and IGSC.
- Presentation of theoretical analysis to incorporate the uncertainty in demand of a single of products under the following scenarios: 1) the distribution of product demand is known, 2) the product demand has unknown distribution with known mean and known variance into calculation of the expected profit in a GSC.
- Presentation of theoretical analysis to provide the optimal decision approach under a situation where the product demand has unknown distribution with known mean and known variance.
- Presentation of theoretical analysis to evaluate how quality of distribution of recyclable parts after disassembly of used product affect the optimal operational for order quantity, collection incentive of used products and lower limit of quality level for recycling and the expected profit of the manufacturer.
- Presentation of theoretical analysis to provide how incorporation of profit sharing approach into IGSC can not only promote the more aggressive eco-activity among all members in the GSC, but also shift to the decision-making under IGSC from that under DGSC.

Concretely, this study proposes an optimal production policy for a GSC with material flows from collection of used products to reuse of recycled parts in production of products. In the GSC, a retailer pays an incentive for collection of used products from customers and hands them over to a manufacturer. In this case, the retailer places an order for an order quantity of the products to the manufacturer, considering the product demand uncertainty. The manufacturer disassembles the used products, and then classifies the recyclable parts into quality levels by the result of the inspection of the used products. The manufacturer remanufactures products using recyclable parts with acceptable quality levels and pays the compensation a part of the retailer's incentive for collection of used products based on the quantity of the recycled parts to the retailer.

Here, the uncertainty in demand of a single product which a retailer faces to sell the product in a market are assumed as the following scenarios: 1) the distribution of product demand is known, 2) the product demand has unknown distribution with known mean and known variance.

For each situation, this paper develops two types of mathematical models and conducts the theoretical analysis in order to find how order quantity, collection incentive of used products and lower limit of quality level for recycling of used products affect the expected profits of each member and the whole supply chain (SC) under both DGSC and IGSC. Specifically, two types of optimal decisions are proposed for product quantity, collection incentive of used products and lower limit of quality level for recycling in GSC. One is under DGSC whose objective is to maximize the expected profit of each member based on optimal decision approach of the Stackelberg game. The other is under IGSC whose objective is to maximize the whole SC's expected profit.

The analysis numerically investigates how the following factors: i) available distribution information of product demand, ii) the quality of the recyclable parts after disassembly of used products affect the optimal operation and the expected profits a retailer, a manufacturer and the whole SC under DGSC and IGSC. Additionally, the results of optimal operation under DGSC is compared with that under IGSC under above factors (i) and (ii). Moreover, as supply chain coordination, the effects of three profit sharing approaches on each members' profit are investigated under IGSC: I) Adoption of profit ratios between members, II) Adoption of Nash bargaining solution (Nagarajan and Sosic, 2008; Du *et al*., 2011), III) Adoption of the combination of (I) and (II).

The contribution of this paper is to provide the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to academic researchers and real-world policymakers regarding operations in a GSC:

- The optimal order quantity in the scenario 2 where the product demand has unknown distribution with known mean and known variance is determined as a lower value than the scenario 1 where the distribution of product demand is known. This is due to the situation where the optimal decision in the scenario 2 is made under the worst situation where a retailer obtains the lowest expected profit.
- It is possible to guarantee to bring more profits to all members (a retailer and a manufacturer) in a GSC by taking the more aggressive eco-activity where not only a retailer pays incentive to customers in order to collect the more used products from customers, but also a manufacturer compensate some parts of incentive the retailer paid. Therefore, incorporation of the optimal collection incentive into a GSC can encourage both activities of the collection and the recycling of used products, guaranteeing the expected profits of a retailer, a manufacturer and the whole SC in the GSC.
- It is profitable to determine optimally the lower level of quality of recyclable parts after disassembly of the used products when the quality of recyclable parts is distributed several quality level.
- The optimal lower level of quality of recyclable parts under IGSC can be determined as a lower value than that under DGSC. Also, the optimal collection incentive under IGSC can be determined as a higher value than that under DGSC. Therefore, the optimal decisions under IGSC can encourage the more aggressive eco-activity among a retailer, a manufacturer and the whole SC in GSC.
- From the aspect of profit, incorporation of profit sharing approach into IGSC can not only promote the more aggressive eco-activity among all members in a GSC, but also shift to the decision-making under IGSC, guaranteeing the more expected profits of all members and the whole SC in a GSC.

The rest of our paper is organized as follows: in Section 2, notation used in our model is defined. In Section 3, operational flows of a GSC and the model assumptions are described. Section 4 formulates the expected profits in GSC. Section 5 proposes the optimal decision-making under DGSC and IGSC. Section 6 discusses incorporation of profit sharing approach into IGSC as supply chain coordination. Section 7 shows the results of numerical examples to illustrate managerial insights for the optimal operation of the GSC proposed in our paper. In Section 8, conclusions, managerial insights and future researches for this paper are summarized.

2. NOTATION

The following notations are used to formulate a GSC addressed in this paper.

General notations

Q : order quantity of product, referred to order quantity

t : collection incentive per used product (purchasing cost), referred to collection incentive

u : lower limit of quality level to remanufacture recyclable parts after disassembly of used products, referred to lower limit of quality level ($0 \le u \le 1$)

$A(t)$: collection quantity of product for collection incentive t

$R(t)$: compensation per used product paid to a retailer from a manufacturer for the amount of used products which are remanufactured

c_a : disassembly and inspection cost per used product

ℓ : quality level of recyclable parts ($0 \le \ell \le 1$)

$g(\ell)$: probability density function of quality level ℓ

$c_r(\ell)$: remanufacturing cost per a recyclable part in the case of quality level ℓ

c_d : disposal cost per un-reused part

c_n : procurement cost per new part

c_m : production cost per product

m_a : margin obtained from wholesale per product

w : wholesale price of product, referred to unit wholesale price

p : sales price per product, referred to unit sales price
t_U : upper limit of collection incentive t
s : shortage penalty cost per product of which demand is unsatisfied
h_r : inventory holding cost per unsold products
x : demand of product in a market
$f(x)$: probability density function of demand x

Notations for a DGSC

Q_D^* : optimal order quantity under DGSC
t_D^* : optimal collection incentive under DGSC
$u_D(t)$: provisional lower limit of quality level determined for a given collection incentive t under DGSC
u_D^* : optimal lower limit of quality level under DGSC

Notations for an IGSC

Q_C^* : optimal order quantity under IGSC
t_C^* : optimal collection incentive under IGSC
u_C^* : optimal lower limit of quality level under IGSC

3. MODEL DESCRIPTIONS

3.1 Operational Flows of a GSC

(1) A GSC consisting of a retailer and a manufacturer is considered. Also, it is assumed that a single product such as consumer electronics (mobile phone, personal computer), semiconductor and electronic component is produced and is sold in a market.

(2) A retailer pays the unit collection incentives t to collect used products from a market and delivers the collection quantity $A(t)$ of the used products with the unit cost c_t to the manufacturer.

(3) A manufacturer disassembles the used products, inspects all the recyclable parts with the unit cost c_a. After the disassembly, the manufacturer classifies the recyclable parts into the quality level $\ell(0 \le \ell \le 1)$. The manufacture determines optimally the lower limit of quality level $u(0 \le u \le 1)$ for the recyclable parts. The manufacturer remanufactures all the recyclable parts with quality level ℓ more than the lower limit of quality level u. The manufacturer disposes all the recyclable parts with lower quality level than u with the unit cost c_d.

(4) The manufacturer pays the compensation to the retailer for the cooperation to collection of the used products. Concretely, the manufacture pays the compensation $R(t)$ to the retailer who paid the unit collection incentive t to collect the quantity $A(t)$ of the used products.

(5) The retailer determines optimally the unit collection incentive t and the order quantity Q of the product under the uncertainty in product demand so as to maximize the retailer's expected profit. The retailer places an order of the quantity Q of a single product with the manufacturer.

(6) The manufacturer produces the same quantity Q of the product ordered from the retailer with the unit cost c_m, and sells the product to retailer at the unit wholesale price w.

(7) The manufacturer produces the required quantity of new parts with the unit cost c_n if the quantity of the recycled parts is unsatisfied with the required quantity of parts for the order quantity Q.

(8) The retailer sells the product in a market with the unit sales price p during a single period. The retailer incurs the unit inventory holding h_r of the unsold products, while the retailer incurs the unit shortage penalty cost s of the unsatisfied product demand.

3.2 Model Assumptions

(1) In the scenario $1\,(i=1)$ for the product demand, the demand x follows a probabilistic distribution and the probability density function (PDF) of x, $f(x)$, is known. In the scenario $2\,(i=2)$, the product demand has unknown PDF with known mean μ and known variance σ^2 for the demand x. Here, $\mu > 0$, $\sigma > 0$ and $\sigma^2 > 0$.

(2) A single recyclable part is extracted from the unit of used products. The manufacturer remanufactures products using a single type of recyclable parts with acceptable quality levels.

(3) Regarding collecting the used products, a retailer pays the unit collection incentive t to collect the used products from a market. Here, the collection quantity of the used products $A(t)$ varies according to the unit collection incentive t. In general, the higher the unit collection incentive t is, the more a retailer can collect the used products from a market, where the unit collection incentive t has the upper limit t_U ($0 \le t \le t_U < p$). The manufacturer pays the compensation to cooperation of collecting the used products to the retailer. Concretely, the manufacturer pays the compensation $R(t)$ to the retailer who paid the unit collection incentive t according to the quantity of the recycled parts from the used products. Here, the collection quantity $A(t)$ is not enough to satisfy the expected demand of product even if retailer pays the upper limit t_U of t.

(4) The unit wholesale price w is calculated from the unit procurement cost c_n of new parts, the unit production cost c_m of product and the unit margin m_a obtained from wholesales per product.

(5) The variability of quality level ℓ of the recyclable parts is modeled as a probabilistic distribution with the PDF $g(\ell)$.

(6) The unit remanufacturing cost $c_r(\ell)$ to a recycled part from a recyclable part with ℓ varies as to the quality level $\ell\ (0 \le \ell \le 1)$. The lower the quality level ℓ is, the higher the unit remanufactured cost $c_r(\ell)$ is. Here, $\ell = 0$ indicates the worst quality level of the

recyclable parts, meanwhile $\ell = 1$ indicates the best quality level of the recyclable products. Thus, $c_r(\ell)$ is a monotone decreasing function in terms of quality level ℓ. Note that each quality of the recycled parts produced from recyclable parts is as good as that of new parts produced from new materials.

4. EXPECTED PROFITS IN GSC

First, the retailer's expected profit in scenario 1 ($i = 1$) of the product demand is discussed. From 2, the retailer's profit consists of the collection cost and the delivery cost of the used products, the procurement cost of product, the compensation revenue, the product sales, the inventory holding cost of the unsold products and the shortage penalty cost for unsatisfied product demand in a market. The retailer's expected profit in $i = 1$ for the order quantity Q, the unit collection incentive t and the lower limit of quality level u, $E^1[\pi_R(Q, t, u)]$, is formulated as

$$\begin{aligned} E^1[\pi_R(Q,t,u)] &= -t\,A(t) - c_t A(t) \\ &\quad -wQ + R(t)\int_u^1 g(\ell)A(t)d\ell \\ &\quad +\left\{p\int_0^Q xf(x)dx + pQ\int_Q^\infty f(x)dx\right\} \\ &\quad -h_r\int_0^Q (Q-x)f(x)dx \\ &\quad -s\int_Q^\infty (x-Q)f(x)dx. \end{aligned} \tag{1}$$

In Eq. (1), the first term is the collection cost of the used products, the second term is the delivery cost of the used products, the third term is the procurement cost of product, the fourth term is the expected compensation revenue from a manufacturer, the fifth term is the expected product sales of product, the sixth term is the expected inventory holding cost of the unsold products the final term is the expected shortage penalty cost for unsatisfied product demand in a market.

The manufacturer's profit consists of the product wholesales, the disassembly and the inspection costs of the used products, the remanufacturing cost of recyclable parts after disassembly of the used products, the compensation cost to a retailer, the disposal cost of unrecycled parts, the procurement cost of new parts and the production cost of product. The manufacturer's expected profit for Q, t and u, $E[\pi_M(u, t, Q)]$, is formulated as

$$\begin{aligned} E[\pi_M(Q,t,u)] &= wQ - c_a A(t) - A(t)\int_u^1 c_r(\ell)g(\ell)d\ell \\ &\quad -R(t)\int_u^1 g(\ell)A(t)d\ell - c_d A(t)\int_0^u g(\ell)d\ell \\ &\quad -c_n\left\{Q - A(t)\int_u^1 g(\ell)d\ell\right\} - c_m Q \end{aligned} \tag{2}$$

In Eq. (2), the first term is the product wholesales, the second term is the disassembly and the inspection costs of the used products, the third term is the remanufacturing cost of recyclable parts after disassembly of the used products, the fourth term is the compensation cost to a retailer, the fifth term is the disposal cost of unrecycled parts, the sixth term is the procurement cost of new parts, and the final term is the production cost of product. Therefore, it can be seen from Eq. (2) that the manufacturer's expected profit is unaffected by any scenario of the product demand.

The whole SC's profit is calculated from the sum of the retailer's profit and the manufacturer's profit. In this case, the whole SC's profit consists of the collection cost and the delivery cost of the used products, the disassembly and the inspection costs of the used products, the remanufacturing cost of recyclable parts after disassembly of the used products, the disposal cost of un-recycled parts, the procurement cost of new parts and the production cost of product, the product sales, the inventory holding cost of the unsold products and the shortage penalty cost for unsatisfied product demand in a market. Therefore, the whole SC's expected profit in $i = 1$ for Q, t and u, $E^1[\pi_S(u, t, Q)]$, is obtained as the sum of both members' expected profits in Eqs. (1) and (2), corresponding to

$$\begin{aligned} E^1[\pi_S(Q,t,u)] &= E^1[\pi_R(Q,t,u)] + E[\pi_M(Q,t,u)] \\ &= -tA(t) - c_t A(t) - c_a A(t) \\ &\quad -A(t)\int_u^1 c_r(\ell)g(\ell)d\ell - c_d A(t)\int_0^u g(\ell)d\ell \\ &\quad -c_n\left\{Q - A(t)\int_u^1 g(\ell)d\ell\right\} - c_m Q \\ &\quad +p\int_0^Q xf(x)dx + pQ\int_Q^\infty f(x)dx \\ &\quad -h_r\int_0^Q (Q-x)f(x)dx \ -h_r\int_0^Q (Q-x)f(x)dx \\ &\quad -s\int_Q^\infty (x-Q)f(x)dx \end{aligned} \tag{3}$$

In Eq. (3), the first term is the collection cost of the used products, the second term is the delivery cost of the used products, the third term is the disassembly and the inspection costs of the used products, the fourth term is the remanufacturing cost of recyclable parts after disassembly of the used products, the fifth term is the disposal cost of un-recycled parts, the sixth term is the procurement cost of new parts, and the seventh term is the production cost of product. The eighth term is the expected product sales, the ninth term is the expected inventory holding cost of the unsold products the final term is the expected shortage penalty cost for unsatisfied product demand in a market. From Eq. (3), it can be seen that the terms regarding the wholesales of products and the compensation for the collection incentive occurring between a retailer and a manufacturer are canceled out. Therefore, the whole SC's expected profit is unaffected by the compensation for the collection incentive.

Next, the retailer's expected profit in scenario 2 $(i=2)$ of the product demand is discussed. Here, the retailer's expected profit $E^1[\pi_R(Q,t,u)]$ in scenario 1 $(i=1)$ of the product demand for Q, t and u in Eq. (3) can be rewritten as follows:

$$\begin{aligned}E^1[\pi_R(Q,t,u)] &= -tA(t)-c_tA(t) \\ &+R(t)\int_u^1 g(\ell)A(t)d\ell+(p-w)Q \\ &-(p+h_r)\int_0^Q (Q-x)f(x)dx \\ &-s\int_Q^\infty (x-Q)f(x)dx\end{aligned} \tag{4}$$

The elicitation process of Eq. (4) is shown in Appendix A.

When mean μ and variance σ^2 of the demand x are known, the upper limit of the expected excessive inventory quantity and the upper limit of the shortage quantity which demand x is unsatisfied with order quantity Q are derived using the distribution-free approach (DFA: Gallego and Moon, 1993; Moon and Gallego, 1994; Moon and Choi, 1995; Alfares and Elmorra, 2005) as

$$E[x-Q]^+ \le \left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2, \tag{5}$$

$$E[Q-x]^+ \le \left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)/2\right\}. \tag{6}$$

The elicitation processes of Eqs. (5) and (6) are shown in Gallego and Moon (1993) and Alfares and Elmorra (2005).

The lower limit of the retailer's expected profit in $i=2$ can be obtained by applying DFA into the retailer's expected profit in $i=1$. By substituting Eqs. (5) and (6) into the retailer's expected profit $E^1[\pi_R(Q,\ t,\ u)]$ in scenario 1 $(i=1)$ of the product demand for Q, t and u in Eq. (4), the lower limit of the retailer's expected profit in $i=2$ for Q, t and $u, E^2[\pi_R(Q,t,u)]$, can be derived as

$$\begin{aligned}E^2[\pi_R(Q,t,u)] &= -tA(t)-c_tA(t) \\ &+R(t)\int_u^1 g(\ell)A(t)d\ell+(p-w)Q \\ &-(p+h_r)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2 \\ &-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2\end{aligned} \tag{7}$$

The whole SC's expected profit in $i=2$, $E^2[\pi_S(Q,\ t\ u)]$, is obtained as the sum of both members' expected profits in Eqs. (7) and (2), corresponding to

$$\begin{aligned}E^2[\pi_S(Q,t,u)] &= E^2[\pi_R(Q,t,u)]+E[\pi_M(Q,t,u)] \\ &= -tA(t)-c_tA(t)-c_aA(t) \\ &-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell-c_dA(t)\int_0^u g(\ell)d\ell \\ &-c_n\left\{Q-A(t)\int_u^1 g(\ell)d\ell\right\}-c_mQ \\ &-(p+h_r)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2 \\ &-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2\end{aligned} \tag{8}$$

5. OPTIMAL DECISIONS-MAKING FOR GSC

5.1 Decentralized Green Supply Chain

For the optimal decisions are made under DGSC, the optimal decision approach for the Stackelberg game (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Sun, 2012; Watanabe *et al*., 2013) is adopted. The reason why the Stackelberg game is adopted under DGSC of this paper is shown as follows: the optimal decision in the Stackelberg game is made under a situation consisting of one leader of the decision-making and one (multiple) follower(s). First, a leader of the decision-making makes the optimal decision so as to the leader's profit. Next, one (multiple) follower(s) make(s) the optimal decision(s) so as to maximize the follower(s)' profit(s) under the optimal decision made by the leader of the decision-making. Suppose that decision variable(s) of supply chain members affect(s) not only the optimal decision so as to maximize the profit of a supply chain member, but also that (those) of the other supply chain member(s), interacting between supply chain members' profit. Under the situation, the optimal decision approach in the Stackelberg game is adopted effectively among supply chain members (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Sun, 2012; Watanabe *et al*., 2013). This paper regards a retailer as the leader of the decision-making under DGSC and regards a manufacture as the follower of the decision-making of the retailer under DGSC. The reason is due to the following situation: a retailer not only pays the unit collection incentives t to collect used products from a market so as to cooperate the encouragement the manufacturer's recycling activity of used products, but also faces stochastic demands of products in a market, sells the products in the market and earns the most profit in the entire supply chain.

The retailer determines the optimal order quantity $Q_D^{i*}(i=1,2)$ in scenario $i(=1,2)$ of the product demand and the optimal unit collection incentive t_D^* so as to maximize the retailer's expected profit. The manufacturer determines the optimal lower limit of quality level u_D^* so as to maximize the manufacturer's expected profit under the optimal order quantity Q_D^{i*} and the optimal unit collection incentive t_D^* determined by the retailer. Next, the manufacturer produces the same quality of the optimal order quantity Q_D^{i*} and sells the product to the retailer at the unit wholesale price w. The procedure for the optimal decision-making (Q_D^{i*}, t_D^*, u_D^*) under DGSC is explained as follows. First, the optimal order quantity Q_D^{1*} in $i=1$ under DGSC is discussed. The optimal order quantity under DGSC Q_D^{1*} in $i=1$ is determined under t and u so as to maximize the expected profit of a retailer who is the leader of the decision-making under DSC. A manufacturer follows the optimal order quantity under DGSC Q_D^{1*} in $i=1$ determined by the retailer.

Proposition 1: The retailer's expected profit in $i=1$ in Eq. (1) is the concave function in terms of the order quantity Q under t and u.

Proof: The first- and second-order differential equations between the order quantity Q and the expected profit $E^1\left[\pi_R\left(Q \middle| t, u\right)\right]$ of the retailer in $i=1$ in Eq. (1) under t and u are derived as follows:

$$dE^1\left[\pi_R\left(Q \middle| t, u\right)\right] \big/ dQ$$

$$= -w + p + s - (p + h_r + s)\int_0^Q f(x)dx\,, \tag{9}$$

$$d^2E^1\left[\pi_R\left(Q \middle| t, u\right)\right] \big/ dQ^2 = -\left(p + h_r + s\right) f\left(Q\right). \tag{10}$$

The elicitation processes of Eq. (9) is shown in Appendix B. It is derived that Eq. (10) is negative since it is natural to satisfy the condition $p > 0,\ h_r > 0,\ s > 0$. The theoretical analysis results in Proposition 1.

Proposition 2: The optimal order quantity Q_D^{1*} in $i=1$ can be obtained as the following unique solution to maximize Eq. (1):

$$Q_D^{1*} = F^{-1}\left(\frac{-w + p + s}{p + h_r + s}\right). \tag{11}$$

Proof: The solution of $dE^1\left[\pi_R\left(Q \middle| t, u\right)\right] \big/ dQ = 0$ substituting 0 into Eq. (9) results in Proposition 2.

Next, the optimal order quantity Q_D^{2*} in $i=2$ under DGSC is discussed. The optimal order quantity under DGSC Q_D^{2*} in $i=2$ is determined under t and u so as to maximize the expected profit of a retailer who is the leader of the decision-making under DSC. A manufacturer follows the optimal order quantity under DGSC Q_D^{2*} in $i=2$ determined by the retailer.

Proposition 3: The retailer's expected profit in $i=2$ in Eq. (7) is the concave function in terms of the order quantity Q under t and u.

Proof: The first- and second-order differential equations between the order quantity Q and the retailer's expected profit $E^2\left[\pi_R\left(Q \middle| t, u\right)\right]$ in $i=2$ in Eq. (7) under t and u are derived as follows:

$$dE^2\left[\pi_R\left(Q \middle| t, u\right)\right] \big/ dQ$$

$$= \frac{1}{2}\left\{(p + s - h_r - 2w) - (p + s + h_r)\frac{(Q - \mu)}{\left[\sigma^2 + (Q - \mu)^2\right]^{\frac{1}{2}}}\right\}, \tag{12}$$

$$\frac{d^2E^2\left[\pi_R(Q, t, u)\right]}{dQ^2} = -\frac{\sigma^2\left(p + s + h_r\right)}{2\left[\sigma^2 + (Q - \mu)^2\right]^{\frac{3}{2}}}. \tag{13}$$

The elicitation processes of Eqs. (12) and (13) are shown in Appendix C. It is derived that Eq. (13) is negative since it is natural to satisfy the condition $p > 0,\ h_r > 0,\ s > 0$. The theoretical analysis results in Proposition 3.

Proposition 4: The optimal order quantity Q_D^{2*} in $i=2$ can be obtained as the following unique solution to maximize Eqs. (14) and (15):

$$Q_D^{2*} = \frac{\mu + \sigma y_D}{\sqrt{1 - y_D^2}}, \tag{14}$$

$$y_D = \frac{p + s - h_r - 2w}{p + h_r + s}. \tag{15}$$

Proof: The solution of $dE^2\left[\pi_R\left(Q \middle| t, u\right)\right] \big/ dQ = 0$ substituting 0 into equation (12) results in Proposition 4.

Next, under the optimal order quantity in each scenario i of the product demand, $Q_D^{i*}(i = 1, 2)$, in Eqs. (11), (14), and (15), the optimal unit collection incentive t_D^* and the optimal lower limit of quality level u_D^* under DGSC are determined independently from standpoints where the retailer is the leader of the decision-making and the manufacturer is the follower of the decision-making.

The following first-order differential equation between the lower limit of quality level u and the expected profit $E\left[\pi_M(u) \middle| Q_D^{i*}, t\right](i = 1, 2)$ of the manufacturer in Eq. (2) under the optimal order quantity $Q_D^{i*}(i = 1, 2)$ and the unit collection incentive t is obtained as

$$\frac{dE\left[\pi_M(u) \middle| Q_D^{i*}, t\right]}{du}(i = 1, 2)$$

$$= A(t)d(u)\left\{c_r(u) + R(t) - c_d - c_n\right\}. \tag{16}$$

The elicitation process of Eq. (16) is shown in Appendix D.

Here, Eq. (16) is zero if and only if to satisfy the following condition:

$$c_r(u) + R(t) - c_d - c_n = 0 \tag{17}$$

Here, from Eq. (6) in Section 3.2 model assumptions, it can be seen that that there is the unique lower limit of quality level u to satisfy Eq. (17) under t. We define the lower limit of quality level u to satisfy Eq. (17) as the provisional lower limit of quality level $u_D(t)$ determined under t. It can be seen that $u_D(t)$ maximizes the expected profit $E\left[\pi_M(u)\middle|Q_D^{i*}, t\right](i=1, 2)$ under $Q_D^{i*}(i=1, 2)$ and t. By varying t within the range where $0 \le t \le t_U$, t and $u_D(t)$ are substituted into the Eq. (1) under Q_D^{1*} and Eq. (7) under Q_D^{2*}. The optimal combination (t_D^*, u_D^*) is determined as the combination $(t, u_D(t))$ to maximize the retailer's expected profit $E\left[\pi_R(t, u_D(t))\middle|Q_D^{i*}\right]$ $(i=1, 2)$ Therefore, the optimal unit collection incentive t_D^* and the optimal lower limit of quality level u_D^* are determined mutually between members. The expected profits of each member and the whole SC under DGSC can be obtained by using the optimal decisions $(Q_D^{i*}, t_D^*, u_D^*)(i=1, 2)$.

5.2 Integrated Green Supply Chain

In IGSC, the optimal decisions regarding order quantity $Q_C^{i*}(i=1, 2)$ in scenario $i(=1, 2)$ of the product demand, the unit collection incentive t_C^* and the lower limit of quality level u_C^* are made so as to maximize the whole SC's expected profit. First, the optimal order quantity Q_C^{1*} in $i=1$ under IGSC is discussed.

The optimal order quantity under IGSC Q_C^{1*} in $i=1$ is determined under t and u so as to maximize the expected profit of the whole SC in Eq. (3). A retailer and a manufacturer follow the optimal order quantity under IGSC Q_C^{1*} in $i=1$.

Proposition 5: The whole SC's expected profit in $i=1$ in Eq. (3) is the concave function in terms of the order quantity Q under t and u.

Proof: The first- and second-order differential equations between the order quantity Q and the whole SC's expected profit $E^1\left[\pi_S\left(Q\middle|t, u\right)\right]$ in $i=1$ in Eq. (3) under t and u are derived as follows:

$$dE^1\left[\pi_S\left(Q\middle|t, u\right)\right]\Big/dQ$$

$$= -c_n - c_m + p + s - (p + h_r + s)\int_0^Q f(x)dx, \tag{18}$$

$$d^2E^1\left[\pi_R\left(Q\middle|t, u\right)\right]\Big/dQ^2 = -(p + h_r + s)f(Q). \tag{19}$$

The elicitation processes of Eq. (18) is shown in Appendix E.

It is derived that Eq. (19) is negative since it is natural to satisfy the condition $p > 0,\ h_r > 0,\ s > 0$. The theoretical analysis results in Proposition 5.

Proposition 6: The optimal order quantity Q_C^{1*} in $i=1$ can be obtained as the following unique solution to maximize Eq. (3):

$$Q_C^{1*} = F^{-1}\left(\frac{-c_n - c_m + p + s}{p + h_r + s}\right). \tag{20}$$

Proof: The solution of $dE^1\left[\pi_R\left(Q\middle|t, u\right)\right]\Big/dQ = 0$ substituting 0 into Eq. (18) results in Proposition 6.

Next, the optimal order quantity Q_C^{2*} in $i=2$ under IGSC is discussed. The optimal order quantity under IGSC Q_C^{2*} in $i=2$ is determined under t and u so as to maximize the whole SC's expected profit in Eq. (8). A retailer and a manufacturer follows the optimal order quantity under IGSC Q_C^{2*} in $i=2$.

Proposition 7: The whole SC's expected profit in $i=2$ in Eq. (8) is the concave function in terms of the order quantity Q under t and u.

Proof: The first- and second-order differential equations between the order quantity Q and the whole SC's expected profit $E^2\left[\pi_S\left(Q\middle|t, u\right)\right]$ in $i=2$ in Eq. (8) under t and u are derived as follows:

$$dE^2\left[\pi_S\left(Q\middle|t, u\right)\right]\Big/dQ$$

$$= \frac{1}{2}\left\{\{p + s - h_r - 2(c_m + c_n)\} - (p + s + h_r)\frac{(Q-\mu)}{\left[\sigma^2 + (Q-\mu)^2\right]^{\frac{1}{2}}}\right\} \tag{21}$$

$$\frac{d^2E^2\left[\pi_R(Q, t, u)\right]}{dQ^2} = -\frac{\sigma^2(p + s + h_r)}{2\left[\sigma^2 + (Q-\mu)^2\right]^{\frac{3}{2}}}. \tag{22}$$

The elicitation processes of Eqs. (21) and (22) are shown in Appendix F.

It is derived that Eq. (22) is negative since it is natural to satisfy the condition: $p > 0,\ h_r > 0,\ s > 0$. The theoretical analysis results in Proposition 7.

Proposition 8: The optimal order quantity Q_C^{2*} in $i=2$ can be obtained as the following unique solution to maximize Eqs. (23) and (24):

$$Q_C^{2*} = \mu + \sigma\frac{y_C}{\sqrt{1 - y_C^2}}, \tag{23}$$

$$y_C = \frac{p + s - h_r - 2(c_n + c_m)}{p + h_r + s}. \tag{24}$$

Proof: The solution of $dE^2\left[\pi_S\left(Q\middle|t, u\right)\right]\Big/dQ = 0$ substituting 0 into Eq. (21) results in Proposition 8.

Next, under the optimal order quantity in $i(=1, 2)$, $Q_C^{i*}(i=1, 2)$, in Eqs. (20), (23), and (24), the optimal unit collection incentive t_C^* and the optimal lower limit of quality level u_C^* are determined under IGSC. As the similar way to determine the optimal lower limit of quality level u_D^* under DGSC, the optimal lower limit of quality level u_C^* is obtained so as to satisfy generally the following condition:

$$dE[\pi_s(u\,|\,Q_C^{i*},t)]\big/du\big(i=1,2\big)$$
$$=A(t)g(u)\{c_r(u)-c_d-c_n\}=0 \tag{25}$$
$$\Leftrightarrow c_r(u)=c_d+c_n \tag{26}$$

The elicitation processes of Eq. (25) is shown in Appendix G.

The optimal unit collection incentive t_C^* is determined so as to maximize the expected profit of the whole SC in $i(=1,2)$ under IGSC, $E\left[\pi_s\left(t\,\middle|\,Q_C^{i*},u_C^*\right)\right](i=1,2)$, in Eqs. (3) and (8) under $Q_C^{i*}\,(i=1,2)$ in Eqs. (20), (23), and (24), and the optimal lower limit of quality level u_C^* in Eq. (25). Substituting $Q_C^{i*}\,(i=1,2)$ and u_C^* into the whole SC's expected profit in $i(=1,2)$ in Eqs. (3) and (8), t_C^* is determined so as to maximize the whole SC's expected profit $E\left[\pi_s\left(t\,\middle|\,Q_C^{i*},u_C^*\right)\right](i=1,2)$ by varying t within the range where $0\le t\le t_U$. The expected profits of each member and the whole SC under IGSC can be obtained by using the optimal decisions $\left(Q_C^{i*},t_C^*,u_C^*\right)(i=1,2)$.

6. INCORPORAITON OF PROFIT SHARING APPROACH INTO IGSC AS SUPPLY CHAIN COORPORATION

As supply chain coordination to guarantee the profit improvement for each member under IGSC, the effects of three profit sharing approaches on the expected profit of each member under IGSC in scenario $i(=1,2)$ of the product demand are discussed. First, the profit sharing 1 adopting the profit ratio between both members is discussed. In the profit sharing 1, the expected profit of each member under IGSC with supply chain coordination in $i(=1,2)$ is obtained by adding the amount of profit sharing of each member in $i(=1,2)$, φ_R^i and φ_M^i, to the expected profit of each member for the optimal decisions in $i(=1,2)$ under DGSC as

$$\tilde{E}^i\left[\pi_R(Q_C^{i*},u_C^*,t_C^*)\right]=E^i\left[\pi_R(Q_D^{i*},u_D^*,t_D^*)\right]+\varphi_R^i \tag{27}$$
$$\tilde{E}^i\left[\pi_M(Q_C^{i*},u_C^*,t_C^*)\right]=E^i\left[\pi_M(Q_D^{i*},u_D^*,t_D^*)\right]+\varphi_M^i$$
$$\varphi_R^i=\Delta ES^i\times E\left[\pi_R\left(Q_C^{i*},u_C^*,t_C^*\right)\right]\Big/E\left[\pi_S\left(Q_C^{i*},u_C^*,t_C^*\right)\right] \tag{28}$$
$$\varphi_M^i=\Delta ES^i\times E\left[\pi_M\left(Q_C^{i*},u_C^*,t_C^*\right)\right]\Big/E\left[\pi_S\left(Q_C^{i*},u_C^*,t_C^*\right)\right] \tag{29}$$
$$\Delta ES^i=E\left[\pi_S\left(Q_C^{i*},u_C^*,t_C^*\right)\right]-E\left[\pi_S\left(Q_D^{i*},u_D^*,t_D^*\right)\right]. \tag{30}$$

Second, the profit sharing 2 adopting the Nash bargaining solutions (Nagarajan and Sosic, 2008; Du *et al.*, 2011) is discussed in order to coordinate the unit wholesale price w^i and compensation per used product collected by a retailer at the incentive t, $R^i(t)$, in $i(=1,2)$ between both members. Here, w and $R(t)$ are set as $R(t)=(1+\alpha)t$ and $w=w(m_a)=c_n+c_m+m_a$. For simplicity, the degree α of compensation for the retailer's collection incentive t and the margin m_a for wholesale per product are coordinated as the Nash bargaining solutions α^{iN} and m_a^{iN} in $i(=1,2)$. Substituting α^{iN} and m_a^{iN} in i $(=1,2)$ into w^i and $R^i(t)$, the unit wholesale price w^i and the compensation $R^i(t)$ in $i(=1,2)$ are calculated. The coordinated α^{iN} and m_a^{iN} in $i(=1,2)$ are determined so as to satisfy the following equations:

$$Max\ T\left(\alpha^{iN},m_a^{iN}\right)(i=1,2) \tag{31}$$
$$=\left\{\pi_R^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_R^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right)\right\}$$
$$\times\left\{\pi_M^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_M^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right)\right\}$$

subject to

$$\pi_R^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_R^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right)>0 \tag{32}$$
$$\pi_M^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_M^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right)>0 \tag{33}$$

where Eqs. (32) and (33) are the constraint conditions to guarantee that the expected profit of each member with supply chain coordination is always higher than that without supply chain coordination.

Third, the profit sharing 3 combining profit sharing 1 with profit sharing 2 is discussed. Here, a retailer is the leader of the decision-making under DGSC and a manufacturer is the follower of the decision-making made by the retailer. Under the situation, this paper considers that it is necessary that the increment of the retailer's expected profit in Eq. (32) is larger than that of the manufacturer's expected profit in Eq. (33) when the optimal decision is shifted from DGSC to IGSC. Therefore, the profit sharing 3 adds the following new constraint condition:

$$\pi_R^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_R^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right) \tag{34}$$
$$>\pi_M^{iN}\left(\alpha^{iN},m_a^{iN}\,\middle|\,Q_C^{i*},t_C^*,u_C^*\right)-\pi_M^i\left(\alpha,m_a\,\middle|\,Q_D^{i*},t_D^*,u_D^*\right)$$

to the constraint conditions in profit sharing 2.

7. NUMERICAL EXPERIMENTS

In this section, the results of the optimal decisions under DGSC are compared with those under IGSC as to two scenarios of the product demand; scenario 1 $(i=1)$: the distribution of product demand is known and scenario 2 $(i=2)$: the product demand has unknown distribution with known mean and known variance. The optimal order quantity, the optimal unit collection incentive, the optimal lower limit of quality level and the expect profits of a retailer, a manufacturer and the whole

SC under DGSC are compared with those under IGSC as to the scenario $i(=1, 2)$ of the product demand. Also, the effect of the quality of the recyclable parts in used products on the optimal decisions and the expected profits is shown. Moreover, as supply chain coordination, the effects of three profit sharing approaches: I) Adoption of profit ratio between members, II) Adoption of Nash bargaining solution, III) Adoption of the combination of profit sharing approaches (I) and (II) on each members' profit under IGSC are shown. We used the following system parameter values for numerical examples: $p=150$, $s=175$, $h_r=15$, $c_a=1$, $c_d=1$, $c_t=1$, $c_n=35$, $c_m=2$, $m_a=15$. Regarding the scenario of the product demand x, x follows the normal distribution with the mean $\mu=1000$ and variance $\sigma^2=300$ in scenario 1 $(i=1)$, meanwhile mean and variance of x are knows as $\mu=1000$ and $\sigma^2=300$ in scenario 2 $(i=2)$. Further, $A(t)$, w and $c_r(\ell)$ are set as $A(t)=100+50t\left(0\le t\le t_U, t_U=p\right)$, $=c_n+c_m+m_a$, $c_r(\ell)=40(1-0.9\ell)$, satisfying the conditions of Eqs. (3), (4), and (6) in model assumptions. Moreover, $R(t)$ is defined as $R(t)=(1+\alpha)t$, where α denotes degree of compensation for the retailer's unit collection incentive t without any supply chain coordination. Here, $\alpha=0.7$ is set in aspect of a manufacturer's profit.

As shown in Figure 1, we assume some shapes of the distribution of quality level $\ell\ \left(0\le\ell\le1\right)$ of recyclable parts in used products. We model each shape of the distribution of quality level $\ell\ \left(0\le\ell\le1\right)$ of recyclable parts by using the beta distribution. This is the reason why we use the beta distribution is not only because it's possible to express various shapes, but more important, it's widely used to measure relative parameters like level l, or anything that is between 0–1. Concretely, the beta distribution can express various shapes of distribution of recyclable parts in used products such as the uniform distribution-type shape, the normal distribution-type shape, the exponential distribution-type shape, the left-biased distribution shape, the right-biased distribution shape, by using the following probability density function with parameters (a, b):

$$f\left(\ell|a,b\right)=\frac{\Gamma\left(a+b\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\ell^{m-1}\left(1-\ell\right)^{n-1}, \tag{35}$$

where $\Gamma(\cdot)$ denotes the gamma function. As shown in Figure 1, we provide four cases of the beta distribution:

Case 1 $B(\ell|1,1)$: the situation where each quality of recyclable parts are uniformly distributed, corresponding to the uniform distribution-type shape for quality level $\ell\ \left(0\le\ell\le1\right)$,
Case 2 $B(\ell|2,2)$: the situation where there are the more recyclable parts with the middle quality and each quality of recyclable parts are symmetrically distributed, corresponding to the normal distribution-type shape for quality level $\ell\ \left(0\le\ell\le1\right)$,
Case 3 $B(\ell|3,2)$: the situation where there are the more recyclable parts with the relatively high quality, corresponding to the right-biased distribution shape for quality level $\ell\ \left(0\le\ell\le1\right)$,
Case 4 $B(\ell|2,3)$: the situation where there are the more recyclable parts with the relatively low quality, corresponding to the left-biased distribution shape for quality level $\ell\ \left(0\le\ell\le1\right)$.

By changing parameters (a, b) of the probability density function of the beta distribution in Eq. (35), we can see how the results of the optimal operations in the GSC change.

Table 1 shows the comparisons of the optimal order quantity $Q_j^{i*}(i=1, 2.\ j=D, C)$ under DGSC and IGSC as to scenario $i(=1, 2)$ of the product demand. From Table 1, the optimal order quantities, Q_D^{2*} and Q_C^{2*}, in $i=2$ are smaller than those in $i=1$. This is the reason why the available demand information is limited in $i=2$, so the optimal order quantities are determined more carefully in $i=2$ than those are done in $i=1$. Moreover, in scenario $i(=1, 2)$, the optimal order quantity Q_C^{i*} under IGSC are larger than those under DGSC. This reason is clear from the analysis results regarding the optimal order quantity in Eqs. (11) and (20) and in $i=1$, Eqs. (14), (15), (23) and (24) in $i=2$ under the general condition where $w>c_n+c_m$. That is, the optimal order quantity Q_D^{i*} under DGSC are affected by the unit price w, meanwhile the optimal order quantity Q_C^{i*} under IGSC are affected by the sum of the procurement cost and the production cost of product, c_n+c_m, satisfying the general condition $w>c_n+c_m$.

Table 2 shows the comparisons of the optimal lower limit of quality level $u_j^{i*}(i=1, 2.\ j=D, C)$ under DGSC and IGSC and the optimal unit collection incentive t_j^{i*} under DGSC and IGSC. From Eqs. (17) and (25), u_j^{i*} is unaffected by Q_j^{i*} in each scenario of the product demand. From numerical search, it is confirmed that t_j^{i*} was unaffected by any scenario of the product demand.

Table 3 shows the comparisons of the expected

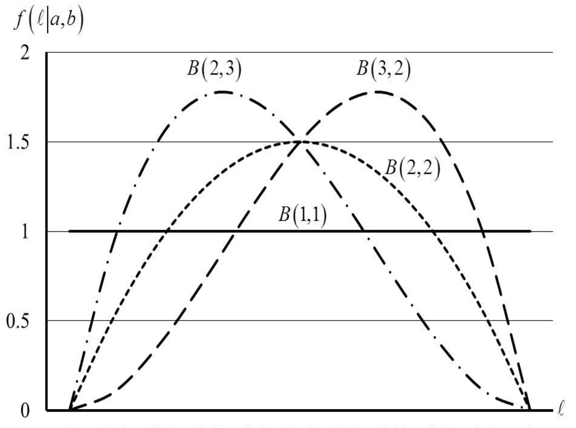

Figure 1. Four cases of distribution of quality level $\ell\left(0\le\ell\le1\right)$ of recyclable parts in used proucts modeled as the beta distribution $B(\ell|a, b)$.

Table 1. Comparisons of optimal order quantity under DGSC and IGSC as to available demand information $(i = 1, 2)$

Conditions of distribution of the quality level ℓ $(0 \le \ell \le 1)$ of recyclable parts	Optimal order quantity			
	DGSC	IGSC	DGSC	IGSC
	Q_D^{1*}	Q_D^{2*}	Q_C^{1*}	Q_C^{2*}
Case 1 $B(\ell \mid 1, 1)$	1256	1228	1307	1289
Case 2 $B(\ell \mid 2, 2)$	1256	1228	1307	1289
Case 3 $B(\ell \mid 3, 2)$	1256	1228	1307	1289
Case 4 $B(\ell \mid 2, 3)$	1256	1228	1307	1289

DGSC: decentralized green supply chain, IGSC: integrated green supply chain.

Table 2. Comparisons of the optimal unit collection incentive and the optimal lower limit of quality level of recyclable parts under DGSC and IGSC

Conditions of distribution of the quality level ℓ $(0 \le \ell \le 1)$ of recyclable parts	Optimal unit collection incentive		Optimal unit collection incentive	
	DGSC	IGSC	DGSC	IGSC
	t_D^*	t_C^*	u_D^*	u_C^*
Case 1 $B(\ell \mid 1, 1)$	2.94	4.61	0.25	0.11
Case 2 $B(\ell \mid 2, 2)$	3.94	4.52	0.30	0.11
Case 3 $B(\ell \mid 3, 2)$	6.13	6.30	0.40	0.11
Case 4 $B(\ell \mid 2, 3)$	2.27	2.74	0.22	0.11

DGSC: decentralized green supply chain, IGSC: integrated green supply chain.

Table 3. Comparisons of expected profits of retailer and the whole SC under DGSC for scenarios 1 and 2 of product demand

Conditions of distribution of the quality level of recyclable parts	Retailer's expected profit		The whole SC's expected profit	
	Scenario 1 of product demand	Scenario 2 of product demand	Scenario 1 of product demand	Scenario 2 of product demand
Case 1 $B(\ell \mid 1, 1)$	69582	57380	91003	78472
Case 2 $B(\ell \mid 2, 2)$	69729	57526	90946	78415
Case 3 $B(\ell \mid 3, 2)$	70205	58003	92265	79734
Case 4 $B(\ell \mid 2, 3)$	69583	57380	89943	77412

SC: supply chain, DGSC: decentralized green supply chain.

profits of retailer and the whole SC under DGSC for scenario 1 and scenario 2 of product demand. It can be seen that all the expected profits with DFA under scenario 2 are guaranteed to be lower than those under scenario 1.

Next, the optimal unit collection incentive t_D^* under DGSC are compared with the optimal unit collection incentive t_C^* under IGSC in cases 1–4 for the distribution of the quality level ℓ of recyclable parts. From Table 2, it can be seen that t_D^* and t_C^* are affected by for cases 1–4 of ℓ. In case 3, the more parts tend to be remanufactured, since ℓ is relatively high quality. This is the reason why t_D^* and t_C^* are determined as higher values indicating that more used products tend to be collected under the higher unit collection incentives. Meanwhile, in case 4, the less parts tend to be remanufactured, since ℓ is relatively low quality. This is the reason why t_D^* and t_C^* are determined as lower values indicating that less used products tend to be collected under the lower unit collection incentives .

Moreover, the optimal unit collection incentive t_D^* under DGSC are compared with the optimal unit collection incentive t_C^* under IGSC. From Eq. (1) in $i = 1$ and Eq. (7) in $i = 2$ regarding the retailer's expected profit, t_D^* is affected by the compensation income. Meanwhile, from Eq. (3) in $i = 1$ and Eq. (8) in $i = 2$, regarding the whole SC's expected profit, t_C^* is unaffected by the compensation income, but t_C^* is affected by the disassembly and inspection cost of the used products, the remanufacturing cost of the recyclable parts, the disposal cost of un-reused parts and the procurement cost of new parts. This is the reason why t_C^* is determined as higher value than t_D^* in Table 2. It implies that the collection quantity of used products under IGSC is more than that under DGSC. In Table 2, we also compare u_D^* with u_C^* From the Eq. (17), u_D^* is affected by t_D^* determined by the retailer. Meanwhile, from the Eq. (25), u_C^* is unaffected by t_D^*, since the term of compensation is canceled out between both members under IGSC. Also, from Eqs. (17) and (25), u_C^* can be determined as lower values than u_D^*. This fact implies that the recycling of the used products can be encouraged under IGSC. This feature can be confirmed from the results of numerical analysis, u_D^* and u_C^*, in Table 2. Also, u_D^* is compared with u_C^* under cases 1–4 for the distribution of the quality level ℓ of recyclable parts. It can be seen that u_C^* is unaffected by any condition of distribution of ℓ of them. This is the reason why the compensation relevant to the probability distribution of ℓ of them is canceled out between members under IGSC. Meanwhile, u_D^* is affected by each condition of distribution of ℓ of them.

Furthermore, the expected profits of the retailer, the manufacturer under DGSC are compared with those under IGSC as to scenario $i(=1, 2)$ of the product demand. Table 4 shows the results for $i=1$ in cases 1–4 of the distribution of the quality level ℓ of recyclable parts. From Table 4, the expected profit of the manufacturer in each case under IGSC is higher than that under DGSC. Only in case 4 under $i=1$ in Table 4, the expected profit of the retailer under IGSC is lower than that under DGSC. In other cases under $i=1$ in Table 4, the expected profits of both members under IGSC are higher than those under DGSC. However, the manufacturer has the more increment of the profit obtained under IGSC than the retailer has under IGSC. This result is same in scenario $i=2$. This implies that the increment of the expected profit for each member under IGSC does not reflect the size of the expected profit of each member. It is difficult for the retailer to shift the optimal decisions under IGSC which can enhance the expected profit of the whole SC.

Under the situation, any reasonable profit sharing is necessary between members under IGSC so as to shift to the optimal decisions under IGSC from those under DGSC, guaranteeing more profits to members under IGSC than those under DGSC. The effects of profit sharing approach under IGSC on the expected profits of the retaler and the manufacturer are investigated. Table 5 shows the effects of supply chain coordination adopting each profit sharing approach described in Section 6. for Case 1 of the distribution of the quality level $\ell(0 \le \ell \le 1)$ of recyclable parts in scenario $1\,(i=1)$ where the expected profit of the retailer is lower under IGSC. From the results of Table 5, it can be seen that the expected profits of members under IGSC with all profit sharing approaches are higher than those under DGSC in any condition of distribution of the quality level ℓ of them. Moreover, the superiority is compared between three profit sharing approaches. In the profit sharing approach 1, the increment of the expected profit obtained under IGSC is shared between members without any adjustment of parameters regarding supply chain coordination contract between both members. It may be difficult to accept the

Table 4. Comparisons of expected profits of both members under DGSC and IGSC in available demand information $i=1$

Conditions of distribution of the quality level of recyclable parts	Retailer's expected profit		Manufacturer's expected profit	
	DGSC	IGSC	DGSC	IGSC
Case 1 $B(\ell \vert 1, 1)$	69582	69659 (+77)	20841	21345 (+504)
Case 2 $B(\ell \vert 2, 2)$	69729	69830 (+101)	20648	21116 (+468)
Case 3 $B(\ell \vert 3, 2)$	70205	70603 (+398)	21226	21661 (+435)
Case 4 $B(\ell \vert 2, 3)$	69583	69358 (-225)	19931	20585 (+654)

DGSC: decentralized green supply chain, IGSC: integrated green supply chain.

Table 5. Effects of supply chain coordination in available demand information $i=1$ in case 1 of distribution of $\ell(0 \le \ell \le 1)$ of recyclable parts

Expected profits of GSC	No profit sharing	Profit sharing approach incorporated into IGSC		
	DGSC	I	II	III
Retailer	69582	70029 (+447)	69872 (+290)	69873 (+291)
Manufacturer	20841	20975 (+134)	21131 (+290)	21130 (+289)
Whole supply chain	90424	91003 (+579)	91003 (+579)	91003 (+579)
Coordinated degree of compensation α^{1N}	0.7	0.7	1.63	1.34
Coordinated margin m_a^{1N}	15	15	15.8	15.5

GSC: green supply chain, DGSC: decentralized GSC, IGSC: integrated GSC.

profit sharing approach 1 between members as supply chain coordination. In the profit sharing approaches 2 and 3, the increment of the expected profit obtained under IGSC is shared between members, using reasonable parameters regarding the unit whole sales price and the compensation per used product collected by a retailer at the incentive in supply chain coordination contract adjusted by Nash bargaining solutions. Moreover, in the profit sharing approach 3, it is possible to reflect the size of the expected profit of each member on the amount of profit sharing for each member. For the retailer who is the leader of the decision-making under DGSC, the profit sharing approach 3 is the most reasonable one to encourage all members in GSC to shift to the optimal decisions under IGSC from those under DGSC.

8. CONCLUSIONS

This study proposed an optimal production policy for a GSC with material flows from the collection of used products to the reuse of recycled parts in production of products. In the GSC, a retailer paid an incentive for collection of used products from customers and hands them over to a manufacturer. In this case, the retailer placed an order for an order quantity of the products to the manufacturer, considering the product demand uncertainty. The manufacturer disassembled the used products, and then classified the recyclable parts into quality levels by the result of the inspection of the used products. The manufacturer remanufactured products using recyclable parts with acceptable quality levels and paid for compensation a part of the retailer's incentive for collection of used products based on the quantity of the recycled parts to the retailer.

Here, the uncertainty in demand of a single product which a retailer faced in selling the product in a market was assumed as the following scenarios: 1) the distribution of product demand was known, 2) the product demand has unknown distribution with known mean and known variance. For each situation, this paper developed two types of mathematical models and conducted the theoretical analysis in order to find the effect of order quantity, collection incentive of used products and lower limit of quality level for recycling of used products on the expected profits of each member and the whole SC under both DGSC and IGSC. Concretely, two types of optimal decisions were proposed for product quantity, collection incentive of used products and lower limit of quality level for recycling in GSC. One was under DGSC whose objective was to maximize the expected profit of each member. The other was under IGSC whose objective was to maximize the whole SC's expected profit.

The analysis numerically investigated how the following factors: i) available distribution information of product demand, ii) the quality of the recyclable parts after disassembly of used products affected the optimal operation and the expected profits a retailer , a manufacturer and the whole SC under DGSC and IGSC. Additionally, the results of optimal operation under DGSC were compared with those under IGSC under above factors (i) and (ii). Moreover, as supply chain coordination, the effects of three profit sharing approaches on each member's profit were investigated under IGSC: I) adoption of profit ratios between members, II) adoption of Nash bargaining solution, III) adoption of the combination of (I) and (II).

This paper contributed the following managerial insights from outcomes obtained from both the theoretical research and the numerical analysis to both academic researchers and real-world policymakers regarding operations in a GSC:

- The optimal order quantity in the scenario 2 where the product demand had unknown distribution with known mean and known variance was determined as a lower value than the scenario 1 where the distribution of product demand was known. This was due to the situation where the optimal decision in the scenario 2 was made under the worst situation where a retailer obtained the lowest expected profit.
- It was possible to guarantee to bring more profits to all members (a retailer and a manufacturer) in a GSC by taking the more aggressive eco-activity where not only a retailer paid incentive to customers in order to collect the more used products from customers, but also a manufacturer compensated some parts of incentive the retailer paid. Therefore, incorporation of the optimal collection incentive into a GSC could encourage both activities of the collection and the recycling of used products, increasing the expected profits of all members and the whole SC in the GSC.
- It was profitable to determine optimally the lower level of quality of recyclable parts after disassembly of the used products when the quality of recyclable parts was distributed several quality level.
- The optimal lower level of quality of recyclable parts under IGSC could be determined as a lower value than that under DGSC. Also, the optimal collection incentive under IGSC could be determined as a higher value than that under DGSC. Therefore, the optimal decisions under IGSC could encourage the more aggressive eco-activity regarding the collection and the remanufacturing of used products among all members and the whole SC in a GSC.
- From the aspect of profit, incorporation of profit sharing approach into IGSC could promote not only the more aggressive eco-activity among all members in the GSC, but also shift the decision-making under IGSC, guaranteeing the more expected profits of all members and the whole SC in a GSC.

Therefore, it is highly expected that research outcomes in this paper would provide not only the optimal solution and its practices to construct a GSC to encourage both aggressive eco-activities of the collection and the remanufacturing of used products to firms, but also informative motivations for researchers and policymakers regarding operations in a GSC.

In the optimal operation for a GSC proposed in this paper, the optimal collection incentive of used products can be determined as a lower value indicating the less aggressive collection activity of used products. In the situation, there are used products with low quality. So, a manufacturer tends to take an inactive remanufacturing activity. In this case, a retailer tends to obtain the less compensation for the collection inventive of used products from the manufacturer.

As the extendable consideration including the above issue, it will be necessary to discuss the following issues to analyze the optimal operation for a GSC:

- Framework of a GSC to encourage the aggressive eco-activities regarding the collection and remanufacturing of used products even if there are used products with low quality
- A situation of uncertainty in the collection quantity of the used products
- Limitation of information regarding quality distribution of used products/ recyclable parts
- Alternative supply chain coordination between a retailer and a manufacturer to evaluate the profit balance and cost effectiveness of each member in GSC.

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APPENDIX A

$$
\begin{aligned}
E^2\left[\pi_R(Q,t,u)\right] = & -tA(t) - c_tA(t) \\
& -wQ + R(t)\int_u^1 g(\ell)A(t)d\ell \\
& +p\int_0^Q \left\{Q-(Q-x)\right\} f(x)dx + pQ\int_Q^\infty f(x)dx \\
& -h_r\int_0^Q (Q-x)f(x)dx \\
& -s\int_Q^\infty (x-Q)f(x)dx \\
= & -tA(t) - c_tA(t) + R(t)\int_u^1 g(\ell)A(t)d\ell \\
& -wQ + pQ\int_0^Q f(x)dx + pQ\int_Q^\infty f(x)dx \\
& -p\int_0^Q (Q-x)f(x)dx \\
& -h_r\int_0^Q (Q-x)f(x)dx \\
& -s\int_Q^\infty (x-Q)f(x)dx \\
= & -tA(t) - c_tA(t) + R(t)\int_u^1 g(\ell)A(t)d\ell - wQ + pQ \\
& -(p+h_r)\int_0^Q (Q-x)f(x)dx \\
& -s\int_Q^\infty (x-Q)f(x)dx
\end{aligned}
\tag{4}
$$

Therefore, the elicitation process of Eq. (4) can be shown.

APPENDIX B

$$
\begin{aligned}
\frac{dE^1\left[\pi_R(Q,t,u)\right]}{dQ} = & -w \\
& +p\frac{d}{dQ}Q\int_Q^\infty f(x)dx + p\frac{d}{dQ}\int_0^Q x f(x)dx \\
& -h_r\frac{d}{dQ}\int_0^Q (Q-x)f(x)dx - s\frac{d}{dQ}\int_Q^\infty (x-Q)f(x)dx \\
= & -w \\
& +p\frac{d}{dQ}Q\int_Q^\infty f(x)dx + p\frac{d}{dQ}\int_0^Q \left\{Q-(Q-x)\right\} f(x)dx \\
& -h_r\frac{d}{dQ}\int_0^Q Qf(x)dx + h_r\frac{d}{dQ}\int_0^Q xf(x)dx \\
& -s\frac{d}{dQ}\int_Q^\infty xf(x)dx + s\frac{d}{dQ}\int_Q^\infty Qf(x)dx \\
= & -w \\
& +p\frac{d}{dQ}\int_Q^\infty Qf(x)dx + p\frac{d}{dQ}\int_0^Q Qf(x)dx \\
& -p\frac{d}{dQ}\int_0^Q (Q-x)f(x)dx \\
& -h_r\frac{d}{dQ}\int_0^Q Qf(x)dx + h_r\frac{d}{dQ}\int_0^Q xf(x)dx \\
& -s\frac{d}{dQ}\int_Q^\infty xf(x)dx + s\frac{d}{dQ}\int_Q^\infty Qf(x)dx \\
= & -w + p\frac{d}{dQ}\int_0^\infty Qf(x)dx \\
& -p\frac{d}{dQ}\int_0^Q Qf(x)dx + p\frac{d}{dQ}\int_0^Q xf(x)dx \\
& -h_r\frac{d}{dQ}\int_0^Q Qf(x)dx + h_r\frac{d}{dQ}\int_0^Q xf(x)dx \\
& -s\frac{d}{dQ}\int_Q^\infty xf(x)dx + s\frac{d}{dQ}\int_Q^\infty Qf(x)dx \\
= & -w + p \\
& -p\int_0^Q f(x)dx - pQf(Q) + pQf(Q) \\
& -h_r\int_0^Q f(x)dx - h_rQf(Q) + h_rQf(Q) \\
& +sQf(Q) + s\int_Q^\infty f(x)dx - sQf(Q) \\
= & -w + p \\
& -p\int_0^Q f(x)dx - h_r\int_0^Q f(x)dx + s\left\{1 - \int_0^Q f(x)dx\right\} \\
= & -w + p + s \\
& -p\int_0^Q f(x)dx - h_r\int_0^Q f(x)dx - s\int_0^Q f(x)dx \\
= & -w + p + s - (p + h_r + s)\int_0^Q f(x)dx
\end{aligned}
\tag{9}
$$

Therefore, the elicitation process of Eq. (9) can be shown.

APPENDIX C

$$
\begin{aligned}
E^2\left[\pi_R(Q,t,u)\right] = & -tA(t) - c_tA(t) \\
& +R(t)\int_u^1 g(\ell)A(t)d\ell + (p-w)Q \\
& -(p+h_r)\frac{\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}} - (\mu-Q)}{2} \\
& -s\frac{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}} - (Q-\mu)}{2}
\end{aligned}
\tag{7}
$$

Eq. (7) can be rewritten as

$$
\begin{aligned}
E^2\left[\pi_R(Q,t,u)\right] = & -tA(t) - c_tA(t) + R(t)\int_u^1 g(\ell)A(t)d\ell \\
& +\frac{1}{2}\left\{2(p-w)Q - (p+h_r)\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right. \\
& \left. +(p+h_r)(\mu-Q) - s\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}} + s(Q-\mu)\right\} \\
= & -tA(t) - c_tA(t) + R(t)\int_u^1 g(\ell)A(t)d\ell
\end{aligned}
$$

$$+\frac{1}{2}\{(2p-2w)Q+(p+h_r-s)(\mu-Q)$$

$$\left.-(p+h_r+s)\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right\}$$

$$=-t\,A(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)\,d\ell$$

$$+\frac{1}{2}\{(p+s-h_r-2w)Q+(p+h_r-s)\mu$$

$$\left.-(p+s+h_r)\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}.$$

$$\frac{dE^2\left[\pi_R(Q,t,u)\right]}{dQ}$$

$$=\frac{1}{2}\left\{(p+s-h_r-2w)-\frac{d}{dQ}(p+s+h_r)\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}$$

$$=\frac{1}{2}\left\{(p+s-h_r-2w)-(p+s+h_r)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\} \qquad (12)$$

Therefore, the elicitation process of Eq. (12) can be shown.

$$\frac{d^2E^2\left[\pi_{RL}(Q,t,u)\right]}{dQ^2}$$

$$=\frac{1}{2}\cdot\frac{d}{dQ}\{(p+s-h_r-2w)$$

$$\left.-(p+s+h_r)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\}$$

$$=-\frac{d}{dQ}\frac{(p+s+h_r)(Q-\mu)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}$$

$$=-\frac{\sigma^2(p+s+h_r)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}} \qquad (13)$$

Therefore, the elicitation process of Eq. (13) can be shown.

APPENDIX D

$$\frac{dE\left[\pi_M\left(u\,\middle|\,Q_i^*,t\right)\right]}{du}(i=1,2)$$

$$=R(t)g(u)A(t)+A(t)c_r(u)g(u)$$

$$-c_dA(t)g(u)-c_nA(t)g(u)$$

$$=A(t)g(u)\{R(t)+c_r(u)-c_d-c_n\} \qquad (16)$$

Therefore, the elicitation process of Eq. (16) can be shown.

APPENDIX E

$$\frac{dE^1\left[\pi_S(Q,t,u)\right]}{dQ}$$

$$=-c_n-c_m+p\frac{d}{dQ}Q\int_Q^\infty f(x)dx+p\frac{d}{dQ}\int_0^Q xf(x)dx$$

$$-h_r\frac{d}{dQ}\int_0^Q(Q-x)f(x)dx-s\frac{d}{dQ}\int_Q^\infty(x-Q)f(x)dx$$

$$=-c_n-c_m+p+s-(p+h_r+s)\int_0^Q f(x)dx \qquad (18)$$

Therefore, the elicitation process of Eq. (18) can be shown.

APPENDIX F

Eq. (8) can be rewritten as by replacing w in Eq. (7) with (c_m+c_n),

$$E^2\left[\pi_S(Q,t,u)\right]=-tA(t)-c_tA(t)$$

$$+\{p-(c_m+c_n)\}Q$$

$$-(p+h_r)\frac{\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}-(\mu-Q)}{2}$$

$$-s\frac{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}-(Q-\mu)}{2}$$

$$=-tA(t)-c_tA(t)$$

$$+\frac{1}{2}\left\{2\{p-(c_m+c_n)\}Q-(p+h_r)\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right.$$

$$\left.+(p+h_r)(\mu-Q)+s(Q-\mu)-s\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}$$

$$=-tA(t)-c_tA(t)$$

$$+\frac{1}{2}\{2\{p-(c_m+c_n)\}Q+(p+h_r-s)(\mu-Q)$$

$$\left.-(p+h_r+s)\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right\}$$

$$=-tA(t)-c_tA(t)$$

$$+\frac{1}{2}\{(p+s-h_r-2(c_m+c_n))Q+(p+h_r-s)\mu$$

$$\left.-(p+s+h_r)\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}.$$

By replacing w in Eq. (12) with $(c_m + c_n)$,

$$\begin{aligned}
&\frac{dE^2\left[\pi_S(Q,t,u)\right]}{dQ}\\
&=\frac{1}{2}\left\{\left\{p+s-h_r-2(c_m+c_n)\right\}\right.\\
&\left.-\frac{d}{dQ}(p+s+h_r)\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}\\
&=\frac{1}{2}\left\{\left\{p+s-h_r-2(c_m+c_n)\right\}-(p+s+h_r)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\}
\end{aligned} \tag{21}$$

Therefore, the elicitation process of Eq. (21) can be shown.

By replacing w in Eq. (13) with $(c_m + c_n)$,

$$\begin{aligned}
&\frac{d^2E^2\left[\pi_S(Q,t,u)\right]}{dQ^2}\\
&=\frac{1}{2}\cdot\frac{d}{dQ}\left\{\left\{p+s-h_r-2(c_m+c_n)\right\}\right.\\
&\left.-(p+s+h_r)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\}\\
&=-\frac{d}{dQ}\frac{(p+s+h_r)(Q-\mu)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\\
&=-\frac{\sigma^2(p+s+h_r)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}}
\end{aligned} \tag{22}$$

Therefore, the elicitation process of Eq. (22) can be shown.

APPENDIX G

$$\begin{aligned}
&\frac{dE\left[\pi_S(Q,t,u)\right]}{du}\\
&=A(t)c_r(u)g(u)-c_dA(t)g(u)-c_nA(t)g(u)\\
&=A(t)g(u)\left\{c_r(u)-c_d-c_n\right\}
\end{aligned} \tag{25}$$

Therefore, the elicitation process of Eq. (25) can be shown.