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A Simple Equivalent Circuit for Efficiency Calculation of Brushless DC Motors

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Abstract – This paper shows a calculation method of several types of loss and the efficiency of brushless DC motors coupled with a load system by using a simple equivalent circuit, in which copper loss, eddy current loss, hysteresis loss, friction loss, viscous loss, and inverter loss are taken into account. We clarify each loss and motor efficiency at different motor speeds and different output torques by using the Microsoft-Excel. Moreover, the calculated results are in good agreement with the measured ones.

Keywords: Brushless DC motor, Equivalent circuit, Loss, Efficiency

1. Introduction

Brushless DC motors are widely used in industrial applications and computer peripheral devices, because of no brush and no commutator, wide speed range, and relatively high efficiency. When the motor is connected to a load system, there are several types of loss, namely, copper loss, eddy current loss, hysteresis loss in the motor, and friction loss and viscous loss in the motor and load system, and inverter loss. These losses change according to the operating speed and the load condition. For example, when the motor speed becomes high, the iron loss increases, which reduces the motor performance. Hence, the analysis and prediction of numerous types of loss in the motors are essential. The conventional analysis method used to assess the iron loss especially the eddy-current loss is a three-dimensional finite element method [1]-[4]. However this method is very time consuming. It is important to calculate easily the motor efficiency at different operating speeds and load conditions. An equivalent circuit method is very useful for easy calculation of the losses and efficiency of the motor. A resistance corresponding to eddy-current loss was introduced in the d- and q-axis equivalent circuits [5]. An equivalent circuit for DC motor considering numerous types of losses was proposed, and the relationship between the maximum current and the currents at no-load and locked conditions was expressed when the equivalent circuit was applied to calculate the brushless DC motors [6].

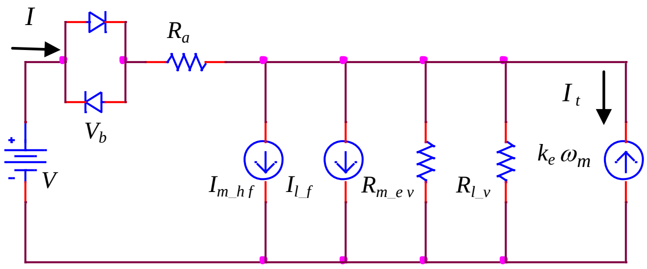

Fig. 1. A simple equivalent circuit for brushless DC motors.

This paper proposes a simple equivalent circuit for the calculation of losses and efficiency of brushless DC motors. The equivalent circuit is an improved one, which is described in [6]. This paper shows how to decide the parameters of the simple equivalent circuit for copper loss, eddy current loss and hysteresis loss in the motor, friction loss and viscous loss in the mechanical system, and voltage-drop loss in the inverter circuit. This paper clarifies each loss and the efficiency of the brushless DC motor at different motor speeds and different output torques using the Microsoft-Excel. Moreover, the comparison with experimental results is discussed.

2. Equivalent Circuit

2.1 Equivalent Circuit for Brushless DC motors

A simple equivalent circuit for brushless DC motors is shown in Fig. 1. A well-known steady state equivalent circuit for DC motors is given by the input voltage V, the armature resistance R_a, and the electromotive force $k_e \omega_m$. This paper proposes to improve it for brushless DC motors. V and I are the input voltage and input current to the

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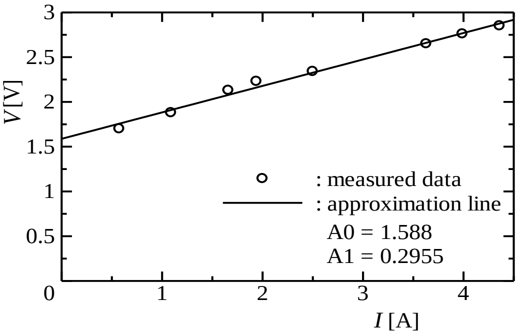


Fig. 2. *V-I* characteristic at the lock test.

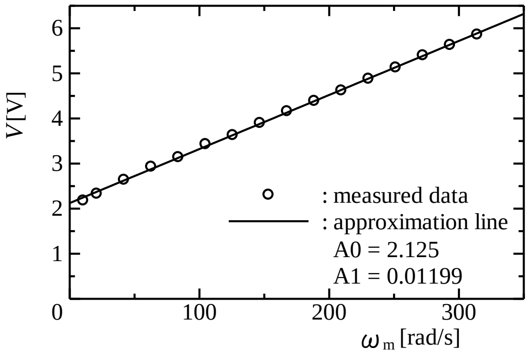


Fig. 3. V- ω_m characteristic at the no-load test.

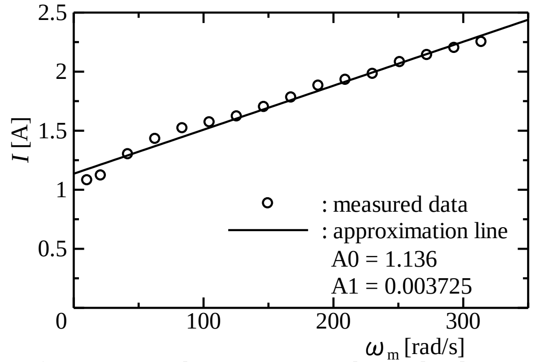


Fig. 4. I- ω_m characteristic at the no-load test.

Inverter, respectively. The voltage drop of transistor and diode in the inverter circuit is introduced by a forward voltage V_b of two diodes connected in reversely parallel. The resistance of transistor and diode is also included into R_a, namely, R_a is the summation of stator resistance of the motor and the resistance of transistor and/or diode. Eddy current loss is proportional to the square of the motor speed, and the mechanical viscous loss is also proportional to the square of the motor speed. Hence, the loss can be represented by resistances R_{m_ev} and R_{l_v}. R_{m_ev} corresponds to eddy current loss plus viscous loss of the motor, and R_{l_v} corresponds to viscous loss of the load system. Hysteresis loss is proportional to the motor speed, and the mechanical friction loss is also proportional to the motor speed. Hence, the losses can be represented by current sources I_{m_hf} and I_{l_f}. I_{m_hf} corresponds to hysteresis loss plus friction loss of the motor, and I_{l_f} corresponds to friction loss of the load system. They are connected in parallel to $k_e\omega_m$.

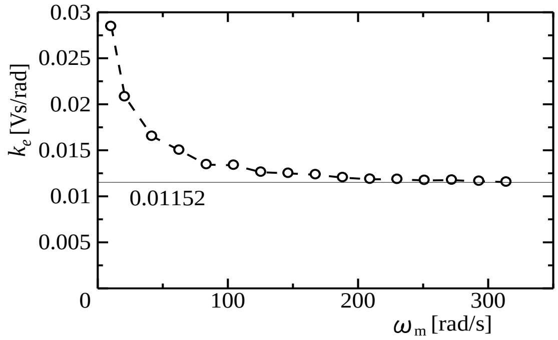


Fig. 5. k_e - ω_m characteristic.

2.2 Measurement of Parameters in Equivalent Circuit for Brushless DC Motor

Parameters in the circuit can be decided by two simple experiments. One is a lock test. When the motor speed ω_m is 0, the input voltage is given by

$$V = |V_b| + R_a I \ . \tag{1}$$

Therefore, V_b and R_a can be decided by *V-I* characteristic. Fig. 2 shows the *V-I* characteristic of the lock test. Using the approximation line shown in this figure, two parameters are estimated as $R_a = 0.2955\,\Omega$, $V_b = 1.588$ V.

The other is a no-load test. When the output power is 0, the current flowing through $k_e\omega_m$ is 0. Hence, the input voltage and current are given by

$$V = |V_b| + R_a I + k_e\omega_m \tag{2}$$

$$I = k_e\omega_m / \left(R_{m\_ev} \,//\, R_{l\_v}\right) + I_{m\_hf} + I_{l\_f} \ . \tag{3}$$

Therefore, k_e can be decided by V- ω_m characteristic, and $\left(R_{m\_ev} \,//\, R_{l\_v}\right)$ and $\left(I_{m\_hf} + I_{l\_f}\right)$ can be decided by I- ω_m characteristic. Figs. 3 and 4 show V- ω_m and I- ω_m characteristics, when the motor coupled with a brake is rotating without developing output torque. We estimate that $\left(R_{m\_ev} \,//\, R_{l\_v}\right) =$ 3.108 Ω and $\left(I_{m\_hf} + I_{l\_f}\right) =$ 1.136 A. We also measured the V- ω_m and I- ω_m characteristics, when the motor is rotating without the brake, that is, motor alone. From these experiments, we obtained

$$R_{m\_ev} = 6.03\Omega$$

$$R_{l\_v} = 10.59\Omega$$

$$I_{m\_hf} = 0.378\text{A}$$

$$I_{l\_f} = 0.758\text{A}.$$

k_e is given as a function of ω_m as shown in Fig. 5. In order to check k_e, we measured the electromotive force (EMF) of the motor. Fig. 6 shows the waveform of EMF and the rectified waveform of three-phase EMF. The mean value of the rectified EMF gives k_e = 0.01152 [Vs/rad]. It is found from Fig. 5 the value of k_e calculated by the no-load test approaches to the value given by EMF.

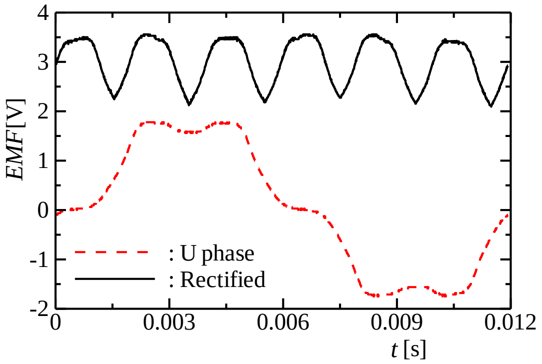


Fig. 6. Electromotive force of the experimental motor.

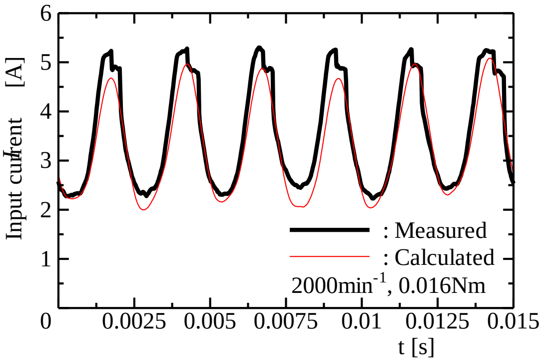


Fig. 7. Measured and calculated input currents.

It is also found that the EMF includes large ripple. This ripple produces a ripple in the input current as shown in Fig. 7. The ripple in the input current increases the copper loss and the eddy current loss. Fig. 8 shows an equivalent circuit for the calculation of ripple current. In this figure, V, V_b, I_{m_hf}, I_{l_f} and R_{l_v} are omitted, and $k_e\omega_m$ is replaced by ΔE and L is inserted. Here, ΔE is the harmonic components of EMF, and L is the inductance between two terminals of the motor. The measured value of L is 84.5μH. We can take into account the copper loss and the eddy current loss in the motor by the harmonic components of EMF by using the equivalent circuit. Note that hysteresis loss by the harmonic components of EMF is not considered. It means that the area of minor magnetic hysteresis loop is 0.

3. Motor Characteristics Using Equivalent Circuit

3.1 Loss and Efficiency

When the torque T_e and the motor speed ω_m are specified as independent variables, the input voltage V, the input current I, the output power P_o, the copper loss W_c, the eddy current and mechanical viscous loss W_e, the hysteresis and mechanical friction loss W_h, the voltage-drop loss of transistor and diode W_t, and the efficiency η are given by as follows using the simple equivalent circuit,

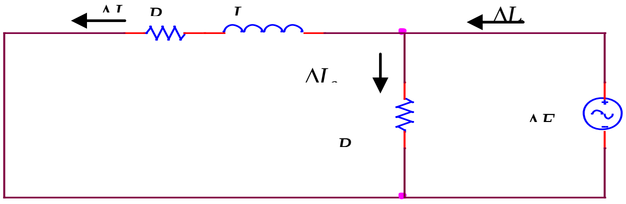


Fig. 8. Equivalent circuit for the calculation of ripple component.

$$I_t = T_e / k_e \tag{4}$$

$$I = k_e\omega_m / \left(R_{m\_ev} // R_{l\_v}\right) + I_{m\_hf} + I_{l\_f} + I_t \tag{5}$$

$$P_o = T_e\omega_m \tag{6}$$

$$W_c = R_a I^2 \tag{7}$$

$$W_e = \left(k_e\omega_m\right)^2 / \left(R_{m\_ev} // R_{l\_v}\right) \tag{8}$$

$$W_h = k_e\omega_m\left(I_{m\_hf} + I_{l\_f}\right) \tag{9}$$

$$W_t = V_b I \tag{10}$$

$$\eta = P_o / \left(P_o + W_c + W_t + W_e + W_h\right). \tag{11}$$

If the ripple of EMF is taken into account, the following components should be included

$$\Delta T_e = -\sum_{i=1}^{\infty} \mathrm{Re}(\Delta E_i \Delta I_i{}^*) \tag{12}$$

$$\Delta W_c = \sum_{i=1}^{\infty} R_a (\Delta I_i)^2 \tag{13}$$

$$\Delta W_e = \sum_{i=1}^{\infty} R_{m\_ev} (\Delta I_{e\_i})^2 \tag{14}$$

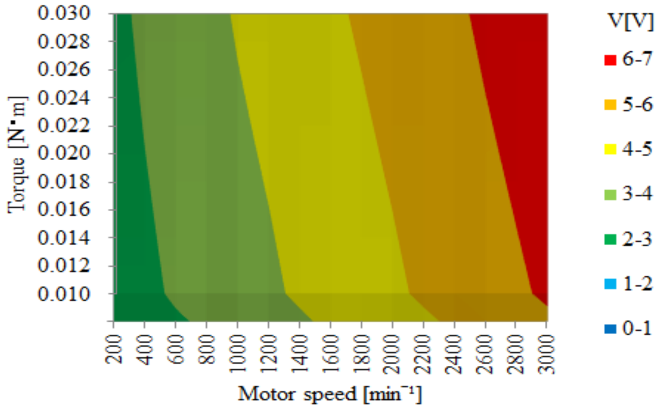
Therefore, the efficiency including the ripple of EMF is given by

$$\begin{aligned}\eta' = {} & (T_e + \Delta T_e)\omega_m \\ & /\left((T_e + \Delta T_e)\omega_m + W_c + \Delta W_c + W_t + W_e + \Delta W_e + W_h\right).\end{aligned} \tag{15}$$

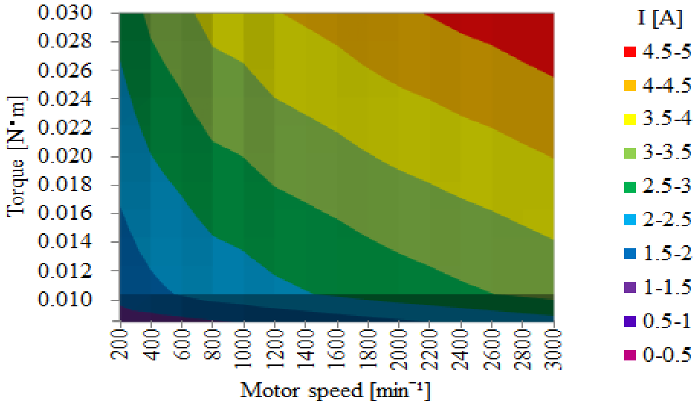
Maps of each variable are calculated by Microsoft-Excel, and are drawn in Fig. 9, where the vertical axis is not $T_e + \Delta T_e$ but T_e for easy drawing.

3.2 Comparison with Measured Data

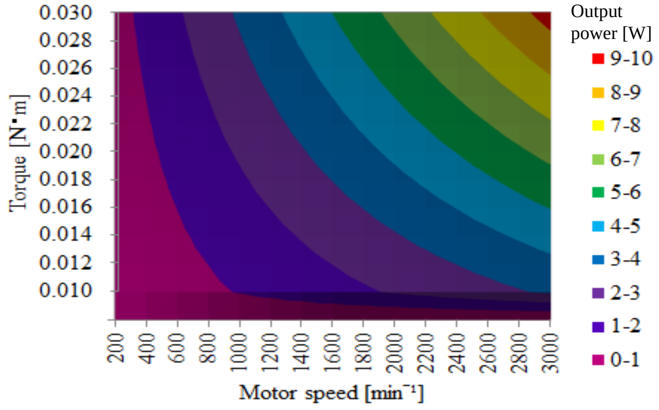
We have measured input voltage, input current, torque, motor speed and input power of an experimental motor of 100W, 12V and 2,500min^{-1}. Maps of each variable are drawn in Fig. 10. By comparing the input voltages shown in



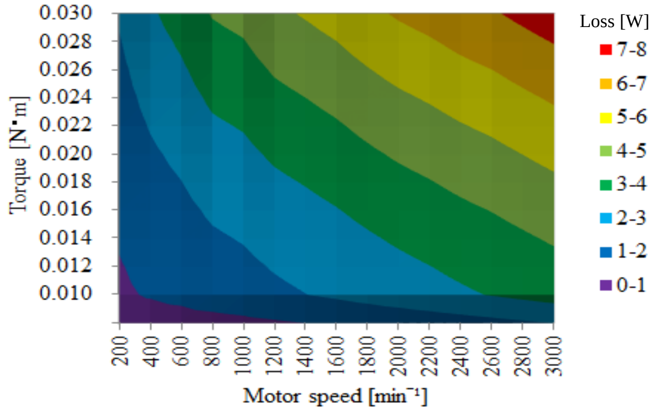
(a) Input voltage V



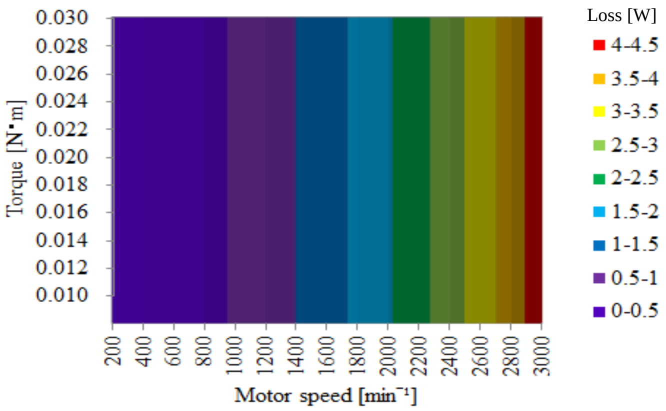
(b) Input current I



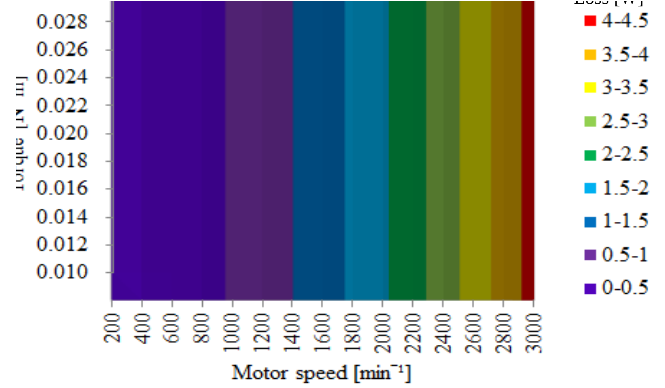
(c) Output power P_o



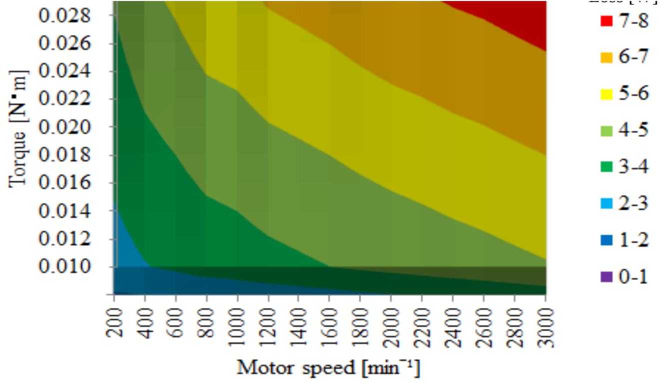
(d) Copper loss W_c



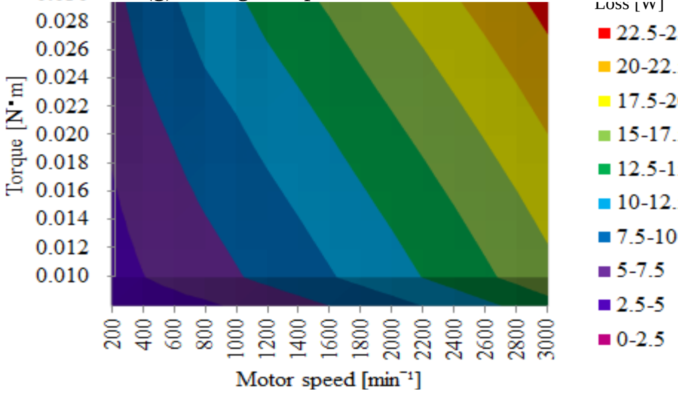
(e) Eddy current and mechanical viscous loss W_e



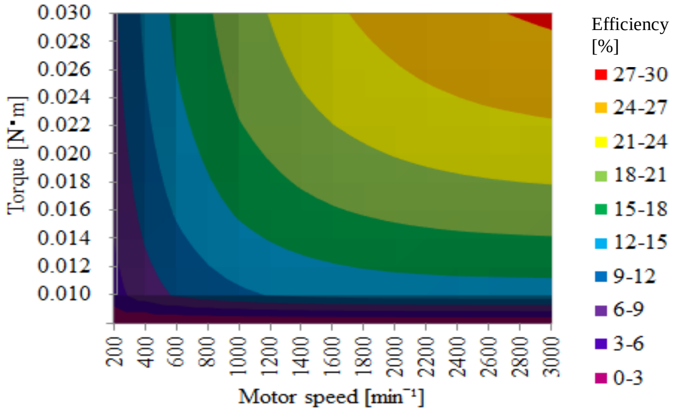
(f) Hysteresis and mechanical friction loss W_h



(g) Voltage drop loss of transistor and diode W_t



(h) Entire loss



(i) Efficiency η

Fig. 9. Maps for each variable.

Figs. 9(a) and 10(a), they show vertical stripes, and the maximum difference is 0.18 V. The input currents drawn in Figs. 9(b) and 10(b) show sloped stripes, and the maximum difference is 1.2 A. The entire loss also shows sloped stripes, and the maximum difference is 3.6 W. The efficiency drawn in Figs. 9(i) and 10(d) shows striped pattern of hyperbola, and the maximum difference is 2.7 %. Therefore, it is verified that the motor characteristics calculated by the equivalent circuit shown in Fig. 1 are in good agreement with the measured ones.

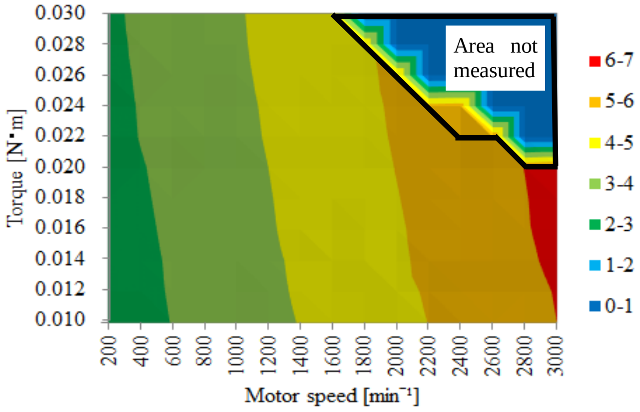
3.3 Comparison with Conventional Equivalent Circuit

The well-known steady state equivalent circuit for DC motors is represented by the armature resistance R_a and the electromotive force $k_e\omega_m$, whose values are the same as those shown in Fig. 1, namely, V_b, R_{m_ev}, R_{l_v}. R_{m_ev}, I_{m_hf} and I_{l_f} are ignored. Maps of each variable calculated by the conventional circuit are shown in Figs. 11. The input current and the copper loss, which is equal to the entire loss, give perfectly horizontal stripes. The motor efficiency give a higher value because the other losses are ignored.

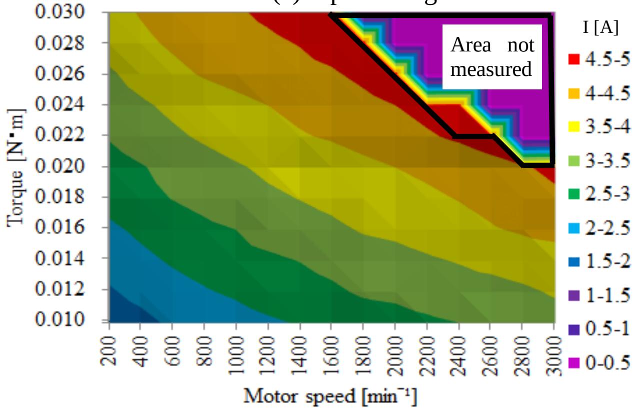
4. Simulation of Voltage and Current waveforms

The efficiency of the experimental brushless DC motor is low as shown above, because the loss of an experimental inverter, a torque meter and a hysteresis brake is larger than the loss of the experimental motor. For example, since the stator resistance of the experimental motor is 30mΩ, the resistance of transistor and/or diode, which is 0.296-0.030 = 0.266Ω, is larger than the stator resistance.

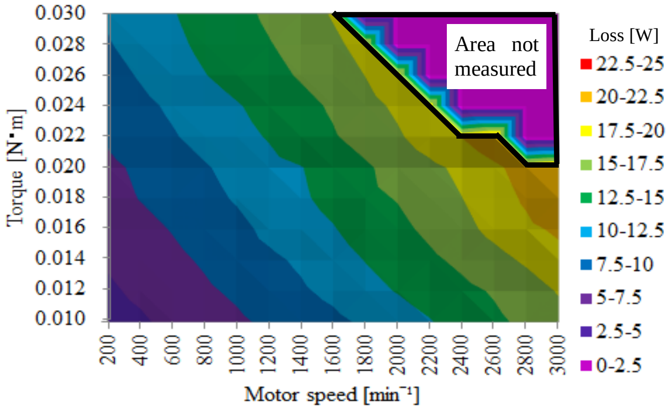
In order to verify the stator voltage and current in the system with large inverter loss, we calculate their waveforms. The voltage equations for the brushless DC



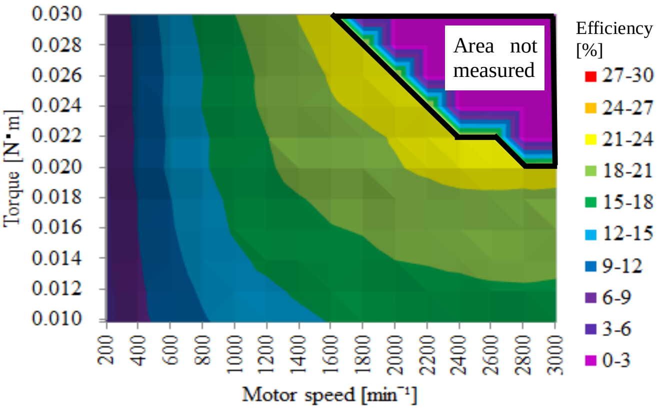
(a) Input voltage V



(b) Input current I

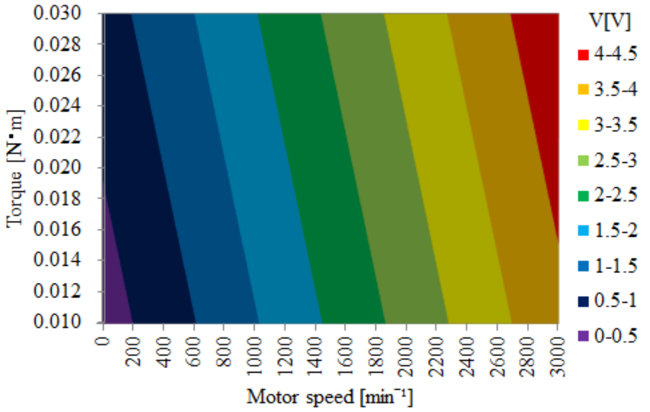


(c) Entire loss

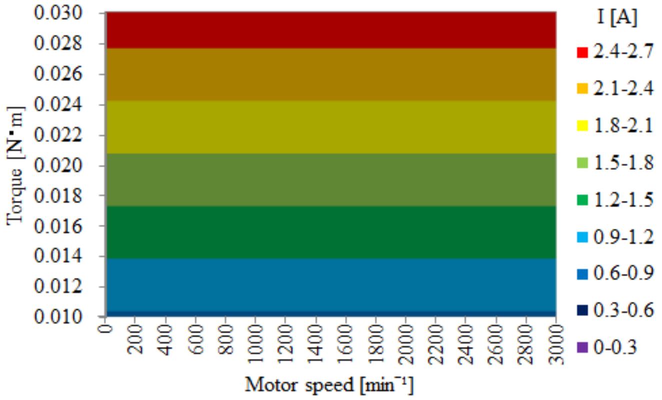


(d) Efficiency η

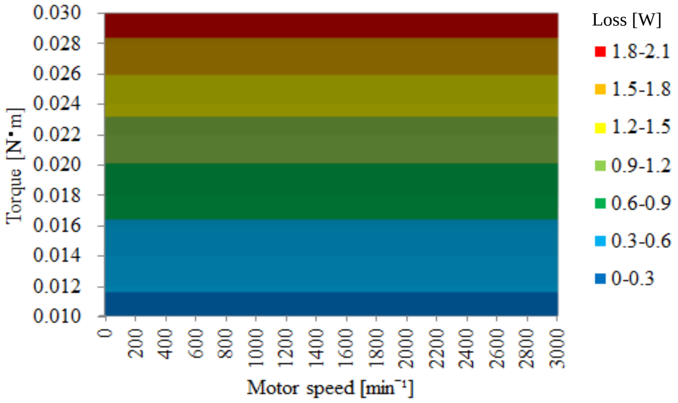
Fig. 10. Maps of the measured variable.



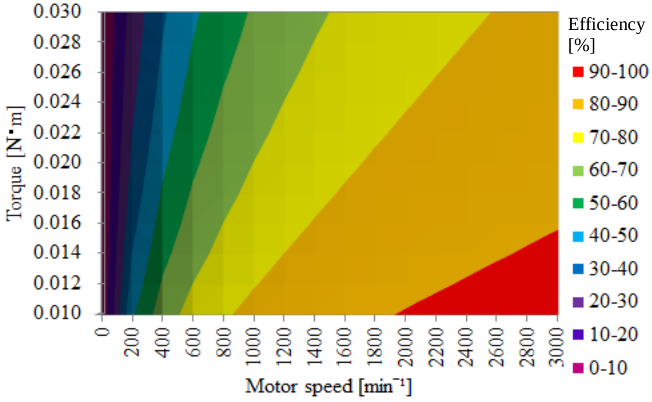
(a) Input voltage *V*



(b) Input current *I*



(c) Entire loss



(d) Efficiency η

Fig. 11. Maps calculated with conventional equivalent

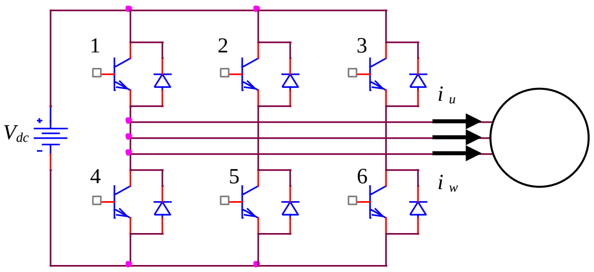


Fig. 12. Drive circuit for brushless DC motors.

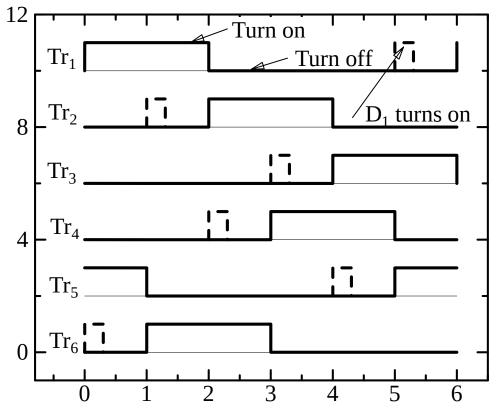


Fig. 13. Turn-on signal of each transistor.

motor shown in Fig. 12 are represented as follows by considering the summation of three stator currents $i_u + i_v + i_w = 0$,

$$\begin{aligned}(L-M)\frac{di_u}{dt}+R_a i_u &= 1/3(-2e_u+e_v+e_w+2V_u-V_v-V_w)\\(L-M)\frac{di_v}{dt}+R_a i_v &= 1/3(e_u-2e_v+e_w-V_u+2V_v-V_w)\end{aligned}\tag{16}$$

where L, M and e are the self-inductance, mutual inductance, and the electromotive force. The stator voltage V_u is expressed by considering the voltage drop and the resistance of transistor and diode V_{tr}, V_d, R_{tr} and R_d, when the turn-on signal of an upper transistor 2 is active as shown in Fig. 13,

$$\begin{aligned}V_u &= V_{dc}-V_{tr}-R_{tr}i_u &&\text{if } i_u>0\\V_u &= V_{dc}+V_d+R_d\left|i_u\right| &&\text{if } i_u<0.\end{aligned}\tag{17}$$

When the turn-on signal of a lower transistor 5 is active,

$$\begin{aligned}V_u &= 0-V_d-R_d i_u &&\text{if } i_u>0\\V_u &= 0+V_{tr}+R_{tr}\left|i_u\right| &&\text{if } i_u<0.\end{aligned}\tag{18}$$

Here, eqs. (16), (17) and (18) do not take into account the eddy current loss, hysteresis loss, friction loss and viscous loss. The simulated result and the measured one are shown in Figs. 14 and 15. It is found that the simulated result agree well with the measured one. The stator current flows in the period of 120 degree. Even if the upper transistor turns on, the waveform of the stator voltage is not a DC voltage V_{dc}, but approximately same value as the electromotive force because of the large inverter resistance.

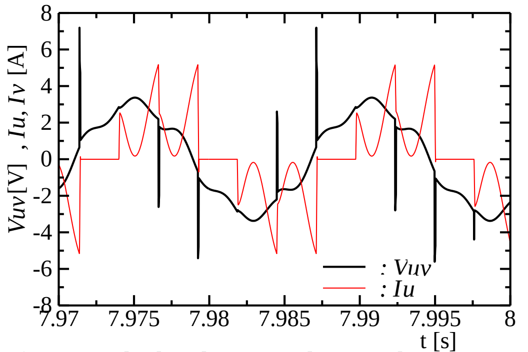


Fig. 14. Calculated stator voltage and current.

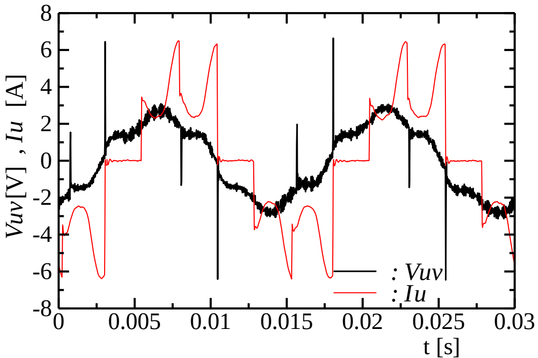


Fig. 15. Measured stator voltage and current.

5. Conclusion

This paper has proposed a simple equivalent circuit for the calculation method of several losses and the efficiency of brushless DC motors. The copper loss, eddy current loss, hysteresis loss, friction loss and viscous loss, and inverter loss are taken into account in the circuit. This circuit can calculate easily the maps of the motor efficiency and losses by using the Microsoft Excel, even if the motor is built in the system with a large inverter loss and/or mechanical loss.

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