# Optimal Power Allocation and Outage Analysis for Cognitive MIMO Full Duplex Relay Network Based on Orthogonal Space-Time Block Codes

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## Abstract

This paper investigates the power allocation and outage performance of MIMO full-duplex relaying (MFDR), based on orthogonal space-time block codes (OSTBC), in cognitive radio systems. OSTBC transmission is used as a simple means to achieve multi-antenna diversity gain. Cognitive MFDR systems not only have the advantage of increasing spectral efficiency through spectrum sharing, but they can also extend coverage through the use of relays. In cognitive MFDR systems, the primary user experiences interference from the secondary source and relay simultaneously, owing to full duplexing. It is therefore necessary to optimize the transmission powers at the secondary source and relay. In this paper, we propose an optimal power allocation (OPA) scheme based on minimizing the outage probability in cognitive MFDR systems. We also analyse the outage probability of the secondary user in noise-limited and interference-limited environments in Nakagami-m fading channels. Simulation results show that the proposed schemes achieve performance improvements in terms of reducing outage probability.


***Keywords:*** Cognitive radio, orthogonal space-time block codes, MIMO full-duplex relay, power allocation.

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## 1. Introduction

Cognitive radio (CR) [1] is becoming one of the most promising technologies for efficient spectrum utilization. Spectrum sharing methods in CR can be classified in two categories: spectrum overlay and spectrum underlay. Recently, spectrum underlay sharing protocols have drawn increasing interest [2][3]. The underlay paradigm allows cognitive (secondary) users to utilize the licensed spectrum if the interference caused to the primary users is below a prescribed interference threshold. Because of the constraint on transmitted power, the performance of cognitive underlay protocols degrades significantly in fading environments. One efficient method to improve the performance of the secondary network is to use a cooperative relay [4][5][6]. It is well known that the cooperative cognitive relay is able to mitigate signal fading arising from multipath propagation and, at the same time, improve the outage performance of wireless networks.

In essence, the relay systems can be classified into two categories: half-duplex relaying (HDR) systems, where the relay receives and retransmits the signal on orthogonal channels, and full-duplex relaying (FDR) systems, where the reception and retransmission at the relay occur at the same time on the same channel. FDR systems have been examined for their ability to effectively prevent capacity degradation due to additional use of time slots [7][8].

The multiple-input multiple-output (MIMO) technology provides another approach to combat fading. It can offer antenna diversity without requiring additional bandwidth or transmitting power. OSTBC transmission [9][10] is a simple way to obtain a multi-antenna diversity gain. It reduces complexity and only requires linear processing at the receiver end. Performance analysis for OSTBC transmission in a non-spectrum sharing scenario with decode-and-forward (DF) and amplify-and- forward (AF) relays has been presented in [9] and [10], respectively. The authors in [11] consider cognitive AF relaying networks for spectrum sharing, based on distributed OSTBC. In particular, they derive the exact closed-form expression for outage probability. On the other hand, [12] reports on the outage performance of MIMO cognitive DF relaying systems that use OSTBC transmission over Rayleigh fading channels. It has been verified that the cognitive relay network using OSTBC can achieve full degree of diversity. However, these prior works only consider cognitive MIMO half-duplex relaying (CogMHDR-D ). To the best of our knowledge, no work has been carried out on cognitive MIMO full-duplex relaying (CogMFDR). Furthermore, the use of FDR nodes introduces interference problems that are inherent to the full duplex approach [8]. The primary user receives interference from the secondary source and relay, simultaneously. Consequently, in order to satisfy an interference constraint, the transmission powers at the secondary source and relay have to be lower than the transmission power of the CogMHDR-D . Arbitrarily reducing the transmission power, however, deteriorates performance for the secondary user (SU). Thus, to improve the performance of the SU in a CogMFDR, optimal power allocation is essential.

Our goal in this paper is to study an optimal power allocation scheme and evaluate the outage performance of the CogMFDR based on OSTBC. We build upon the work of [12], in which the secondary source ( $SU_{TX}$ ), relay ( $SU_R$ ), and the destination ( $SU_D$ ) are assumed to be equipped with multiple antennas. This configuration corresponds to the scenario where the base station (i.e., $SU_{TX}$ ) communicates with the user (i.e., $SU_D$ ) with the help of relay nodes (i.e., $SU_R$ ); because of the system's size and complexity, the base station and relay nodes can have multiple antennas for better performance. To minimize outage probability, an optimal power allocation scheme is proposed based on this scenario. The corresponding probability of the secondary system is also evaluated in a Nakagami-m fading channel. Such models have been extensively studied in various wireless communication systems because they capture physical channel phenomena more accurately than Rayleigh and Rician models. In summary, the major contributions of this paper are as follows:

1) We propose an OPA scheme to minimize the outage probability of the secondary user network. This concerns the power allocation problem for the CogMFDR based on a OSTBC system.

2) Exact outage probabilities are derived in noise-limited and interference-limited environments in Nakagami-m fading channels. These are validated through simulations.

3) The performance of the CogMFDR system deteriorates in the presence of interference because of full duplexing. The proposed OPA scheme can help to alleviate this issue and achieve maximum performance gain.

In the sequel, the paper is organized as follows. In Section 2, the system model is described. In Section 3, an OPA scheme is proposed and the exact outage probability is derived for two restrictive environments. Simulation results are provided in Section 4 and Section 5.

## 2. System Model

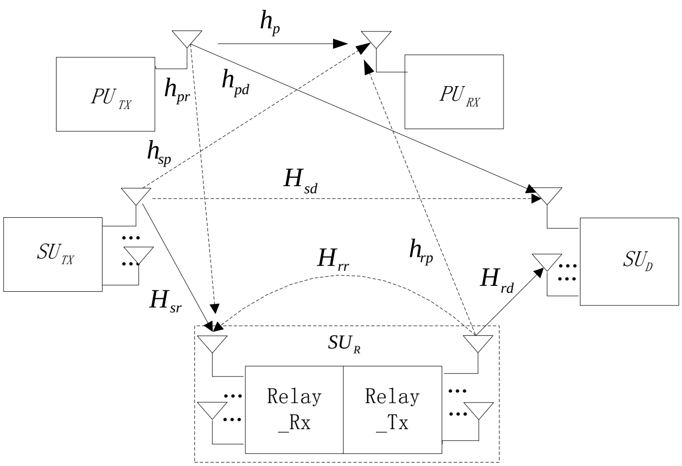


**Fig. 1.** Cognitive MIMO Full-Duplex Relay Network based on Orthogonal Space-Time Block Codes

The system model is shown in **Fig. 1**. In this figure, the primary transmitter $PU_{TX}$ and receiver $PU_{RX}$ are equipped with only one antenna. The secondary source node $SU_{TX}$, the secondary relay node $SU_R$, and secondary destination node $SU_D$ are equipped with $N_S$ antennas, $N_R$ antennas and $N_D$ antennas, respectively. Secondary user coexists with the primary user (PU) in an underlay approach. Additionally, $SU_R$ adopts the DF cooperative protocol to assist the $SU_{TX}$ with data transmission, in FDR node. Similar to [12], we assume that $SU_{TX}$ and $SU_R$ use the same OSTBC(s), which means $N_S = N_R = N_D = N$. The entire transmission is accomplished in two phases.

In the first phase, the $SU_{TX}$ encodes the $K$ symbols, $x_1, x_2, \ldots x_K$, selected from a signal constellation using the OSTBC matrix, $G_s = \{c_s^{li}\}_{L\times N}$. Letter $L$ denotes the block length and each codeword $c_s^{li}$ contains the signal transmitted from the ***i-th*** antenna at the ***l-th*** symbol interval. $c_s^{li}$ is a linear combination of $x_1, x_2, \ldots x_K$ and their conjugates $x_1^*, x_2^*, \ldots x_K^*$. According to a property of the coding matrix, any pair of columns taken from $G_s$ is orthogonal. The $SU_{TX}$ transmits the encoded signals to the $SU_R$ over $N$ antennas and $L$ symbol intervals. The power of each codeword at $SU_{TX}$ is $E_s = \xi\left[\left|c_s^{li}\right|^2\right]$, where $\xi[\cdot]$ denotes the expectation operator. We assume that channel fading is quasi-static, i.e., the fading coefficients are constant during the block length of an OSTBC codeword, and will change independently every $L$ intervals. The signal matrices received at $SU_R$ are

$$Y_r = G_s H_{sr} + N_r \tag{1}$$

where $H_{sr} = \{h_{sr}^{ij}\}_{N\times N}$ denotes the $SU_{TX} \to SU_R$ channel matrices. Symbol $N_r = \{n_r^{ij}\}_{L\times N}$ denotes the noise matrices at $SU_R$ in the first phase.

If a relay node, $SU_R$, cannot decode the source message correctly, transmission is not performed during the second phase. Otherwise, during the second phase, $SU_R$ encodes the source message using the same OSTBC and forwards the encoded signal matrix $G_r = \{c_r^{lj}\}_{L\times N}$ to $SU_D$ over $N$ antennas and $L$ symbol intervals. The power of each codeword at $SU_R$ is $E_r = \xi\left[\left|c_r^{lj}\right|^2\right]$. The signal received at $SU_D$ is expressed as

$$Y_d = G_r H_{rd} + N_d \tag{2}$$

where $H_{rd} = \{h_{rd}^{ij}\}_{N\times N}$ denotes the $SU_R \to SU_D$ channel matrices. The noise matrices at $SU_D$ during the second phase are denoted by $N_d = \{n_d^{ij}\}_{L\times N}$.

As shown in **Fig. 1**, there are three sources of interference at $SU_R$ and $SU_D$ in CogMFDR. Firstly, because of full duplexing, the retransmission signal at the FDR node interferes with the signal received via the $H_{rr}$ channel. This is called echo interference [8]. Meanwhile, $SU_D$ also receives interference from $SU_{TX}$ over the $H_{sd}$ channel. Secondly, in a spectrum sharing system, nodes $SU_R$ and $SU_D$ experience interference from $PU_{TX}$ as well. By applying a noise whitening filter at the nodes, the effective noise approximates white Gaussian [13]-[16]. Thirdly, $PU_{RX}$ in CogMFDR also receives interference from $SU_{TX}$ and $SU_R$ over channels $h_{sp}$ and $h_{rp}$, respectively. In summary, the channel gains are as follows:

$$\begin{aligned} &g_{sr} = \|H_{sr}\|_F^2, g_{rd} = \|H_{rd}\|_F^2; && \textit{datalink} \\ &b_{rr} = \|H_{rr}\|_F^2, b_{sd} = \|H_{sd}\|_F^2; && \textit{interference links to } SU_R \textit{ and } SU_D \\ &a_{sp} = \|h_{sp}\|_F^2, a_{rp} = \|h_{rp}\|_F^2; && \textit{interference links to } PU_{RX} \end{aligned} \tag{3}$$

where $\| \; \|_F^2$ denotes the squared Frobenius norm. In order to control the power of the secondary nodes, we assume that the secondary system can obtain perfect channel state information (CSI) about the interference link between $SU_{TX}$ and $PU_{RX}$. The secondary user can share the primary user's spectrum, as long as the amount of interference inflicted on the primary receiver is below a predetermined interference power threshold (IPT) value $Q$.

For a comparison, we consider the CogMHDR with diversity (CogMHDR-D). In CogMHDR-D, $SU_{TX} \to SU_D$ link is considered for transmission in the first phase. Similarly to [12], the signal matrices received at the $SU_D$, i.e., $Y_{sd} = \left\{ y_{sd}^{ij} \right\}_{L\times N}$ is given by

$$Y_{sd} = G_s H_{sd} + N_{sd} \tag{4}$$

where $H_{sd} = \left\{ h_{sd}^{ij} \right\}_{N\times N}$ denotes the $SU_{TX} \to SU_D$ channel matrices. Symbol $N_{sd} = \left\{ n_{sd}^{ij} \right\}_{L\times N}$ denotes the noise matrices at $SU_D$. Meanwhile, the transmission powers of $SU_{TX}$ and $SU_R$ are constrained by

$$I_{sp} = \sum_{i=1}^{N} \left| h_{sp}^{i} \right|^2 \xi \left[ \left| c_s^{li} \right|^2 \right] = \frac{P_s}{N} \left\| h_{sp} \right\|_F^2 \le Q \tag{5}$$

$$I_{rp} = \sum_{i=1}^{N} \left| h_{rp}^{i} \right|^2 \xi \left[ \left| c_r^{li} \right|^2 \right] = \frac{P_r}{N} \left\| h_{rp} \right\|_F^2 \le Q \tag{6}$$

where $h_{sp}^i$, $h_{rp}^i$ are the channel gains between $PU_{RX}$ and the $i$-$th$ transmit antenna at $SU_{TX}$, and the channel gains between $PU_{RX}$ and the $i$-$th$ transmit antenna at $SU_R$, respectively. $P_s = NE_s$ and $P_r = NE_r$ are the total transmission powers of $SU_{TX}$ and $SU_R$ respectively.

However, in CogMFDR, $SU_{TX}$, and FDR node $SU_R$, transmit their signals simultaneously on the same spectrum. Thus, $PU_{RX}$ receives interference from $SU_{TX}$ and $SU_R$ simultaneously. In this case, the transmission powers of $SU_{TX}$ and $SU_R$ should be constrained by

$$\frac{P_s}{N} \left\| h_{sp} \right\|_F^2 + \frac{P_r}{N} \left\| h_{rp} \right\|_F^2 \le Q \tag{7}$$

## 3. Optimal Power Allocation And Outage Probability Analysis in CogMFDR

In this section, we study an optimal power allocation (OPA) scheme for CogMFDR. We begin by comparing the performances of CogMHDR-D without joint power allocation and CogMFDR with the EPA scheme. Building on this result, we propose an OPA scheme capable of minimizing the SU outage probability of CogMFDR. We then analyse the outage probabilities in noise-limited and interference-limited environments.

The fading coefficients of all channels are identical and independently distributed (i.i.d.) Nakagami-m random variables. Therefore, $\left\| h_{sp} \right\|_F^2$ and $\left\| h_{rp} \right\|_F^2$ follow Gamma distributions with parameters $\left( 1/\beta_{sp}, m_{sp} N \right)$ and $\left( 1/\beta_{rp}, m_{rp} N \right)$, respectively. Similarly, $\left\| H_{sd} \right\|_F^2, \left\| H_{rd} \right\|_F^2, \left\| H_{sr} \right\|_F^2$ and $\left\| H_{rr} \right\|_F^2$ follow Gamma distributions with parameters $\left( 1/\beta_{sd}, m_{sd} N^2 \right), \left( 1/\beta_{rd}, m_{rd} N^2 \right), \left( 1/\beta_{sr}, m_{sr} N^2 \right)$ and $\left( 1/\beta_{rr}, m_{rr} N^2 \right)$. The probability density function (PDF) and cumulative distributed function (CDF) of a gamma random variable $g$

with parameter $(1/\beta, m)$ are given by

$$f_g(x) = \frac{\beta^m}{\Gamma(m)} x^{m-1} e^{-\beta x} \tag{8}$$

$$F_g(x) = \frac{\gamma(m, \beta x)}{\Gamma(m)} \tag{9}$$

where $\gamma(\alpha, x)$ is the incomplete gamma function [17] and $\beta = m/\Omega$. In order to investigate the effect of interference due to full duplexing, we assume that the parameters of all channel gains, with the exception of $b_{rr}$ and $b_{sd}$, are unity. This ensures a fair comparison between CogMFDR and CogMHDR-D .

### 3.1 Comparison of Outage Performance of CogMHDR-D and CogMFDR with EPA Scheme

**A. Outage Analysis for CogMHDR-D**

In this type of system, $SU_{TX}$ and $SU_R$ must satisfy the interference power constraints (5) and (6). The maximum allowed transmission powers at $SU_{TX}$ and $SU_R$ can be expressed as

$$P_s^H = \frac{QN}{a_{sp}}, \qquad P_r^H = \frac{QN}{a_{rp}} \tag{10}$$

where $P_s^H$ and $P_r^H$ are the transmission powers of $SU_{TX}$ and $SU_R$ in the CogMHDR-D, respectively. Similarly to [12], the squaring method proposed in [18] is used to decode OSTBCs. We can then obtain the received signal-to-noise ratios (SNRs) at $SU_R$ and $SU_D$ from (1), (2), (4) and (10) as follows

$$\gamma_{SR}^H = \frac{P_s^H \|H_{sr}\|_F^2}{Nr\delta_R^2} = \frac{\lambda_R g_{sr}}{a_{sp}} \tag{11}$$

$$\gamma_{RD}^H = \frac{P_r^H \|H_{rd}\|_F^2}{Nr\delta_D^2} = \frac{\lambda_D g_{rd}}{a_{rp}} \tag{12}$$

$$\gamma_{SD}^H = \frac{P_s^H \|H_{sd}\|_F^2}{Nr\delta_D^2} = \frac{\lambda_D b_{sd}}{a_{sp}} \tag{13}$$

where $\gamma_{SR}^H, \gamma_{RD}^H$ and $\gamma_{SD}^H$ are SNRs for the link $SU_{TX} \to SU_R, SU_R \to SU_D$ and $SU_{TX} \to SU_D$ respectively. $\lambda_R = \frac{Q}{r\delta_R^2}, \lambda_D = \frac{Q}{r\delta_D^2}$ , and the code rate of OSTBC is $r = K/L$ . The noise powers $\delta_R^2$ and $\delta_D^2$ are defined by

$$\delta_R^2 = \delta_{RN}^2 + \delta_{PR}^2, \qquad \delta_D^2 = \delta_{DN}^2 + \delta_{PD}^2 \tag{14}$$

where $\delta_{RN}^2$ and $\delta_{DN}^2$ are the noise variances at $SU_R$ and $SU_D$, respectively, $\delta_{PR}^2$ and $\delta_{PD}^2$ are the noise variances from $PU_{TX}$ to $SU_R$ and $SU_D$, respectively. Consider that $\gamma_{RD}^H = \frac{\lambda_D g_{rd}}{a_{rp}}$, where $g_{rd}$ and $a_{rp}$ are Gamma distributions with parameters $(1, N^2)$ and $(1, N)$, respectively. Therefore, the PDF of $\gamma_{RD}^{H}$ can be developed as

$$
\begin{aligned}
f_{\gamma_{RD}^{H}}(x) &= \int_0^\infty y f_{g_{rd}}\left(\frac{xy}{\lambda_D}\right) f_{a_{rp}}(y)\,dy \\
&= \frac{x^{N^2-1}}{\lambda_D{}^{N^2-1}\Gamma(N^2)\Gamma(N)} \int_0^\infty y^{N^2+N-1} e^{-y\left(\frac{x}{\lambda_D}+1\right)} dy \\
&= \frac{\lambda_D{}^{N+1}\Gamma(N^2+N)\,x^{N^2-1}}{\Gamma(N^2)\Gamma(N)(x+\lambda_D)^{N^2+N}}
\end{aligned} \tag{15}
$$

We define $\eta_H$ as the required SNR in CogMHDR-D. Similarly to [12], the outage probability of the secondary system when the $SU_R$ cannot correctly decode the source message is given by

$$
\begin{aligned}
p_{out}^{uns} &= \Pr\left\{\gamma_{SR}^{H} < \eta_H, \gamma_{SD}^{H} < \eta_H\right\} \\
&= \int_0^\infty F_{g_{sr}}\left(\frac{\eta_H x}{\lambda_R}\right) F_{b_{sd}}\left(\frac{\eta_H x}{\lambda_D}\right) f_{\|a_{sp}\|^2}(x)\,dx \\
&= \frac{1}{\Gamma^2(N^2)\Gamma(N)} \int_0^\infty \gamma\left(N^2, \frac{\eta_H x}{\lambda_R}\right)\gamma\left(N^2, \frac{\eta_H x}{\lambda_D}\right) x^{N-1} e^{-x} dx \\
&\overset{(eq1)}{=} \frac{1}{\Gamma(N)}\left[\Gamma(N) - \sum_{j=0}^{N^2-1}\frac{\eta_H^j \lambda_D{}^N \Gamma(j+N)}{j!(\eta_H+\lambda_D)^{j+N}} - \sum_{i=0}^{N^2-1}\frac{\eta_H^i \lambda_R{}^N \Gamma(i+N)}{i!(\eta_H+\lambda_R)^{i+N}} \right. \\
&\left. + \sum_{i=0}^{N^2-1}\sum_{j=0}^{N^2-1}\frac{\eta_H^{i+j}\Gamma(i+j+N)\lambda_R^N \lambda_D^{i+j+N}}{i!j!(\eta_H\lambda_D+\eta_H\lambda_R+\lambda_R\lambda_D)^{i+j+N}}\right]
\end{aligned} \tag{16}
$$

where (eq1) uses expression $\gamma(L,x) = \Gamma(L)\left(1 - e^{-x}\sum_{i=0}^{L-1}\frac{x^i}{i!}\right)$. Accordingly, the outage probability of the secondary system when the $SU_R$ can correctly decode the source message is given by

$$
\begin{aligned}
p_{out}^{suc} &= \Pr\left\{\gamma_{SR}^{H} > \eta_H, \gamma_{SD}^{H} + \gamma_{RD}^{H} < \eta_H\right\} \\
&= \int_0^{\eta_H}\int_0^\infty \left(1 - F_{g_{sr}}\left(\frac{x\eta_H}{\lambda_R}\right)\right) F_{b_{sd}}\left(\frac{(\eta_H - y)x}{\lambda_D}\right) f_{a_{sp}}(x)\,dx\, f_{\gamma_{RD}^{H}}(y)\,dy \\
&= m_1\int_0^{\eta_H}\frac{y^{N^2-1}}{(y+\lambda_D)^{N^2+N}}dy - m_2\int_0^{\eta_H}\frac{y^{N^2-1}(\eta_H - y)^i}{(y+\lambda_D)^{N^2+N}\left((\eta_H - y)\lambda_R + \lambda_D(\eta_H+\lambda_R)\right)^{i+j+N}}dy \\
&\overset{(eq2)}{=} \frac{m_1\eta_H^{N^2}}{N^2\lambda_D{}^{N^2+N}}\, {}_2F_1\left(N^2+N, N^2; 1+N^2; -\frac{\eta_H}{\lambda_D}\right) \\
&- \frac{m_2}{(-\lambda_R)^{i+j+N}}\varphi\left(N^2+N, i+j+N, N^2-1, i, -\lambda_D, \eta_H + \frac{\lambda_D(\eta_H+\lambda_R)}{\lambda_R}\right)
\end{aligned} \tag{17}
$$

Where $m_1 = \frac{\Gamma(N^2+N)\lambda_D^{N+1}}{\Gamma(N^2)\Gamma^2(N)} \sum_{j=0}^{N^2-1} \frac{\eta_H^j \lambda_R^N \Gamma(j+N)}{j!(k+\lambda_R)^{j+N}}$, $m_2 = \frac{\Gamma(N^2+N)}{\Gamma(N^2)\Gamma^2(N)} \sum_{i=0}^{N^2-1}\sum_{j=0}^{N^2-1} \frac{\eta_H^j \lambda_D^{j+2N+1} \lambda_R^{i+N} \Gamma(i+j+N)}{i!j!}$,

(eq2) uses [17, eq. (3.194.1)], and $\varphi(a,b,c,d,y_0,y_1)$ is defined as follows

$$\varphi(a,b,c,d,y_0,y_1) = \int_0^{\eta_H} \frac{y^c(\eta_H - y)^d}{(y-y_0)^a (y-y_1)^b} dy = \sum_{i=1}^{d} C_d^i \eta_H^{d-i} (-1)^i \int_0^{\eta_H} \frac{y^{c+i}}{(y-y_0)^a (y-y_1)^b} dy$$

$$\overset{(eq3)}{=} \sum_{i=1}^{d} C_d^i \eta_H^{d-i} (-1)^i \left[ \sum_{j=1}^{a} \frac{d^{(a-j)}(y-y_1)^{-b}}{(a-j)!dy^{(a-j)}}\Big|_{y=y_0} \times \int_0^{\eta_H} \frac{y^{c+i}}{(y-y_0)^j} dy + \sum_{k=1}^{b} \frac{d^{(b-k)}(y-y_0)^{-a}}{(b-k)!dy^{(b-k)}}\Big|_{y=y_1} \times \int_0^{\eta_H} \frac{y^{c+i}}{(y-y_1)^k} dy \right]$$

$$\overset{(eq4)}{=} \sum_{i=1}^{d} C_d^i \eta_H^{d-i} (-1)^i \left[ \sum_{j=1}^{a} \frac{d^{(a-j)}(y-y_1)^{-b}}{(a-j)!dy^{(a-j)}}\Big|_{y=y_0} \times \frac{\eta_H^{c+i+1}\, {}_2F_1\left(j, c+i+1; c+i+2; \frac{\eta_H}{y_0}\right)}{(-y_0)^j (c+i+1)} \right. \tag{18}$$

$$\left. \sum_{k=1}^{b} \frac{d^{(b-k)}(y-y_0)^{-a}}{(b-k)!dy^{(b-k)}}\Big|_{y=y_1} \times \frac{\eta_H^{c+i+1}\, {}_2F_1\left(k, c+i+1; c+i+2; \frac{\eta_H}{y_1}\right)}{(-y_1)^k (c+i+1)} \right]$$

With (eq3) follows the partial fraction in [17, eq.(3.326.2)] and (eq4) uses [17, eq. (3.194.1)]. Therefore, the total outage probability $O_{out}^H$ of CogMHDR-D, which is the sum of (16) and (17), is given by

$$O_{out}^H = p_{out}^{uns} + p_{out}^{suc} \tag{19}$$

**B. Outage Analysis for CogMFDR with EPA Scheme**

As discussed above, $SU_{TX}$ and $SU_R$ in CogMFDR must satisfy the interference constraint (7). A simple way (not optimal) to ensure satisfaction of the interference constraint is to set the predetermined interference threshold to half the value of the CogMHDR-D threshold. The transmission powers of $SU_{TX}$ and $SU_R$, written as $P_s^E$ and $P_r^E$, are given by

$$P_s^E = \frac{QN}{2a_{sp}}, \qquad P_r^E = \frac{QN}{2a_{rp}} \tag{20}$$

This scheme is referred to as 'Equal Power Allocation' (EPA). The received SNR at the $SU_R$ and $SU_D$ can be expressed as

$$\gamma_{SR}^F = \frac{\frac{P_s^E}{N} g_{sr}}{r\delta_R^2 + b_{rr}\frac{P_r^E}{N}} = \frac{\frac{g_{sr}}{a_{sp}}}{\frac{2}{\lambda_R} + \frac{b_{rr}}{a_{rp}}} \tag{21}$$

$$\gamma_{RD}^F = \frac{\frac{P_r^E}{N} g_{rd}}{r\delta_D^2 + b_{sd}\frac{P_s^E}{N}} = \frac{\frac{g_{rd}}{a_{rp}}}{\frac{2}{\lambda_D} + \frac{b_{sd}}{a_{sp}}} \tag{22}$$

Similarly to (15), we can calculate $f_{\frac{g_{sr}}{a_{sp}}}(x)$ and $f_{\frac{b_{rr}}{a_{rp}}}(x)$ as follows

$$f_{\frac{g_{sr}}{a_{sp}}}(x)=\frac{\Gamma\left(N^2+N\right)x^{N^2-1}}{\Gamma\left(N^2\right)\Gamma(N)(x+1)^{N^2+N}} \tag{23}$$

$$f_{\frac{b_{rr}}{a_{rp}}}(x)=\frac{\Gamma\left(m_{rr}N^2+N\right)\beta_{rr}^{m_{rr}N^2}x^{m_{rr}N^2-1}}{\Gamma\left(m_{rr}N^2\right)\Gamma(N)\left(\beta_{rr}x+1\right)^{m_{rr}N^2+N}} \tag{24}$$

The outage probability for the $SU_{TX}\rightarrow SU_R$ link can be derived as

$$\begin{aligned}
&\Pr\left\{\gamma_{SR}^F<\eta_F\right\}\\
&=\int_0^\infty f_{\frac{b_{rr}}{a_{rp}}}(y)\int_0^{\frac{2\eta_F}{\lambda_R}+y\eta_F} f_{\frac{g_{sr}}{a_{sp}}}(x)\,dxdy\\
&=\rho_1\int_0^\infty\frac{y^{m_{rr}N^2-1}}{\left(\beta_{rr}y+1\right)^{m_{rr}N^2+N}}\int_0^{\frac{2\eta_F}{\lambda_R}+y\eta_F}\frac{x^{N^2-1}}{(x+1)^{N^2+N}}dxdy\\
&\overset{(eq5)}{=}\rho_1\sum_{i=0}^{N^2-1}\frac{C_{N^2-1}^i(-1)^i}{N+i}\times\left[\int_0^\infty\frac{y^{m_{rr}N^2-1}}{\left(\beta_{rr}y+1\right)^{m_{rr}N^2+N}}dy-\int_0^\infty\frac{\lambda_R^{i+N}y^{m_{rr}N^2-1}}{\left(\beta_{rr}y+1\right)^{m_{rr}N^2+N}\left(2\eta_F+\lambda_R y\eta_F+\lambda_R\right)^{i+N}}dy\right]\\
&\overset{(eq6)}{=}\rho_1\sum_{i=0}^{N^2-1}\frac{C_{N^2-1}^i(-1)^i}{N+i}\times\left[\frac{B\left(m_{rr}N^2,N\right)}{\beta_{rr}^{m_{rr}N^2}}-\frac{\lambda_R^{i+N}B\left(m_{rr}N^2,2N+i\right)}{\beta_{rr}^{m_{rr}N^2}\left(2\eta_F+\lambda_R\right)^{i+N}}\times{}_2F_1\left(c_1,c_2;c_3;c_4\right)\right]
\end{aligned} \tag{25}$$

where $c_1=N+i,c_2=m_{rr}N^2,c_3=m_{rr}N^2+2N+i,c_4=1-\frac{\lambda_R\eta_F}{\left(2\eta_F+\lambda_R\right)\beta_{rr}},\rho_1=\frac{\beta_{rr}^{m_{rr}N^2}\Gamma\left(m_{rr}N^2+N\right)\Gamma\left(N^2+N\right)}{\Gamma\left(m_{rr}N^2\right)\Gamma\left(N^2\right)\Gamma^2(N)}$

and $\eta_F$ is the required SNR in CogMFDR. (eq5) uses the binomial theorem under the integral, while (eq6) uses [17, eq.(3.251.11)] and [17, eq. (3.259.3.11)]. Similarly, we can get the outage probability for the $SU_R\rightarrow SU_D$ link as follows

$$\begin{aligned}
&\Pr\left\{\gamma_{RD}^F<\eta_F\right\}\\
&=\rho_2\sum_{j=0}^{N^2-1}\frac{C_{N^2-1}^j(-1)^j}{N+j}\times\left[\frac{B\left(m_{sd}N^2,N\right)}{\beta_{sd}^{m_{sd}N^2}}-\frac{\lambda_D^{j+N}B\left(m_{sd}N^2,2N+j\right)}{\beta_{sd}^{m_{sd}N^2}\left(2\eta_F+\lambda_D\right)^{j+N}}\times{}_2F_1\left(d_1,d_2;d_3;d_4\right)\right]
\end{aligned} \tag{26}$$

where $\rho_2=\frac{\beta_{sd}^{m_{sd}N^2}\Gamma\left(m_{sd}N^2+N\right)\Gamma\left(N^2+N\right)}{\Gamma\left(m_{sd}N^2\right)\Gamma\left(N^2\right)\Gamma^2(N)},d_1=N+j,d_2=m_{sd}N^2,d_3=m_{sd}N^2+2N+j,$

and $d_4=1-\frac{\lambda_D\eta_F}{\left(2\eta_F+\lambda_D\right)\beta_{sd}}$. The outage probability is

$$O_{out}^E=1-\left[1-\Pr\left\{\gamma_{SR}^F<\eta_F\right\}\right]\left[1-\Pr\left\{\gamma_{RD}^F<\eta_F\right\}\right] \tag{27}$$

where $\Pr\left\{\gamma_{SR}^F<\eta_F\right\}$ and $\Pr\left\{\gamma_{RD}^F<\eta_F\right\}$ are defined in (25) and (26).

### 3.2 Outage Analysis for CogMFDR with OPA Scheme

To derive the OPA values $P_s^{opt}$ and $P_r^{opt}$ at $SU_{TX}$ and $SU_R$ in CogMFDR, the outage probability of $SU$ is obtained by solving the optimization problem.

$$\begin{aligned} \min_{P_s, P_r \geq 0} \quad & O_{out}^F = \Pr\left\{\min\left(\gamma_{SR}^F, \gamma_{RD}^F\right) < \eta_H\right\} \\ s.t \quad & \frac{P_s^{opt} a_{sp}}{N} + \frac{P_r^{opt} a_{rp}}{N} \leq Q \end{aligned} \tag{28}$$

In (28), the received SNRs at $SU_R$ and $SU_D$ are

$$\gamma_{SR}^F = \frac{P_s^{opt} g_{sr}}{r\delta_R^2 N + P_r^{opt} b_{rr}} \tag{29}$$

$$\gamma_{RD}^F = \frac{P_r^{opt} g_{rd}}{r\delta_D^2 N + P_s^{opt} b_{sd}} \tag{30}$$

As shown in (29) and (30), the interference due to full duplexing is added to the received signals at $SU_R$ and $SU_D$. Because the outage probability is determined by the worst instantaneous received SNR in (28), the optimization problem can be reformulated as

$$\begin{aligned} \max_{P_s, P_r \geq 0} \quad & \gamma = \min\left(\gamma_{SR}^F, \gamma_{RD}^F\right) \\ s.t \quad & \frac{P_s^{opt} a_{sp}}{N} + \frac{P_r^{opt} a_{rp}}{N} \leq Q \end{aligned} \tag{31}$$

When the sum of the transmission powers at $SU_{TX}$ and $SU_R$ is constrained, the outage probability is minimized when the SNRs at $SU_R$ and $SU_D$ are identical, i.e., $\gamma_{SR}^F = \gamma_{RD}^F$ as in [19]. Thus, the transmission powers of $SU_{TX}$ and $SU_R$, which minimize the outage probability, satisfy

$$\begin{aligned} & \frac{P_s^{opt} g_{sr}}{r\delta_R^2 N + P_r^{opt} b_{rr}} = \frac{P_r^{opt} g_{rd}}{r\delta_D^2 N + P_s^{opt} b_{sd}} \\ s.t \quad & P_s^{opt} a_{sp} + P_r^{opt} a_{rp} \leq QN \end{aligned} \tag{32}$$

The OPA values $P_s^{opt}$ and $P_r^{opt}$ in (32), are the roots of a quadratic equation. Because $P_s^{opt}, P_r^{opt} \geq 0$, it follows that

$$\begin{aligned} P_s^{opt} = & \frac{1}{2\left(a_{sp}^2 g_{rd} b_{rr} - a_{rp}^2 g_{sr} b_{sd}\right)}\Big(a_{rp}^2 g_{sr} \tau_D^2 + 2a_{sp} g_{rd} b_{rr} QN + a_{sp} a_{rp} g_{rd} \tau_R^2 \\ & - a_{rp}\sqrt{a_{rp}^2 g_{sr}^2 \tau_D^4 + 2a_{rp} g_{sr} g_{rd} \tau_R^2\left(a_{sp}\tau_D^2 + 2b_{sd} QN\right) + g_{rd}\left(4g_{sr} b_{rr} QN\left(a_{sp}\tau_D^2 + b_{sd} QN\right) + a_{sp}^2 g_{rd} \tau_R^4\right)}\Big) \end{aligned} \tag{33}$$

$$\begin{aligned} P_r^{opt} = & \frac{1}{2\left(a_{rp}^2 g_{sr} b_{sd} - a_{sp}^2 g_{rd} b_{rr}\right)}\Big(a_{sp} a_{rp} g_{sr} \tau_D^2 + 2a_{rp} g_{sr} b_{sd} QN + a_{sp}^2 g_{rd} \tau_R^2 \\ & - a_{sp}\sqrt{a_{rp}^2 g_{sr}^2 \tau_D^4 + 2a_{rp} g_{sr} g_{rd} \tau_R^2\left(a_{sp}\tau_D^2 + 2b_{sd} QN\right) + g_{rd}\left(4g_{sr} b_{rr} QN\left(a_{sp}\tau_D^2 + b_{sd} QN\right) + a_{sp}^2 g_{rd} \tau_R^4\right)}\Big) \end{aligned} \tag{34}$$

where $\tau_R^2 = r\delta_R^2 N, \tau_D^2 = r\delta_D^2 N$. In the above, $P_s^{opt}$ and $P_r^{opt}$ consist of the channel gains for all links, the noise powers at the $SU_R$ and $SU_D$, and the interference threshold. The EPA scheme does not satisfy the SNR balancing, because it only considers $a_{sp}$ and $a_{rp}$. The OPA scheme, however, ensures that the SNR condition is satisfied at $SU_R$ and $SU_D$. This minimizes the outage probability of $SU$ subject to the interference constraint (7). To verify the performance improvement using the OPA scheme, we record the outage probability in two environments that vary in the presence of noise and interference: the noise-limited and the interference-limited ones.

**A. Outage Probability in a Noise- Limited Environment**

In a noise-limited environment, the interference terms in (29) and (30) are negligible. The SNRs at $SU_R$ and $SU_D$ are respectively approximated as follows:

$$\gamma_{SR}^F \approx \gamma_R^N = \frac{P_s^N g_{sr}}{r\delta_R^2 N}, \qquad \gamma_{RD}^F \approx \gamma_D^N = \frac{P_r^N g_{rd}}{r\delta_D^2 N} \tag{35}$$

where $\gamma_R^N$ and $\gamma_D^N$ are the SNRs at $SU_R$ and $SU_D$, and $P_s^N$ and $P_r^N$ are the OPA values at $SU_{TX}$ and $SU_R$. It follows from relations (33) and (34) that

$$P_s^N = \frac{g_{rd} QN\delta_R^2}{a_{rp} g_{sr}\delta_D^2 + a_{sp} g_{rd}\delta_R^2} \tag{36}$$

$$P_r^N = \frac{g_{sr} QN\delta_D^2}{a_{rp} g_{sr}\delta_D^2 + a_{sp} g_{rd}\delta_R^2} \tag{37}$$

In (36) and (37), $P_s^N$ and $P_r^N$ consist of the channel gains $g_{sr}, g_{rd}, a_{sp}$ and $a_{rp}$, the noise powers at the $SU_R$ and $SU_D$, and the interference threshold. The SNR balancing condition is satisfied, and that guarantees that the outage probability of the secondary user is minimized in the noise-limited environment. Therefore, the received SNRs at $SU_R$ and $SU_D$ are given by

$$\gamma_R^N = \gamma_D^N = \frac{g_{sr} g_{rd} Q}{a_{rp} g_{sr} r\delta_D^2 + a_{sp} g_{rd} r\delta_R^2} = \left(\frac{a_{sp}}{g_{sr}\lambda_R} + \frac{a_{rp}}{g_{rd}\lambda_D}\right)^{-1} \tag{38}$$

The outage probability of the secondary user in the noise-limited environment can be written as

$$O_{out}^N = \Pr\left\{\min\left(\gamma_R^N, \gamma_D^N\right) < \eta_F\right\} = \Pr\left\{\left(\frac{a_{sp}}{g_{sr}\lambda_R} + \frac{a_{rp}}{g_{rd}\lambda_D}\right)^{-1} < \eta_F\right\} \tag{39}$$

The overall outage probability can thus be obtained in closed form as

$$O_{out}^N = \rho_3 \sum_{i=0}^{N-1} \frac{(-1)^i C_{N-1}^i \eta_F^{i+N^2}}{\left(i+N^2\right)\lambda_D^{i+N^2}} \times B\left(N, 2N^2+i\right) \times {}_2F_1\left(i+N^2, N; N+2N^2+i; 1+\frac{\eta_F}{\lambda_R}\right) \tag{40}$$

where $\rho_3 = \dfrac{\Gamma^2\left(N+N^2\right)}{\Gamma^2(N)\Gamma^2\left(N^2\right)}$. For a detailed derivation of (40), please refer to Appendix A.

**B. Outage Probability in an Interference-Limited Environment**

In the interference-limited environment, the received interference powers at $SU_R$ and $SU_D$

are higher than the noise powers. Similar to (29) and (30), the noise powers are negligible and the SNRs are approximated by

$$\gamma_R \approx \gamma_R^I = \frac{P_s g_{sr}}{P_r b_{rr}}, \qquad \gamma_D \approx \gamma_D^I = \frac{P_r g_{rd}}{P_s b_{sd}} \tag{41}$$

where $\gamma_R^I$ and $\gamma_D^I$ are the SNRs at $SU_R$ and $SU_D$, respectively. It follows that (33) and (34) can be rewritten as

$$P_s^I = \frac{\sqrt{g_{rd} b_{rr}}\, QN}{a_{sp}\sqrt{g_{rd} b_{rr}} + a_{rp}\sqrt{g_{sr} b_{sd}}} \tag{42}$$

$$P_r^I = \frac{\sqrt{g_{sr} b_{sd}}\, QN}{a_{sp}\sqrt{g_{rd} b_{rr}} + a_{rp}\sqrt{g_{sr} b_{sd}}} \tag{43}$$

where $P_s^I$ and $P_r^I$ are the OPA values at $SU_{TX}$ and $SU_R$ in the interference-limited environment. These power allocations minimize the outage probability of the secondary user by satisfying the SNR balancing condition. The SNRs at $SU_R$ and $SU_D$ can then be expressed as

$$\gamma_R^I = \gamma_D^I = \left(\frac{g_{sr} g_{rd}}{b_{rr} b_{sd}}\right)^{\frac{1}{2}} \tag{44}$$

It is interesting to note that as shown in (44), the IPT value Q, the channel gains $a_{sp}$ and $a_{rp}$ do not affect the SNRs at $SU_R$ and $SU_D$. This is because the transmission powers of $SU_{TX}$ and $SU_R$, which satisfy the interference constraint (7) in CogMFDR, directly interfere with $SU_D$ and $SU_R$. Thus, the overall outage probability is given by

$$\begin{aligned} O_{out}^I &= \rho_4 \sum_{i=0}^{N^2+m_{sd}N^2-1} (-1)^i C_{N^2+m_{sd}N^2-1}^i \\ &\times \left[ \sum_{j=1}^{N^2+m_{rr}N^2} \frac{(-1)^{N^2+m_{rr}N^2-j} C_{N^2+m_{rr}N^2+i-j}^i \varphi(e_1,e_2)}{(j-1)(\beta_{rr}\beta_{sd})^{j-1}} + \sum_{j=1}^{1+i} \frac{(-1)^{N^2+m_{rr}N^2} C_{N^2+m_{rr}N^2+i-j}^{1+i-j} \varphi(e_3,e_2)}{(j-1)} \right] \end{aligned} \tag{45}$$

where $\rho_4 = \frac{\beta_{rr}^{m_{rr}N^2} \beta_{sd}^{m_{sd}N^2} \Gamma(N^2+m_{rr}N^2)\Gamma(N^2+m_{sd}N^2)}{\Gamma(N^2)\Gamma(m_{rr}N^2)\Gamma(m_{sd}N^2)}, e_1 = N^2+i-1, e_2 = N^2+m_{rr}N^2+(1+i-j)$ $, e_3 = N^2+i-j$, and $\varphi(a,b)$ is defined as in (54). The derivation process of (45) is provided in Appendix B.

## 4. Numerical Simulations

In this section, Monte-Carlo simulations are executed and the impact of factors on outage performance is examined. Based on the above analysis, the factors which affect the outage performance are as follows: a. the IPT value $Q$; b. the mean interference channel fading exponents $m_{rr}$ and $m_{sd}$; c. the additive noise variance $\delta_{RN}^2$ and $\delta_{DN}^2$, and the interference power $\delta_{PR}^2$ and $\delta_{PD}^2$; d. the number $N$ of antennas in the SU nodes.

We set the rate threshold of cognitive relay networks to $R_{th} = 1bit/s/Hz$. The OSTBC rate is assumed to be the greatest achievable value, see e.g., [20], which is given by $r = \frac{M+1}{2M}$, where $N = 2M$ (when $N$ is even) or $N = 2M - 1$ (when $N$ is odd). Furthermore, we set the channel parameters $\Omega_{rr} = \Omega_{sd} = 1$.

**Fig. 2** shows the outage probabilities in CogMHDR-D and CogMFDR using the EPA scheme with respect to $m_{rr}$ and $m_{sd}$, where $N = 3$. In this figure, we set $\eta_F = 2^{R_{th}} - 1$ in CogMFDR and $\eta_H = 2^{2R_{th}} - 1$ in CogMHDR-D. This is because only half the resources are utilized in CogMHDR-D. The noise power is set at $\delta_R^2 = \delta_D^2 = 0dB$. As shown in Fig.2, the outage performance of CogMFDR improves as the interference channel gains ($m_{rr}$ and $m_{sd}$) decrease as we would expect. Because, the worse the interference channel gains are, the smaller interference power introduced by FDR would be added at the receive nodes, therefore, the better outage performance would be. When compared with CogMFDR, the outage probability of CogMHDR-D is inversely proportional to the channel gain $m_{sd}$ due to the $SU_{TX} \rightarrow SU_D$ link is considered for transmission. Meanwhile, the performance gain between CogMFDR and CogMHDR-D would decrease when Q increases. This is because the interference power, which has a bad effect on the outage performance, would be larger when Q increases. However, the proposed EPA scheme cannot alleviate the effect of this interference.

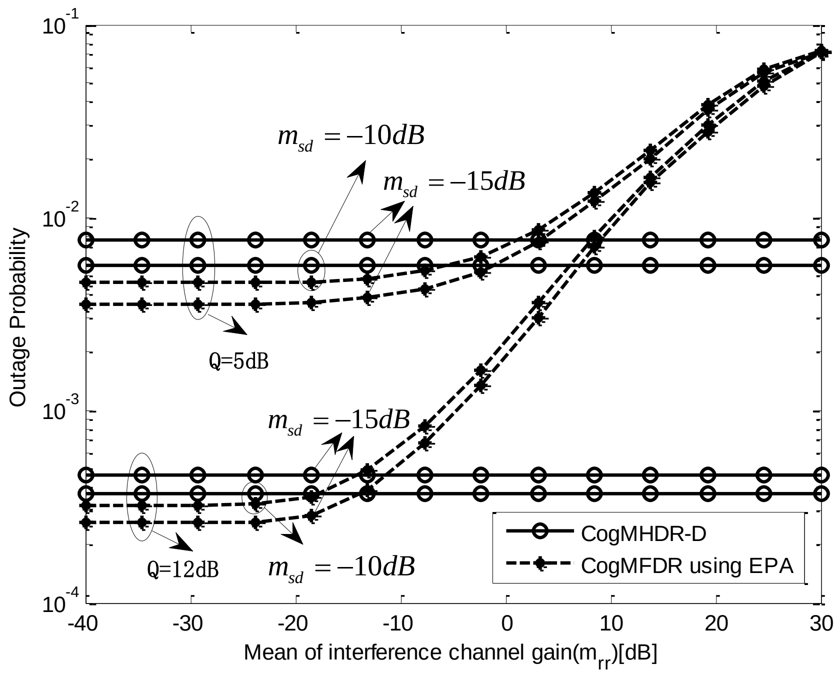


**Fig. 2.** SU outage probabilities in CogMHDR-D and CogMFDR using the EPA scheme with respect to $m_{rr}$ and $m_{sd}$

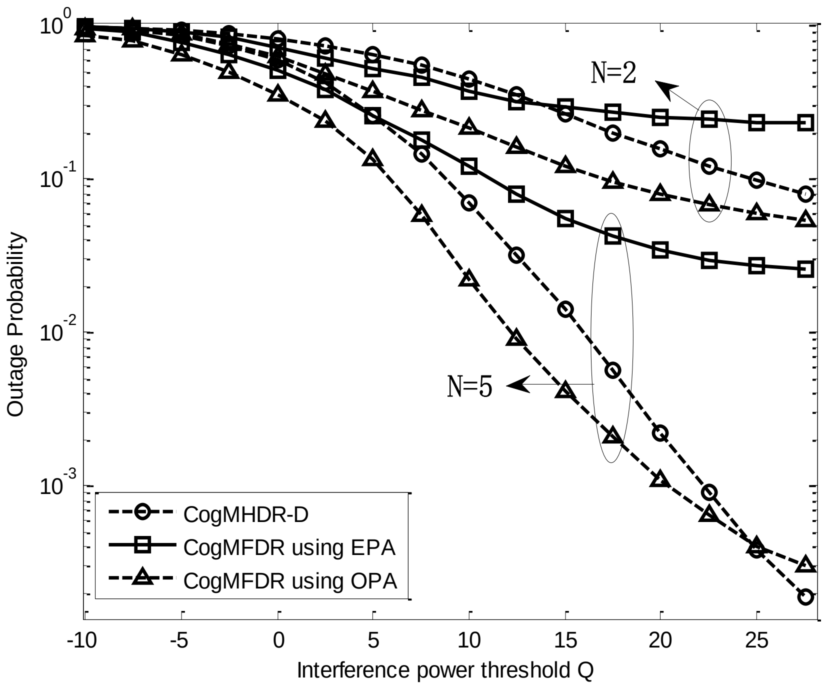


**Fig. 3.** SU outage probabilities of CogMFDR using the OPA and EPA schemes.

**Fig.3** shows the secondary user outage probabilities in CogMFDR using the OPA and EPA schemes, respectively. We set $m_{rr} = m_{sd} = 20dB$ and $\delta_R^2 = \delta_D^2 = 0dB$. As can be seen in the figure, the outage probabilities decrease as the number of antenna increases for both schemes. The outage probability of CogMFDR with EPA scheme decreases more slowly than that of CogMFDR with OPA scheme or CogMHDR-D as Q increases, it even has a worse performance compared to the CogMHDR-D when the SNR is high. This is because the increase in SNR causes the interference effect to become dominant relative to the noise effect. It denotes CogMFDR only has a better performance than CogMHDR-D in low SNR region due to the interference introduced by full-duplex. These results further validate the investigation of FDR and HDR in ref [8]. However, despite the fact that a floor occurs in the high SNR region owing to local interference, the floor in the OPA scheme occurs at a higher SNR region than in the EPA scheme. This is because the OPA scheme balances the SNRs at $SU_R$ and $SU_D$. Therefore, the proposed OPA scheme can help to achieve full performance gain in CogMFDR.

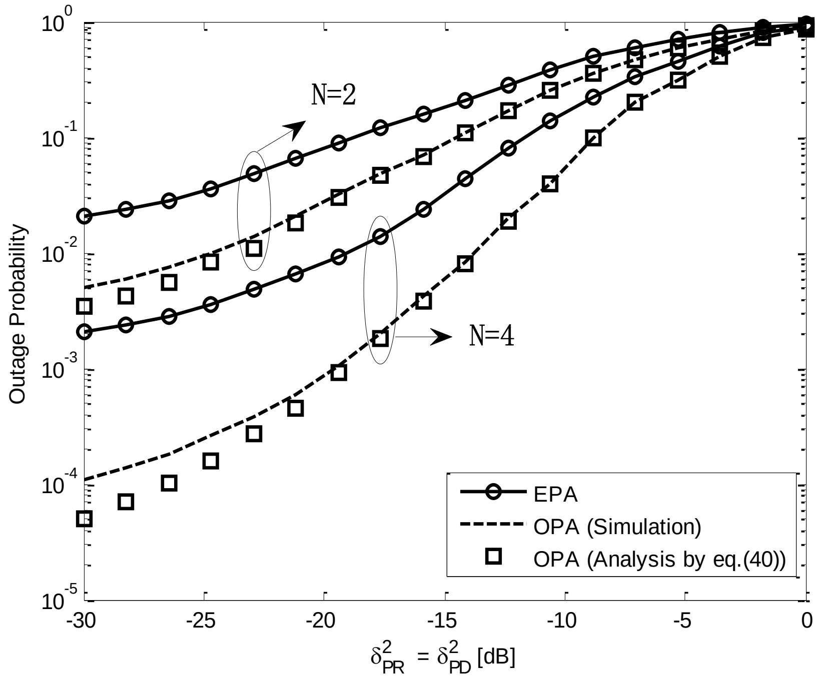


**Fig. 4.** SU outage probabilities of CogMFDR using the OPA and EPA schemes with respect to $\delta_{PR}^2$ and $\delta_{PD}^2$

**Fig. 4** shows the outage probabilities of CogMFDR using the OPA and EPA schemes, with respect to the interference power from $PU_{TX}$ . In this figure, we assume additive noise variance, $\delta_{RN}^2 = \delta_{DN}^2 = 0dB$, $Q = -10dB$ and $m_{rr} = m_{sd} = -20dB$. It can be observed that the outage performance of the OPA scheme is superior to that of the EPA scheme, as expected. This performance gain becomes more evident with increasing the number of antennas. Meanwhile, it is verified that the simulated outage probability perfectly matches the theoretical one derived for the noise-limited environment, when the interference power is large.

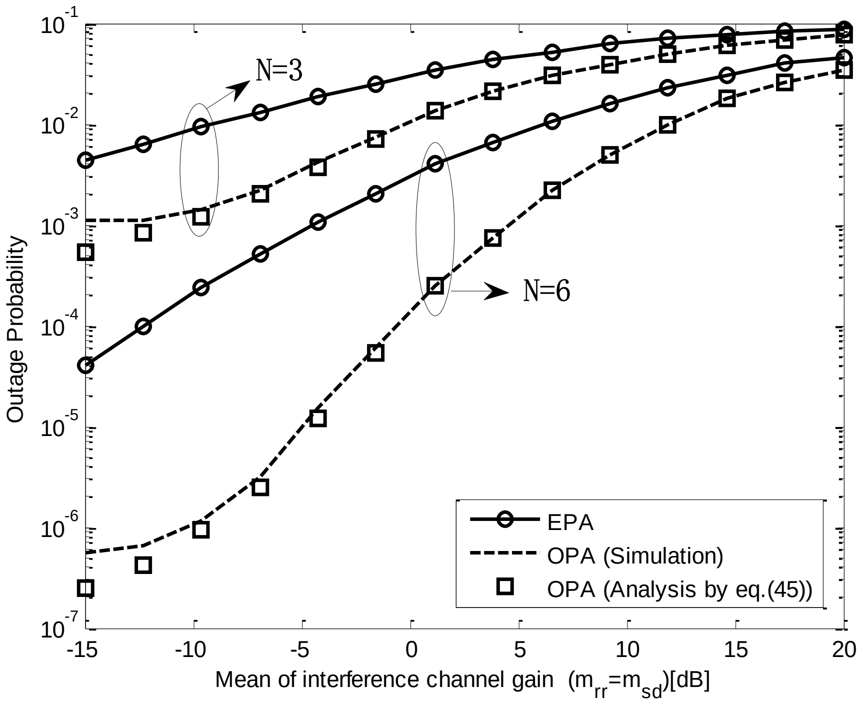


**Fig. 5.** SU Outage probabilities of CogMFDR using the OPA and EPA schemes with respect to $m_{rr}$ and $m_{sd}$

**Fig. 5** shows the outage probabilities of CogMFDR as functions of the mean interference channel gain, $m_{rr} = m_{sd}$, when the OPA and EPA schemes are used. It is assumed that $\delta_R^2 = \delta_D^2 = -10dB$ and $Q = 10dB$. As shown in **Fig. 5**, the outage performance using the EPA scheme is more vulnerable to the interference channel gain than using the OPA one. This can be mainly attributed to the fact that only the channel gains $a_{sp}$ and $a_{rp}$ are considered in the EPA scheme. On the other hand, the OPA scheme in CogMFDR performs SNR balancing which makes it robust to such interference. It can also be observed that the simulated outage probability with the OPA scheme agrees with the theoretical outage probability of the interference limited environment when the interference channel gain is large.

**Fig. 6** shows the outage probabilities of the secondary user with respect to the interference threshold. We set $\delta_R^2 = \delta_D^2 = -10dB$, $m_{rr} = m_{sd} = -15dB$ and $N = 2$. As mentioned before, when the interference threshold increases, $SU_{TX}$ and $SU_R$ can raise their transmission powers so that the secondary user's throughput increases. However, in CogMFDR, any increase in the transmission power leads to an increase in interference observed at $SU_R$ and $SU_D$. Hence, in **Fig. 6**, a high interference threshold indicates an interference-limited environment. This demonstrates that the outage probability using the OPA scheme follows closely the outage analysis of the interference-limited environment. Otherwise, when the interference threshold is small, the outage probability using the OPA follows the outage analysis of the noise-limited environment.

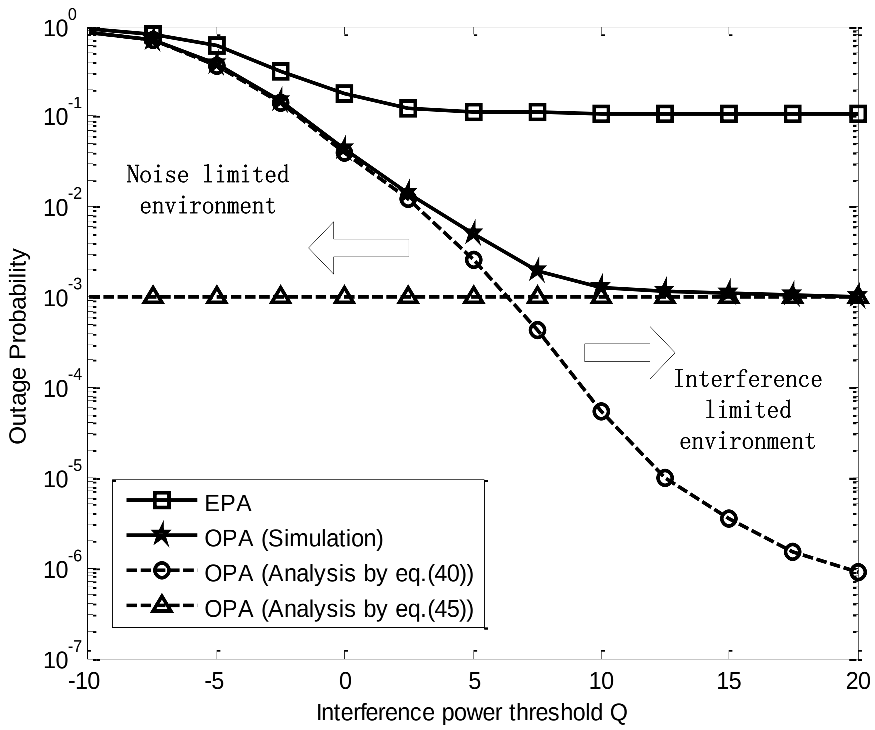


**Fig. 6.** SU outage probabilities of CogMFDR using the OPA scheme with respect to Q

## 5. Conclusion

In this paper, we proposed an OPA scheme to minimize the overall outage probability of CogMFDR and then derived the outage probabilities for the secondary user in noise-limited and interference-limited environments. The results obtained were further used to confirm that the proposed CogMFDR outperform the conventional CogMHDR-D with regards to the outage probability when the SNR or the interference power is low. Simulation results demonstrated that using the OPA scheme in CogMFDR can improve the performance gain that full duplexing offers. Moreover, it also confirmed that the simulated outage probability values agree perfectly with the theoretical ones, in both the noise-limited and interference-limited environments.

In reality, as we all know, the system is more likely to operate in low SNR region than in high SNR region, especially for the power limited system where the transmit power is limited by the IPT value. Therefore, the proposed CogMFDR and OPA scheme are more valuable for the practical application than conventional schemes.

## APPENDIX A

This appendix details the derivation of the secondary user outage probability in a CogMFDR using the OPA scheme, in a noise-limited environment. Using (38), the outage probability can be derived as

$$\begin{aligned} O_{out}^{N} &= \Pr\left\{\left(\frac{a_{sp}}{g_{sr}\lambda_R}+\frac{a_{rp}}{g_{rd}\lambda_D}\right)^{-1}<\eta_F\right\} \\ &= \int_0^{\infty} f_{\frac{a_{sp}}{g_{sr}}}(y)\int_{\lambda_D\left(\frac{1}{\eta_F}-\frac{y}{\lambda_R}\right)}^{\infty} f_{\frac{a_{rp}}{g_{rd}}}(x)\,dxdy \end{aligned} \tag{46}$$

As discussed in the main text, to investigate the presence of interference due to full duplexing, we assume that the parameters of all channel gains, with the exception of $b_{rr}$ and $g_{sd}$, are unity. Therefore, similarly to (15), we obtain

$$f_{\frac{a_{sp}}{g_{sr}}}(x)=f_{\frac{a_{rp}}{g_{rd}}}(x)=\frac{\Gamma\left(N+N^2\right)x^{N-1}}{\Gamma(N)\Gamma\left(N^2\right)(x+1)^{N+N^2}} \tag{47}$$

The outage probability of the secondary user can be derived as

$$\begin{aligned} O_{out}^{N} &= \int_0^{\infty} f_{\frac{a_{sp}}{g_{sr}}}(y)\int_{\lambda_D\left(\frac{1}{\eta_F}-\frac{y}{\lambda_R}\right)}^{\infty} f_{\frac{a_{rp}}{g_{rd}}}(x)\,dxdy \\ &= \rho_3\int_0^{\infty}\frac{y^{N-1}}{(y+1)^{N+N^2}}\int_{\lambda_D\left(\frac{1}{\eta_F}-\frac{y}{\lambda_R}\right)}^{\infty}\frac{x^{N-1}}{(x+1)^{N+N^2}}\,dxdy \\ &\overset{(eq7)}{=} \rho_3\sum_{i=0}^{N-1}\frac{(-1)^i C_{N-1}^{i}\eta_F^{i+N^2}}{\left(i+N^2\right)\lambda_D^{i+N^2}}\int_0^{\infty}\frac{y^{N-1}}{(y+1)^{N+N^2}\left(1-\frac{\eta_F y}{\lambda_R}\right)^{i+N^2}}\,dy \\ &\overset{(eq8)}{=} \rho_3\sum_{i=0}^{N-1}\frac{(-1)^i C_{N-1}^{i}\eta_F^{i+N^2}}{\left(i+N^2\right)\lambda_D^{i+N^2}}\times B\left(N,2N^2+i\right)\times {}_2F_1\left(i+N^2,N;N+2N^2+i;1+\frac{\eta_F}{\lambda_R}\right) \end{aligned} \tag{48}$$

where $\rho_3=\frac{\Gamma^2\left(N+N^2\right)}{\Gamma^2(N)\Gamma^2\left(N^2\right)}$. (eq7) uses the binomial theorem and (eq8) uses [17, eq.(3.259.3.11)].

## APPENDIX B

This appendix details the derivation of the outage probability of CogMFDR using the OPA scheme in an interference-limited environment. In (44), when $M_1=g_{sr}/b_{rr}$ and $M_2=g_{rd}/b_{sd}$, the PDF of $Y=(M_1M_2)^{\frac{1}{2}}$ is derived as [21]

$$f_Y(y)=\int_0^{\infty}\frac{1}{z}f_{M_1}(z)f_{M_2}\left(\frac{y}{z}\right)dz \tag{49}$$

Channel gains $g_{sr}, g_{rd}, b_{rr}$, and $b_{sd}$ follow Gamma distributions with parameters $\left(1,N^2\right),\left(1,N^2\right),\left(1/\beta_{rr},m_{rr}N^2\right),\left(1/\beta_{sd},m_{sd}N^2\right)$, respectively. Therefore, as in (15), we can get

$$f_{M_1}(x)=\frac{\beta_{rr}^{m_{rr}N^2}x^{N^2-1}\Gamma\left(N^2+m_{rr}N^2\right)}{\Gamma\left(N^2\right)\Gamma\left(m_{rr}N^2\right)\left(x+\beta_{rr}\right)^{N^2+m_{rr}N^2}} \tag{50}$$

$$f_{M_2}(x) = \frac{\beta_{sd}^{m_{sd}N^2} x^{N^2-1} \Gamma\left(N^2 + m_{sd}N^2\right)}{\Gamma\left(N^2\right)\Gamma\left(m_{sd}N^2\right)\left(x+\beta_{sd}\right)^{N^2+m_{sd}N^2}} \tag{51}$$

Substituting (50) and (51) into (49), yields

$$\begin{aligned}
f_Y(y) &= \int_0^\infty \frac{1}{z} f_{M_1}(z) f_{M_2}\left(\frac{y}{z}\right) dz \\
&= \rho_4 y^{N^2-1} \times \int_0^\infty \frac{z^{N^2+m_{sd}N^2-1}}{\left(z+\beta_{rr}\right)^{N^2+m_{rr}N^2}\left(y+\beta_{sd}z\right)^{N^2+m_{sd}N^2}} dz \\
&\overset{(eq9)}{=} \rho_4 \sum_{i=0}^{N^2+m_{sd}N^2-1} (-1)^i C^i_{N^2+m_{sd}N^2-1} y^{N^2+i-1} \times \int_{\beta_{sd}\beta_{rr}}^\infty \frac{1}{m^{N^2+m_{rr}N^2}\left[\left(m-\beta_{sd}\beta_{rr}\right)+y\right]^{1+i}} \\
&\overset{(eq10)}{=} \rho_4 \sum_{i=0}^{N^2+m_{sd}N^2-1} (-1)^i C^i_{N^2+m_{sd}N^2-1} \times \left[\sum_{j=1}^{N^2+m_{rr}N^2} \frac{(-1)^{N^2+m_{rr}N^2-j} C^i_{N^2+m_{rr}N^2+i-j} y^{N^2+i-1}}{(j-1)\left(\beta_{sd}\beta_{rr}\right)^{j-1}\left(y-\beta_{sd}\beta_{rr}\right)^{N^2+m_{rr}N^2+1+i-j}} + \sum_{j=1}^{1+i} \frac{(-1)^{N^2+m_{rr}N^2} C^{1+i-j}_{N^2+m_{rr}N^2+i-j} y^{N^2+i-j}}{(j-1)\left(y-\beta_{sd}\beta_{rr}\right)^{N^2+m_{rr}N^2+1+i-j}}\right]
\end{aligned} \tag{52}$$

where $\rho_4 = \frac{\beta_{rr}^{m_{rr}N^2} \beta_{sd}^{m_{sd}N^2} \Gamma\left(N^2+m_{rr}N^2\right)\Gamma\left(N^2+m_{sd}N^2\right)}{\Gamma\left(N^2\right)\Gamma\left(m_{rr}N^2\right)\Gamma\left(m_{sd}N^2\right)}$. (eq9) uses the binomial theorem and (eq10) uses the partial fraction in [17, eq.(3.326.2)]. Therefore, the outage probability of a secondary user using the OPA scheme in the interference-limited environment is given by

$$\begin{aligned}
O^I_{out} &= \Pr\left\{Y < \eta_F\right\} = \int_0^{\eta_F} f_Y(y) dy \\
&= \rho_4 \sum_{i=0}^{N^2+m_{sd}N^2-1} (-1)^i C^i_{N^2+m_{sd}N^2-1} \times \\
&\left[\sum_{j=1}^{N^2+m_{rr}N^2} \frac{(-1)^{N^2+m_{rr}N^2-j} C^i_{N^2+m_{rr}N^2+i-j}}{(j-1)\left(\beta_{rr}\beta_{sd}\right)^{j-1}} \times \int_0^{\eta_F} \frac{y^{N^2+i-1}}{\left(y-\beta_{rr}\beta_{sd}\right)^{N^2+m_{rr}N^2+1+i-j}} dy \right. \\
&\left. + \sum_{j=1}^{1+i} \frac{(-1)^{N^2+m_{rr}N^2} C^{1+i-j}_{N^2+m_{rr}N^2+i-j}}{(j-1)} \times \int_0^{\eta_F} \frac{y^{N^2+i-j}}{\left(y-\beta_{rr}\beta_{sd}\right)^{N^2+m_{rr}N^2+1+i-j}} dy \right] \\
&= \rho_4 \sum_{i=0}^{N^2+m_{sd}N^2-1} (-1)^i C^i_{N^2+m_{sd}N^2-1} \times \left[\sum_{j=1}^{N^2+m_{rr}N^2} \frac{(-1)^{N^2+m_{rr}N^2-j} C^i_{N^2+m_{rr}N^2+i-j} \varphi\left(e_1,e_2\right)}{(j-1)\left(\beta_{rr}\beta_{sd}\right)^{j-1}} + \sum_{j=1}^{1+i} \frac{(-1)^{N^2+m_{rr}N^2} C^{1+i-j}_{N^2+m_{rr}N^2+i-j} \varphi\left(e_3,e_2\right)}{(j-1)}\right]
\end{aligned} \tag{53}$$

where $e_1 = N^2 + i - 1, e_2 = N^2 + m_{rr}N^2 + 1 + i - j, e_3 = N^2 + i - j$. Function $\varphi(a,b)$ is defined as follows

$$\begin{aligned}
\varphi(a,b) &= \int_0^{\eta_F} \frac{x^a}{\left(x-\beta_{rr}\beta_{sd}\right)^b} dx \\
&\overset{(eq11)}{=} \left(-\beta_{rr}\beta_{sd}\right)^{-b} \times \frac{\eta_F^{a+1}}{(a+1)} {}_2F_1\left(b, a+1; a+2; \frac{\eta_F}{\beta_{rr}\beta_{sd}}\right)
\end{aligned} \tag{54}$$

where (eq11) uses [17, eq.( 3.194.1)].

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