

# Analysis of Improved Cyclostationary Spectrum Sensing with SLC Diversity over Composite Multipath Fading-Lognormal Shadowing Channels

Ying Zhu<sup>1,2</sup>, Jia Liu<sup>1</sup>, Zhiyong Feng<sup>1</sup> and Ping Zhang<sup>1</sup>

<sup>1</sup>Key Laboratory of Universal Wireless Communications, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing, 100876, China

<sup>2</sup>Department of Teaching and Practising, Guilin University of Electronic Technology  
Guilin, 541004, China

[e-mail: zhuyingbupt@gmail.com, liujia0406@126.com, {fengzy, pzh}@bupt.edu.cn]

\*Corresponding author: Ying Zhu

*Received August 3, 2013; revised January 9, 2014; revised January 26, 2013; accepted January 29, 2014;  
published March 31, 2014*

---

## Abstract

Spectrum sensing is a key technical challenge for cognitive radio (CR). It is well known that multi-cycle cyclostationarity (MC) detector is a powerful method for spectrum sensing. However, conventional MC detector is difficult to implement due to its high computational complexity. This paper pays attention to the fact that the computation complexity can be reduced by simplifying the test statistic of conventional MC detector. Based on this simplification process, an improved MC detector is proposed. Compared with the conventional one, the proposed detector has the low-computational complexity and sufficient-accuracy on sensing performance. Subsequently, the sensing performances are further investigated for the cases of Rayleigh, Nakagami- $m$ , Rician, composite Rayleigh fading-lognormal shadowing and composite Nakagami fading-lognormal shadowing channels, respectively. Furthermore, the square-law combining (SLC) is introduced to improve the detection capability over fading-shadowing channels. The corresponding closed-form expressions of average detection probability are derived for each case by the moment generation function (MGF) approach. Finally, illustrative and analytical results show that the efficiency and reliability of proposed detector and the improvement on sensing performance by SLC over composite fading-shadowing channels.

---

**Keywords:** Spectrum sensing, improved MC detector, Rayleigh, Nakagami- $m$ , Rician, composite Rayleigh fading-lognormal shadowing, composite Nakagami fading-lognormal shadowing, SLC, MGF.

---

This work is supported by the National Natural Science Foundation of China (61227801, 61121001), the National Key Technology Research and Development Program of China (2012ZX03003006, 2012ZX03006003-003, 2013ZX03003014-003, 2011ZX03004-003), the Fundamental Research Funds for the Central Universities (2013RC0106) and the Program for New Century Excellent Talents in University (NCET-01-0259).

<http://dx.doi.org/10.3837/tiis.2014.03.005>

## 1. Introduction

**R**adio spectrum, an expensive and limited resource, is surprisingly underutilized by licensed users (primary users, PU) [1]. Such spectral under-utilization has motivated cognitive radio (CR) technology which has built-in radio environment awareness and spectrum intelligence [2][3]. Cognitive radio enables opportunistic access to unused licensed bands. For instance, CR users first sense the activities of primary users and access the spectrum holes if no primary activities are detected. While sensing accuracy is important for avoiding interference to the PU, reliable spectrum sensing is not always guaranteed, due to the multipath fading and shadowing [4-7].

Different approaches for spectrum sensing for CR applications have been proposed, e.g., [8-11]. The commonly considered approaches are based on power spectrum estimation, energy detection and multi-cycle cyclostationary (MC) detection. Power spectrum estimation may not work reliably in the low signal-to-noise ratio (SNR) regime. Energy detection, on the other hand, is subject to uncertainty in noise and interference statistics. The MC detection, which are inherent properties of digital modulated signals, has been proposed in the literature to overcome the problems above [11-13]. In fact, it is well known that if a signal has strong cyclostationary properties, it can be detected at low SNRs. In addition, MC detector can inherently distinguish PUs from secondary users as well as interferers, if they have dissimilar cyclic features.

Although the conventional MC detectors operate much better than energy and power ones, it requires extensive computation to provide sufficiently low error probability, which causing high computational complexity [12][13]. The high computational complexity leads to a long detection time, which seriously degrades spectrum efficiency of the CRs, since all communications should be stopped during detection. Therefore, it's necessary to reduce the computational complexity of conventional MC detector. An improved MC detector with low-computational complexity and sufficient-sensing accuracy is needed.

Furthermore, since the channel from PU to CR is multipath fading and shadowing in practical CR networks, the PU's signal might be severely attenuated before it reaches to CR [14]. That is to say, the analysis of sensing performance in fading and shadowing environments could help quantify the exact sensing performance of detector. However, the existing researches are mainly restricted to Rayleigh fading, which is not comprehensive enough [15][16]. To our best knowledge, none of the previous work has addressed the issue of MC detector over Nakagami- $m$  and Rician fading channels. Despite focus on the sensing performance over fading channels, all the previous researches omitted the effect of shadowing. In practical implementation, many detectors will be found either in stationary positions or with low mobility in environment with varying degree of vegetation covers [17]. These foliage and temporal stationary behavior introduce another degradation known as shadowing which cannot be mitigated by averaging the received signal strength [18]. Hence, if we wish to examine the exact sensing performance of MC detector, the effect of shadowing needs to be investigated.

To improve the sensing performance in the severe fading and shadowing conditions, multiple antennas for spectrum sensing can be deployed [17][19]. The diversity reception technology are then used to improve the detection reliability. The need for multiple antennas is also driven by the promise of a high data rate and high efficiency broadband services by standards such as the Long Term Evolution (LTE), WiMax and IMT-Advanced. The notion

of using a multiple antenna CR for detecting the spectrum holes has thus attracted much interest.

In order to reduce the complexity of conventional MC detector, we proposed an improved MC detector, which is low-computational complexity and high accuracy. Subsequently, the sensing performance is further investigated by employing multiple antennas in fading and shadowing environments. The remainder of this paper is organized as follows. In Section 2, we present related work. The improved MC detector with low computational complexity and high accuracy on sensing performance is proposed in Section 3. In Section 4, the SLC diversity reception is employed to improve detection capability. Subsequently, the sensing performance is investigated over Rayleigh, Nakagami- $m$ , Rician fading, composite Rayleigh fading-lognormal shadowing and the composite Nakagami fading-lognormal shadowing channels, respectively. The closed-form expressions of detection probability are then derived by MGF approach, correspondingly. Section 5 presents simulation results and the concluding remarks are given in Section 6.

## 2. Related Work

Many papers focus on reducing the computation complexity of conventional MC detector. In [20], a cooperative MC detector was proposed in which the detector combines distinct single-cycle (SC) detectors for different cycle-frequencies (CFs) and the final decision is obtained by OR-rule (a hard decision rule of cooperative sensing) the primitive decisions of the SC detectors. An adaptive cooperative cyclostationary beamforming-based spectrum sensing method with affordable complexity was introduced for multiple antenna CR in [21]. In [22], a sequential framework was proposed to apply for collaborative MC detector in order to reduce the average detection time. In [23-25], cooperative cyclostationary methods were proposed to improve the performance. The cooperative and collaborative MC detectors above were both introduced to reduce computation complexity. In the proposed cooperative and collaborative schemes, the computation of test statistics is done by a group of cooperative CRs with a distributed manner, so that the complexity burden on a single CR is reduced. Nevertheless, a major drawback of these schemes is the need for a very large number of cooperative CRs to make MC detection reliable, which is impractical in CR system sometimes. Therefore, it's still necessary to improve the conventional MC detector for single CR in local spectrum sensing scheme.

For multiple antenna spectrum sensing, the diversity reception technology of multiple antenna such as maximum ratio combiner (MRC) and square-law combining (SLC) are used to improve the detection reliability [16][19]. The detection performance of MRC has been analyzed in [16]. Study [19] deduces the exact detection performance analysis by utilizing SLC. When MRC is used, channel state information (CSI) from the primary user to CRs and from each CR to the fusion center is needed. When SLC is used with fixed amplification factor at each secondary user, only CSI from CRs to the fusion center is needed. Since MRC requires complete knowledge of the channel state information (CSI), a simpler technique is the SLC, which does not require the complete knowledge of CSI.

Moreover, previous performance analysis studies [17-25] primarily utilize the probability density function (PDF) based approach, i.e. the conditional detection probability is integrated over the PDF of the output signal-to-noise-ratio (SNR). In the processing of derivation of detection performance, the MGF approach in [16] provides a more flexible, general framework for analyzing the sensing performance than PDF approach. This new approach can avoid some difficulties of the PDF method by using a contour integral representation of

the Marcum-Q function [26].

In summary, the major contributions of this paper include: 1) an improved MC detector with low computational complexity and high accuracy is proposed. In the proposed method, we present a reliable simplification for the test statistic of conventional MC detector. The benefits of the simplification are able to reduce the computational complexity which caused by compute the test statistic. From analysis, we find that the test statistic of proposed detector follows a chi-square distribution. The closed-form expressions of detection probability and false-alarm probability are then derived; 2) since complete knowledge of CSI is hard to obtain in practical CR, SLC is introduced to improve detection capability and its contribution is demonstrated by comparing it with the case without SLC; 3) based on the SLC, the sensing performance of improved MC detector is investigated over Rayleigh, Nakagami- $m$ , Rician fading channels, composite Rayleigh fading-lognormal shadowing and the composite Nakagami fading-lognormal shadowing channels, respectively. The corresponding closed-form average detection probability is derived by using the moment generation function (MGF) approach. The effect of fading and shadowing on sensing performance is then demonstrated by simulation results. These results help quantify and understand the achievable detection performance of the proposed detector.

### 3. Multi-cyclostationary (MC) Detector

In this section, we start by analyzing the conventional MC detector. Then, we propose an improved MC detector to reduce the computational complexity based on simplifying the test statistic of the conventional one. Subsequently, the closed-form expressions of detection probability and false alarm probability of proposed detector are derived.

A typical signal detection problem is usually formulated as a binary hypothesis testing problem:

$$\begin{cases} H_0: r(t) = n(t) \\ H_1: r(t) = hs(t) + n(t) \end{cases} \quad (1)$$

where  $H_0$  denotes the absence of a primary user (PU) and  $H_1$  denotes its presence,  $r(t)$  is the CR user's received signal,  $h$  is the gain of channel between the PU and the CR user,  $n(t)$  is the additive white Gaussian noise (AWGN),  $s(t)$  is the PU's transmitted signal. In spectrum sensing, CR user measures the sufficient statistics at first, and then compares it with a threshold which is determined with a desirable false alarm probability in order to decide between two hypotheses.

#### 3.1 Conventional MC Detector

The sufficient statistic of Maximum Likelihood (ML) detector in an AWGN channel is given by [27, Ch. 4]

$$Y_{ML} = \int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} \int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} R_s(u, v) r(u) r^*(v) du dv \quad (2)$$

where  $r(t)$  is the received signal,  $R_s(u, v) = E[s(u)s^*(v)]$  is the autocorrelation function of the PU's transmitted signal  $s(t)$  and  $T$  is the observation interval. If the signal is cyclostationary, then the sufficient statistic of ML detector can be expressed as

$$Y_{ML} = \sum_{k=1}^{N_\alpha} \int_{-\infty}^{+\infty} S_s^{\alpha_k}(f) S_r^{\alpha_k}(f) df, \quad \alpha_k = \frac{k}{T_c} \quad (3)$$

where  $S_s^{\alpha_k}(f)$  and  $S_r^{\alpha_k}(f)$  are the Spectral Correlation Functions (SCFs) of  $s(t)$  and  $r(t)$ , respectively, at the  $k$ -th cycle frequency (CF)  $\alpha_k = k/T_c$ ,  $N_\alpha$  is the number of CFs,  $S_r^{\alpha_k}(f)$  is the Fourier transform of the cyclic autocorrelation function  $R_r^{\alpha_k}(\tau)$ , and the time independent function  $R_r^{\alpha_k}(\tau)$  is calculated as the Fourier-series coefficient of the periodic autocorrelation function  $R_r(t, \tau) = \sum_k R_r^{\alpha_k}(\tau) e^{j2\pi\alpha_k t}$ ,  $\alpha_k = k/T_c$ , with period  $T_c \neq 0$  [28][29]. Since the PU's transmitted signal is unknown, the  $S_s^{\alpha_k}(f)$  can be assumed as a rectangular function with  $\Delta f$  bandwidth. Thus,

$$Y_{MC} = \sum_{k=1}^{N_\alpha} \int_{f-\frac{\Delta f}{2}}^{f+\frac{\Delta f}{2}} S_r^{\alpha_k}(v) dv \quad (4)$$

Consequently, the sum of cyclostationary signal powers at all CFs is the sufficient statistic for the optimum detector in ML criterion for cyclostationary signals. This detector is called the MC detector. Moreover, the total power can be calculated as [25]

$$Y_{MC} = \sum_{k=1}^{N_\alpha} R_r^{\alpha_k}(\tau=0) \quad (5)$$

where  $R_r^{\alpha_k}(\tau=0)$  represents the cyclostationary signal power at the  $k$ -th CF.

### 3.2 Improved MC Detector

#### 3.2.1 Simplified Test Statistic

Since  $Y_{MC} = \text{Re}(Y_{MC}) + j\text{Im}(Y_{MC})$  is a complex random variable, the test statistic of conventional MC detector is given by

$$T_{MC} = |Y_{MC}|^2 = \left| \sum_{k=1}^{N_\alpha} R_r^{\alpha_k}(\tau=0) \right|^2 = \sum_{k=1}^{N_\alpha} \left| R_r^{\alpha_k}(\tau=0) \right|^2 + \sum_{k=1}^{N_\alpha} \sum_{n=1, n \neq k}^{N_\alpha} R_r^{\alpha_k}(\tau=0) R_r^{\alpha_n^*}(\tau=0) \quad (6)$$

In order to reduce computational complexity, we will make a reliable approximation to  $T_{MC}$ . Since the second term of  $T_{MC}$  brings a large of computation, here is omitted. The first term of  $T_{MC}$  is used as the test statistic of the proposed detector. Thus, the simplified test statistic of proposed detector can be defined as

$$T_{sim} = \sum_{k=1}^{N_\alpha} \left| R_r^{\alpha_k}(\tau=0) \right|^2 \quad (7)$$

#### 3.2.2 Computational complexity analysis

In order to investigate the computational complexity of  $T_{MC}$  and  $T_{sim}$ , we make analysis as follows:

- Computational complexity analysis of computing  $T_{MC}$  : since  $T_{MC} = \sum_{k=1}^{N_\alpha} \left| R_r^{\alpha_k}(\tau=0) \right|^2 + \sum_{k=1}^{N_\alpha} \sum_{n=1, n \neq k}^{N_\alpha} R_r^{\alpha_k}(\tau=0) R_r^{\alpha_n^*}(\tau=0)$ , the conventional detector needs to do complex multiplication for  $N_\alpha^2$  times and complex addition for  $N_\alpha^2 - 1$  times to compute  $T_{MC}$ , therefore, the total computation of  $T_{MC}$  is  $2N_\alpha^2 - 1$ . Denote  $\Theta(N_\alpha^2)$  as the computational complexity of conventional detector.
- Computational complexity analysis of computing  $T_{sim}$  : since  $T_{sim} = \sum_{k=1}^{N_\alpha} \left| R_r^{\alpha_k}(\tau=0) \right|^2$ , the proposed detector needs to do complex multiplication for  $N_\alpha$  times and complex

addition for  $N_\alpha - 1$  times to compute  $T_{sim}$ , thus, the total computation of  $T_{sim}$  is  $2N_\alpha - 1$ .

Denote  $\Theta(N_\alpha)$  as the computational complexity of proposed detector.

Since  $N_\alpha \gg 1$ , it can be clearly seen that the computational complexity is greatly reduced by the simplification of test statistic, which means that proposed MC detector is of lower computational complexity than the conventional one.

Next, the sensing performance of proposed detector will be analyzed. By simulation results in Section IV, we will show that there is no appreciable difference in detection performance between proposed detector and conventional detector.

### 3.2.3 Sensing Performance Analysis

Since  $T_{sim}$  is the test statistic of proposed detector, the structure of detector can be defined as

$$T_{sim} = \sum_{k=1}^{N_\alpha} \left| R_r^{\alpha_k}(\tau=0) \right|^2 \begin{matrix} > \\ < \end{matrix} \begin{matrix} H_1 \\ H_0 \end{matrix} \lambda \quad (8)$$

where  $\lambda$  is detection threshold.

Consequently, the false-alarm and detection probability of proposed detector can be represented as

$$P_f = \int_{\lambda}^{+\infty} P(T_{sim} | H_0) dT_{sim} \quad (9)$$

$$P_d = \int_{\lambda}^{+\infty} P(T_{sim} | H_1) dT_{sim} \quad (10)$$

In order to obtain the conditional probability density functions (pdfs)  $P(T_{sim} | H_0)$  and  $P(T_{sim} | H_1)$ , we should focus on the  $R_r^{\alpha_k}(\tau=0)$  in (7) first. Denote

$$Y_1 = R_r^{\alpha_k}(\tau=0) \quad (11)$$

The cyclic autocorrelation function of  $Y_1$  can be shown as

$$R_r^{\alpha_k}(\tau) = \lim_{T \rightarrow \infty} \int_{\frac{T}{2}-\tau}^{\frac{T}{2}+\tau} r(t + \frac{\tau}{2}) r^*(t - \frac{\tau}{2}) e^{-j2\pi\alpha_k t} dt \quad (12)$$

From (11) and (12), the discrete-time counterpart of  $Y_1$  is given by

$$Y_1 = \frac{1}{N_S} \sum_{w=1}^{N_S} |r[w]|^2 e^{-j2\pi\alpha_k w} \quad (13)$$

Hence, the  $H_0$  and  $H_1$  in an AWGN channel according to (1) can be represented as

$$\begin{cases} H_0 : Y_1 = \frac{1}{N_S} \sum_{w=1}^{N_S} |n[w]|^2 e^{-j2\pi\alpha_k w} \\ H_1 : Y_1 = \frac{1}{N_S} \sum_{w=1}^{N_S} |s[w] + n[w]|^2 e^{-j2\pi\alpha_k w} \end{cases} \quad (14)$$

where  $n[w] = \text{Re}(n[w]) + j \text{Im}(n[w])$  a complex AWGN sample with zero mean and variance is  $\sigma_0^2$ ,  $s[w]$  is a PU signal sample. The proposed detector can be viewed as measures the signal power for single cycle frequency  $\alpha_k$ , then the structure of detector can be defined as

$$T_1 = |Y_1|^2 = \left| R_r^{\alpha_k}(\tau=0) \right|^2 \begin{matrix} > \\ < \end{matrix} \begin{matrix} H_1 \\ H_0 \end{matrix} \lambda \quad (15)$$

Therefore,

$$P_{f,\alpha_k} = \int_{\lambda}^{+\infty} P(T_1|H_0) dT_1 \quad (16)$$

$$P_{d,\alpha_k} = \int_{\lambda}^{+\infty} P(T_1|H_1) dT_1 \quad (17)$$

The conditional probability density functions (pdf)  $P(T_1|H_0)$  and  $P(T_1|H_1)$  of  $T_1$  can be derived from the conditional pdfs  $P(Y_1|H_0)$  and  $P(Y_1|H_1)$  of  $Y_1$ , where  $(T_1|H_0)$ ,  $(T_1|H_1)$ ,  $(Y_1|H_0)$  and  $(Y_1|H_1)$  means that the  $T_1$  and  $Y_1$  are a random variables under hypothes  $H_0$  and  $H_1$ , respectively. Base on the central limit theorem for large  $N_s, N_\alpha \gg 1$ ,  $Y_1$  is approximately Gaussian.

Since  $\text{var}\{|n[w]|^2\} = \text{var}\{\text{Re}^2(n[w]) + \text{Im}^2(n[w])\} = \sigma_0^4$  and  $E\{|n[w]|^2\} = \sigma_0^2$ , the expectation and variance of  $\{Y_1|H_0\}$   $Y_1$  under  $H_0$  are

$$E\{Y_1|H_0\} = \frac{\sigma_0^2}{N_s} \sum_{w=1}^{N_s} e^{-j2\pi\alpha_k w} = 0 \quad (18)$$

$$\text{var}\{Y_1|H_0\} = \frac{1}{N_s} \sum_{w=1}^{N_s} |e^{-j2\pi\alpha_k w}|^2 \text{var}\{|n[w]|^2\} = \frac{\sigma_0^4}{N_s} \quad (19)$$

where  $E\{\cdot\}$  and  $\text{var}\{\cdot\}$  are the expectation and variance, respectively. Since  $\{Y_1|H_0\} = \text{Re}(Y_1|H_0) + j\text{Im}(Y_1|H_0)$  is a complex Gaussian random variable, then  $(T_1|H_0) = \{Y_1|H_0\}^2 = [\text{Re}(Y_1|H_0)]^2 + [\text{Im}(Y_1|H_0)]^2$  follows a central chi-square distribution with two degrees of freedom, which probability density function can be obtained by

$$P(T_1|H_0) = \frac{1}{2\sigma_1^2} e^{-\frac{T_1}{2\sigma_1^2}}, \quad \sigma_1^2 = \frac{\sigma_0^4}{N_s} \quad (20)$$

Thus, the false alarm probability is

$$P_{f,\alpha_k} = e^{-\frac{\lambda}{2\sigma_1^2}} \quad (21)$$

Since  $E\{|s[w] + n[w]|^2\} = |s[w]|^2 + \sigma_0^2$  and  $\sum_{w=1}^W e^{-j2\pi\alpha_k w} = 0$ , the mean of  $(Y_1|H_1)$  is

$$E\{Y_1|H_1\} = \frac{1}{N_s} \sum_{w=1}^{N_s} (|s[w]|^2 + \sigma_0^2) e^{-j2\pi\alpha_k w} = \frac{1}{N_s} \sum_{w=1}^{N_s} |s[w]|^2 e^{-j2\pi\alpha_k w} = P_{\alpha_k} \quad (22)$$

where the complex-valued  $P_{\alpha_k}$  is the signal power at the  $k$ -th of CF  $\alpha_k$ . Since noise samples are statistically independent, the variance of  $(Y_1|H_1)$  is

$$\text{var}\{Y_1|H_1\} = \frac{1}{N_s^2} \sum_{w=1}^{N_s} |e^{-j2\pi\alpha_k w}|^2 \text{var}\{|s[w]|^2 + |n[w]|^2\} = \frac{2\sigma_0^2 P}{N_s} + \frac{\sigma_0^4}{N_s} \quad (23)$$

where  $P = \frac{1}{N_s} \sum_{w=1}^{N_s} |s[w]|^2$ . Therefore, the  $(T_1|H_1) = \{Y_1|H_1\}^2 = [\text{Re}(Y_1|H_1)]^2$

$+[\text{Im}(Y_1|H_1)]^2$  follows a non central chi-square distribution with two degrees of freedom, its pdf is given by

$$P(T_1|H_1) = \frac{1}{2\sigma_2^2} e^{-\frac{T_1+u_1}{2\sigma_2^2}} I_0\left(\frac{\sqrt{T_1}u_1}{\sigma_2^2}\right) \quad (24)$$

where  $u_1 = \sqrt{\{E[\text{Re}(Y_1|H_1)]\}^2 + \{E[\text{Im}(Y_1|H_1)]\}^2} = |P_{\alpha_k}|$  and  $\sigma_2^2 = 2\sigma_0^2 P/N_s + \sigma_0^4/N_s$ .

Then the detection probability can be given by

$$P_{d,\alpha_k} = Q_1\left(\frac{u_1}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2}\right) \quad (25)$$

where  $Q_1(\cdot, \cdot)$  is the first degree generalized Marcum-Q function.

The derivation of above shows that the  $(T_1|H_0)$  follows a central chi-square distribution with two degrees of freedom and  $(T_1|H_1)$  follows a non-central chi-square distribution with two degrees of freedom. Since  $T_{sim} = \sum_{k=1}^{N_\alpha} |R_r^{\alpha_k}(\tau=0)|^2 = \sum_{k=1}^{N_\alpha} T_1$ , it can be easily conclude that  $(T_{sim}|H_0)$  follows a central chi-square distribution with  $2N_\alpha$  degrees of freedom and  $(T_{sim}|H_1)$  follows a non central chi-square distribution with  $2N_\alpha$  degrees of freedom. Then, the conditional pdfs of  $T_{sim}$  can be represented as

$$P(T_{sim}|H_0) = \frac{1}{2^{N_\alpha} \sigma_1^{2N_\alpha} \Gamma(N_\alpha)} e^{-\frac{T_{sim}}{2\sigma_1^2}} T_{sim}^{N_\alpha-1}, \sigma_1^2 = \frac{\sigma_0^4}{N_s} \quad (26)$$

$$P(T_{sim}|H_1) = \frac{1}{2\sigma_2^2} e^{-\frac{T_{sim}+u_2}{2\sigma_2^2}} \left(\frac{T_{sim}}{u_2}\right)^{\frac{N_\alpha-1}{2}} I_{N_\alpha-1}\left(\frac{\sqrt{T_{sim}}u_2}{\sigma_2^2}\right), \sigma_2^2 = \frac{2\sigma_0^2 P}{N_s} + \frac{\sigma_0^4}{N_s} \quad (27)$$

where  $u_2 = \sum_{k=1}^{N_\alpha} u_1 = \sum_{k=1}^{N_\alpha} |P_{\alpha_k}|$ ,  $I_\nu(\cdot)$  is the  $\nu$ -th-order modified Bessel function of the first kind and  $\Gamma(\cdot)$  is the gamma function. The corresponding false alarm and detection probabilities are

$$P_f = \int_{\lambda}^{+\infty} P(T_{sim}|H_0) dT_{sim} = \frac{\Gamma\left(N_\alpha, \frac{\lambda}{2\sigma_1^2}\right)}{\Gamma(N_\alpha)} \quad (28)$$

$$P_d = \int_{\lambda}^{+\infty} P(T_{sim}|H_1) dT_{sim} = Q_{N_\alpha}\left(\frac{\sqrt{u_2}}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2}\right) \quad (29)$$

where  $Q_{N_\alpha}(\cdot, \cdot)$  is the generalized Marcum-Q function with  $N_\alpha$  degrees. Since MC detector is in AWGN channel (i.e.  $h_i = 1$ ),  $u_2 = \sum_{k=1}^{N_\alpha} |P_{\alpha_k}|$  is signal power and  $\sigma_0^2$  is noise power,  $\gamma = u_2/\sigma_0^2$  denotes SNR. Then, (29) can be rewritten as

$$P_d = Q_{N_\alpha}\left(\frac{\sigma_0\sqrt{\gamma}}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2}\right) \quad (30)$$



## 4. Sensing Performance of Proposed Detector with SLC diversity over Composite Multipath Fading-Lognormal Shadowing Channels

As shown in Fig. 1, we consider a CR with  $L$  antennas and assume that the PU is of a single antenna. Here, Square-law combining (SLC) is employed to combine the outputs of  $L$  antennas. We also assume a narrowband wireless system [30].

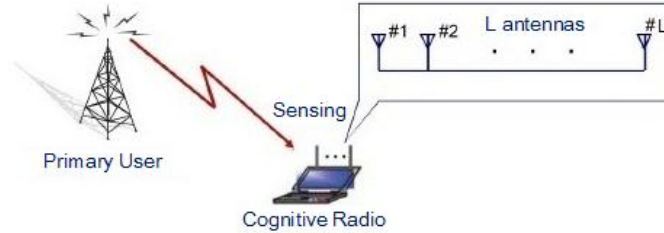


Fig. 1.  $L$  antennas based spectrum sensing.

In this section, we first derive the detection probability and false alarm probability of the proposed MC detector with SLC over AWGN channel, then, the multipath fading channels such as Rayleigh, Nakagami- $m$  and Rician are considered. Finally, the sensing performance over composite Rayleigh fading-lognormal shadowing and composite Nakagami fading-lognormal shadowing channels are investigated.

### 4.1 Square-law Combining

In SLC scheme, the outputs of the square-law devices (square-and-integrate operation per antenna) are added to yield a new test statistic  $T_{sim\_Σ}$ . Therefore, the new test statistic  $T_{sim\_Σ}$  can be written by

$$T_{sim\_Σ} = \sum_{i=1}^L T_{sim\_i} \quad (31)$$

where  $T_{sim\_i}$  denotes the test statistic from the  $i$ -th square-law device.

If SLC is employed to improve the sensing performance of proposed detector over AGWN channel, the test statistic  $T_{sim\_Σ}$  follows a centrality chi-square distribution with  $LN_\alpha$  degrees of freedom under  $H_0$ . Therefore, the false alarm probability becomes

$$P_{f,SLC} = P(T_{sim\_Σ} > \lambda | H_0) = \frac{\Gamma(LN_\alpha, \frac{\lambda}{2\sigma_1^2})}{\Gamma(LN_\alpha)} \quad (32)$$

Similarly, under  $H_1$ ,  $T_{sim\_Σ}$  has a non-central chi-square distribution with  $LN_\alpha$  degree of freedom, and non-centrality parameter  $2\gamma_t = 2\sum_{i=1}^L \gamma_i$ . Thus, the detection probability can be obtained as

$$P_{d,SLC} = P(T_{sim\_Σ} > \lambda | H_1) = Q_{LN_\alpha} \left( \frac{\sigma_0 \sqrt{\gamma_t}}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2} \right) \quad (33)$$

### 4.2 Average Detection Probability over Composite Fading-shadowing Channels

In composite fading-shadowing fading channels,  $P_f$  of (32) will remain the same since  $P_f$  is considered for the case of no signal transmission and as such is independent of SNR.

Therefore, we will derive the average detection probability ( $\overline{P_d}$ ) by using MGF over Rayleigh, Nakagami- $m$ , Rician, composite Rayleigh fading-lognormal shadowing and composite Nakagami fading-lognormal shadowing channels, respectively.

#### 4.2.1 Average Detection Probability - MGF Approach

The generalized Marcum-Q function in (33) can be written as a circular contour integral within the contour radius  $r \in [0,1)$ . Then, the expression (33) can be represented as

$$P_{d,SLC} = \frac{e^{-\frac{\lambda}{2\sigma_2^2}}}{2\pi j} \oint_{\Delta} \frac{e^{-\frac{\sigma_0^2}{2\sigma_2^2} \left(\frac{1-z}{z}\right) \gamma_t + \frac{\lambda}{2\sigma_2^2} z}}{z^{LN_\alpha} (1-z)} dz \quad (34)$$

where  $\Delta$  is a circular contour of radius  $r \in [0,1)$ .

The moment generating function (MGF) of average SNR  $\overline{\gamma_t}$  is  $M_{\overline{\gamma_t}}(s) = E(e^{-s\overline{\gamma_t}})$ . Thus, the average detection probability,  $\overline{P_{d,SLC}}$ , is given by

$$P_{d,SLC} = \frac{e^{-\frac{\lambda}{2\sigma_2^2}}}{2\pi j} \oint_{\Delta} f(z) dz \quad (35)$$

where  $f(z) = M_{\overline{\gamma_t}} \left( \frac{\sigma_0^2}{2\sigma_2^2} \left(1 - \frac{1}{z}\right) \right) \frac{e^{\frac{\lambda}{2\sigma_2^2} z}}{z^{LN_\alpha} (1-z)}$ .

In our study, the expression in (35) is fairly general and holds for any case where the MGF is available in suitable form.

#### 4.2.2 Average Detection Probability over Rayleigh Fading Channels

The MGF of Rayleigh fading combined SLC is

$$M_{\overline{\gamma_t},SLC}^{Ray}(s) = (1 + \overline{\gamma_t} s)^{-L} \quad (36)$$

After substituting this MGF in (35), the average detection probability,  $\overline{P_{d,SLC}}$ , over Rayleigh channel can be written in the form of expression (35) with

$$f(z) = \frac{e^{\frac{\lambda}{2\sigma_2^2} z}}{(1 + \mu_1)^L (z - \theta_1)^L z^{\beta_1} (1-z)}, \text{ where } \mu_1 = \sigma_0^2 \overline{\gamma_t} / 2\sigma_2^2, \theta_1 = \mu_1 / (1 + \mu_1) \text{ and } \beta_1 = L(N_\alpha - 1).$$

In radius  $r \in [0,1)$ , there are  $\beta_1$  poles at the origin  $z=0$  and  $L$  pole at  $z=\theta_1$ . By applying the residue theorem to (35), the detection probability over Rayleigh fading is obtained as follows

$$\overline{P_{d,SLC}}^{Ray} = \begin{cases} e^{-\frac{\lambda}{2\sigma_2^2}} [\text{Re}s(f; 0, \beta_1) + \text{Re}s(f; \theta_1, L)] & N_\alpha > L \\ e^{-\frac{\lambda}{2\sigma_2^2}} \text{Re}s(f; \theta_1, L) & N_\alpha \leq L \end{cases} \quad (37)$$

where  $\text{Res}(f;0,\beta_1) = \frac{D^{\beta_1-1} \left[ \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)(z-\theta_1)^L} \right] \Big|_{z=0}}{(1+\mu_1)^L (\beta_1-1)!}$  and  $\text{Res}(f;\theta_1,L) = \frac{D^{L-1} \left[ \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)z^{\beta_1}} \right] \Big|_{z=\theta_1}}{(1+\mu_1)^L (L-1)!}$ .

$D^n(f(z))$  denotes the  $n$ th derivative of  $f(z)$  with respect to  $z$ .

### 4.2.3 Average Detection Probability over Nakagami- $m$ Fading Channels

The MGF of Nakagami- $m$  fading combined SLC is

$$M_{\gamma_t, Nak}^-(s) = (1 + \overline{\gamma_t} s)^{-Lm} \tag{38}$$

Using (35), detection probability over Nakagami- $m$  channel,  $\overline{P_{d,SLC}^{Nak}}$ , can be written in the

form of expression (35) with  $f(z) = \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1+\mu_2)^{Lm} (z-\theta_2)^{Lm} z^{\beta_2} (1-z)}$ , where  $\mu_2 = \sigma_0^2 \overline{\gamma_t} / 2\sigma_2^2 m$ ,  $\theta_2 = \mu_2 / (1 + \mu_2)$  and  $\beta_2 = L(N_\alpha - m)$ .

Following the similarly procedure as in Rayleigh fading case, the detection probability over Nakagami- $m$  channel is given by

$$\overline{P_{d,SLC}^{Nak}} = \begin{cases} e^{-\frac{\lambda}{2\sigma_2^2}} [\text{Res}(f;0,\beta_2) + \text{Res}(f;\theta_2,Lm)] & N_\alpha > mL \\ e^{-\frac{\lambda}{2\sigma_2^2}} \text{Res}(f;\theta_2,Lm) & N_\alpha \leq mL \end{cases} \tag{39}$$

where  $\text{Res}(f;0,\beta_2) = \frac{D^{\beta_2-1} \left[ \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)(z-\theta_2)^{Lm}} \right] \Big|_{z=0}}{(1+\mu_2)^{Lm} (\beta_2-1)!}$  and  $\text{Res}(f;\theta_2,Lm) = \frac{D^{Lm-1} \left[ \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)z^{\beta_2}} \right] \Big|_{z=\theta_2}}{(1+\mu_2)^{Lm} (Lm-1)!}$ .

From analysis above, we notice that the results are limited to an integer of  $Lm$  and allow us to compute  $\overline{P_{d,SLC}^{Nak}}$  for certain non-integer values of  $m$ . For example,  $\overline{P_{d,SLC}^{Nak}}$  over 2 branches can be computed for  $1/2$  multiples of  $m$  values. We also notice that Rayleigh fading channel is a special case of Nakagami- $m$  channel, which  $\overline{P_{d,SLC}^{Ray}}$  can be obtained by substitute  $m=1$  in (38).

### 4.2.4 Average Detection Probability over Rician Fading Channels

The MGF of Rician fading combined SLC is given by

$$M_{\gamma_t, SLC}^{Ric}(s) = \left( \frac{1+K}{1+K+s\overline{\gamma_t}} \right)^L \exp \left( -\frac{LKs\overline{\gamma_t}}{1+K+s\overline{\gamma_t}} \right) \tag{40}$$

where  $K$  is the Rice factor. Notice that for the special case of  $K = 0$  (Rayleigh fading), (40) reduces to Rayleigh in (36). Hence, using (40), the average detection probability over Rician fading can be obtained as

$$\overline{P_{d,SLC}^{Ric}} = \frac{A}{2\pi j} \oint_{\Delta} \frac{e^{\frac{a}{z-\theta_3}}}{(z-\theta_3)} f(z) dz \quad (41)$$

where  $f(z) = \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1+K+\mu_2)^L (z-\theta_3)^{L-1} z^{\beta_2} (1-z)}$ ,  $A = e^{-\frac{\lambda}{2\sigma_2^2} [(1+K)e^{-K\theta_3}]^L}$ ,  $a = LK\theta_3(1-\theta_3)$  and  $\theta_3 = \mu_2/(\mu_2 + K + 1)$ .

Applying Laurent series expansion for  $\frac{e^{\left(\frac{a}{z-\theta_3}\right)}}{(z-\theta_3)}$  in (41) when  $K \neq 0$  and using the residue theorem to integrate term by term, the average detection probability over Rician fading can be derived as

$$\overline{P_{d,SLC}^{Ric}} = \begin{cases} A \sum_{n=1}^{a^{n-1}} [\text{Res}(f; 0, \beta_2) + \text{Res}(f; \theta_3, L+n-1)] & N_\alpha > L \\ A \sum_{n=1}^{a^{n-1}} \text{Res}(f; \theta_3, L+n-1) & N_\alpha \leq L \end{cases} \quad (42)$$

where  $\text{Res}(f; 0, \beta_2) = \frac{D^{\beta_2-1} \left( \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)(z-\theta_3)^{L+n-1}} \right) \Big|_{z=0}}{(1+K+\mu_2)^L (\beta_2-1)!}$  and  $\text{Res}(f; \theta_3, L+n-1) = \frac{D^{L+n-2} \left( \frac{e^{\frac{\lambda}{2\sigma_2^2}z}}{(1-z)z^{\beta_2}} \right) \Big|_{z=\theta_3}}{(1+K+\mu_2)^L (L+n-2)!}$ .

#### 4.2.5 Average Detection Probability over Composite Rayleigh Fading-lognormal Shadowing Channels

Since shadowing process is typically modeled as a lognormal distribution, the composite Rayleigh fading-lognormal shadowing channels model follows a mixture Rayleigh fading-lognormal distribution as [17]

$$g_{rl}(x) = \sum_{i=1}^N \phi_i e^{-\varepsilon_i x}, x \geq 0, \phi_i \geq 0, \varepsilon_i \geq 0 \quad (43)$$

where  $\phi_i = \rho_i e^{-(\sqrt{2}\delta\eta_i + \psi)} / (\sqrt{\pi} \sum_{i=1}^N \rho_i)$ ,  $\varepsilon_i = e^{-(\sqrt{2}\delta\eta_i + \psi)}$ ,  $N$  is the number of terms in the mixture,  $\eta_i$  and  $\rho_i$  are abscissas and weight factors for the Gaussian-Laguerre integration.  $\eta_i$  and  $\rho_i$  for different  $N$  values can be calculated by a simple MATLAB program.  $\psi$  and  $\delta$  are the mean and the standard deviation of the lognormal distribution, respectively. Therefore, the MGF of composite Rayleigh fading-lognormal shadowing combined SLC can be expressed as

$$M_{SLC}^{rl}(s) = \sum_{i=1}^N \left( \frac{\phi_i}{\varepsilon_i + s} \right)^L \quad (44)$$

Thus, the average detection probability over composite Rayleigh fading-lognormal shadowing channels,  $\overline{P_{d,SLC}^{rl}}$ , can be evaluated in closed-form as

$$\overline{P_{d,SLC}^{rl}} = \begin{cases} e^{-\frac{\lambda}{2\sigma_2^2} \sum_{i=1}^N \left(\frac{\phi_{li}}{\varepsilon_{li}}\right)^L} [\text{Re}s(f_{li}; 0, \beta_1) + \text{Re}s(f_{li}; \theta_{li}, L)] & N_\alpha > L \\ e^{-\frac{\lambda}{2\sigma_2^2} \sum_{i=1}^N \left(\frac{\phi_{li}}{\varepsilon_{li}}\right)^L} \text{Re}s(f_{li}; \theta_{li}, L) & N_\alpha \leq L \end{cases} \quad (45)$$

where  $f_{li}(z) = \frac{e^{-\frac{\lambda}{2\sigma_2^2} z}}{(1 + \mu_{li})^L (z - \theta_{li})^L z^{\beta_1} (1 - z)}$  and  $\mu_{li} = \sigma_0^2 / 2\sigma_2^2 \varepsilon_{li}$ ,  $\theta_{li} = \mu_{li} / (1 + \mu_{li})$ . Following a similar procedure as in Rayleigh fading case, the residues  $\text{Re}s(f_{li}; 0, \beta_1)$  and  $\text{Re}s(f_{li}; \theta_{li}, L)$  can be obtained.

#### 4.2.6 Average Detection Probability over Composite Nakagami Fading-lognormal Shadowing Channels

For a composite Nakagami fading-lognormal shadowing channel, the composite Nakagami-lognormal distribution is a mixture of Nakagami fading and lognormal shadowing. Then, the SNR distribution  $f_{nl}(x)$  can be expressed as [31]

$$f_{nl}(x) = \sum_{i=1}^N \phi_{2i} x^{m-1} e^{-\varepsilon_{2i} x}, x \geq 0, \phi_{2i} \geq 0, \varepsilon_{2i} \geq 0 \quad (46)$$

where  $\phi_{2i} = m^m \rho_i e^{-m(\sqrt{2}\delta\eta_i + \psi)} / (\Gamma(m) \sqrt{\pi} \sum_{i=1}^N \rho_i)$ ,  $\varepsilon_{2i} = m e^{-(\sqrt{2}\delta\eta_i + \psi)}$  and  $f_{nl}(x)$  is equal to a mixture of gamma distributions [32]. Since the SNR's possibility density function in this case follows gamma-lognormal distribution, we can accurately approximate the gamma-lognormal distribution through a mixtures of gamma distribution. Then, the MGF of composite Nakagami fading-lognormal shadowing combined SLC can be expressed as

$$M_{SLC}^{nl}(s) = \sum_{i=1}^N \left( \frac{\phi_{2i}}{(\varepsilon_{2i} + s)^m} \right)^L \quad (47)$$

Thus, the average detection probability over composite Nakagami fading-lognormal shadowing channels,  $\overline{P_{d,SLC}^{nl}}$ , can be evaluated in closed-form as

$$\overline{P_{d,SLC}^{nl}} = \begin{cases} e^{-\frac{\lambda}{2\sigma_2^2} \sum_{i=1}^N \left(\frac{\phi_{2i}}{\varepsilon_{2i}}\right)^{Lm}} [\text{Re}s(f_{2i}; 0, \beta_2) + \text{Re}s(f_{2i}; \theta_{2i}, Lm)] & N_\alpha > Lm \\ e^{-\frac{\lambda}{2\sigma_2^2} \sum_{i=1}^N \left(\frac{\phi_{2i}}{\varepsilon_{2i}}\right)^{Lm}} \text{Re}s(f_{2i}; \theta_{2i}, Lm) & N_\alpha \leq Lm \end{cases} \quad (48)$$

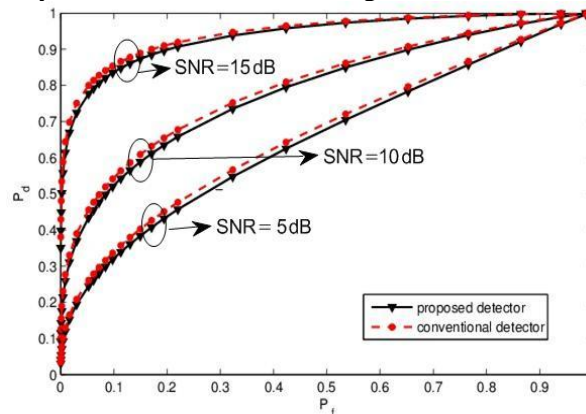
where  $f_{2i}(z) = \frac{e^{-\frac{\lambda}{2\sigma_2^2} z}}{(1 + \mu_{2i})^{Lm} (z - \theta_{2i})^{Lm} z^{\beta_2} (1 - z)}$ ,  $\theta_{2i} = \mu_{2i} / (1 + \mu_{2i})$ ,  $\mu_{2i} = \sigma_0^2 / 2\sigma_2^2 \varepsilon_{2i}$  and

$\beta_2 = L(N_\alpha - m)$ . Following a similar procedure as in Nakagami fading case, the residues  $\text{Re}s(f_{2i}; 0, \beta_2)$  and  $\text{Re}s(f_{2i}; \theta_{2i}, Lm)$  can be obtained, we omit the expressions here for brevity. We also notice that composite Rayleigh fading-lognormal shadowing channel is a special case of composite Nakagami fading-lognormal shadowing channel, which  $\overline{P_{d,SLC}^{rl}}$  can be obtained by substitute  $m=1$  in (48).

## 5. Numerical Results and Analysis

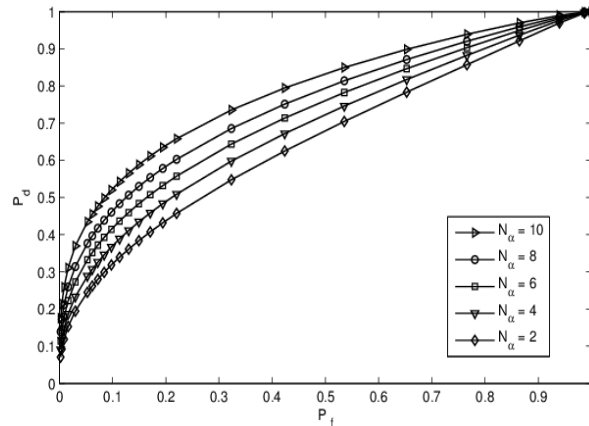
In this section, we provide analytical and simulation results to verify the analytical framework. In order to show the sensing performance of the proposed detector, we plot receiver operating characteristic (ROC) curves (plots of detection probability  $P_d$  versus false alarm probability  $P_f$ ), complementary ROC curves (plots of miss detection probability  $P_m$  versus false alarm probability  $P_f$ ),  $P_d$  vs. average SNR curves and  $P_d$  vs.  $L$  curves. Note that each of the following figures contains both analytical result and simulation result, which are represented by lines and discrete marks, respectively.

To verify the reliability and efficiency of proposed detector, we make analysis as follows: 1) reliability analysis: Fig. 2 shows the ROC curves of proposed MC detector and conventional MC detector over AWGN channel, for different SNR  $\gamma = \{5, 10, 15\}$  dB and  $N_\alpha = 10$ . It can be observed that the analytical results match well with their simulation analysis, confirming the accuracy of the analysis. It can be seen that there is a slight difference on sensing performance between the proposed and conventional detector. Although this simplification slightly degrades sensing accuracy, it still maintains a satisfactory sensing capability; 2) computational complexity analysis: based on the analysis in Section 3.2, the computational complexity of conventional MC detector is  $\Theta(10^2)$  while the proposed detector is  $\Theta(10)$  for  $N_\alpha = 10$ . The analysis above clearly illustrated that the proposed detector is more efficient than the conventional one. In all, Fig.2 and the computational complexity analysis verify that the proposed MC detector can reduce the computational complexity while still maintains enough detection sensitivity.



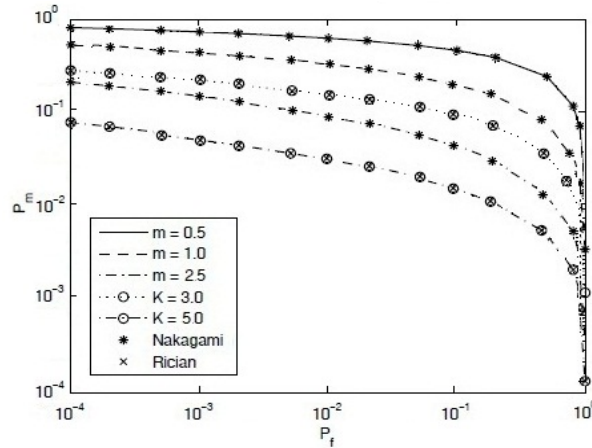
**Fig. 2.** ROC curves for the proposed and conventional MC detector with different SNRs ( $\gamma = \{5, 10, 15\}$  dB,  $N_\alpha = 10$ ).

For further investigated the sensing performance of proposed detector, we plot the ROC curves with different numbers of  $N_\alpha$ , for  $N_\alpha = 2, 4, 6, 8, 10$ . In Fig. 3, with the increase value of  $N_\alpha$ , the sensing performance improves, which is similar to conventional detector. Since MC detector measures the sum of  $N_\alpha$  spectral correlation function, the more  $N_\alpha$  is, the more accurate binary decision the detector makes.



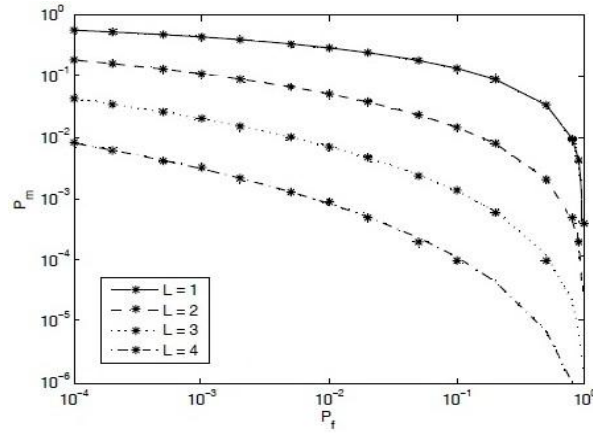
**Fig. 3.** ROC curves for the proposed detector with different  $N_\alpha$  ( $N_\alpha = \{2, 4, 6, 8, 10\}$ ,  $\gamma = 10\text{dB}$ ).

**Fig. 4** depicts the complementary ROC curves of the proposed detector over Rayleigh, Nakagami- $m$  and Rician fading channels. Rayleigh and Rician  $K=0$  curves coincide with the Nakagami  $m=1$  curve and therefore not shown. The fading index quantifies the detection performance for several Nakagami parameter  $m$  and Rician factor  $K$  values. As expected, higher values of the fading index  $m$  and  $K$  imply a relatively less degraded received signal and thus lead to a lower miss detection probability  $P_m$ .



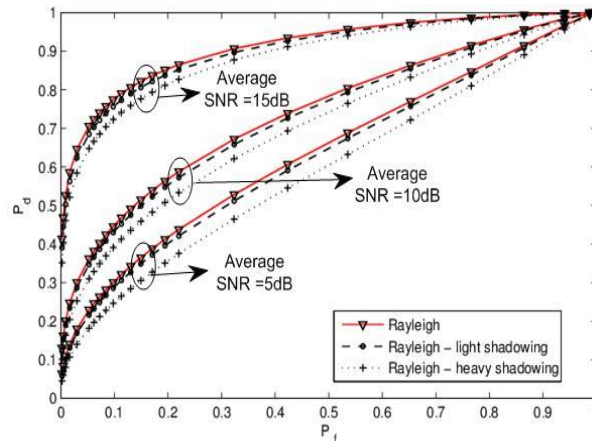
**Fig. 4.** Complementary ROC curves over Rayleigh, Nakagami- $m$  and Rician fading channels (average SNR  $\bar{\gamma}_t = 10\text{dB}$ ,  $L = 2$ ,  $N_\alpha = 10$ ).

**Fig. 5** shows the complementary ROC curves of the proposed detector over Rician fading channel with SLC, for  $K=4$  and  $\bar{\gamma}_t = 10\text{dB}$ . The number of diversity branches varies for one to four. It can be observed that the sensing capability of 1-branch ( $L=1$ , non-SLC) is the lowest bound of the sensing ability of the proposed detector. With increasing in the number of SLC diversity branch, there is much improvement on sensing capability of the proposed detector.



**Fig. 5.** Complementary ROC curves with SLC diversity over Rician fading channel ( average SNR  $\overline{\gamma}_i = 10\text{dB}$ ,  $L = \{1, 2, 3, 4\}$ ,  $K = 4$ ,  $N_\alpha = 10$ ).

**Fig. 6** depicts the ROC curves of proposed detector over composite Rayleigh fading-lognormal shadowing channel without SLC diversity. The shadowing effects considered here were introduced by Loo in [11] and are (i) light shadowing ( $\psi = 0.115$  and  $\delta = 0.115$ ), which corresponds to sparse tree cover and (ii) heavy shadowing ( $\psi = 3.914$  and  $\delta = 0.806$ ), which corresponds to dense tree cover. We take  $N = 10$  in (44), which makes the mean square error (MSE) between the exact gamma-lognormal channel model and the approximated mixture gamma channel model in (44) less than  $10^{-4}$ . The numerical results match well with their theoretical analysis, confirming the accuracy of the analysis. It can be seen that the performance of proposed detector degrades with increase in composite Rayleigh fading-lognormal shadowing environments, and improves at higher SNR. Further, for Rayleigh and composite Rayleigh fading-light shadowing environments, there is a slight difference in detector's performance. However, there is a significant performance degradation of the proposed detector due to the heavy shadowing effect in lower average SNR (e.g. 5dB).

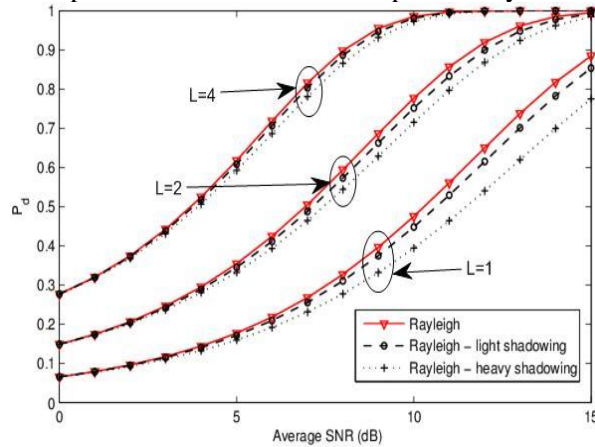


**Fig. 6.** ROC curves of proposed detector without SLC over Rayleigh and composite Rayleigh fading-lognormal shadowing channels (average SNR  $\overline{\gamma}_i = \{5, 10, 15\text{dB}\}$ ,  $N_\alpha = 10$ ).

Next, we consider the effect of SLC diversity in alleviating the composite Rayleigh fading-lognormal shadowing. **Fig. 7** demonstrated that SLC diversity improves the detection performance, even in the serious composite Rayleigh fading-lognormal shadowing environments. For example, in composite Rayleigh-heavy shadowing environment with the

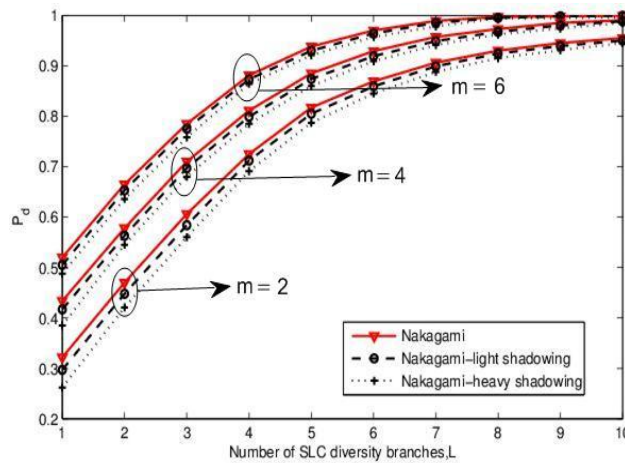


low average SNR (e.g. 5dB), we find that the detection probability for 4-branches SLC case ( $L=4$ ) is almost six times than that for 1-branch non-SLC case ( $L=1$ ). Furthermore, for 2-branches SLC case ( $L=2$ ), there is approximately 3 dB performance gain compared with non-SLC case. Therefore, the SLC diversity mitigates the impact of fading-shadowing and introduces a significant improvement in the detection probability.



**Fig. 7.**  $P_d$  vs. average SNR with SLC diversity over Rayleigh and composite Rayleigh fading-lognormal shadowing channels ( $P_f = 0.01, L = \{1, 2, 4\}, N_\alpha = 10$ ).

**Fig. 8** generate the  $P_d$  vs.  $L$  curves for Nakagami and composite Nakagami fading-lognormal shadowing conditions with different Nakagami fading parameter  $m$ , for  $m = 2, 4, 6$ . The shadowing effects considered here are (i) light shadowing ( $\psi = 0.115$  and  $\delta = 0.115$ ) and (ii) heavy shadowing ( $\psi = 3.914$  and  $\delta = 0.806$ ). We also take  $N = 10$  in (47). As we can see, there is an improvement in the performance with increasing in  $m$ . A higher value of  $m$  means better conditions of the sensing channels between the CR and PU. Furthermore, there is a great improvement in detection performance by employing SLC diversity, even in heavy shadowing environments. Clearly the greater is the number of SLC diversity branch, the better the detector works. That is to say, the SLC diversity mitigate the impact of composite Nakagami fading-shadowing, and introduce a significant improvement on sensing performance.



**Fig. 8.**  $P_d$  vs.  $L$  over Nakagami and composite Nakagami fading-lognormal shadowing channels ( $P_f = 0.01, m = \{2, 4, 6\}, N_\alpha = 10$ ).

## 6. Conclusion

In this paper, the sensing performance of improved MC detector based SLC diversity in fading and shadowing environments has been studied. We first proposed an improved MC detector, which is computational efficiency and sufficient accuracy on sensing performance. Subsequently, the sensing performance of proposed detector is investigated in fading and shadowing environments. For Rayleigh, Nakagami- $m$ , Rician, composite Rayleigh fading-lognormal shadowing and composite Nakagami fading-lognormal shadowing channels, the closed-form expressions of detection probability by employing SLC are derived by using MGF approach, respectively. Illustrative and analytical results show that although the composite fading and shadowing degrade the sensing performance of proposed detector, the gain is significantly improved by SLC. Although we investigate the sensing performance over composite Rayleigh fading-lognormal shadowing channel and composite Nakagami fading-lognormal shadowing channel in this paper, the same analytical framework can be extended to composite Rician fading-lognormal shadowing channel in future work. These results help quantify the performance gains for MC detection with diversity reception over composite fading and shadowing channels, which can help emerging applications such as cognitive radio and ultra wide-band radio.

## References

- [1] Mitola J. "Cognitive radio: an integrated agent architecture for software defined radio". PhD dissertation, *KTH Royal Institute of Technology*, Sweden, May 2000. [Article \(CrossRef Link\)](#)
- [2] Sridhara K., Chandra A., Tripathi P.S.M, "Spectrum challenges and solutions by cognitive radio: an overview," *Wireless Personal Communications*, 2008, vol.45, no.3, pp. 281-291. [Article \(CrossRef Link\)](#)
- [3] El-Hajj W., Safa H., Guizani M, "Survey of security issues in cognitive radio networks," *Journal of Internet Technology*, 2011, vol.12, no.2, pp. 181-198.
- [4] Chen Y., Cho C., You I., Chao, H. "A cross-layer protocol of spectrum mobility and handover in cognitive lte networks," *Simulation Modeling Practicing and Theory*, 2011, vol.19, no.8, pp. 1723-1744. [Article \(CrossRef Link\)](#)
- [5] Arslan, H. "Cognitive radio, software defined radio, and adaptive wireless systems," Springer, 2007. [Article \(CrossRef Link\)](#)
- [6] Huang, L., Gao, Z., Guo, D., Chao, H., Park. J, "A sensing policy based on the statistical property of licensed channel in cognitive network," *International Journal of Internet Protocol Technology*, 2010, vol.5, no.4, pp. 219-229. [Article \(CrossRef Link\)](#)
- [7] Gao, Z., Huang, L., Yao, Y., Wu. T, "Performance analysis of a busycognitive multi-channel mac protocol," *International Journal of Internet Protocol Technology*, 2011, vol.11, no.3, pp. 299-306. [Article \(CrossRef Link\)](#)
- [8] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal Select Areas Communications*, vol. 23, no. 2, pp. 201-220, Feb. 2005. [Article \(CrossRef Link\)](#)
- [9] A. Ghasemi, E. S. Sousa, "Spectrum sensing in cognitive radio networks: The cooperation-processing tradeoff," *Wireless Communications and Mobile Computing*, vol.7, no.9, pp. 1049-1060, Nov. 2007. <http://onlinelibrary.wiley.com/doi/10.1002/wcm.480/abstract>
- [10] S. M. Mishra, A. Sahai, and R. W. Brodersen, "Cooperative sensing among cognitive radios," in Proc. of International Conference of Communications, Istanbul, Turkey, Jun. 11-15, 2006. [Article \(CrossRef Link\)](#)
- [11] A. V. Dandawaté and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Transaction on Signal Processing*, vol. 42, no. 9, pp.2355-2369, Sep. 1994. [Article \(CrossRef Link\)](#)

- [12] W. A. Gardner, "Signal interception: A unifying theoretical framework for feature detection". *IEEE Transaction on Communications*, 1988, vol.36, pp. 897 - 906. [Article \(CrossRef Link\)](#)
- [13] J.Wang, T. Chen, B. Huang, "Cyclo-period estimation for discrete time cyclo-stationary signals", *IEEE Transaction on Signal Processing*, 2006, vol.54, no.1, pp. 83-94. [http://dx.doi.org/doi:Article \(CrossRef Link\)](http://dx.doi.org/doi:Article (CrossRef Link))
- [14] R. Tandra and A. Sahai, "Fundamental Limits on Detection in Low SNR Under Noise Uncertainty," in *Proc. of Wireless Communications Symp. on Signal Processing*, 2005. [Article \(CrossRef Link\)](#)
- [15] M. Derakhshani, M. Nasiri-Kenari, T. Le-Ngoc., "Cooperative cyclostationary spectrum sensing in cognitive radios at low SNR regimes," *IEEE Transaction on Wireless Communications*, 2012, vol.10, no.11, pp. 3754-3764. [Article \(CrossRef Link\)](#)
- [16] A. Pandharipande and J.P. Linnartz, "Performance analysis of primary user detection in multiple antenna cognitive radio," in *Proc. of IEEE International Conference on Communications Conference*, 2007, pp. 6482-6486. [Article \(CrossRef Link\)](#)
- [17] Saman Atapattu, Chintha Tellambura, Hai Jiang, "Energy Detection Based Cooperative Spectrum Sensing in Cognitive Radio Networks", *IEEE Transaction on Wireless Communications*, vol.10, no.4, pp.1232-1242, 2011. [Article \(CrossRef Link\)](#)
- [18] C. Loo, "Digital Transmission through a Land Mobile Satellite Channel," *IEEE Transaction on Wireless Communications*, vol. 38, pp. 693-697, 1990. [Article \(CrossRef Link\)](#)
- [19] S. P. Herath, N. Rajatheva, and C. Tellambura, "Energy detection of unknown signals in fading and diversity reception," *IEEE Transaction on Wireless Communications*, 2011, vol.59, no.9, pp. 2443-2453. [Article \(CrossRef Link\)](#)
- [20] Z. Quan, S. Cui, and A. H. Sayed, "Feature detection based on multiple cyclic frequencies in cognitive radios," in *Proc of IEEE Microwave Conference*, Sep. 2008. [Article \(CrossRef Link\)](#)
- [21] K. W. Choi, W. S. Jeon, and D. G. Jeong, "Sequential detection of cyclostationary signal for cognitive radio systems," *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 4480-4485, Sep. 2009. [Article \(CrossRef Link\)](#)
- [22] K. L. Du and W. H. Mow, "Affordable cyclostationarity-based spectrum sensing for cognitive radio with smart antennas," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1877-1886, May 2010. [Article \(CrossRef Link\)](#)
- [23] J. Lunden, V. Koivunen, A. Huttunen, H. V. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 11, pp. 4182-4195, Nov. 2009. [Article \(CrossRef Link\)](#)
- [24] H. Sadeghi and P. Azmi, "Cyclostationarity-based cooperative spectrum sensing for cognitive radio networks," in *Proc. of IEEE IST*, Aug. 2008. [Article \(CrossRef Link\)](#)
- [25] M. Derakhshani, M. Nasiri-Kenari, and T. Le-Ngoc. "Cooperative cyclostationary spectrum sensing in cognitive radios at low SNR regimes," in *Proc. of IEEE International Conference on Communications*, 2012, vol.10, no.11, pp. 3754-3764. [Article \(CrossRef Link\)](#)
- [26] C. Tellambura, A. Annamalai, V. K. Bhargava, "Closed Form and Infinite Series Solutions for the MGF of a Dual-Diversity Selection Combiner Output in Bivariate Nakagami-m Fading," *IEEE Transaction on Wireless Communications*, 2003, vol. 51, no.4, pp. 539-542. [Article \(CrossRef Link\)](#)
- [27] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*, Part III. Wiley, 2001. [Article \(CrossRef Link\)](#)
- [28] W. A. Gardner, "Cyclostationarity in Communications and Signal Processing," *IEEE Press*, 1994. [Article \(CrossRef Link\)](#)
- [29] W. A. Gardner and M. S. Spooner, "Signal interception: performance advantages of cyclic-feature detectors," *IEEE Transaction on Wireless Communications*, 1992, vol.40, no.1, pp. 149-159. [Article \(CrossRef Link\)](#)
- [30] A. Goldsmith, "Wireless Communications", Cambridge University Press, 2005. [Article \(CrossRef Link\)](#)
- [31] Saman Atapattu, Chintha Tellambura and Hai Jiang, "Representation of Composite Fading and Shadowing Distributions by using Mixtures of Gamma Distributions," in *Proc. of IEEE*

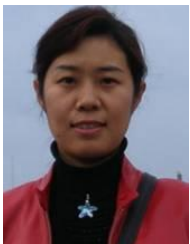
- Conference on Wireless Communications and Networking Conference*. IEEE Press, 2010, pp. 1-5. [Article \(CrossRef Link\)](#)
- [32] M. Wiper, D. R. Insua, and F. Ruggeri, "Mixtures of gamma distributions with applications," *Journal of Computational and Graphical Statistics*, 2001, vol. 10, no. 3, pp. 440–454. [Article \(CrossRef Link\)](#)



**Ying Zhu**, is currently a Ph.D. candidate at Beijing University of Posts and Telecommunications (BUPT), Beijing, China. Her research interests include spectrum sensing, resource sharing and resource management in cognitive radio networks.



**Jia Liu**, received the M.S. degree in electronics and information engineering from Guilin University of Electronic Technology, Guilin, China, in 2008. He is currently working toward the Ph.D. degree in Communication and Information System at the School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, China. His research in-terests include cognitive radio, relay, wireless networkcoding, et al.



**Zhiyong Feng**, received her M.S. and Ph.D. degrees from BUPT, China. She is a professor at BUPT, and is currently leading the Ubiquitous Network Lab in the Wireless Technology Innovation (WTI) Institute. She is a member of IEEE and active in standards development such as ITU-R WP5A/WP5D, IEEE 1900, ETSI and CCSA. Her main research interests include the cognitive wireless networks, convergence of heterogeneous wireless networks, spectrum sensing, dynamic spectrum management, cross-layer design,.



**Ping Zhang**, is a professor at BUPT. Prof. Zhang is the director of Key Lab. of Universal Wireless Communication (BUPT) of Ministry of Education and currently leading the Wireless Technology Innovation (WTI) Institute. He is a member of IEEE and vice director of Sino-Germany Joint Software Institute. He is one of three draftsmen of National Key Program, Member of Experts and Consultants Committee of NSFC. His major research interests include cognitive radio network, wireless communication theory and signal processing.